VOLUME ONE

RADIO ENGINEERING

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NOTE

ALL figures are in the text except the following, which are *folding plates* :—

INTRODUCTION

One of the objects of this book is to enable a non-technical person to acquire a sufficient practical knowledge of principles, without necessarily understanding underlying causes. The most fundamental treatment has ultimately to deal with facts of observation which do not submit of proof, and in practice an engineer who accepts a simple system of facts, provided he knows how to use them, may find himself at an advantage compared with one who delves more deeply into final causes. When a housewife makes a suet pudding, she does not concern herself in very great detail with the individual paths of the component atoms, nor are sonnets written in terms of differential equations.

A second object is to provide a book of reference for use of engineers with some experience. The beginner must not, therefore, be alarmed by descriptions of techniques which he is not expected to master and which he will probably never need.

Capacity for understanding depends on an attitude of mind and not in any way on a high degree of intelligence. To understand a description it is necessary to be interested, it is necessary to pay attention, and it is necessary to appreciate what kind of thing the description is trying to convey. Interest can be cultivated even in a dull story, and in listening to a story the end of the narrative loses all point if certain essential details of the beginning have been lost. This is particularly true in understanding the technique of wireless engineering where descriptions are given in terms and symbols and physical effects with which the reader must be familiar. The present account assumes complete ignorance on the part of the reader and develops the required terminology as it goes along.

Imagine a lecturer starting off to explain magnetism by saying, "A magnet is a piece of steel which has been treated in a certain way and attracts iron." He then goes on to describe how a magnet is surrounded by a magnetic field, how wires moving in this field have odd effects induced in them, and so on. The earnest student may still say, "Yes, but what *is* a magnet?" "*Why* does it attract iron?" The real answer is that although a more detailed description of the ultimate structure of a magnet can be given, by saying, for instance, that all iron molecules are little magnets which in ordinary iron are arranged at random and in magnetic iron all point the same way, ultimately the lecturer will be forced

to admit that he doesn't know why the molecules are magnetic. It is just a fact which must be accepted.

Any description of electricity is just a description of effects which happen and have to be accepted. If an unfortunate student feels that the whole subject is incomprehensible because no one knows why anything happens, he is really adopting an artificial attitude quite inconsistent with his normal everyday attitude. He has contrived to live for quite a number of years without knowing and probably without caring what is the ultimate cause of his existence, why he breathes, why his heart beats, why he likes sweet things and feels uncomfortable in intense cold, why pigs haven't got wings, and whether the egg came before the hen, or vice versa. He probably has one magic word to explain all this: he may call it "Life", or he may call it "Evolution". Isn't he perhaps being a little unfair if he expects a revelation of ultimate divine truth in an explanation given by someone else?

Another difficulty occurs which doesn't usually arise in everyday life. If a child asks, "What is that animal eating grass?" and his father says, "That is a cow," what is his unfortunate sire to say if the child asks "Why?" The answer of course is that the noise "cow" is always used to describe a cow. The use of the noise "cow" is a *convention*. If a grown-up child asks, "Why does a magnetic field of one line per square centimetre act on unit magnetic pole with a force of one dyne?" the answer is that it is a convention to represent a field, which exerts such a force on a unit magnetic pole, by one line per square centimetre. If he asks, "Why does one volt drive a current of one ampere through a resistance of one ohm?" the answer is that the ohm is chosen or defined as that size of resistance through which one volt drives a current of one ampere.

Why is unit current (one ampere) defined as a tenth of the current which exerts a force of 1 dyne on a unit magnetic pole under certain special conditions? The student is not usually bothered with the special conditions, or with the factor of $\frac{1}{10}$, both of which he recognizes as being artifices inserted for some conceivably explicable reason. What bothers him is why a fundamental definition should not immediately give an answer to the obvious question: what is the rate of flow of electrons in the circuit expressed by the number of electrons passing a given point per second? Ultimately it does, of course, but that is not the point. The answer that the electron had not been discovered when the definition was made is out of court because there is still the question: what quantity of electricity

is flowing along the circuit per second? The proper answer is, that by using the definition engineers and experimenters can exchange information and issue specifications which will enable the same magnitude of current to be reproduced whenever required, even if the number of electrons per second is not known.

In this book a number of statements are made without proof, and these must be accepted in the same way as natural laws: but when proof can be given in simple terms it is supplied. Further, the definitions of fundamental quantities are not always the original fundamental definitions but those which lend themselves most readily to explanation.

Magneto motive force, for instance, is treated as a fundamental concept instead of being expressed as a mathematical function of a still more elusive quantity.

Another stumbling-block to the beginner is mathematics. The writer has an infallible method of dealing with all mathematical descriptions. He glances at the type of mathematics involved, and if it is of a type that he can understand, there is evidently no point in reading it. He therefore looks at the conclusions reached to see if these are presented in a useful form. If they are not, the treatise is not very much use, anyway! If, on the other hand, the mathematical argument is incomprehensible, there is also no point in reading it. But provided the problem is clearly formulated and the conclusions are clearly stated in explicit form, they may still be of full value to the practical man if he is capable of substituting in a simple formula. This may be illustrated by an example.

Formulation. What is the volume of a sphere of radius R?

Solution (which need not be read!). Taking three dimensional axes with the origin at the centre of the sphere and using polar co-ordinates r, θ and ϕ; if V is the volume of the sphere

$$dV = \tfrac{1}{3} R^3 . d\theta . d\phi . \cos\theta$$

$$\therefore\ V = 2\int_0^{2\pi}\int_0^{\pi/2} \frac{1}{3} R^3 \cos\theta \, d\theta \, d\phi$$

$$= \frac{2R^3}{3}\int_0^{\pi/2}\Big[\sin\theta\Big]_0^{\pi/2} d\phi$$

$$= \frac{4\pi R^3}{3}$$

$$\textit{Result}\ V = \frac{4\pi R^3}{3}$$

In this case the formulation and the result are clearly stated, but the solution is typical of the slipshod methods beloved by mathematicians. Having invoked the aid of occult powers, they write down a series of expressions corresponding to some of the steps in the correct logical argument, often jumping two or three steps at a time, with little or no explanation as to how each step is derived. The practical man has no time to read the several textbooks which may be necessary to carry him from one step to the next, but this detracts in no way from the usefulness of the final result, if clearly stated. The final result may be expressed in terms of a simple formula as in the above case, or in terms of a graph. Instead of showing a graph, the result may be expressed in terms of *a special function* of one or more of the known quantities supplied in the formulation of the problem. This is usually done only in cases where the functions in question are standard functions, the values of which are given in standard tables or curves. This may sound very complicated, but it is impossible to conceive anything simpler than the practical application which is described below, after the discussion on substitution in a simple formula.

Method of Substituting in a Simple Formula. In the case of the formula $V = \frac{4\pi R^3}{3}$ no statement is given as to the units in which V and R are measured, and none is necessary. If R is the radius in inches, V is the volume in cubic inches; if R is in metres, V is in cubic metres. R should therefore be entered in the formula in terms of the unit in which the answer is required. In general, if no values of units are given in a formula the result will be correct, provided any relevant self-consistent units are inserted in the formula, and the answer will be in terms of the relevant unit in that system. For instance, the distance a train can travel in time t at velocity v is $d = vt$. If velocity is expressed in miles per hour, time t *must* be expressed in hours, and the answer will appear in miles; if v is expressed in feet per second, t *must* be expressed in seconds, and the answer will be in feet.

Formulae are often given in terms of units which are not self-consistent, in which case the values of the units for each symbol are stated, e.g. $d = 60\,vt$ where d = distance in feet, v = velocity in feet per second, and t = time in minutes.

The kind of mistake which is usually made is similar to that made in converting from feet per minute to feet per second (supposing that v is initially given in feet per minute), by multiplying by 60 instead of dividing by 60.

INTRODUCTION

The examples below will be better understood by beginners after the corresponding sections of this book have been read, and should be omitted in a first reading.

1. Given that $I = \frac{E}{R}$, where I is current, E is p.d., and R is resistance, what is the current which flows when a potential difference of 10 kilovolts is applied across 1,000 ohms? The corresponding self-consistent set of units is amperes for current, volts for p.d., and ohms for resistance. Hence $E = 10{,}000$ volts, $R = 1{,}000$ ohms and

$$I = \frac{E}{R} = \frac{10{,}000}{1{,}000} = 10 \text{ amps.}$$

2. Given that the reactance of a condenser is $X_c = \frac{1}{C\omega}$, where C is the capacity of the condenser in Farads, and $\omega = 2\pi f$, where f is the frequency in cycles per second, what is the reactance of 300 micro-microfarads at 1,122 kilocycles per second?

(1 micro-microfarad $= 10^{-12}$ Farad $= \frac{1}{10^{12}}$ Farad

$= \frac{1}{1{,}000{,}000{,}000{,}000}$ Farad)

$C = 300$ micro-microfarads $= 300 \times 10^{-12}$ Farads

$\omega = 2\pi \times 1{,}122 \times 1{,}000$

$$\therefore X_c = \frac{1}{300 \times 10^{-12} \times 2\pi \times 1\cdot122 \times 10^6} = \frac{10^6}{300 \times 2\pi \times 1\cdot122}$$
$$= 473 \text{ ohms}$$

3. $N = 10 \log_{10} \frac{P_1}{P_2}$ is the formula defining N the number of decibels difference between two powers, P_1 and P_2. In this case it is not necessary to specify the units in which the powers are measured, since, provided both powers are expressed in terms of the same unit, the ratio $\frac{P_1}{P_2}$ will be independent of the unit chosen. If $P_1 = 10$ watts and $P_2 = 1$ watt, $\frac{P_1}{P_2} = 10$.

Hence: $N = 10 \log_{10} 10 = 10 \times 1 = 10$ decibels.

Example 3 typifies the general case where a formula contains a special function of a quantity. The function in this case is $\log_{10}$

and the value of $log_{10} \frac{P_1}{P_2}$ can be found from tables of common logarithms once the numerical value of the ratio $\frac{P_1}{P_2}$ is known.

Other functions which will be met are *sin*, *cos*, *tan* and J_0, J_1, J_2, J_3, etc. It is evident that if the change of the name of the functions from log_{10} to *cos* or J_2 merely means that the value of the function has to be looked up in a table or graph of *cos* or J_2, instead of a table of common logarithms, no extraordinary feat of intelligence is required.

The process of substituting in a formula therefore involves the following steps:

A. *Where no values of units are given.*

(1) Make certain that the quantities to be entered in the formula are all in terms of the same system of units.
(2) If they are not, choose a system of units and transform all quantities to numbers defining their magnitude in terms of the units of that system.
(3) Enter the quantities in the formula and work out the answer arithmetically, looking up the values of any functions in appropriate tables or graphs.
(4) Remember that the answer will be in terms of the unit belonging to the same system.

B. *Where values of units are given.*

(1) Transform all the quantities to be entered in the formula to numbers defining their magnitude in terms of the units prescribed for each quantity (i.e. each symbol in the formula).
(2) Enter the quantities in the formula and work out the answer arithmetically, looking up the values of any functions in appropriate tables or graphs.
(3) Remember that the answer is in terms of the prescribed units.

The average maintenance engineer has to use only a very few simple formulae, so few in fact that the summarizing of the above obvious steps hardly appears necessary. Mistakes which occur are however almost invariably due to failure to carry these out properly.

The units which are usually employed and which all belong to the same system are:

Potential Difference . . .	Volts
Current	Amperes
Resistance, Reactance and Impedance	Ohms
Inductance	Henrys
Capacity	Farads
Frequency	Cycles per second
Power	Watts
Electrical Energy	Joules = Watt-seconds
Time	Seconds

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CHAPTER I

HOW BROADCASTING IS DONE

1. Requirements to be met in a Broadcast Transmission.

BRIEFLY, the problem involved in broadcasting is to reproduce a replica of sound, occurring in one place, at a number of distant places substantially simultaneously with the occurrence of the original sound.

Sound may be identified with one or more travelling waves of pressure variation, which, arriving at the ear, cause pressure variations to enter the ear and impinge on the drum. Since the drum can only be in one position at any one time, it is evident that the form of the sound is completely described by drawing a graph of pressure at the ear against time.

A series of such graphs are shown in Fig. 1 for different vowel sounds. It will incidentally be noted that the wave form of a given vowel varies, not only from speaker to speaker, but also according to the position in which the mouth and throat have been left by the preceding consonant.

It will be evident that the important information contained by the sound wave resides in its form, and not in the kind of medium in which the wave exists. It is well known that sound can exist as a vibration in water, in telegraph wires or in the wires of a fence, but, of course, such vibrations cannot be heard by the ear until they give rise to similar (or probably distorted) copies of themselves travelling in air.

2. Transmission from Studio to Aerial.

The first step in broadcasting a sound is to make the sound impinge on a *microphone*, which is nothing more than a device for converting the variations of sound pressure into variations of an electric current. These electric currents are usually very weak and, since the studio is usually remote from the transmitting station, they are caused to operate relay devices called *amplifiers*, which deliver at their output magnified replicas of the current supplied at their input. These magnified replicas are then transmitted over underground circuits, similar to telephone circuits, to the trans-

mitting station where they are *amplified* further before undergoing a second transformation into a form of current variation (carrying

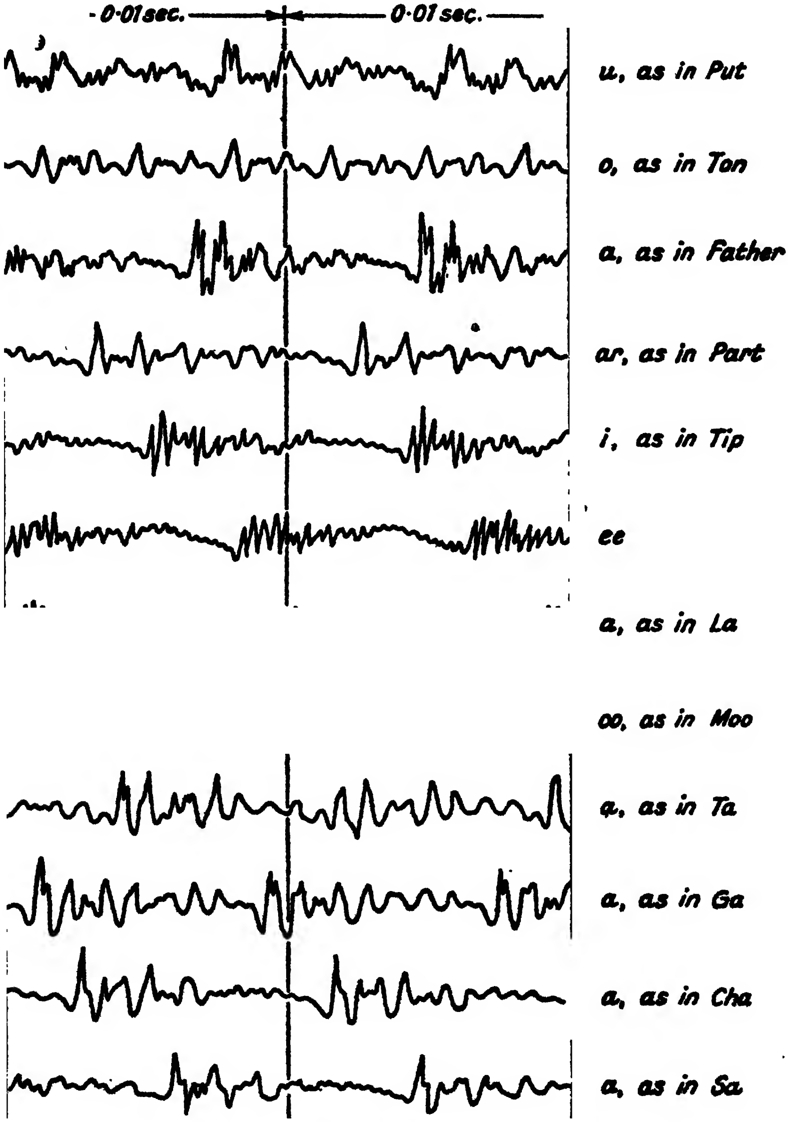

FIG. 1/I:1.—Graphs of Pressure Variation due to a Sound Wave.
(By courtesy of I. B. Crandall and *Bell System Technical Journal.*)

a replica of the original sound wave) which is suitable for *radiation*. The process that produces this final transformation is called *modulation*, and its effect is to transfer the sound-frequency or

audio-frequency vibrations which may be regarded as a mixture of sinusoidal vibrations of frequencies varying from 30 to 10,000 vibrations per second, to a higher range of frequencies. A "*sinusoidal*" vibration is an elementary form of movement similar to that of a pendulum swinging freely at a small amplitude: see Chapter III—The Sine Wave. "*Frequency*" is a loose term, used properly to describe the number of sinusoidal vibrations per second of an alternating current or voltage, but in common practice used to indicate a wave or a current, e.g. a frequency of a thousand cycles per second refers to a wave form consisting of a thousand sinusoidal vibrations per second.

3. Reasons for Use of High-Frequency Currents in Aerial.

There are two main reasons for transferring the sound vibrations, or television signals, which are respectively referred to as audio frequencies and video frequencies, to a higher band of frequencies.

1. The efficiency of aerials as radiators of electro-magnetic waves depends on a relation between size of aerial and frequency. The higher the frequency the smaller is the aerial structure required to give efficient radiation.
2. By stepping the frequency of the waves up to a higher band of frequencies, it is possible to convey the whole of the intelligence contained in the original band of frequencies within a band of frequencies of width equal to a comparatively small percentage of the mean frequency. The result is that circuits which are designed to have a maximum response to the mean frequency have sufficient response to all the frequencies in the band to enable intelligence and musical quality to be preserved without serious distortion. Such circuits are called *tuned circuits* and constitute the means by which it is possible to pick up a transmission "on any one wavelength", excluding transmissions on other wavelengths.

The exact location of the range of radiated frequencies in the frequency spectrum is determined by considerations relating to propagation conditions, area to be served and international agreements relating to allocation of wavelengths.

4. Modulation.

The process of modulation consists in causing the amplified sound or signal currents to operate on or interact with a sinusoidal high-frequency current, of frequency equal to the mean frequency

in the required high-frequency band, in such a way that the envelope of the high-frequency current is of the same form as the original sound wave.

The high-frequency current so operated on is called the carrier frequency, carrier wave, or the carrier, and is said to be modulated

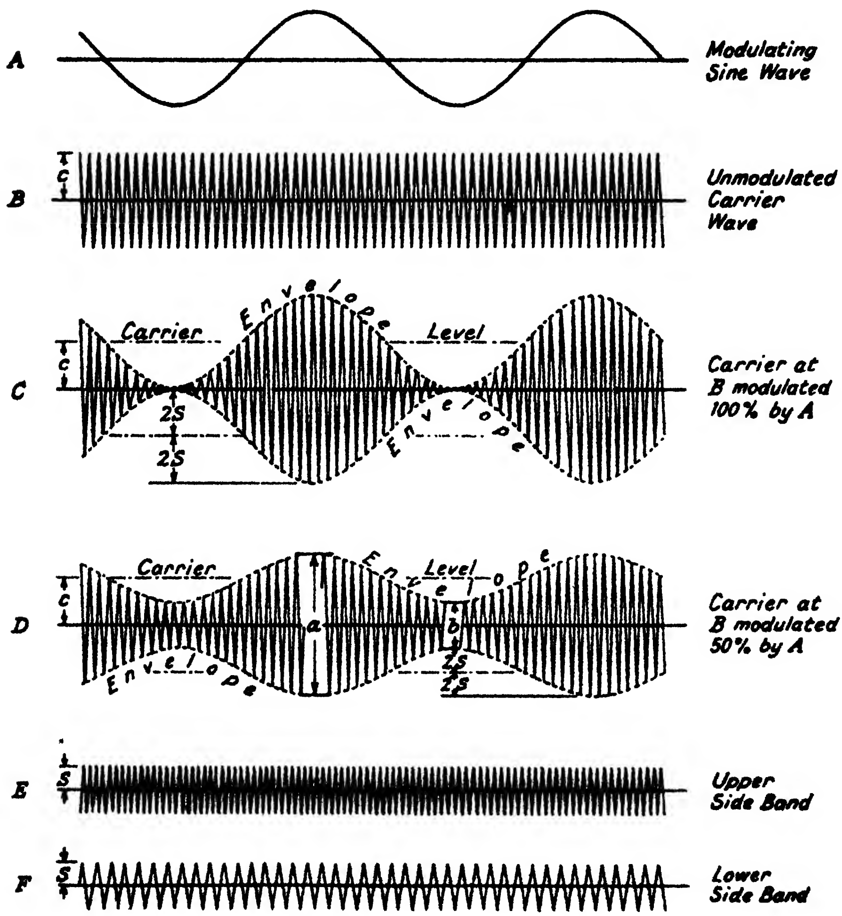

FIG. 1/I:4.—Modulation of a Carrier Wave by a Sine Wave.

with the *form* of the sound wave in question. This process will be made clearer by reference to Fig. 1 which illustrates the process of modulation of a carrier wave by a single sinusoidal wave. This corresponds to the case where the original sound was of sinusoidal form. The sound wave is shown at *A*, the carrier wave at *B*, and two resultant modulated waves at *C* and *D*. The curve drawn

through the peaks of the carrier wave, shown dotted, is called the *envelope* of the modulated wave and is of the same form as the *modulating* wave shown at A. The wave at C in which the carrier wave goes down to zero amplitude in the troughs is said to be 100% modulated, while the wave at D shows a lower percentage modulation. In the case of D the *modulation* m is defined by the amplitude of the *envelope* of the modulated wave $\left(\frac{a-b}{4}\right)$ divided by the amplitude of the original carrier wave $\left(\frac{a+b}{4}\right)$, i.e. the modulation $m \quad \frac{a-b}{a+b}$. For instance, if $\frac{a-b}{a+b} = 0{\cdot}5$, $m = 0{\cdot}5$ the percentage modulation is then equal to $100m = 50\%$.

5. Production of Sidebands by Modulation.

The waves at C and D in Fig. 1/I:4 may be regarded as being constituted in one of two ways. The first way has already been considered: that is they consist of a carrier wave of which the peak amplitude varies sinusoidally. Another way of looking at these waves is to say that they are built up of a number of sinusoidal waves of constant amplitude and different frequency. Analysis shows that the wave forms at C and D can be built up from three constant amplitude sinusoidal waves of different frequency. One of these waves is of the same amplitude and frequency as the original carrier frequency, while two of these waves are of frequency respectively equal to the sum and difference of the carrier-wave frequency and the frequency of the original modulating wave. These two waves, therefore, exist one above the carrier frequency and one below it, separated from it by a distance equal to the frequency of the original modulating wave. The last two waves are called sideband frequencies, because they occur in bands of frequencies located each side of the carrier wave. They are indicated in Fig. 1/I:4 at E and F (not to scale either in frequency or in amplitude). Another way of representing them is shown in Fig. 1/I:5, where the horizontal scale is an axis of frequency and the vertical scale is an axis of amplitude. f_c is then the carrier wave and f_a and f_b are the sideband frequencies. If f_s is the original sound frequency, shown at A, which produced these sideband frequencies by modulating the carrier wave shown at B, then

$$f_a = f_c - f_s \quad \text{and} \quad f_b = f_c + f_s.$$

The amplitudes S of the side frequencies are equal and are

related to the amplitude C of the carrier wave by the modulation, as follows

$$\frac{2S}{C} = m, \text{ the modulation.}$$

The percentage modulation = $100m$.

If the spectrum of the original sound wave, which in practice of course does not normally consist of a single frequency, is of the form indicated at the left of Fig. 1/I:5 by the curved line, it is evident from the above considerations that the process of modulation will give rise to bands of frequency situated each side of the carrier frequency with spectra of the form indicated by the curved lines drawn each side of f_c. At any one time there may be and generally is a number of sideband frequencies present which add up in amplitude, and of which the overall peak amplitude must not exceed that of the carrier. If it does, *over-modulation* is said to

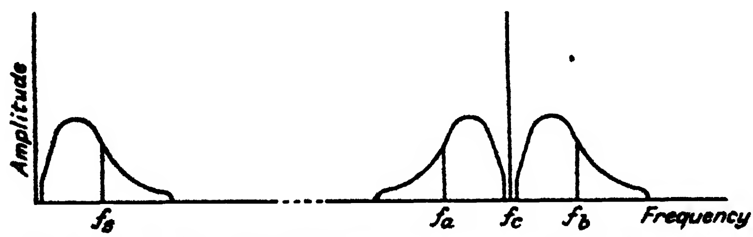

FIG. 1/I:5.—Location of Audio-Frequency Spectrum and Sidebands in Frequency.

occur, and *distortion* of the *audio-frequency* wave form results. This is the reason why continual control of the programme level has to be maintained.

Sometimes modulation is effected at high power, in which case the modulated currents are fed directly to an aerial, while in other cases modulation is effected at low power, in which case the modulated currents must be amplified before being fed to the aerial. The device which receives the amplified programme currents from the studio line, amplifies them further, employs them to modulate a carrier, and supplies the modulated carrier to the aerial with or without still further amplification, is called a *transmitter*.

6. Radiation.

The modulated currents are usually fed from the transmitter to the aerial through high-frequency *feeders*, of which a number of types exist. The most common types in use at the moment are *open wire* feeders which consist simply of a pair of wires supported

on insulators above ground, and *concentric* feeders which consist of a metal tube usually about 6 ins. in diameter with a single conductor in the centre. These feeders generally run to an ***aerial tuning house*** immediately below the aerial where suitable circuits are provided for coupling between the feeder and the aerial. For alternative types of feeder, see XVI:1.

By this means a high-frequency alternating current is set up in the aerial as the result of which energy is radiated into space in the form of an electromagnetic wave. In the case of a vertical aerial, this wave is mainly radiated in a horizontal direction, and when it strikes another aerial, sets up currents in it of a similar form to those in the transmitting aerial. These can be detected by suitable means and transformed so as to reproduce a sufficiently faithful copy of the original waves supplied to the transmitter, and finally, by means of a loudspeaker, into a copy of the original sound wave in the studio.

7. Relation between Frequency and Wavelength.

If you create a disturbance in the surface of a still pond by oscillating movements of your hand at regular intervals, a series of waves will travel away from your hand with a certain velocity, say V ft. per second. If, for instance, your hand strikes the water f times per second, f waves will be created per second, and by the time the first wave has travelled V ft. you will have struck the water f times, and there will, therefore, be f waves in a distance of V ft. The length of each wave will, therefore, be V/f ft. If we call the length of each wave λ, then we can write the simple equation

$$\lambda = \frac{V}{f}$$

Exactly the same relationship between wavelength, velocity and frequency holds in the case of radio waves, the only difference being that the velocity of propagation of radio waves is very much higher than that of the waves in a pond, being almost exactly 300,000,000 metres per second. The relation between ***wavelength*** in metres and frequency in cycles per second in the case of a radio wave is therefore given by

$$\lambda = \frac{300{,}000{,}000}{f}$$

It is, therefore, evident that the wavelength is determined merely by choosing the frequency of the carrier wave.

8. Frequency Nomenclature and Tolerances.

The following nomenclature was laid down by the International Telecommunication and Radio Conference at Atlantic City, 1947: Radio Regulations, Chapter II, Section III, para. 85.10.

Frequency Sub-Division	Frequency Range	Metric Sub-Division
VLF (Very Low Frequency)	Below 30 kc/s	Myriametric Waves
LF (Low Frequency)	30 to 300 kc/s	Kilometric Waves
MF (Medium Frequency)	300 to 3,000 kc/s	Hectometric Waves
HF (High Frequency)	3,000 to 30,000 kc/s	Decametric Waves
VHF (Very High Frequency)	30,000 kc/s to 300 Mc/s	Metric Waves
UHF (Ultra High Frequency)	300 to 3,000 Mc/s	Decimetric Waves
SHF (Super High Frequency)	3,000 to 30,000 Mc/s	Centimetric Waves
EHF (Extremely High Frequency	30,000 to 300,000 Mc/s	Millimetric Waves

Frequencies shall be expressed in kilocycles per second (kc/s) at and below 30,000 kilocycles per second and in megacycles per second (Mc/s) above this frequency.

8.1. Atlantic City Frequency Tolerances.

Low to 1,605 kc/s: ±20 c/s
1,605 kc/s to 4,000 kc/s: ±0·005%
4,000 kc/s to 30,000 kc/s: ±0·003%
30 Mc/s to 500 Mc/s: ±0·003%

9. Allocation of Wavelength and Frequency Bands for Broadcasting.

At the Atlantic City Conference in 1947 the following frequency bands were allocated for broadcasting in zone 1: Europe, Africa and U.S.S.R.

Kilocycles/sec.	*Megacycles/sec.*
160–255	41–68
525–1,605	87·5–100
5,950–6,200	174–216
7,100–7,300	470–585
9,500–9,775	610–960
11,700–11,975	
15,100–15,450	
17,700–17,900	
21,450–26,100	

10. B.B.C. Call Signs, Frequencies and Wavelengths used in Short-wave Services.

In order of Frequency as in February 1953

Call Sign	*Frequency in kc/s*	*Wavelength in Metres*	*Waveband*
GRC	2,880	104·2	
GRB	6,010	49·92	
GWS	6,035	49·71	
GSY	6,040	49·67	
GSA	6,050	49·59	
GSX	6,060	49·50	
GRR	6,070	49·42	
GWM	6,090	49·26	49
GSL	6,110	49·10	
GWA	6,125	48·98	
GRW	6,150	48·78	
GWK	6,165	48·66	
GSZ	6,170	48·62	
GRO	6,180	48·54	
GRN	6,195	48·43	
GRS	7,075	42·40	
MCS	7,110	42·19	
GRM	7,120	42·13	
MCM	7,135	42·05	
GRT	7,150	41·96	
GRK	7,185	41·75	
GWZ	7,200	41·67	
GWL	7,210	41·61	41
GSW	7,230	41·49	
GWI	7,250	41·38	
GSU	7,260	41·32	
GWN	7,280	41·21	
GRJ	7,320	40·98	
GRI	9,410	31·88	
GSB	9,510	31·55	
GWJ	9,525	31·50	
GWB	9,550	31·41	
GWX	9,570	31·35	
GSC	9,580	31·32	
GRY	9,600	31·25	
GWO	9,625	31·17	31
GVZ	9,640	31·12	
GWP	9,660	31·06	
GWT	9,675	31·01	
GRX	9,690	30·96	
GWY	9,700	30·93	
MCP	9,735	30·82	
MCR	9,760	30·74	
MCN	9,770	30·71	
GRH	9,825	30·53	
GRU	9,915	30·26	

Call Sign	*Frequency in kc/s*	*Wavelength in Metres*	*Waveband*
GRG	11,680	25·68	
GVW	11,700	25·64	
GVV	11,730	25·58	
GSD	11,750	25·53	
GVU	11,770	25·49	
GWV	11,790	25·45	
GWH	11,800	25·42	
GSN	11,820	25·38	
GWQ	11,840	25·34	25
GSE	11,860	25·30	
GRE	11,880	25·25	
GWW	11,890	25·23	
MCO	11,910	25·19	
GVX	11,930	25·15	
MCQ	11,945	25·12	
GVY	11,955	25·09	
GRV	12,040	24·92	
GRF	12,095	24·80	
GWC	15,070	19·91	
GWG	15,110	19·85	
GSF	15,140	19·82	
GSO	15,180	19·76	
GWU	15,210	19·72	19
GWD	15,230	19·70	
GSI	15,260	9·66	
GWR	15,300	9·61	
GSP	15,310	9·60	
GWE	15,435	9·44	
GRD	15,450	9·42	
GVP	7,700	16·95	
GRA	7,715	16·93	
GVQ	7,730	16·92	
GSG	7,790	16·86	
GSV	7,810	16·84	16
GRP	7,870	16·79	
GRQ	18,025	16·64	
GVO	18,080	16·59	
GSH	21,470	13·97	
GSJ	21,530	13·93	
GST	21,550	13·92	
GRZ	21,640	13·86	13
GVR	21,675	13·84	
GVS	21,710	13·82	
GVT	21,750	13·79	
GSQ	25,750	11·65	
GSK	26,100	11·49	
GSR	26,400	11·36	11
GSS	26,550	11·30	

B.B.C. Call Signs, Frequencies and Wavelengths used in Short-wave Services.

In Alphabetical Order as in February 1953

Call Sign	*Frequency in kc/s*	*Wavelength in Metres*	*Waveband*
GRA	17,715	16·93	16
GRB	6,010	49·92	49
GRC	2,880	104·2	—
GRD	15,450	19·42	19
GRE	11,880	25·25	25
GRF	12,095	24·80	25
GRG	11,680	25·68	25
GRH	9,825	30·53	31
GRI	9,410	31·88	31
GRJ	7,320	40·98	41
GRK	7,185	41·75	41
GRM	7,120	42·13	41
GRN	6,195	48·43	49
GRO	6,180	48·54	49
GRP	17,870	16·79	16
GRQ	18,025	16·64	16
GRR	6,070	49·42	49
GRS	7,075	42·40	41
GRT	7,150	41·96	41
GRU	9,915	30·26	31
GRV	12,040	24·92	25
GRW	6,150	48·78	49
GRX	9,690	30·96	31
GRY	9,600	31·25	31
GRZ	21,640	13·86	13
GSA	6,050	49·59	49
GSB	9,510	31·55	31
GSC	9,580	31·32	31
GSD	11,750	25·53	25
GSE	11,860	25·30	25
GSF	15,140	19·82	19
GSG	17,790	16·86	16
GSH	21,470	13·97	13
GSI	15,260	19·66	19
GSJ	21,530	13·93	13
GSK	26,100	11·49	11
GSL	6,110	49·10	49
GSN	11,820	25·38	25
GSO	15,180	19·76	19
GSP	15,310	19·60	19
GSQ	25,750	11·65	11
GSR	26,400	11·36	11
GSS	26,550	11·30	11
GST	21,550	13·92	13
GSU	7,260	41·32	41
GSV	17,810	16·84	16
GSW	7,230	41·49	41
GSX	6,060	49·50	49
GSY	6,040	49·67	49
GSZ	6,170	48·62	49

Call Sign	*Frequency in kc/s*	*Wavelength in Metres*	*Waveband*
GVO	18,080	16·59	16
GVP	17,700	16·95	16
GVQ	17,730	16·92	16
GVR	21,675	13·84	13
GVS	21,710	13·82	13
GVT	21,750	13·79	13
GVU	11,770	25·49	25
GVV	11,730	25·58	25
GVW	11,700	25·64	25
GVX	11,930	25·15	25
GVY	11,955	25·09	25
GVZ	9,640	31·12	31
GWA	6,125	48·98	49
GWB	9,550	31·41	31
GWC	15,070	19·91	19
GWD	15,230	19·70	19
GWE	15,435	19·44	19
GWG	15,110	19·85	19
GWH	11,800	25·42	25
GWI	7,250	41·38	41
GWJ	9,525	31·50	31
GWK	6,165	48·66	49
GWL	7,210	41·61	41
GWM	6,090	49·26	49
GWN	7,280	41·21	41
GWO	9,625	31·17	31
GWP	9,660	31·06	31
GWQ	11,840	25·34	25
GWR	15,300	19·61	19
GWS	6,035	49·71	49
GWT	9,675	31·01	31
GWU	15,210	19·72	19
GWV	11,790	25·45	25
GWW	11,890	25·23	25
GWX	9,570	31·35	31
GWY	9,700	30·93	31
GWZ	7,200	41·67	41
MCM	7,135	42·05	41
MCN	9,770	30·71	31
MCO	11,910	25·19	25
MCP	9,735	30·82	31
MCQ	11,945	25·12	25
MCR	9,760	30·74	31
MCS	7,110	42·19	41

11. Chain of Apparatus involved in a Broadcast Transmission.

The essential chain of apparatus involved in a broadcast transmission is indicated in Fig. 1.

Audio-frequency currents from the microphone are amplified by the control amplifier and fed to line. The amplification or *gain* of the control amplifier is continually adjusted by hand so that

the outgoing volume as observed on the programme meter is kept within prescribed limits. On arriving at the transmitter the line currents pass through an audio-frequency amplifier to the *modulator*. This audio-frequency amplifier is usually divided into two parts: a low-power amplifier, usually called a speech input or B amplifier located in the transmitter control room; and a higher power amplifier driven by the speech-input amplifier, which constitutes part of the transmitter proper.

The modulator is an audio-frequency amplifier applying audio-frequency voltages to the *modulated amplifier* in such a way as to produce at the output of the *mod. amp.* a modulated carrier of frequency equal to the frequency of the high-frequency currents

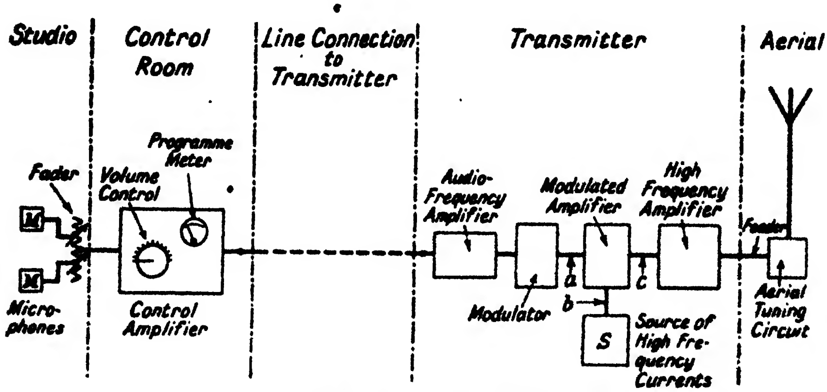

FIG. 1/I:11.—Broadcast Chain from Microphone to Aerial.

fed into the mod. amp. from *S*. From the output of the mod. amp. the modulated currents are sometimes fed directly to the aerial through a feeder and aerial tuning or coupling circuits which are contained in an aerial tuning house (A.T.H.) at the base of the aerial. In other cases a high-frequency amplifier is interposed between the mod. amp. and the feeder. In the case of short-wave transmitters A.T.H.'s are usually unnecessary.

If the audio-frequency wave is assumed to be sinusoidal the currents at (*a*), (*b*) and (*c*) then correspond respectively to the waves at *A*, *B* and *D* in Fig. 1/I:1. At 100% modulation wave *D* is evidently identical with wave *C*.

Studio Arrangements. In the B.B.C. the general arrangement of studio and (studio) control-room apparatus is that all studios and all control positions are led to a control switching point; it is therefore infinitely flexible both with regard to the number of studios and control positions and also the interconnection

of studios and control positions; any control position can be connected to any studio.

The *Faders* are devices for gradually fading in and fading out programmes. A number of programme sources are connected to the input of one control amplifier: each source through a separate fader. By operation of the faders any one programme can be picked up, and changeover can be made rapidly from one programme to another. The outputs of the control amplifiers are fed to *trap valves* which are "amplifiers" of zero gain (zero amplification) with their inputs in parallel and their outputs connected individually to different outgoing lines to transmitters as required.

Equalizers are devices inserted to compensate for the fact that lines do not transmit all frequencies with equal efficiency and have such efficiency frequency (loss-frequency) characteristics that the overall efficiency of line plus equalizer is the same at all frequencies. They are inserted after the balancing and screening transformers (called repeating coils) in all incoming lines of any appreciable length, whether from control rooms at other studio centres or from outside broadcast points. *Attenuators* are inserted after the equalizers to reduce the level of the incoming programme currents to the same level as the microphone currents; -70 db. The meaning of *level* and the method of expressing it is explained in VII:7 and 8.

Arrangements at Transmitters. Incoming lines at transmitters are also equipped with repeating coils and line equalizers, and sometimes attenuators are inserted after the equalizers to provide a standard level, which, however, is normally higher than microphone level.

The detailed circuit arrangements vary considerably at different transmitters. A simple arrangement in use at early type transmitters is shown in Fig. 2, where there are two incoming music lines, one in use and one spare. The outputs of the line equalizers are connected through fine volume-control potentiometers to the input of the speech-input amplifier which contains a coarse volume control. These potentiometers are used in conjunction with line-up tone to adjust the level of the input to the transmitter to its correct value. (See VII:8 and XV:4 for the use of line-up tone to adjust input level to the transmitter.)

They are adjusted to their correct position at the beginning of each transmission and remain in the same position throughout each programme, unless exceptional conditions arise.

The output of each speech input amplifier is connected to a

jack *J* so that either of the two incoming chains can be used to feed the transmitter.

The peak programme meter is a level-indicating device which is bridged across the circuit jack before the input to the transmitter.

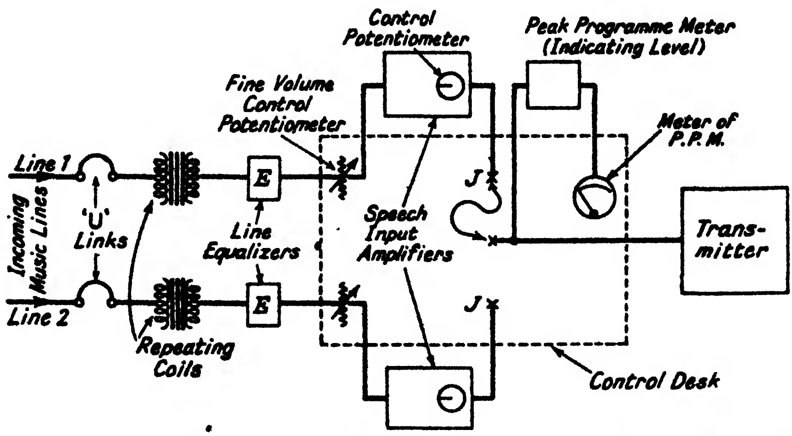

FIG. 2/I:11.—One Type of Arrangement of Incoming Lines at a Transmitter.

12. Limiting Amplifier or Limiter.

An important item in the chain, which is not shown in Figs. 1/I:11 and 2/I:11, is the *limiter* or limiting amplifier. A full description of this is given in Chapter XXII. This is in general use in cases where the importance of keeping the modulation high so as to override noise at the receiving end overweighs requirements of faithful reproduction.

The type of limiter used in this connection usually consists of an audio-frequency amplifier, the amplification of which is arranged to remain constant up to a certain limiting output level and then to fall with increase of input level in such a way that the output level can never exceed the limiting output level. This has the further advantage that the transmitter receives a measure of protection against overload and consequent flashover (sparking across of components and small air gaps).

A preliminary adjustment of levels in the chain is usually made so that peak programme level just reaches the limiting level of the limiter, and the amplification after the limiter is adjusted so that the limiting level (peak programme level) determined by the limiter just modulates the transmitter 100% (or slightly less, e.g. 95% in transmitters where appreciable distortion is experienced at 100% modulation). The amplification before the limiter is then increased

by about 8 db. (e.g. by a power ratio of about six times, see VII:5) so that the range of levels within 8 db. of peak programme level are compressed to constant level, since a programme power which is 8 db. below peak programme level just reaches the roof of the limiter, while higher levels are reduced at the output so that they do not exceed the roof of the limiter. By this means all low programme levels (more than 8 db. below peak programme level) are increased in power six times (i.e. 8 db.). By means of this artifice the observed loudness of the received signal is increased by 8 db. for all low levels, while evidently high levels which are already comparatively strong compared with interference, do not need to be increased.

Limiters of this kind do not introduce distortion of the quality of speech and music but reduce the degree of contrast. They are not to be confused with limiters of a simpler form, e.g. consisting of neon lamps bridged across a high-power part of the circuit which do have the effect of introducing distortion when they operate. Evidently the use of a limiting amplifier in the low-power audio-frequency chain has the effect of making the operation of the high-power limiter less frequent and so avoids distortion.

Since the action of the limiting amplifier is not instantaneous it has become customary to supplement the use of high power limiters by diodes bridged across the circuit and backed off so that they only conduct when the music or speech voltage exceeds a predetermined value.

For details of operation and circuit of a limiter, see XXII:3, 4 and 5.

CHAPTER II

ELECTRICAL EFFECTS AND UNITS

BROADCASTING has been made possible by the commercial application of a number of observed effects in nature. While physicists strive to explain some of these phenomena in terms of others, and to represent some of them in terms of geometrical pictures, the ordinary engineer merely accepts them as implements of his craft.

In choosing units in which to represent the magnitudes of these effects it has naturally been simplest to present them in terms of familiar units of known effects and it has been found possible to relate them to the units of mass, length and time. The units used for this purpose are the French metric units: the *Centimetre*, the *Gramme* and the *Second*. The electrical and magnetic units derived from these are called Centimetre-Gramme-Second units—C.G.S. units. Since C.G.S. units, which are called absolute units, are usually of unsuitable magnitude for everyday use, practical units have been defined which are equal to an arbitrary multiple or submultiple of the C.G.S. unit.

It is possible to derive two sets of C.G.S. units:

(*a*) **Electromagnetic (C.G.S.) Units**: from consideration of the effect of the interaction of magnetic fields.

(*b*) **Electrostatic (C.G.S.) Units**: from consideration of the effect of the interaction of electrostatic fields.

The derivation of the electromagnetic units is most instructive from the point of view of understanding the relation of different effects and *the presentation below is therefore given in terms of electromagnetic units*. The relation between the two kinds of C.G.S. units and their relation to practical units is then given in Tables I to XI in Section 30 of this chapter.

The electromagnetic C.G.S. units are those with the prefix "*ab*" and the electrostatic C.G.S. units are those with the prefix "*stat*". In certain cases units which have not been defined are included in these tables, since sufficient definition for practical purposes is given by their relations to defined units.

Among the most striking effects used in broadcasting are those connected with the thermionic valve. These are of a special nature, are unrelated to the determination of units and are therefore dealt with separately in Chapter IX.

The effects and descriptions below are, as far as possible, presented in a logical sequence, such that each description prepares the ground for certain of those that follow.

1. The Magnetic Field.

The properties of a magnet are well known, and its use in compasses has made everybody familiar with the fact that its poles are of opposite kinds, called North and South, and have the characteristic that like poles repel and unlike poles attract. The North pole is the pole which points to the earth's North magnetic pole when the magnet is freely suspended. A free isolated North pole, assuming such to be a practical possibility, if placed at any

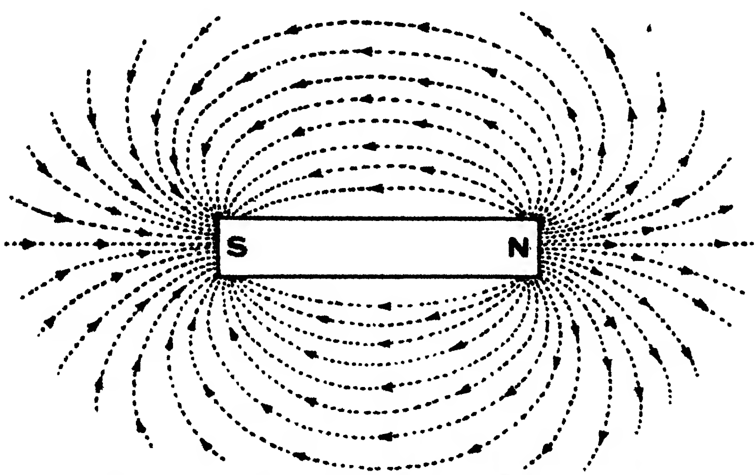

Fig. 1/II:1.—Lines of Force around a Bar Magnet.

point near a magnet would travel along a curved path running from the North pole to the South pole, being propelled with a force dependent on the strength of the *magnetic field* in that neighbourhood. Magnetic fields are therefore represented conventionally by lines of force or *flux* running *from* North *to* South pole outside the magnet, drawn along the paths which would be followed by a free North pole. The general form of these is indicated in Fig. 1. (Inside the magnet the lines of force are continued and then run from South pole to North pole).

A magnetic pole of unit strength is one which, when placed one centimetre from another *unit magnetic pole*, repels it with a force of one dyne.

(A dyne is the force required to produce an increase of velocity

of one centimetre per second each second when acting on a mass of one gramme ; this is very nearly $\frac{1}{981}$ gramme.)

Unit magnetic field is defined as the field of such strength that it acts on a unit magnetic pole in the field, with a force of one dyne. Such a field is said to have a strength of one Maxwell (this name is unimportant) and is represented by one line per square centimetre. The number of lines of force per square centimetre defines the *flux density*, i.e. the field strength.

2. Electrons, Dielectrics (Insulators), and Conductors.

Most electrical phenomena are concerned with two small elementary particles each of about $\frac{1}{1,000}$ the mass of the hydrogen atom : the electron and the positron.

Electrons repel one another, but attract and are attracted by positrons. Electrons and positrons are therefore said to carry or to consist of electrical charges of equal magnitude and opposite sign, e.g. the electron carries a negative charge and the positron a positive charge of $1{\cdot}605 \times 10^{-19}$* coulomb. (The coulomb is defined in section 10 of this chapter.) Normal atoms contain a quantity balance of the two types of particle, such that at distances large compared to atom sizes, atoms do not attract one another and are said to be *uncharged* or *neutral*. Positrons are always bound in the atoms but the electrons sometimes possess a limited degree of freedom.

Certain substances called *dielectrics* behave as if each atom normally preserves its own neutrality in the most strict way, always retaining exactly the correct balance of electrons and positrons. In such substances it is impossible for any appreciable exchange of electrons to take place through the mass of the substance. Such substances are, therefore, said to be *insulators* or *non-conductors* of electricity. Examples of insulators are glass, mica, porcelain, rubber, silk.

Conductors are substances which normally maintain their neutrality as a whole, electrons being free to move from atom to atom within the mass of the substance, in such a way that a stream of electrons can flow through the mass if suitably urged to do so. *An electric current consists of a stream of electrons.* Examples of conductors in order of conductivity are silver, copper, gold, aluminium, iron : copper being in most common use. All metals are conductors.

* Millikan's results recalculated with corrected value for viscosity of air, by Kellstroem. See *Nature*, 1935.

2.1. Convention for Direction of Current. By an unfortunate chance, before the existence of electrons was known, the convention for direction of current was established from the behaviour of a primary cell (e.g. of a battery) having a carbon and zinc plate, as being from carbon to zinc in the circuit external to the cell. As electrons flow from zinc to carbon in the external circuit of a carbon-zinc cell the convention for direction of current is that it is in the *opposite* direction to the flow of electrons. This convention is normally used, and in dealing with certain problems connected with flow of electrons it is necessary to remember that they flow in the opposite direction to the *conventional* direction of current.

The carbon plate is by convention said to be positive and is sometimes painted red, and the negative plate is said to be negative and is painted black. In all electric circuits connected to cells or other sources the current flows through the circuit from the positive or red terminal to the negative or black terminal. (*The electrons flow in the opposite direction.*)

3. Electromotive Force = e.m.f.

One way of providing a motive force to make electrons flow along a wire is to move a magnet past a wire in such a way that the

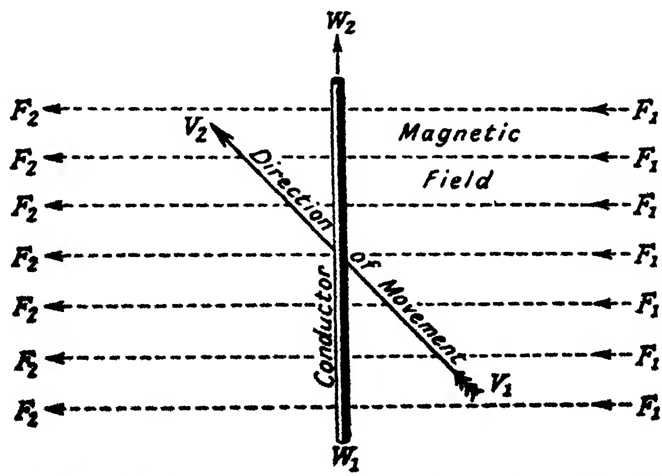

FIG. 1/II:3.—Moving Conductor in a Magnetic Field, showing Relation between Direction of Field, Direction of Motion, and Direction of Induced E.M.F Arrow V_1, V_2 points into paper.

wire cuts the lines of magnetic force. Such a force is called an electromotive force.

Referring to Fig. 1, if a wire W_1W_2 moves into the paper in

the direction V_1V_2 at right angles to the field F_1F_2, the electrons in the wire are induced to flow along the wire in the direction from W_1 to W_2. An electromotive force is then said to have been *induced* in the wire.

If the ends of the wire are connected to a circuit, a current of electrons will flow round that circuit, going out of one end of the wire and coming back into the other end of the wire.

If, however, the wire is just a short piece, as shown in Fig. 1, there will be no circuit in which the electrons can flow, and in that case, as long as the wire is moving through the magnetic field in the same sense, they will pile up towards one end of the wire, increasing in density towards the end of the wire ; the other end of the wire will have a deficit of electrons progressively increasing as that end is approached. The magnitude of an e.m.f. is usually denoted by E or e.

4. The Electric Field.

The wire is therefore said to be charged at its two ends, and, if a free electron were placed in free space near the wire, in other words in the *field* of the wire (neglecting the presence of the magnetic field), it would travel along a curved path joining the end where the electrons are assembled, to the other end, and being impelled with a force dependent on the density of electrons at the end of the wire and the distance apart of the ends. There is, therefore, said to be an *electric field* surrounding the wire. The form of the field depends on the distribution of the charge, but in this case is evidently very similar in form to the field round a bar magnet. If the field is steady it is called an *electrostatic field*.

5. Potential Difference.

The part of a wire in which the e.m.f. is produced is analogous to a pump producing a difference in pressure between its inlet and outlet pipes. In electrical circuits the term pressure is replaced by *potential* and the e.m.f. in the wire is said to create a *potential difference* (p.d.) between the ends of the wire.

Both electromotive force and potential difference are measured in volts. The volt is defined in II:7 below.

According to British Standard Specification No. 423 the magnitude of a p.d. in volts should be represented by V. The practice of using E and e for this purpose is, however, so general that it has been retained except in one or two instances where it is particularly necessary to distinguish between voltage and e.m.f. In

the more advanced chapters, E and e have been reserved for e.m.f., while V and v are used for volts.

6. Field Strength = Strength of Electric Field.

The magnitude of an electric field in space is measured by the *potential gradient* in volts per metre, millivolts per metre or microvolts per metre, and is the p.d. between two points a metre apart along the direction of steepest gradient.

7. Definition of the Volt.

An electromotive force of one absolute unit is generated in each centimetre of a wire moving with a velocity of one centimetre per second through a uniform magnetic field of one Maxwell, i.e. one line per square centimetre, with the wire direction, the direction of the lines of force and the direction of movement all at right angles. In other words, *unit electromotive force is generated in a length of wire which cuts one line of force per second.* This absolute unit of e.m.f. is too small for practical purposes, so that a practical unit, the volt, is used, which is 100 million times as large as the absolute unit.

$$
\begin{aligned}
1 \text{ volt} &= 10^8 \text{ absolute units (C.G.S. units)} \\
&= 1{,}000 \text{ millivolts} = 1{,}000 \text{ mV} \\
&= 10^6 \text{ microvolts} = 10^6\ \mu\text{V}
\end{aligned}
$$

Volts are measured on a voltmeter: millivolts on a millivolt meter.

8. Current and Magnetic Effect of a Current.

The number of electrons per second which flow along a circuit determine the magnitude of the current. The fundamental unit of current is defined by the magnetic effect of the current.

When a current flows through a straight piece of wire a cylindrical magnetic field exists round the wire as indicated in Fig. 1.

If a current is passed through a wire wound in a coil it creates a magnetic field as indicated in Fig. 2. A current flowing

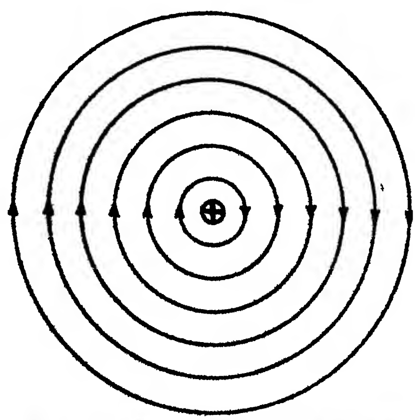

FIG. 1/II:8.—Direction of Magnetic Field round a Conductor carrying a Current flowing into the Paper.

through a coil therefore exerts a force on a magnetic pole in its vicinity.

The sense of the field which is produced in a coil can be determined by the following rule: considering the coil to be a magnet with a North pole and a South pole, the North pole being the one

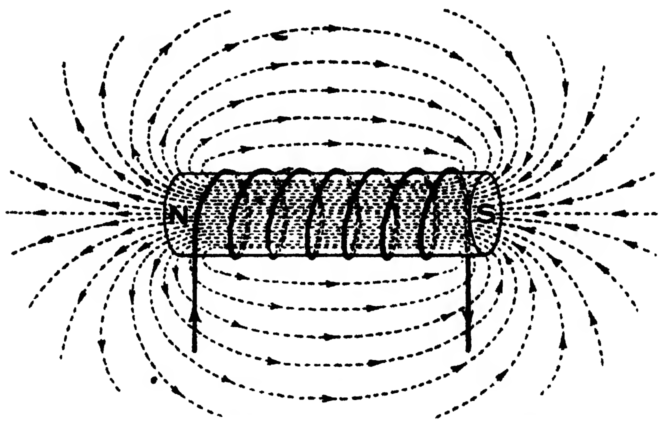

FIG. 2/II:8.—Coil carrying Current showing Relation between Current Direction and Direction of Field.

which would point towards the North if the magnet were freely suspended.

Ampere's Rule. Imagine a man swimming in the same direction as the current (i.e. against the electron stream), facing the axis of the coil, then the North pole will be on his left-hand side.

9. Definition of Ampere, the Practical Unit of Current.

Absolute unit current is defined as that current which, flowing through 1 centimetre of an arc of a circle of 1 centimetre radius, operates on a unit magnetic pole located at the centre of the circle with a force of 1 dyne.

This absolute unit is too large for ordinary practice and the practical unit, the ampere, is one-tenth of the absolute unit.

One ampere = 0·1 absolute unit (C.G.S. unit)
= 1,000 milliamps. = 1,000 mA
= 10^6 microamps. = 10^6 μA

Amperes are measured on an ammeter.
Milliamps. are measured on a milliammeter.
Microamps. are measured on a microammeter.

10. Electrical Quantity: Definition of the Coulomb, the Practical Unit of Electrical Quantity.

Electrical quantity represents a number of electrons; it may be the number of electrons present on an isolated conductor, or the number of electrons which has flowed along a conductor in a circuit, or a number of electrons in free space.

Unit Quantity: the Coulomb. The practical unit of electrical quantity is the *coulomb* and is the quantity of electricity flowing per second past any point in a circuit carrying a current of one ampere. A current of one ampere is a flow of one coulomb per second.

$$1 \text{ coulomb} = 6{\cdot}23 \times 10^{18} \text{ electrons}$$

11. Linkage: E.M.F. induced by Change of Linkage.

If a circuit exists in the form of a loop or a turn of wire (e.g. one turn of a winding) and a line of force passes through the loop or turn, the line of force is said to link the turn and the magnitude of the linkage is said to be unity.

Linkage is the sum of the products of lines of force and turns linked, e.g. if 5 lines link 10 turns, 3 lines link 20 turns, and one line links 30 turns, the linkage

$$N = 5 \times 10+3 \times 20+30 = 140$$

If a loop of wire cuts two lines of force simultaneously, the e.m.f.s will add or subtract according to the relative sense of cutting of the lines of force. A little consideration will show that if the cutting of the two lines of force causes change of linkage, the e.m.f.s add, and if not they subtract.

The e.m.f. induced in one turn in volts is therefore equal to the change of linkage with that turn per second $\times 10^{-8}$. See II:7.

The e.m.f. induced in a complete coil in volts is then equal to the total change of linkage per second $\times 10^{-8}$.

12. Resistance: Definition of the Ohm, the Practical Unit of Resistance.

When a potential difference is applied across the ends of a wire, or when an e.m.f. is induced in a closed loop of wire, a current flows depending on the magnitude of the p.d. or e.m.f. and on the resistance offered by the wire to the flow of electrons.

The unit of resistance is the Ohm, and is so chosen that one volt across one ohm gives rise to a current of one ampere.

Hence *Current* in amperes = *Voltage* ÷ *Ohms.*

One megohm = 10^6 ohms or 1 $M\Omega = 10^6\ \Omega$. One ohm = 10^6 microhms.

Resistance is measured on a Wheatstone bridge, an ohmmeter or a megger, the last being used for resistances of a few megohms and higher.

Conductivity is the reciprocal of resistance.

The unity of conductivity is the Mho.

Hence *Current* in amperes = *Voltage* × *Mhos.*

A resistance of 1 ohm has a conductivity of 1 mho, a resistance of 10 ohms a conductivity of 0·1 mho, a resistance of 0·001 ohm a conductivity of 1,000 mhos, and so on.

13. Wire Resistance.

The resistance of a piece of wire of length l and of area of cross-section A is

$$R = \rho\frac{l}{A} \text{ ohms}$$

Where l = the length of wire in centimetres

A = the area of cross-section of the wire in square centimetres.

ρ = the specific resistance of the wire material in ohms per centimetre cube (Ω/cm.^{3}) = the resistance between opposite faces of a one-centimetre cube of the material.

The specific resistance of a conducting material is sometimes called the *resistivity*.

Resistivity varies with temperature. In Fig. 1 is plotted the relation between the resistivity of hard-drawn copper wire and temperature in degrees Centigrade. For ease of reference a Fahrenheit scale is shown at the top.

Example 1. At 68° Fahrenheit the resistance of a mile (1·609 kilometres) of 25 S.W.G. (Standard Wire Gauge) copper wire which is 20 mils (0·0508 cm.) in diameter

$$= 1{\cdot}726 \times 10^{-6} \times \frac{1{\cdot}609 \times 10^5}{\frac{\pi}{4} \times 0{\cdot}0508^2} = 137\ \Omega$$

(1 mil = 0·001 in.)

The resistances of standard round wire conductor and stranded conductor at 60° F. and 68° F. are given in Tables I and II respectively, which follow immediately. The resistance at any other

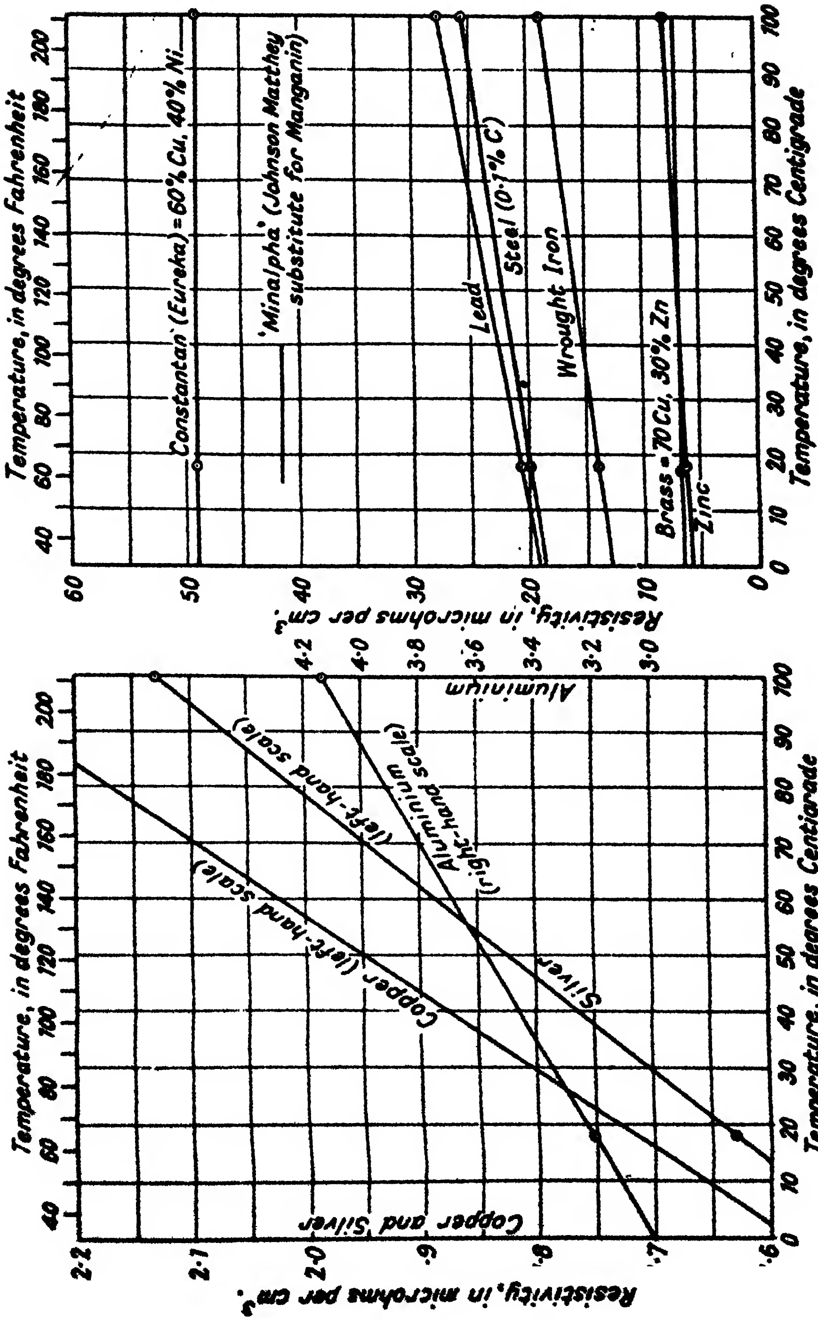

FIG. 1/II:13.—Resistivities of Various Metals and Alloys.

(Extracted from *Tables of Physical and Chemical Constants* by G. W. C. Kaye and T. H. Laby by arrangement with Messrs. Longmans, Green & Co.)

Table I

Diameter, Area, Resistance and Turns per Inch of Annealed Copper Conductors

S.C.C. = Single cotton covered. D.C.C. = Double cotton covered. S.S.C. = Single silk covered. Etc.

S.W.G.	Diameter. Inches	Diameter. Milli-metres	Area. Sq. in.	Resistance at 60° F. Ohms/1,000 ft.	Resistance at 68° F. Ohms/1,000 ft.	Approximate No. of Wires per Inch							
						Bare	Enamel	Enamel and Single Silk	Enamel and Single Cotton Ordinary	Single Silk	Double Silk	Single Cotton Ordinary	Double Cotton Ordinary
7/0	0·500	12·7	0·196	0·0408	0·0417	2·000							
6/0	0·464	11·8	0·169	0·0474	0·0483	2·155							
5/0	0·432	10·9	0·147	0·0547	0·0557	2·315							
4/0	0·400	10·2	0·126	0·0637	0·0650	2·500							
3/0	0·372	9·45	0·109	0·0736	0·0750	2·688							
2/0	0·348	8·84	0·0951	0·0841	0·0857	2·874							
1/0	0·324	8·23	0·0824	0·0971	0·0987	3·086							
1	0·300	7·62	0·0707	0·1132	0·115	3·333							
2	0·276	7·01	0·0598	0·1338	0·135	3·623							
3	0·252	6·4	0·0499	0·1605	0·163	3·968							
4	0·232	5·89	0·0423	0·1893	0·193	4·310							
5	0·212	5·38	0·0353	0·2267	0·231	4·717							
6	0·192	4·88	0·0289	0·2764	0·281	5·208							
7	0·176	4·47	0·0243	0·3289	0·333	5·682							
8	0·160	4·06	0·0201	0·3980	0·407	6·250							
9	0·144	3·66	0·0163	0·4914	0·500	6·944							
10	0·128	3·25	0·0129	0·6219	0·633	7·810	7·5						
11	0·116	2·95	0·0106	0·7570	0·770	8·620	8·2					8·0	7·6
12	0·104	2·64	0·00849	0·9420	0·960	9·620	9·1		8·7			8·8	8·4
13	0·092	2·34	0·00665	1·204	1·23	10·87	9·4		9·7			9·9	9·3
14	0·080	2·03	0·00503	1·592	1·62	12·50	10·8		10·9			11·2	10·5
15	0·072	1·83	0·00407	1·966	2·00	13·89	13·2		12·0			12·5	11·8
16	0·064	1·63	0·00322	2·488	2·53	15·63	14·8	14·1	13·3	14·7	14·5	13·9	13·0
17	0·056	1·42	0·00246	3·249	3·31	17·86	16·9	16·0	15·1	16·7	16·5	15·7	14·5
18	0·048	1·22	0·00181	4·422	4·50	20·83	19·7	18·9	17·2	19·8	19·4	18·0	16·8

												21·0	19·4
20	0·036	0·914	0·00102	7·800	8·00	27·78	20·0	24·0	21·7	20·0	25·3	23·5	21·0
21	0·032	0·813	804×10^{-6}	9·950	10·1	31·25	29·2	27·5	24·4	29·1	28·3	26·1	23·0
22	0·028	0·711	616×10^{-6}	13·00	13·2	35·71	33·0	31·2	27·0	33·0	31·9	29·1	25·4
23	0·024	0·610	452×10^{-6}	17·69	18·0	41·67	38·3	36·1	30·3	39·6	37·8	34·2	28·4
24	0·022	0·559	380×10^{-6}	21·05	21·4	45·45	41·6	39·5	34·4	42·1	36·9	36·7	31·0
25	0·020	0·508	314×10^{-6}	25·47	25·9	50·00	45·5	43·4	37·0	46·0	43·1	39·6	33·1
26	0·018	0·457	254×10^{-6}	31·45	32·0	55·56	50·2	48·1	40·0	51·2	48·3	43·0	35·4
27	0·0164	0·417	211×10^{-6}	37·88	38·3	60·98	55·1	52·3	43·3	55·8	52·3	46·2	37·6
28	0·0148	0·376	172×10^{-6}	46·52	47·3	67·57	61·0	57·4	46·9	61·7	57·4	50·2	38·6
29	0·0136	0·345	145×10^{-6}	55·09	56·0	73·53	66·0	61·7	49·8	66·7	61·7	53·5	40·5
30	0·0124	0·315	121×10^{-6}	66·27	67·3	80·65	72·5	67·1	54·3	72·4	66·6	57·1	44·4
31	0·0116	0·295	106×10^{-6}	75·70	77·0	86·21	77·5	71·4	56·8	76·9	70·4	59·8	46·0
32	0·0108	0·274	$91·6 \times 10^{-6}$	87·40	89·0	92·59	82·7	76·3	59·5	81·9	74·6	62·8	47·8
33	0·0100	0·254	$78·5 \times 10^{-6}$	101·9	104·0	100·0	89·3	81·3	62·5	88·7	79·3	66·2	49·7
34	0·0092	0·234	$66·5 \times 10^{-6}$	120·4	123·0	108·7	97·0	87·7	65·8	94·3	84·7	69·9	51·7
35	0·0084	0·213	$55·4 \times 10^{-6}$	144·4	147·0	119·0	105·0	94·3	74·6	102·0	90·9	80·0	57·1
36	0·0076	0·193	$45·4 \times 10^{-6}$	176·4	180·0	131·6	116·0	102·0	79·4	111·0	97·9	85·4	59·9
37	0·0068	0·173	$36·3 \times 10^{-6}$	220·4	224·0	147·1	128·0	112·0		122·0	104·0	91·7	63·7
38	0·0060	0·152	$28·3 \times 10^{-6}$	283·0	288·0	166·7	145·0	125·0		135·0	113·0	99·0	67·7
39	0·0052	0·132	$21·2 \times 10^{-6}$	376·8	383·0	192·3	164·0	143·0		151·0	125·0	107·0	70·9
40	0·0048	0·122	$18·1 \times 10^{-6}$	442·2	450·0	208·3	178·0	151·0		161·0	131·0	112·0	75·1
41	0·0044	0·112	$15·2 \times 10^{-6}$	526·3	537·0	227·3	192·0	163·0		175·0	149·0		
42	0·0040	0·102	$12·6 \times 10^{-6}$	636·8	650·0	250·0	208·0	175·0		188·0	158·0		
43	0·0036	0·0914	$10·2 \times 10^{-6}$	786·3	800·0	277·8	227·0	192·0		204·0	169·0		
44	0·0032	0·0813	$8·04 \times 10^{-6}$	995·0	1013·0	312·5	256·0	208·0		222·0	181·0		
45	0·0028	0·0711	$6·16 \times 10^{-6}$	1300·0	1323·0	357·1	286·0	227·0		243·0	196·0		
46	0·0024	0·0610	$4·52 \times 10^{-6}$	1769·0	1860·0	416·7	333·0	256·0		270·0	212·0		
47	0·0020	0·0508	$3·14 \times 10^{-6}$	2547·0	2600·0	500·0	385·0	285·0		302·0	232·0		
48	0·0016	0·0406	$2·01 \times 10^{-6}$	3980·0	4067·0								
49	0·0012	0·0305	$1·13 \times 10^{-6}$	7077·0	7200·0								
50	0·0010	0·0254	$0·78 \times 10^{-6}$	10190·0	10367·0								

(By courtesy of Messrs. W. T. Glover & Co., The London Electric Wire Co. and Smith's Ltd., and Sir Isaac Pitman & Sons.

temperature can be obtained by multiplying the value of resistance at 60° by the resistivity at the temperature required and dividing by the resistivity at 60° F.

Example 2. What is the resistance of 50 ft. of No. 40 gauge S.W.G. copper wire at 50° F.?

From Table I the resistance of 1,000 ft. of 40 S.W.G. copper wire at 60° F. is 442·2 Ω, so that the resistance of 50 feet is 22·11 Ω. From Fig. 1 the resistivities at 50° F. and 60° F. are respectively $1{\cdot}656 \times 10^{-6}$ and $1{\cdot}697 \times 10^{-6}$ ohms per cm.³. Hence the required resistance is

$$22{\cdot}11 \times \frac{1{\cdot}656}{1{\cdot}697} = 21{\cdot}6\,\Omega$$

Resistivities of Other Metals and Alloys. Fig. 1 gives approximate values of the resistivities of certain of the more common metals and alloys. With the exception of the resistivity of Minalpha, these values have been obtained from *Tables of Physical and Chemical Constants* by G. W. C. Kaye and T. H. Laby.

13.1. Skin Effect. The resistance of a conductor is a minimum when the current density is uniform over the section of the conductor. This is the condition for D.C. For alternating currents the paths along the conductor near the centre of the conductor are surrounded by more flux lines than those near the outside, so that these paths have greater reactance. The current density is therefore greatest near the surface of the conductor. This phenomenon is called the skin effect and gives rise to an increase in resistance. At radio frequencies the resistance may increase to as much as thirty times the D.C. resistance, or even higher. A very excellent summary of the more important formulae for calculating the high-frequency resistance of conductors appeared in *Electronics* for February 1942, p. 44; this was prepared for the use of the engineers of the General Electric Company of America by J. R. Whinnery. The following are extracts from this summary, but the original is well worth referring to; it deals with special cases which are not given below and quotes references providing documentation. It is, however, purely theoretical and does not give any experimental verification of the methods of calculation.

The high-frequency resistance is expressed in terms of quantities which may appear mysterious to the uninitiated, but which present no difficulty in practice because their values are given in the form of graphs. Definitions of these are given in the list of conventions below, although for practical purposes they need only be

regarded as factors or parameters to be inserted in the formulae given.

Conventions.

d = depth of penetration in centimetres. For a plane solid of infinite depth, this is the depth at which the current density has fallen to $1/e$ (to about 37%) of its value at the surface. It is also the thickness of a plane conductor having a D.C. resistance equal to the h.f. resistance of the plane conductor of infinite depth. The depth of penetration falls as the frequency is increased.

R_s = the skin effect resistance, in ohms, of an element, 1 cm. long and 1 cm. wide, of a plane conductor of infinite depth.

R_0 = the D.C. resistance of any conductor.

r_0 = the radius of a solid round wire, or the outer radius of a hollow tubular conductor in centimetres.

r_i = the inner radius of a hollow tubular conductor in centimetres.

t = the wall thickness of a tubular conductor.

l = length of conductor in centimetres.

R_1 = resistance at low frequencies where the depth of penetration is large compared to the conductor size.

R_2 = resistance at intermediate frequencies where the depth of penetration is comparable with the conductor size.

R_3 = resistance at high frequencies where the depth of penetration is small compared with the conductor size.

Round Wire at (Low) Frequencies where $r_0/d < 1{\cdot}5$, that is the Radius is less than $1{\cdot}5 \times$ the depth of penetration.

The resistance is given by

$$R_1 = R_0\left[1+\frac{1}{48}\left(\frac{r_0}{d}\right)^4\right] \quad . \quad . \quad . \quad (1)$$

The error of this formula is less than $\pm 5\%$ if the conductor radius is less than $1{\cdot}5 \times$ the depth of penetration (i.e. $\frac{r_0}{d} < 1{\cdot}5$) and decreases as the value of $\frac{r_0}{d}$ decreases.

The value of the depth of penetration d is given in Fig. 2 for silver, copper, aluminium, brass and solder, as a function of frequency in megacycles per second. For most purposes Fig. 3 will be found more useful than equation (1).

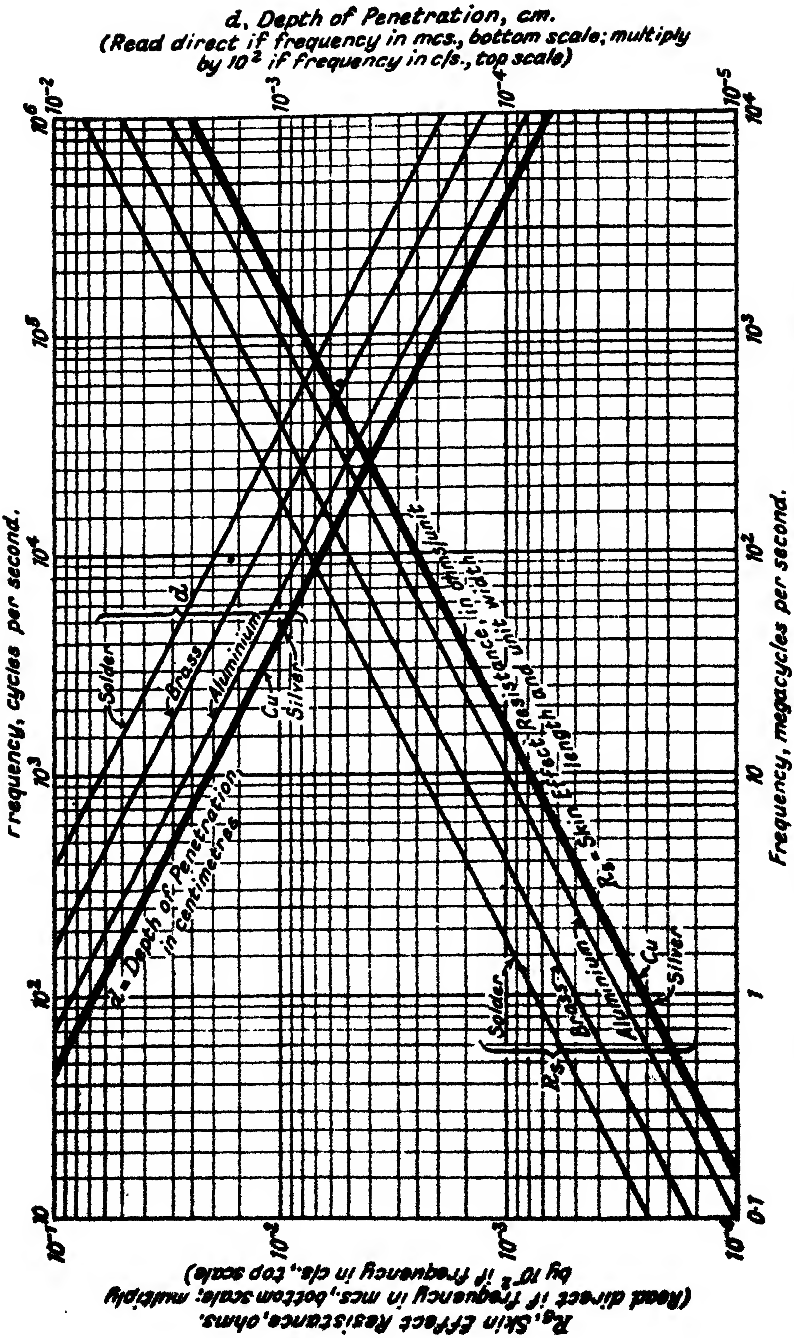

FIG. 2/II:13.—Values of Skin Effect Resistance and Depth of Penetration as defined.

(By courtesy of the Editor of *Electronics*.)

Round Wire at (Intermediate) Frequencies where r_0/d lies between 1·5 and 14.

The ratio R_2/R_0 is plotted in Fig. 3 as a function of r_0/d.

This ratio is derived from a more accurate expression than equation (1).

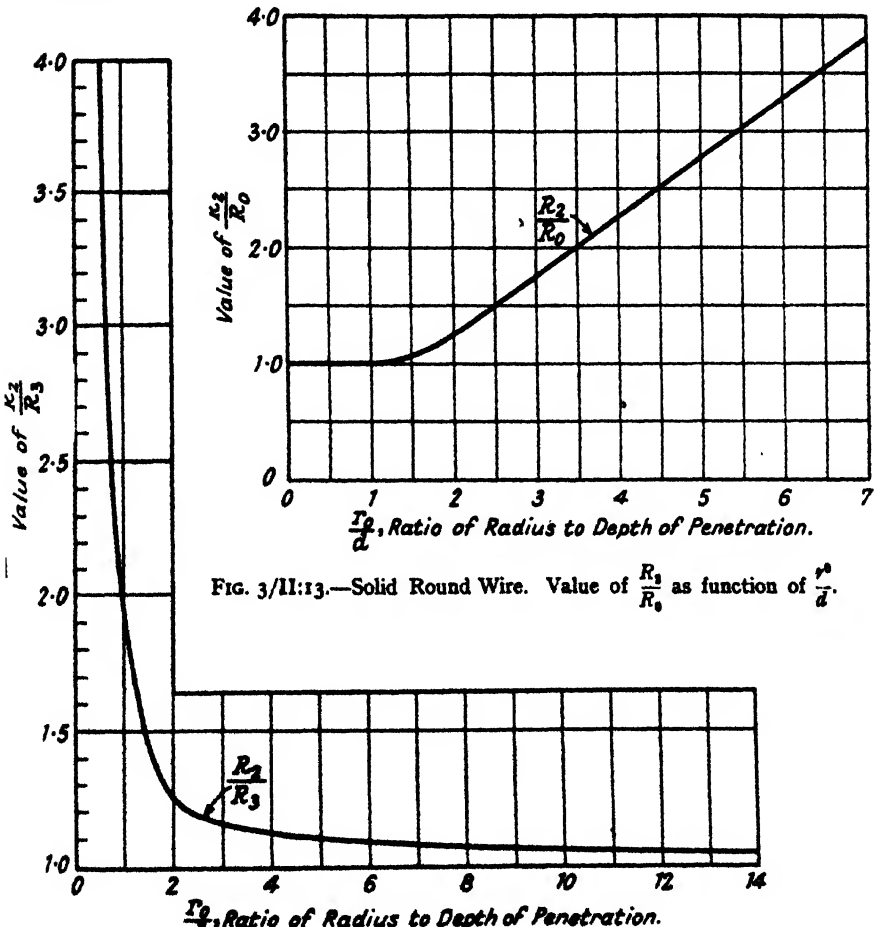

FIG. 3/II:13.—Solid Round Wire. Value of $\frac{R_2}{R_0}$ as function of $\frac{r_0}{d}$.

FIG. 4/II:13.—Solid Round Wire. Value of $\frac{R_2}{R_3}$ as function of $\frac{r_0}{d}$.

(By courtesy of the Editor of *Electronics*.)

Alternatively, the value of R_2 may be obtained from R_3 (see equation (2) below) by use of Fig. 4, which is a plot of the ratio R_2/R_3 against r_0/d. R_3 is given by equation (2) below.

Fig. 3 may be used for values of r_0/d between 1 and 7, while Fig. 4 must be used for values of r_0/d between 7 and 14.

Round Wire at (High) Frequencies where r_0/d is greater than 14.

In this case, since the penetration of the current into the conductor is slight, curvature of the surface is unimportant, and the round wire acts practically as a plane solid of great depth, and width equal to the circumference of the wire. The high-frequency

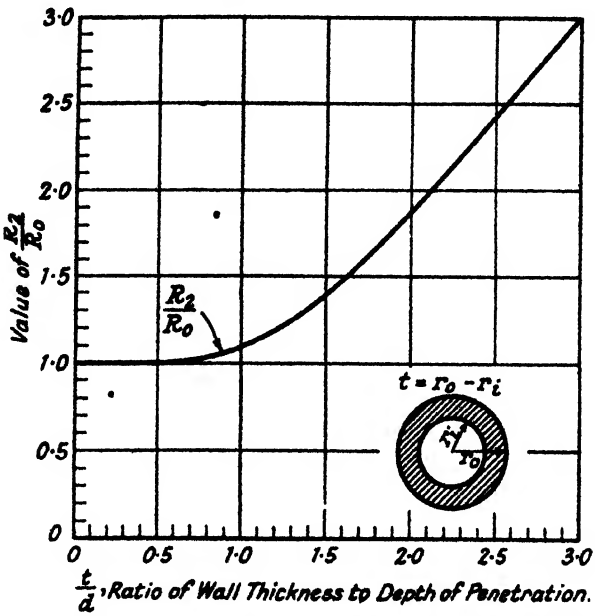

FIG. 5/II:13.—Tubular Conductor. Value of $\frac{R_t}{R_0}$ as function of $\frac{t}{d}$.

(By courtesy of the Editor of *Electronics*.)

resistance is therefore equal to $R_s \times l$ and divided by the circumference of the conductor in centimetres.

$$R_3 = \frac{R_s l}{2\pi r_0} \qquad (2)$$

The value of R_s is given in Fig. 2 for silver, copper, aluminium, brass and solder, as a function of frequency in megacycles per second.

Tubular Conductors.

It is stated that the resistance for very small depths of penetration depends on whether the current is fed to the inner radius or the outer radius of the tube. It appears, however, that in the normal practical case the current is always fed to the outer radius.

For current fed to outer radius

$$R_2 = \frac{R_s l}{2\pi r_o}$$

For current fed to inner radius

$$R_2 = \frac{R_s l}{2\pi r_i}$$

Thin-Walled Tubes at Frequencies where the Depth of Penetration d is Comparable with the Thickness t.

Here the intermediate frequency case will be considered to embrace the low-frequency case.

Fig. 5 shows a plot of R_2/R_0 against t/d, while Fig. 6 shows a plot of R_2/R_3 against t/d.

In most practical tubes the depth of penetration at radio frequencies is negligible compared with the tube thickness and $R_{HF} = R_s$, see "Thick-Walled Tubes" below.

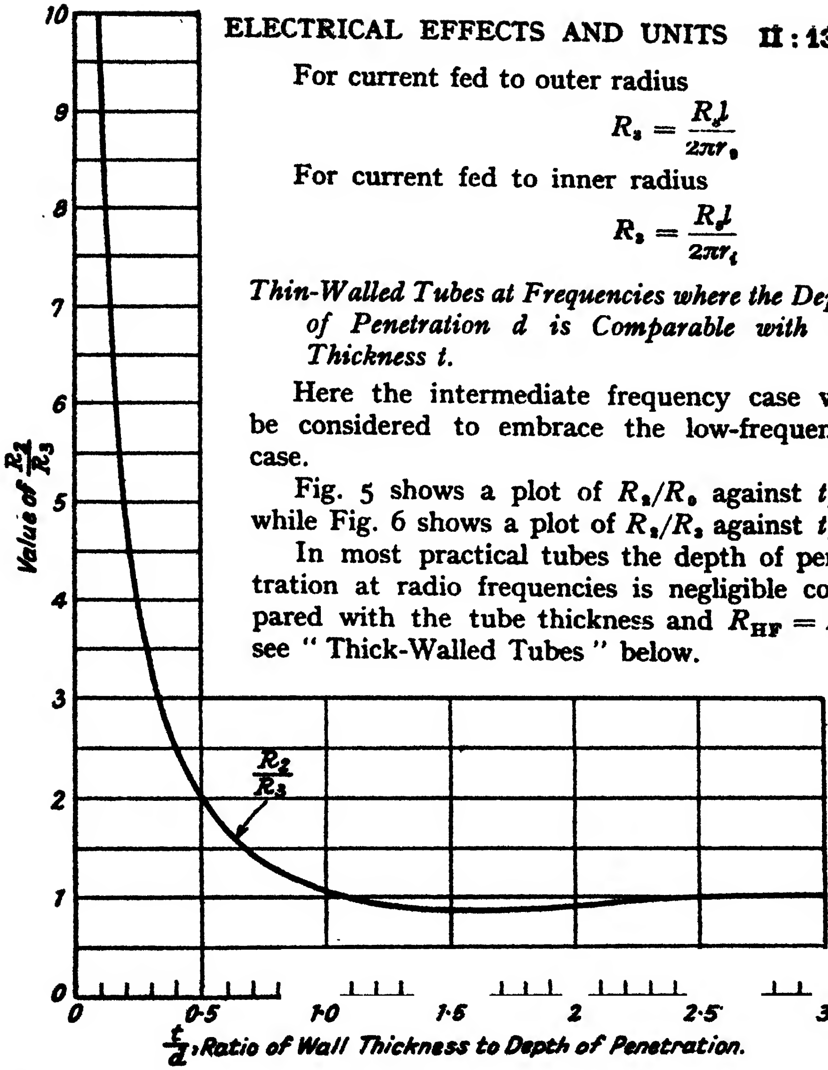

FIG. 6/II:13.—Thin-walled Tubular Conductor. Value of $\frac{R_2}{R_3}$ as function of $\frac{t}{d}$.

(By courtesy of the Editor of *Electronics*.)

Calculation of Skin Effect Resistance R_s and Depth of Penetration d, when not given by Fig. 2.

These may be calculated by the following formulae:

$$d = \frac{1}{2\pi\sqrt{f\mu g \times 10^{-9}}} \text{ centimetres} \quad . \quad . \quad . \quad . \quad (3)$$

$$R_s = \frac{1}{gd} \text{ ohms} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where f = frequency in c/s.

g = conductivity of the material in mhos per centimetre cubed = the reciprocal of the resistivity ρ expressed in ohms per centimetre cubed.

μ = permeability of the material.

General Representation of High-Frequency Resistance of Solid Round Copper Conductors.

From (2), (3) and (4), putting $\mu = 1$

$$R_s = \frac{R_d l}{2\pi r_0} = \frac{l}{2\pi r_0 g d} = \frac{l}{2\pi r_0 g} \times 2\pi\sqrt{fg \times 10^{-9}}$$

$$= \frac{l}{31{,}600 r_0}\sqrt{\frac{f}{g}} = \frac{l\sqrt{f\rho}}{31{,}600 r_0} \text{ ohms} \quad . \quad . \quad . \quad . \quad (5)$$

where l and r_0 may be specified in any unit provided both are expressed in the same units.

If $g = 5{\cdot}8 \times 10^5$ so that $\rho = 1{\cdot}724 \times 10^{-6}$ and $l = 1{,}000$ ft. $= 12{,}000$ ins.

$$R_s = \frac{12{,}000}{31{,}600 r_0}\sqrt{f \times 1{\cdot}724 \times 10^{-6}} \text{ ohms per 1,000 ft.}$$

where r_0 is in inches.

$$\therefore R_s = \frac{4{\cdot}98 \times 10^{-4}}{r_0}\sqrt{f} \text{ ohms per 1,000 ft.} \quad . \quad . \quad . \quad (6)$$

This relation is plotted on Fig. 7 for values of r_0 from 0·001 in. to 10 ins. (and of course $\rho = 1{\cdot}724 \times 10^{-6}$). From equation (5) it is evident that R_s is proportional to the square root of the resistivity so that the H.F. resistance corresponding to any other value of resistivity can be easily determined.

It is useful to divide Fig. 7 into three regions:

(1) in which $R_{HF} = R_1$: the values from Fig. 7 are inapplicable and equation (1) must be used: i.e. r_0/d less than 1·5.

(2) in which $R_{HF} = R_2$ either obtained from Fig. 3 or Fig. 4: i.e. r/d lies between 1·5 and 14: values from Fig. 7 inapplicable.

(3) in which $R_{HF} = R_s$ and the values of R_{HF} may be read directly from Fig. 7: i.e. r_0/d greater than 14.

These limits are evidently determined by plotting on Fig. 7 the relations between R_s and f for which $r_0/d = 1{\cdot}5$ and $r_0/d = 14$.

This may be done very simply as follows:

From equation (6)

$$r_0 = \frac{4{\cdot}98 \times 10^{-4}}{R_s}\sqrt{f} \text{ ins.} = \frac{2{\cdot}54 \times 4{\cdot}98 \times 10^{-4}}{R_s}\sqrt{f} \text{ cm.}$$

$$\therefore \frac{r_0}{d} = \frac{12{\cdot}67 \times 10^{-4}}{R_s}\sqrt{f} \times 2\pi\sqrt{fg \times 10^{-9}}$$

$$\therefore R_s = \frac{1{\cdot}915 \times 10^{-4}f}{r_0/d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

To divide Fig. 7 into the three required regions it is then only necessary to plot equation (7) for values of r_0/d equal to 1·5 and 14. This has been done and the three regions have been indicated. Hence, by entering on Fig. 7 the frequency and the wire radius in inches it is a simple matter to find which equation applies.

For approximate values of resistance lying in the third region the value of R_{HF} per 1,000 ft. may be read directly from Fig. 7.

Thick-Walled Tubes at Frequencies where the Depth of Penetration d is Small Compared to the Thickness t.

For tubes in which the ratio t/d is greater than 3, Fig. 7 may be used to read off the value of R_{HF} directly.

Evidently $$\frac{t}{d} = \frac{t}{r_0} \times \frac{r_0}{d}$$

Hence by entering in Fig. 7 the outer radius r_0 of the tube and he frequency at which the H.F. resistance is required, the value of $_0/d$ may be read off and so the value of t/d can be calculated. If his is greater than 3, the value of R_{HF} is given by the value of R_s orresponding to the plotted point.

Example. What is the H.F. resistance at a megacycle of a opper tube 1 in. in diameter and 0·03 in. thick?

Plotting the point $r_0 = 0{\cdot}5$, frequency $= 10^6$, the value of r_0/d 200 and $R^s = 1$ ohm. Hence, $t/d = 200 \times 0{\cdot}03/0{\cdot}5 = 12$, so ıat the R.F. resistance is given by R_s and is 1 ohm per 1,000 ft. : will be evident that the thickness of the tube can be reduced › a quarter of the value given, i.e. to 0·0075 in., without appreciably fecting the H.F. resistance, provided the outer diameter remains nchanged.

13.2. The R.M.A. Colour Code. For indicating the value resistances which are too small to carry figures conveniently, a lour code is used as follows:

Black	represents	0	or	1	as	a	multiplier
Brown	,,	1	,,	10	,,	,,	,,
Red	,,	2	,,	10^2	,,	,,	,,
Orange	,,	3	,,	10^3	,,	,,	,,
Yellow	,,	4	,,	10^4	,,	,,	,,
Green	,,	5	,,	10^5	,,	,,	,,
Blue	,,	6	,,	10^6	,,	,,	,,

Violet	represents	7 or 10^7	as a multiplier
Grey	,,	8 ,, 10^8	,, ,, ,,
White	,,	9 ,, 10^9	,, ,, ,,
Gold	,,	$10^{-1} = 0{\cdot}1$	,, ,, ,,
Silver	,,	$10^{-2} = 0{\cdot}01$	,, ,, ,,

The body of the resistance element is coloured to define the first figure of the resistance value; one end is coloured to define the second figure; while a band or a dot gives the number of ciphers following the first two figures. The colour of the band or dot, therefore, determines the value of the multiplier according to the right-hand column of the table above.

Examples.

Resistance Value	*A : Body*	*B : End*	*C : Band or Dot*
10 Ω	Brown (1)	Black (0)	Black (× 1)
100 Ω	Brown (1)	Black (0)	Brown (× 10)
5,000 Ω	Green (5)	Black (0)	Red (× 10^2)
25,000 Ω	Red (2)	Green (5)	Orange (× 10^3)
1 MΩ	Brown (1)	Black (0)	Green (× 10^5)
20 MΩ	Red (2)	Black (0)	Blue (× 10^6)

If fitted with axial leads, one end of the body of the resistance is marked with coloured bands which, starting from the band nearest the end, have the same meanings as *A*, *B* and *C* in the above table.

A fourth band is sometimes added to give the tolerance (accuracy) of the resistance value. The colours in the fourth band have the following meanings:

Gold ±5% Silver ±10% No Colour ±20%

Preferred Resistance Values.

In recent years resistances have been made in preferred values based on the following numbers.

10	18	33	56
12	22	39	68
15	27	47	82

Preferred values made are given values in ohms equal to any of the above numbers multiplied by any power of 10 up to the largest size of resistance made.

14. Definition of the Watt; the Unit of Electrical Power.

When a current flows through a resistance the resistance radiates power in the form of heat drawn from the electrical circuit.

The unit of electric power is the *watt*, and is so chosen that

a watt flows in the circuit when the product of the voltage across the circuit and current through it is unity.

1 watt is equivalent to 1 ampere $\times$ 1 volt.

Power in watts = Voltage in volts $\times$ Current in amperes.

15. Mechanical Equivalent of the Watt.

1 watt $= 10^7$ ergs per second.

746 watts = 1 horse-power = 550 ft.-lbs. per second.

16. Thermal Equivalent of the Watt.

When electrical power is converted into heat the amount of heat generated per second per watt is given by:

1 watt = 0·239 mean gramme calories per second.

(The mean gramme calorie is 1/100 the amount of heat required to raise the temperature of one gramme of water from 0° to 100° C.)

17. Inductance: Definition of the Henry, the Unit of Inductance.

If a piece of wire is wound into a coil it is found that when a steady voltage is applied across it the current does not immediately reach its final value (which is determined only by the resistance in the coil and in the remainder of the circuit) but slowly builds up to the final value. The property of the coil which gives rise to the phenomenon is called its inductance, for which the symbol L is used.

The inductance of a coil is due to the building up of a magnetic field in the space constituting the core of the coil. For a given size of coil the intensity of the field is proportional to the current through the coil and the number of turns T; and the total flux of the field is proportional to the intensity multiplied by A, the area of cross-section of the coil. The intensity of the field at any point is represented conventionally by lines indicating the magnitude and direction of the force on a unit magnetic pole at that point. The number of lines per square centimetre indicate the intensity, and the direction of the lines the direction of the force. During the building up of the field these lines of force move so as to travel across, or cut, the windings of the coil and so induce a back e.m.f. in each turn of the coil. The back e.m.f. per turn is proportional to the total flux and so to AT. The total back e.m.f. in T turns is proportional to $AT \times T = AT^2$. Since the inductance is proportional to the back e.m.f. for unit rate of current change,

and the corresponding rate of flux change, the inductance is proportional to AT^2.

For a given core and shape of winding, therefore, the inductance is proportional to the square of the number of turns.

The absolute unit of inductance is defined as the inductance in which unit current produces unit linkage.

The practical unit of inductance—the Henry—is defined as the inductance in which a current of one ampere produces a linkage of 10^8. *Hence if the current through an inductance of one Henry changes at the rate of one ampere per second it produces a change of linkage of 10^8 per second; and so gives rise to a voltage of one volt across the inductance.* Conversely, if a p.d. of one volt is maintained across an inductance of one Henry, the current through the inductance increases at the rate of one ampere per second.

$$1 \text{ Henry} = 1 \text{ H}$$
$$= 10^6 \text{ microhenrys} = 10^6\ \mu\text{H}$$

17.1. Inductance of Air-Core Coils. The values of inductance given in the formulae below are the "d.c." values of inductance, defined by the self-linkage per unit current, and in practice are the values of inductance observed when the coils are measured at audio frequency. Owing to capacity between turns—self-capacity—the inductance measured at radio frequency increases as the frequency of measurement is increased and then falls to zero at self-resonance of the coil (i.e. resonance between the coil and its self-capacity), after which at higher frequencies still the coil behaves as a capacity. For this reason, and more particularly because of inherent circuit capacity, it is evidently not possible to realize air-core coils above a certain value of inductance. At medium waves this value of inductance is in the neighbourhood of 200–300 microhenrys for small coils and 50–100 microhenrys for coils in high-power transmitters. Where the coils are used with a parallel tuning condenser the self-capacity is unimportant, since it can be offset by using a correspondingly smaller value of tuning capacity.

The low-frequency inductance of the single-layer coil at (*a*) in Fig. 1 is given by

$$L = \frac{d \cdot l^2 n^2}{18d + 40l} \text{ microhenrys} \quad . \quad . \quad . \quad (1)$$

where d and l are dimensions in inches as shown and n is the number of turns per inch. This formula is accurate to $\pm 1\%$ when $l > 0{\cdot}4d$.

The inductance of the multi-layer coil at (b) is given by

$$L = \frac{0{\cdot}2d^2n^2}{3d+9l+10h} \text{ microhenrys} \qquad (2)$$

where d, l and h are the dimensions in inches shown in Fig. 1 (b), d being the mean diameter and n the total number of turns. This formula is accurate to about $\pm 1\%$ when the terms in the denominator are approximately equal.

The first formula is embodied in the chart in Fig. 2.

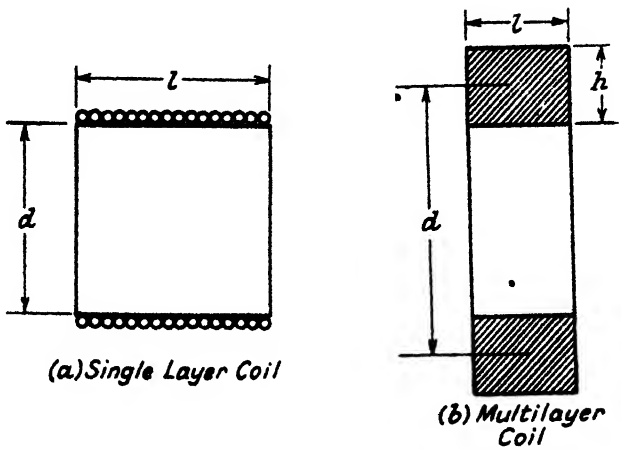

FIG. 1/II:17.—Dimensions of Single Layer and Multi-layer Coils.

USE OF CHART IN FIG. 2

This gives the inductance of solenoids of different length and diameter wound with one turn per inch.

From the diagram, the inductance of any coil, within the dimension limits of the chart, wound with any number of turns per inch can be determined. Conversely, a coil can be designed to have any required inductance.

Example 1. A coil 2 ins. in diameter is wound with 100 turns of No. 26 S.W.G. D.S.C. What is its inductance? From wire tables, 47·6 turns of No. 26 S.W.G. D.S.C. occupy 1 in., so that 100 turns occupy $\frac{100}{47{\cdot}6} = 2{\cdot}1$ ins., i.e. the coil is 2·1 ins. long.

From Fig. 2 the inductance of a coil 2 ins. in diameter, 2·1 ins. long, wound with one turn per inch $= 0{\cdot}147\ \mu H$.

Hence, wound with 47·6 turns per inch, the inductance is $0{\cdot}147 \times 47{\cdot}6^2 = 333\ \mu H$.

It will be noticed that the figure of 47·6 turns per inch does not

agree with the figure obtained from Table I, which is 48·78. This is because the figure was obtained from other tables and illustrates

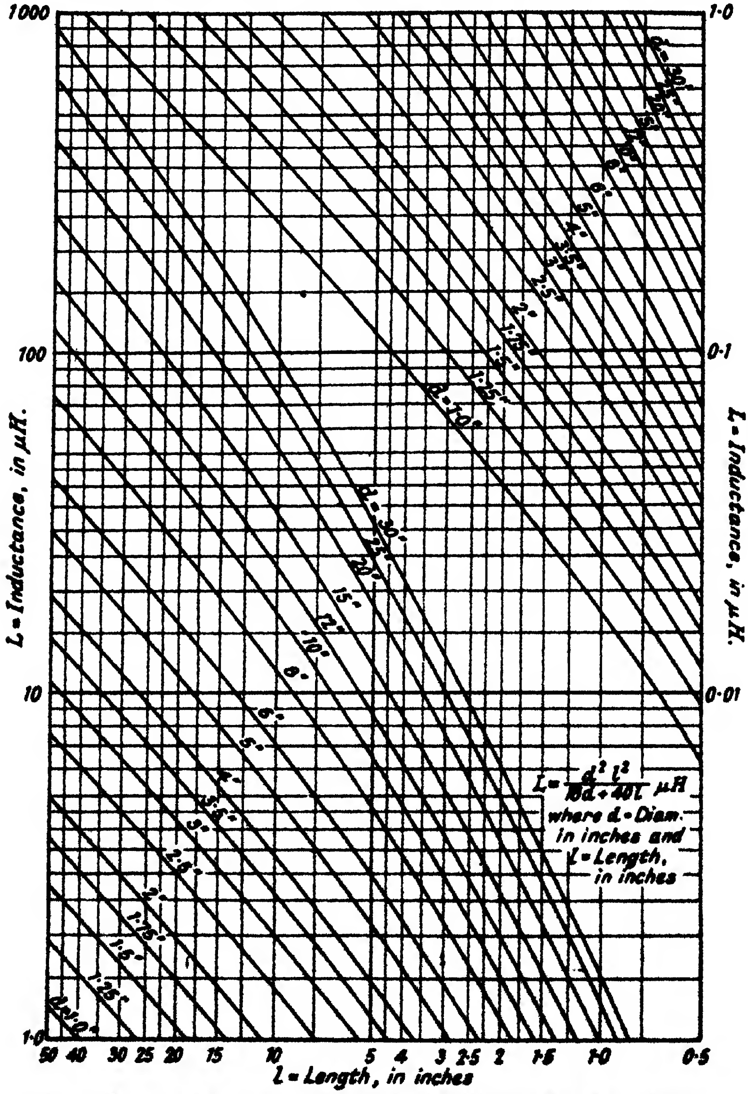

FIG. 2/II:17.—Inductance of Solenoids wound with one turn per inch.

the unavoidable errors in compiling such tables, due to crushing of insulation in winding.

Example 2. Given a former of 4 in. diameter, and a quantity

of No. 14 S.W.G. bare copper wire, to design an inductance of 10 μH. Given also that mechanical or electrical considerations require an air space of 0·12 in. between adjacent turns. The diameter of No. 14 S.W.G. is 0·08, so that one turn and one air gap = 0·08 + 0·12 = 0·2 in. The number of turns per inch is, therefore, 5.

Hence, since the inductance is proportional to the square of the number of turns, the inductance of a coil occupying the same space as the contemplated coil, but having one turn per inch, would be

$$\frac{10}{25} = 0{\cdot}4\ \mu\text{H}$$

From Fig. 2 the length of such a coil, 4 ins. in diameter = 1·85 ins. Hence, the number of turns required at specified spacing

$$= 1{\cdot}85 \times 5 = 9{\cdot}25$$

17.2. Best Coil Form. There are many factors which may influence coil design, but in general the design which gives the highest inductance for a given length of wire is the most desirable. In the case of a single-layer solenoid such as that shown in Fig. 1 (*a*) the maximum inductance for a given length of wire is realized by a coil in which the ratio $\frac{d}{l} = 2{\cdot}46$ approximately.

For the multi-layer coil as shown in Fig. 1 (*b*) the approximate optimum dimensions are given by $d = 3l$ with $h = l$.

The approximate inductance of a multi-layer coil with this form is given by $L = 0{\cdot}0214dn^2$ microhenrys; where n is the number of turns.

17.3. The Coil Figure of Merit: Q. The effective resistance of a coil varies with frequency, and at any frequency f the merit of the coil is defined by $Q = \frac{L\omega}{R_f}$, where $L\omega$ is the reactance of the coil at frequency f and R_f is the effective resistance at that frequency.

Values of Q in radio practice usually lie in the neighbourhood of 100 to 200, but higher values than this are sometimes realized by special design. Since a high Q corresponds to a low value of resistance, it is evidently desirable to have the value of Q as high as possible: in radio transmitters, from the point of view of efficiency, and in radio receivers from the point of view of impedance. (A high Q affords a high impedance in parallel resonant circuits.) For all cases where selective circuits are used, a high value of Q is desirable. Various means are used to improve the Q of coils by reducing the effective resistance.

17.4. Methods of Reducing the Effective Resistance of Coils. Skin effect is sometimes reduced in the case of large-coil transmitters, by constructing the conductor from heavy tape, from hollow tubing, or from a number of separately supported air-insulated strands. The use of tubular conductors provides conducting material only in the part of the conductor where the current density is greatest, while the use of stranded conductors prevents the creation of increased current density at the outer surface of the conductor. To achieve this last effect a type of stranded conductor known as " Litzendraht " is used, in which the insulated strands are so woven or twisted together that over a given length of the conductors each strand runs an equal distance in each part of the whole section of the conductor. The advantage of " Litzendraht " over solid conductors is only effective at frequencies below 1·5 to 2 Mc. *Above 2 Mc " Litzendraht " should not be used,* and it should only be used above 1·5 Mc when its behaviour is known.

By the use of certain alloys of iron in the form of laminations, tape or dust, the cores of inductances may be increased in permeability with the result that fewer turns of wire are required to provide a given inductance, so that the effective resistance is lowered.

17.5. Self-Capacity of Coils. The effective inductance of a coil is modified by the effect of capacity between turns and more particularly by capacity between layers, or in other words, between turns which are at an appreciable difference of potential with regard to one another.

The general effect of self-capacity in a coil is to make it behave like an inductance with a condenser in parallel : a parallel resonant circuit. At frequencies well below the self-resonant frequency of a coil the inductance is approximately equal to the true self-inductance as determined by the number of linkages per unit current. The effective value of inductance then rises towards resonance and falls rapidly towards zero just before resonance, reaching zero at the resonant frequency. The impedance of a coil at self-resonance is a pure resistance, and above resonance is capacitative approaching more and more to that of a pure capacity as the frequency is increased. Since self-capacity gives rise to circulating currents, it increases coil losses and so degrades the Q of the coil.

Self-capacity is reduced :

(*a*) By sectionalizing windings so that parts of the winding with a high potential difference between them are kept apart.

(*b*) By special windings so arranged that the conductors do not lie closely side by side but are separated by an air gap. Further, each layer is not wound parallel with the layer below, but crosses it at a slight angle. An example of this is the honeycomb form of winding.

(*c*) By winding the coils with small-gauge wire and so keeping the dimensions of the coil small. This evidently involves a compromise between the self-capacity and the Q of the soil.

(*d*) By the use of dust, tape or laminated cores of suitable alloys. These enable a given inductance to be obtained with a smaller winding than in the case of an air-cored coil and so with a smaller self-capacity.

17.6 Miscellaneous Inductance Formulae. The following formulae are given by W. H. Nottage in *The Calculation and Measurement of Inductance and Capacity.*

R.F. Inductance of an Air-Core Single Circular Turn.

$$L = 0{\cdot}02893a \log_{10}\left[\frac{8a}{r} - 0{\cdot}855\right] \text{ microhenrys} \qquad (3)$$

where a = radius of circle in centimetres
r = radius of wire ,, ,,

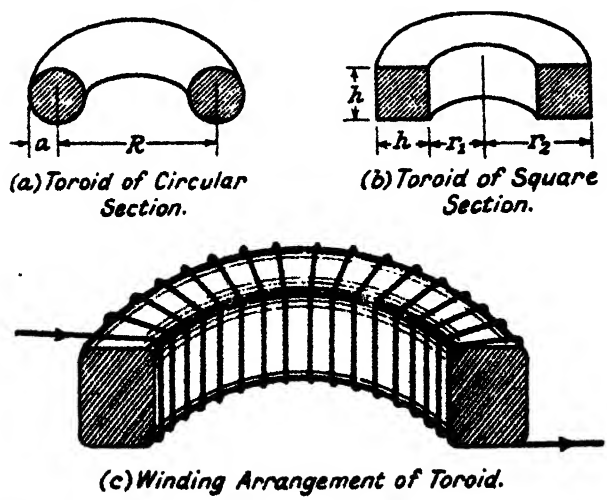

FIG. 3/II:17.—Dimensions of Toroidal Cores and Winding Arrangement.

D.C. Inductance of Air-Core Toroids.

Fig. 3(*c*) shows the winding arrangement on a ring core of square section. The arrangement of the ends of the winding is slightly misleading since normally a toroidal winding would be continued right round the toroid.

Referring to Fig. 3(*a*), the inductance of an air core toroidal coil of circular section and of dimensions as shown is given by

$$L = 0{\cdot}1257n^2(R - \sqrt{R^2 - a^2}) \text{ microhenrys} \qquad (4)$$

where n = the total number of turns, and R and a are in centimetres.

Referring to Fig. 3(*b*), the inductance of an air core toroidal coil of square section and of dimensions as shown is given by

$$L = 0{\cdot}004606n^2h \log_{10} \frac{r_2}{r_1} \text{ microhenrys} \quad . \quad . \quad (5)$$

where n = total number of turns, and h, r, and r_2 are in centimetres.

Inductance of a Straight Wire. (See XVI:13.)

The arguments of purists who object that the inductance of a part of a circuit is a meaningless concept, may be partially defeated by defining such an inductance as the back e.m.f. induced in that part of the circuit per unit rate of current change through it. The defeat is not complete, however, because they will then raise arguments about the effects of capacity, and finally will ask how such a back e.m.f. can be measured. As an engineering approximation the concept is, however, very useful.

The inductance of a straight circular conductor at d.c. and low frequencies is given by

$$L = 0{\cdot}002l\left(2{\cdot}303 \log_{10} \frac{4l}{d} - 0{\cdot}75\right) \text{microhenrys} \qquad (6)$$

where l and d are respectively the length and diameter of the conductor in centimetres.

The inductance of an isolated straight conductor of circular section at high frequency is given by

$$L = 0{\cdot}002l\left(2{\cdot}303 \log_{10} \frac{4l}{d} - 1\right) \text{microhenrys} \quad . \quad . \quad (7)$$

The reduction of inductance is due to skin effect, which reduces the internal inductance (i.e. the inductance due to the flux inside the conductor) substantially to zero. This formula is only applicable at frequencies where the length of conductor is a small fraction of a wavelength, and where a standing wave condition exists such that the current is substantially of the same magnitude all along the conductor. It is further restricted to the case where the conductor has small mutual inductance with, or capacity to, any other part of the circuit, or any other circuit. See XVI:13.

On Fig. 4 the quantity $2{\cdot}54 \times 0{\cdot}002\left(2{\cdot}303 \log_{10} \frac{4l}{d} - 1\right)$ is plotted as a function of l for conductors of different gauge, and is then

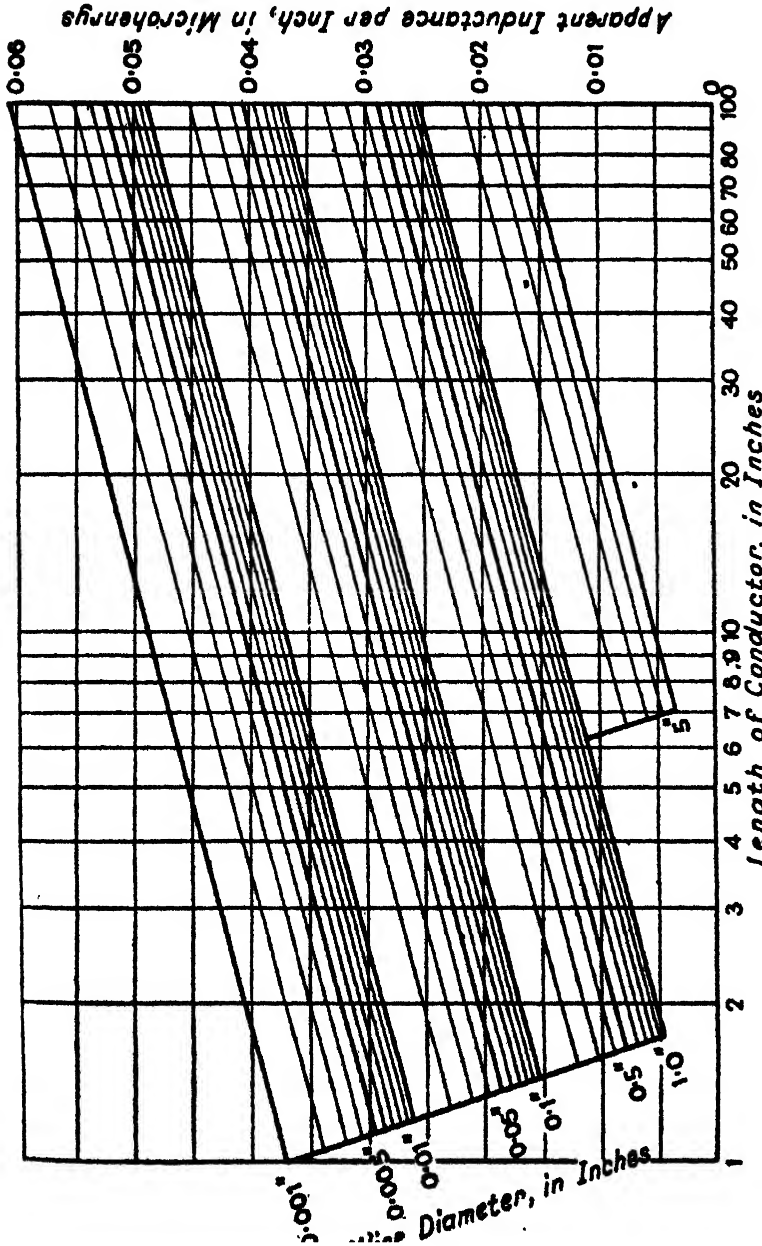

FIG. 4/II:17.—Inductance of Straight Wires.

To obtain inductance of a straight wire: Enter wire diameter and length in inches. Read apparent inductance per inch and multiply this by the wire length in inches. $L = 0{\cdot}00508l\left(2{\cdot}303 \log 4\frac{l}{d} - 1\right)$ μH.

The apparent high-frequency inductance *per foot* of a straight wire is plotted on Fig. 5.

High-Frequency Inductance of Straight Thin Tape of Length l cms., Breadth b cms., and thickness t cms.

When t is negligible:

$$L = 0{\cdot}002l\left(2{\cdot}303 \log_{10}\frac{2l}{b} - \frac{1}{2}\right) \text{ microhenrys} \quad . \quad (8)$$

When t is not negligible:

$$L = 0{\cdot}002l\left(2{\cdot}303 \log_{10}\frac{2l}{t+b} + \frac{1}{2} + \frac{0{\cdot}2235\,(t+b)}{l}\right) \text{ microhenrys} \quad . \quad (9)$$

Loop Inductance of a Balanced Pair of Round Conductors of Length l cms., Diameter d cms., and Separation D cms.: Length large compared to the Separation.

Direct Current.

$$L = 0{\cdot}004l\left(2{\cdot}303 \log_{10}\frac{2D}{d} + \frac{1}{4}\right) \text{ microhenrys} \quad . \quad (10)$$

High Frequency.

$$L = 0{\cdot}00921l \log_{10}\frac{2D}{d} \text{ microhenrys} \quad . \quad (11)$$

The apparent high-frequency loop inductance per foot of a balanced pair is plotted on Fig. 5 as well as the high-frequency loop inductance of an unbalanced single-wire earth return circuit.

Inductance of a Square of Round Wire of Diameter d cms.: Side of Square = a cms.

$$L = 0{\cdot}008a\left(2{\cdot}303 \log_{10}\frac{2a}{d} + 1{\cdot}226 - P\right) \quad . \quad (12)$$

At high frequencies $P = 2{\cdot}0$ and for direct current $P = 1{\cdot}75$.

The apparent high-frequency inductance per foot length of a square is plotted on Fig. 5.

Inductance of a Rectangle of Round Wire of Diameter d cms.: Sides of Rectangle = a cms., and b cms.

$$L = 0{\cdot}004[2{\cdot}303(a+b) \log_{10}\frac{4ab}{d} - 2{\cdot}303\{a \log_{10}(a+D) + b \log_{10}(b+D)\} - P(a+b) + 2D + d] \quad . \quad (13)$$

where $D = \sqrt{a^2+b^2}$ = the length of the diagonal, and P has the same values as in the case of the square.

In Fig. 5 is plotted the apparent high-frequency inductance per foot length of conductor for the following cases: a balanced pair, a single turn of square shape, an isolated straight conductor, a vertical conductor of height h above ground, and a single horizontal conductor constituting an unbalanced earth return circuit. The apparent inductance per foot length has no practical meaning except in the case of the balanced circuit and the earth return circuit, but, in all cases, when multiplied by the length of conductor, it gives the total inductance. In the case of the balanced circuit and the earth return circuit, the resulting inductance is the loop inductance, which is the inductance which would be observed looking into one end of the circuit with the other end short circuited, in the absence of resistance and capacity. This is evidently rather an ideal quantity. In the case of the balanced pair the apparent inductance per unit length must be multiplied by the length of one conductor to give the loop inductance; in all other cases it should be multiplied by the total length of conductor.

Except the line for the vertical conductor, which is discussed in XVI:13, Fig. 5 is derived by plotting the values of inductance given by equations (7), (11) and (12), divided by the length of conductor.

The application of the values of inductance given in Fig. 5 cannot be stated in simple terms. The values of loop inductance can be used for determining the characteristic impedance and propagation constants of balanced and unbalanced lines, see XVI:1.1 to 1.3 and XVI:13. The inductance of the vertical wire can also be used for deriving the nominal characteristic impedance of a vertical wire, see XVI:13.

An approximation to the inductance of short lengths of horizontal conductor, such as the grid leads of valves in short-wave amplifiers, may be reached in several ways.

Consider a horizontal conductor 2 ft. long and 1 ft. above ground, which is a thousandth of a foot in diameter:

Square: The inductance per unit length of a 2-ft. square of the same gauge is given from Fig. 5 by entering $\frac{2a}{d} = 4{,}000$, and is 0·46 μH. (The total true inductance is therefore 0·92 μH.)

Isolated Conductor: The inductance per unit length is given from Fig. 5 by entering $\frac{4l}{d} = 8{,}000$, and is nearly 0·49 μH.

Horizontal Conductor above Ground: The inductance per unit length is given by entering $\frac{4h}{d} = 4{,}000$, and is 0·51 μH.

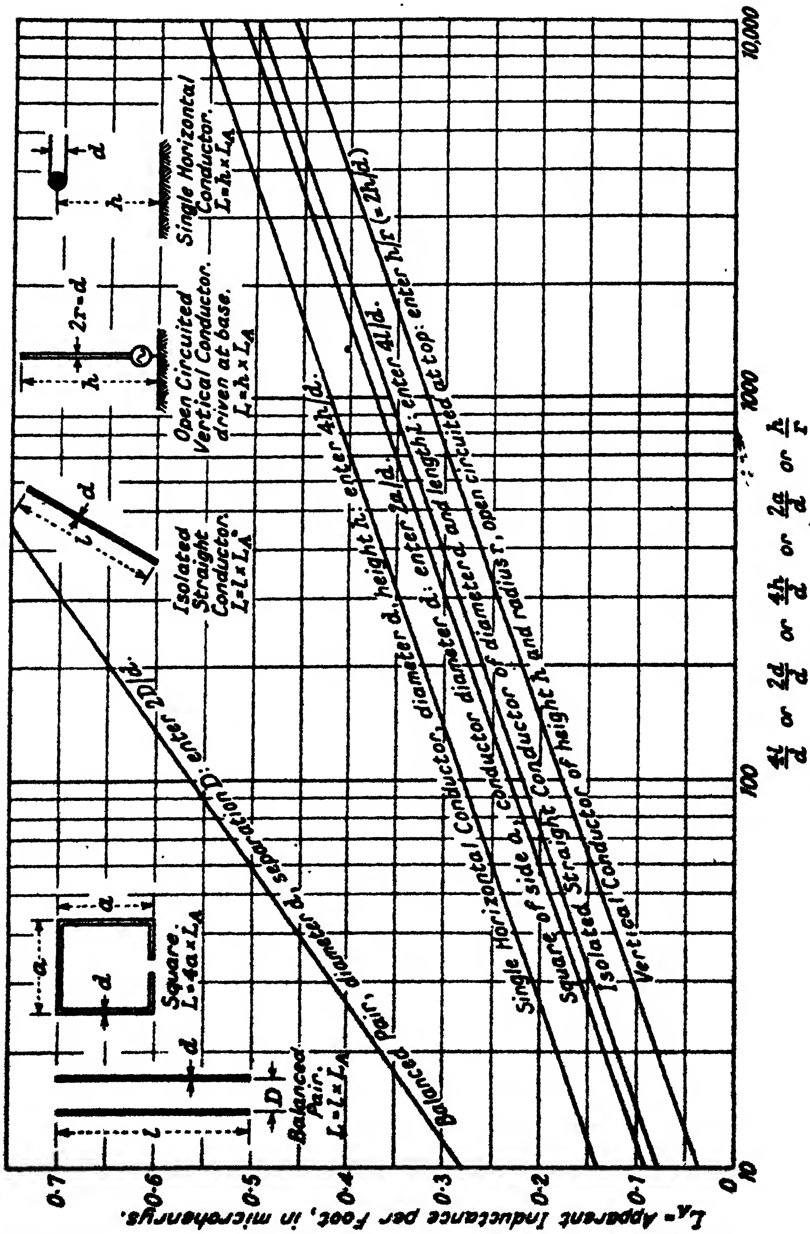

FIG. 5/II:17.—H.F. Inductance of Round Wires = Apparent Inductance per foot × Length of Conductor in feet.

It is evident therefore that the error involved by using the inductance of the isolated conductor is small when the length of the conductor is less than twice its height above ground. In fact, the inductance of the isolated conductor can be used to give an approximation for the inductance of all short leads, whether horizontal or otherwise.

Owing to its capacity, the impedance presented by a conductor is not that which is obtained by multiplying its inductance by the angular frequency. As a corollary, the value of capacity to be placed in series with a grid lead, to neutralize its reactance, is not equal to the reactance of the conductor when so calculated. This is because the drive to the conductor is normally supplied as a potential difference from ground potential, and the conductor therefore behaves like a transmission line. The impedance presented to the driving source is therefore the sending-end impedance of the transmission line, constituted by the conductor, when terminated in the valve input impedance (assuming the case of a grid lead), or other impedance in other cases. See XVI:13.

18. Magnetomotive Force = m.m.f.

The paths followed by the lines of force, in the space round a coil carrying a current, together constitute the magnetic circuit.

Consistent with the notion of current being driven round a circuit by electromotive force, the flux of the magnetic field is said to be driven round the magnetic circuit by a magnetomotive force generated by the current through the coil. Magnetomotive force is measured in Gilberts. The magnitude of this magnetomotive force is proportional to the ampere turns being given by

$$F = \frac{4\pi IT}{10} \text{ Gilberts}$$

where I is the current in amperes
and T is the number of turns.

The flux of magnetic lines for force which flows is given by

$$\Phi = \frac{F}{S} = \text{total number of lines of force} = \text{total flux in } \textit{gauss}$$

where S = the reluctance of the circuit, analogous to resistance in the electrical circuit, see II:19 below.

19. Reluctance of a Magnetic Circuit.

Where a magnetic circuit has the form of a ring, as, for instance, in the case of a toroid, wound on a closed ring of high permeability (see Fig. 3/II:17), since nearly all the lines of force go through

the ring and very few through the air the reluctance of the circuit is given by

$$S = \frac{l}{\mu A}$$

where l = mean length of flux path through the core in centimetres
A = area of cross-section of flux path in square centimetres
μ = permeability of core material.

Reluctances in series and parallel obey the same laws as resistance. It follows that if a core of a high-permeability material has an air gap in it of one or two millimetres, the reluctance of the magnetic circuit is substantially that due to the air gap alone: see II:20.

The object of inserting an air gap in a core is to make the reluctance of the circuit constant and independent of variations in current amplitude, since the reluctance of air is independent of flux density.

20. Permeability.

The specific reluctance of air is unity (i.e. the reluctance of a one-centimetre cube between opposite faces).

Iron, nickel, and in particular certain alloys of iron, such as Permalloy, Mumetal, Rhometal, etc., have very low values of specific reluctance, such that a core of one of these materials under the influence of a given number of ampere turns may give rise to a flux many thousand times the flux which would occur under the same conditions with an air core. Such materials are said to have a high permeability, the permeability μ being defined by the ratio

$$\mu = \frac{\text{Flux through a given path in the material}}{\text{Flux through an identical path in air}}$$

the same m.m.f. being applied in each case.

The permeability of a material varies widely as the flux density through it varies.

Since the inductance of a circuit depends on the rate at which flux linked with the circuit is built up with increase of current through the circuit, increasing the effective permeability of the core of an inductance increases the inductance in the same ratio as the effective permeability is increased.

21. Capacity (Capacitance).

If two plates of metal are placed remote from one another and a potential difference applied between them, for instance by con-

necting them, one to each end of a wire in which an e.m.f. is induced, a current flows from one plate to the other, so that charges of equal and opposite sign collect on each plate. One charge is represented by an excess over the neutral complement of electrons; and the other charge by a deficit below the neutral complement of electrons, the deficit being exactly equal to the excess.

If, now, the two plates are brought closer together, keeping the applied potential difference at the same value, owing to the attraction between the electrons on one plate and the positrons on the other plate which have lost their neutralizing electrons, the charges on the two plates increase still further, i.e. electrons flow from the plate with a deficit to the plate with an excess of electrons.

The characteristic of the two plates by virtue of which they hold a charge when infinitely remote is called their *self-capacity*.

The extra capacity consequent on proximity of the plates is called their *mutual capacity*.

Definition of Condenser. The combination of two plates in close proximity is called a *condenser*, and its total capacity, which is equal to the sum of the mutual capacity and self-capacity, is referred to merely as capacity. The charge held by a condenser is measured by the number of electrons displaced from one plate to the other.

22. Definition of the Practical Units of Capacity; the Farad, the Microfarad and the Micro-Microfarad.

The unit of capacity is the *Farad* and is so chosen that when a potential difference of one volt is applied across a condenser having a capacity of one Farad, the condenser becomes charged with one coulomb.

The Farad is much too large a unit for practical purposes and the practical units in use are the microfarad (μF) and the micro-microfarad ($\mu\mu$F), sometimes called the picofarad (pF).

$$\begin{aligned}\text{One Farad} &= 10^6 \text{ microfarads} = 10^6\ \mu\text{F} \\ &= 10^{12} \text{ micro-microfarads} = 10^{12}\ \mu\mu\text{F} = 10^{12} \text{ pF.}\end{aligned}$$

22.1. Capacity Formulae. *The mutual capacity of two plates of area $A = D^2$ square centimetres and distance d centimetres apart where $\frac{D}{d}$ is large, is given approximately by*

$$C = \frac{1{\cdot}111A\varepsilon}{4\pi d} = 0{\cdot}0884\frac{A\varepsilon}{d} \text{ micro-microfarads } (\mu\mu\text{F})$$

where ε is the dielectric constant.

This formula holds for any shape of plate.

FIG. 1/II:22.—Construction of Condenser Plates.

Condensers are usually made up of a number of vanes or sheets of metal of equal area, every alternate sheet of metal being connected together to constitute one plate of the condenser, and the remaining sheets being connected together to constitute the other plate, as shown in Fig. 1.

The capacity of such a condenser is given by

$$C = 0{\cdot}0 \; \frac{nA\varepsilon}{a}$$

where n is the number of dielectrics.

In Fig. 1, for instance, the number of dielectrics is 7.

The common variable condenser used in radio receivers and elsewhere consists of two sets of vanes capable of relative movement so as to change the amount of overlap and so vary the capacity.

The self-capacity of a sphere of radius r centimetres is

$$C = 1{\cdot}111r \ \mu\mu\text{F}$$

The self-capacity of a circular disc of radius r centimetres is

$C = 0{\cdot}707r \ \mu\mu\text{F}$ (Approximate accuracy: $\pm 10\%$) (Nottage)

The self-capacity of a conductor of irregular shape and surface area S which is not elongated is, roughly

$$C = 1{\cdot}11\left(\frac{S}{4\pi}\right)^{\frac{1}{2}} \mu\mu\text{F}$$

Nottage includes a factor a in this formula which he states to be nearly unity, but does not give its value.

23. Dielectric Constant.

If a dielectric (see II:2) is placed between the plates of a condenser so as to occupy all the space between the plates, the capacity is increased by a factor called the dielectric constant, or specific inductive capacity.

The dielectric constant of vacuum is unity and that of air is almost exactly unity, while that of certain types of mica (Muscovite) is as high as 7·5. Mica is generally used for high-voltage condensers which are required to occupy a very small space since it has high resistance to *voltage breakdown.* Variable condensers for use at high voltage are constructed as variable air condensers and immersed in oil.

Voltage breakdown of a dielectric does not refer to the small current which flows corresponding to the applied voltage and the resistance of the dielectric, but to the disruptive discharge which may flow through the dielectric, disintegrating it, or over the surface of the dielectric, burning it and destroying its insulating properties.

23.1. Dielectric Constants. The dielectric constant of air is unity, that of glass and mica is about 6 and that of paper is just over 2.

General characteristics of commercial dielectrics in common use are given in Table I on the next page.

For further details see the Smithsonian Tables, which give dielectric constants at radio frequencies.

23.2. Characteristics of Shell KB 30 Transformer Oil. This is the oil used by the B.B.C. in transformers, chokes and condensers.

Dielectric Constant	2·18 at 20° C., 2·14 at 50° C.
Resistivity	$1 \times 10^{9}\ \Omega$/cm.[3]
Power Factor at 50 c/s	0·0001 at 20° C., 0·0006 at 50° C.
Power Factor at 10^{6} c/s	Less than 0·05
Voltage breakdown	Withstands 30 kilovolts R.M.S. sinusoidal voltage between 13-mm. diameter spheres 4 mm. apart immersed 50 mm. below the surface of the oil. (During tests made under the above conditions voltage breakdown usually occurs at 35–40 kilovolts.)

24. Condensers.

Evidently a condenser may in principle be constituted by any form of conducting surfaces separated by any type of dielectric.

In practice the most common types of dielectric are:

Air Mica Paper Oil Ceramics.

It appears probable that in future the use of dielectrics with a very high dielectric constant will increase: see, for instance, Rutile in Table I.

In the B.B.C. condensers are used which are filled with nitrogen at a pressure of 200 lbs./sq. in. Such condensers are made in capacities of the order of 0·0002 to 0·0008 μF and will withstand very high voltages: 42,000 volts peak working, and 84,000 volts D.C. test. Such a condenser would withstand, for instance, a 9,550-volt R.M.S. medium frequency carrier modulated 100% plus 10,600-volt R.M.S. in the range 30–10,000 c/s.

24.1. Electrolytic Condensers.

Some metals, such as aluminium and tantalum, when used as anodes in a suitable electrolyte (e.g. solution of a salt in water), become coated with a film capable of holding a charge at potentials below the breakdown voltage of the film.

TABLE

Typical Properties of

tan d = Series Resistance/Serie

	Polythene	Polystyrene	Polymethyl methacrylate	Polyvinyl chloride	Cellulose nitrate	Cellulose acetate	Ethyl cellulose	Phenol No filler
Trade Names	Alkathene	Distrene Trolitul	Perspex Diakon	Welvic P.V.C.	Xylonite Celluloid	Rexoid Dialux	Ethyl Cellulose	Bakelite
Specific gravity .	0·92	1·05	1·19	1·4	1·37	1·32	1·14	1·28
ϵ (1 Mc/s) . . .	2·3	2·56	2·85	4·0	6·5	4·0	3·1	5
tan d (1 Mc/s) .	0·0003	0·0002	0·015	0·05	0·08	0·03	0·02	0·04
ϵ (3,000 Mc/s) .	2·3	2·56	2·65	3·3	4·0	3·2	2·8	3·5
tan d (3,000 Mc/s)	0·0005	0·0003	0·01	0·04	0·10	0·04	0·02	0·07
Water abs. (24 hours) . . .	0·00%	0·00%	0·3%	0·5%	2%	5%	1·5%	0·15
Distortion (temp. °C.)	95	75	75	80	65	60	65	120
Tensile str. (lbs./sq. in.)	1,500	7,000	9,500	3,000	7,000	7,000	7,500	7,5
Comp. str. (lbs./sq. in.)	—	12,000	13,000	—	25,000	15,000	11,000	—
Flexural str. (lbs./sq. in.) .	—	15,000	12,000	—	9,000	10,000	8,000	15,00
Elongation (%) .	400	3·5	3	300	35	30	25	—
Young's Modulus (lbs./sq. in.) $\times 10^5$	—	1·8	5	—	3	2·5	3	8·5
Type	←			Thermo-plastic			→	←
Machining qualities	Poor	Fair	Excellent	Fair	Good	Good	Good	Fair
Forms available .	M, E, S, F	M, E, S, F	M, E, S	M, E, S, L	M, E, S, L, F	M, E, S, L, F	M, E, S, L, F	M, E,
Principal uses . .	Cables, connectors, aerials	H.F. dielectric lenses	Radomes, aircraft windows	Cable sheath	Films, cheap mouldings	Non-flam film	General mouldings	Adhes lacqu base resin

Code: M, Moulding. E, Extru

This effect is used in electrolytic condensers, which usually consist of an aluminium anode and a cathode of some non-film forming metal immersed in an electrolyte. Sometimes the cathode constitutes the container for the electrolyte. In the so-called wet type of electrolytic condensers the electrolyte solution is liquid.

Dielectric Materials

Reactance. ε = dielectric constant

ormaldehyde p.f. laminates				Urea formaldehyde resin	Aniline formaldehyde	Alkyd resin fibre-glass laminate	Steatite	Rutile	Expanded polyvinyl formal
ller	Wood-flour filler	Paper Base	Cotton fabric base						
ol, Paxolin, Mouldrite, Catalin, nite, Micarta, etc.				Beetle Mouldrite	Panilax	CR39 Laminac	Frequentite	Faradex	Plastazote
}	1·35	1·33	1·33	1·48	1·23	1·73	2·7	4·1	0·15
	6	4·5	5·5	6·2	3·5	4·3	6·0	90	1·2
015	0·06	0·03	0·05	0·035	0·009	0·03	0·001	0·0003	0·004
8	3·5	3·4	3·7	4·5	3·4	3·7	5·7	88	1·16
02	0·05	0·03	0·06	0·08	0·005	0·014	0·0006	0·0004	0·003
15%	0·4%	3%	3%	2%	0·06%	0·8%	0·05%	0·05%	0·8%
50	130	150	150	130	100	125	>1,000	>1,000	65
,000	6,000	13,000	10,000	6,000	9,500	37,000	14,000	11,000	500
,000	25,000	30,000	37,000	24,000	23,000	50,000	85,000	80,000	300
,000	12,000	16,000	22,000	11,000	15,000	26,000	20,000	13,000	—
—	—	—	—	—	—	—	—	—	—
12	12	8	8	13	3·5	16	—	—	0·4
Thermo-setting					Thermo-plastic	Thermo-setting	Ceramics		Thermo-plastic
Fair	Good	Good	Excellent	Fair	Excellent	Fair	Poor	Poor	Poor
M, E	M, E	S	S	M, E, S, L	M, S	M, S, F	Cast to shape required		S, M
F. insulators	General mouldings	Panels, etc.	Gear wheels, etc.	General	H.F. insulating board	Radomes, reflectors	Condensers, insulators, coil formers		Life-buoys, refrigeration radom

heet. F, Film. L, Lacquers.

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In the so-called dry type the anode and cathode are made in the form of one or more thin metal sheets rolled up with a layer of electrolyte impregnated gauze between them. The dry type of condenser is usually made up in a paper or metal container, the former of which, however, is inactive electrically. An electrolytic condenser must always be used in a circuit which applies a steady D.C. voltage across it which is greater than any A.C. voltage in the circuit, so that current can never flow through the condenser in the reverse direction and break down the film. Electrolytic condensers are capable of providing capacities of phenomenally large size, e.g. a 50-microfarad condenser to work on 12 volts can be made up to occupy a cylinder about 1 in. diameter and 2 ins. long. Condensers of 1,000 and 2,000 microfarads are quite a practicable proposition for low voltages, a 1,000 microfarad condenser to operate on 12 volts, for instance, could be contained in a space of about 50 cubic inches. As the voltage is increased the size of the condenser also increases. In practice, on account of limitations set by the breakdown voltage of the film, electrolytic condensers are not usually made for voltages much in excess of 500 volts.

Electrolytic condensers are characterized by a leakage current corresponding to a parallel resistance which varies with the applied voltage; this leakage current may rise from zero to 10 or 100 microamps. in a low-voltage type of condenser, and may rise from zero to 4 or 5 milliamps. in a high-voltage type of condenser. The leakage current is usually greater in the condensers used on higher voltages. Because of the leakage current, which will not necessarily be equal in two electrolytic condensers of the same type, electrolytic condensers cannot be placed in series unless shunted with a resistance potentiometer to mask the unbalanced D.C. voltage across the electrolytic condensers which would otherwise occur, causing too large a voltage across one of the condensers and consequent breakdown.

Electrolytic condensers have the anode marked with a plus sign and their cathode with a minus sign and must always be connected in circuit so that the positive terminal is towards the positive source of voltage supply.

25. Interaction Between Magnetic Field and Current.

A current of 10 amperes (absolute unit current) flowing through 1 centimetre of a wire perpendicular to a uniform magnetic field of 1 line per square centimetre is acted upon by a force of 1 dyne impelling the wire in a direction at right angles to both field and

wire. See Inverse Lenz's Law in the next section for the relation between direction of current and direction of movement.

26. Lenz's Law and its Inverse.

Lenz's Law applies to the effect described in Section 3 in which motion of a conductor through a magnetic field gives rise to an induced e.m.f. This is sometimes called electromagnetic induction.

Lenz's Law. In all cases of electromagnetic induction the induced e.m.f.s have such direction that the induced currents exert forces tending to stop the mechanical movement producing them.

Inverse Lenz's Law. In all cases where currents interacting with fields produce mechanical movements, the resultant movements have such direction that they give rise to back e.m.f.s opposing the e.m.f. giving rise to the currents producing the movement. Hence, referring to Fig. 1/II:3, a current flowing from W_1 to W_2 would tend to make the conductor move from V_2 to V_1, i.e. out of the paper towards the reader.

27. Thermo-electric Effect.

When the junction of two dissimilar metals is heated, an e.m.f. of a few millivolts is generated in a direction depending on the kind of metals which are used. The assembly of two conductors of different metals is called a couple or thermo-junction; and in conjunction with a D.C. millivoltmeter provides a means of measuring temperature.

By associating the couple with a heater (in mechanical contact with the junction of the two metals) through which an unknown current is passed, the unknown current can be measured by observing the e.m.f. generated by the couple. The assembly of heater and thermo-junction is called a thermo-couple.

28. Contact Potentials.

When any contact between two metals, such as may constitute the contacts of a switch, is made, small e.m.f.s are generated called contact potentials. Even when the contacts are both of the same metal, contact potentials are observed due to impurities in the metal or deposits.

29. "Dry" and "Wet" Contacts.

In the case of contacts made with very light pressures such as occur in sensitive relays, an unsatisfactory contact is sometimes made unless there is a small steady difference of potential between

the contacts. Contacts without such a difference are called " dry " contacts and contacts with such a difference are called " wet " contacts. With the pressure occurring in switches and most relays this effect is, however, usually unnoticeable.

30. Conversion Tables between Units.

The tables which follow on pp. 67–76 give the relations between practical electrical units and the absolute electromagnetic and electrostatic C.G.S. units.

The absolute electromagnetic C.G.S. units are given the prefix *ab*, e.g. abvolt, abampere, abcoulomb, abohm, abhenry, abfarad, etc.

The absolute electrostatic C.G.S. units are given the prefix *stat-*, e.g. statvolt, statampere, statcoulomb, statohm, stathenry, statfarad, etc.

The statfarad used to be called the " centimetre ", since it is the self-capacity of a sphere of 1 cm. radius. This nomenclature still persists in some parts of Europe. As the abhenry is also sometimes called a " centimetre ", this usage is confusing.

CONVERSION TABLES

TABLE I

Power or Rate of Doing Work *

Multiply Number of → by ↘ to Obtain ↓	British thermal units per minute	Ergs per second	Foot-pounds per minute	Foot-pounds per second	Horsepower *	Kilogram-calories per minute	Kilowatts	Metric horsepower	Watts
British thermal units per minute	1	$5{\cdot}689 \times 10^{-9}$	$1{\cdot}285 \times 10^{-3}$	$7{\cdot}712 \times 10^{-2}$	42·41	3·969	56·89	41·83	$5{\cdot}689 \times 10^{-2}$
Ergs per second	$1{\cdot}758 \times 10^{8}$	1	$2{\cdot}259 \times 10^{5}$	$1{\cdot}356 \times 10^{7}$	$7{\cdot}457 \times 10^{9}$	$6{\cdot}977 \times 10^{8}$	10^{10}	$7{\cdot}355 \times 10^{9}$	10^{7}
Foot-pounds per minute	778·0	$4{\cdot}426 \times 10^{-6}$	1	60	$3{\cdot}3 \times 10^{4}$	3,087	$4{\cdot}426 \times 10^{4}$	$3{\cdot}255 \times 10^{4}$	44·26
Foot-pounds per second	12·97	$7{\cdot}376 \times 10^{-8}$	$1{\cdot}667 \times 10^{-2}$	1	550	51·44	737·6	542·5	0·7376
Horsepower *	$2{\cdot}357 \times 10^{-2}$	$1{\cdot}341 \times 10^{-10}$	$3{\cdot}030 \times 10^{-5}$	$1{\cdot}818 \times 10^{-3}$	1	$9{\cdot}355 \times 10^{-2}$	1·341	0·9863	$1{\cdot}341 \times 10^{-3}$
Kilogram-calories per minute	0·2520	$1{\cdot}433 \times 10^{-9}$	$3{\cdot}239 \times 10^{-4}$	$1{\cdot}943 \times 10^{-2}$	10·69	1	14·33	10·54	$1{\cdot}433 \times 10^{-2}$
Kilowatts	$1{\cdot}758 \times 10^{-2}$	10^{-10}	$2{\cdot}260 \times 10^{-5}$	$1{\cdot}356 \times 10^{-3}$	0·7457	$6{\cdot}977 \times 10^{-2}$	1	0·7355	10^{-3}
Metric horsepower	$2{\cdot}390 \times 10^{-2}$	$1{\cdot}360 \times 10^{-10}$	$3{\cdot}072 \times 10^{-5}$	$1{\cdot}843 \times 10^{-3}$	1·014	$9{\cdot}485 \times 10^{-2}$	1·360	1	$1{\cdot}360 \times 10^{-3}$
Watts	17·58	10^{-7}	$2{\cdot}260 \times 10^{-2}$	1·356	745·7	69·77	1,000	735·5	1

1 Cheval-vapeur = 75 kilogram-metres per second.
1 Poncelet = 100 kilogram-metres per second.

* The "horse-power" used in these tables is equal to 550 foot-pounds per second by definition. Other definitions are one horse-power equals 746 watts (U.S. and Great Britain) and one horse-power equals 736 watts (continental Europe). Neither of these latter definitions is equivalent to the first; the "horse-powers" defined in these latter definitions are widely used in the rating of electrical machinery.

Tables I to XI by courtesy of Messrs. John Wiley and Sons. Taken from "Electrical Engineers' Handbook," by H. Pender and K. McIlwain.

TABLE II

Quantity of Electricity and Dielectric Flux

Multiply Number of → by to Obtain ↓	Abcoulombs	Ampere-hours	Coulombs	Faradays	Stat-coulombs
Abcoulombs	1	360	0·1	9649	$3{\cdot}335 \times 10^{-11}$
Ampere-hours	$2{\cdot}778 \times 10^{-3}$	1	$2{\cdot}778 \times 10^{-4}$	26·80	$9{\cdot}259 \times 10^{-14}$
Coulombs	10	3,600	1	$9{\cdot}649 \times 10^{4}$	$3{\cdot}335 \times 10^{-10}$
Faradays	$1{\cdot}036 \times 10^{-4}$	$3{\cdot}731 \times 10^{-2}$	$1{\cdot}036 \times 10^{-5}$	1	$3{\cdot}457 \times 10^{-15}$
Statcoulombs	$2{\cdot}998 \times 10^{10}$	$1{\cdot}080 \times 10^{13}$	$2{\cdot}998 \times 10^{9}$	$2{\cdot}893 \times 10^{14}$	1

TABLE III

Electric Current

Multiply Number of → by to Obtain ↓	Abamperes	Amperes	Statamperes
Abamperes	1	0·1	$3{\cdot}335 \times 10^{-11}$
Amperes	10	1	$3{\cdot}335 \times 10^{-10}$
Statamperes	$2{\cdot}998 \times 10^{10}$	$2{\cdot}998 \times 10^{9}$	1

Table IV

Electric Potential and Electromotive Force

Multiply Number of → by to Obtain ↓	Abvolts	Microvolts	Millivolts	Statvolts	Volts
Abvolts	1	100	10^5	$2{\cdot}998 \times 10^{10}$	10^8
Microvolts	0·01	1	1,000	$2{\cdot}998 \times 10^8$	10^6
Millivolts	10^{-5}	0·001	1	$2{\cdot}998 \times 10^5$	1,000
Statvolts	$3{\cdot}335 \times 10^{-11}$	$3{\cdot}335 \times 10^{-9}$	$3{\cdot}335 \times 10^{-6}$	1	$3{\cdot}335 \times 10^{-3}$
Volts	10^{-8}	10^{-6}	0·001	299·8	1

Table V

Electric Field Intensity and Potential Gradient

Multiply Number of → by to Obtain ↓	Abvolts per centimetre	Microvolts per metre	Millivolts per metre	Statvolts per centimetre	Volts per centimetre	Kilovolts per centimetre	Volts per millimetre	Volts per inch	Volts per mil
Abvolts per centimetre	1	1	1,000	$2 \cdot 998 \times 10^{10}$	10^{8}	10^{11}	10^{9}	$3 \cdot 937 \times 10^{7}$	$3 \cdot 937 \times 10^{10}$
Microvolts per metre	1	1	1,000	$2 \cdot 998 \times 10^{10}$	10^{8}	10^{11}	10^{9}	$3 \cdot 937 \times 10^{7}$	$3 \cdot 937 \times 10^{10}$
Millivolts per metre	0·001	0·001	1	$2 \cdot 998 \times 10^{7}$	10^{5}	10^{8}	10^{6}	$3 \cdot 937 \times 10^{4}$	$3 \cdot 937 \times 10^{7}$
Statvolts per centimetre	$3 \cdot 335 \times 10^{-11}$	$3 \cdot 335 \times 10^{-11}$	$3 \cdot 335 \times 10^{-8}$	1	$3 \cdot 335 \times 10^{-3}$	3·335	$3 \cdot 335 \times 10^{-2}$	$1 \cdot 313 \times 10^{-3}$	1·313
Volts per centimetre	10^{-8}	10^{-8}	10^{-5}	299·8	1	1,000	10	0·3937	393·7
Kilovolts per centimetre	10^{-11}	10^{-11}	10^{-8}	0·2998	0·001	1	0·01	$3 \cdot 937 \times 10^{-4}$	0·3937
Volts per millimetre	10^{-9}	10^{-9}	10^{-6}	29·98	0·1	100	1	$3 \cdot 937 \times 10^{-2}$	39·37
Volts per inch	$2 \cdot 540 \times 10^{-8}$	$2 \cdot 540 \times 10^{-8}$	$2 \cdot 540 \times 10^{-5}$	761·6	2·540	2,540	25·40	1	1,000
Volts per mil	$2 \cdot 540 \times 10^{-11}$	$2 \cdot 540 \times 10^{-11}$	$2 \cdot 540 \times 10^{-8}$	0·7616	$2 \cdot 540 \times 10^{-3}$	2·540	$2 \cdot 540 \times 10^{-2}$	0·001	1

Table VI

Electric Resistance

Multiply Number of → by to Obtain ↓	Abohms	Megohms	Microhms	Ohms	Statohms
Abohms	1	10^{15}	1,000	10^{9}	$8{\cdot}988 \times 10^{20}$
Megohms	10^{-15}	1	10^{-12}	10^{-6}	$8{\cdot}988 \times 10^{5}$
Microhms	0·001	10^{12}	1	10^{6}	$8{\cdot}988 \times 10^{17}$
Ohms	10^{-9}	10^{6}	10^{-6}	1	$8{\cdot}988 \times 10^{11}$
Statohms	$1{\cdot}112 \times 10^{-21}$	$1{\cdot}112 \times 10^{-6}$	$1{\cdot}112 \times 10^{-18}$	$1{\cdot}112 \times 10^{-12}$	1

Electrical Conductance

1 mho = 1 ohm^{-1} = 10^{-6} megmho = 10^{6} micromho

Table VII

(*Capacity*) (*Capacitance*)

Multiply Number of → by to Obtain ↓	Abfarads	Farads	Microfarads	Statfarads
Abfarads	1	10^{-9}	10^{-15}	$1{\cdot}112 \times 10^{-21}$
Farads	10^{9}	1	10^{-6}	$1{\cdot}112 \times 10^{-12}$
Microfarads	10^{15}	10^{6}	1	$1{\cdot}112 \times 10^{-6}$
Statfarads	$8{\cdot}988 \times 10^{20}$	$8{\cdot}988 \times 10^{11}$	$8{\cdot}988 \times 10^{5}$	1

TABLE VIII

Inductance

to Obtain ↓ / Multiply Number of → by	Abhenrys *	Henrys	Microhenrys	Millihenrys	Stathenrys
Abhenrys *	1	10^{9}	1,000	10^{6}	$8{\cdot}988 \times 10^{20}$
Henrys	10^{-9}	1	10^{-6}	0·001	$8{\cdot}988 \times 10^{11}$
Microhenrys	0·001	10^{6}	1	1,000	$8{\cdot}988 \times 10^{17}$
Millihenrys	10^{-6}	1,000	0·001	1	$8{\cdot}988 \times 10^{14}$
Stathenrys	$1{\cdot}112 \times 10^{-21}$	$1{\cdot}112 \times 10^{-12}$	$1{\cdot}112 \times 10^{-18}$	$1{\cdot}112 \times 10^{-15}$	1

* An abhenry is sometimes called a " centimetre ".

TABLE IX

Magnetic Flux

to Obtain ↓ / Multiply Number of → by	Kilolines	Maxwells (or lines)	Webers
Kilolines	1	0·001	10^{5}
Lines of Force (Maxwells)	1,000	1	10^{8}
Webers	10^{-5}	10^{-8}	1

TABLE X

Magnetic Flux Density

Multiply Number of → by ↘ to Obtain ↓	Gausses (or lines per square centimetre)	Lines per square inch	Webers per square centimetre	Webers per square inch
Gausses * (or lines per square centimetre)	1	0·1550	10^8	$1·550 \times 10^7$
Lines per square inch	6·452	1	$6·452 \times 10^8$	10^8
Webers per square centimetre	10^{-8}	$1·550 \times {}^{-9}$	1	0·1550
Webers per square inch	$6·452 \times 10^{-8}$	10^{-8}	6·452	1

* The name "gauss" is sometimes used for the unit of magnetic field intensity (1 gauss = 1 gilbert per cm). Since flux density = permeability × field intensity ($B = \mu$H) these two quantities have the same units if μ is considered dimensionless, just as 1 abhenry = $\mu \times$ 1 cm; hence the occasional name centimetre for an abhenry. The A.I.E.E. sanctions "gauss" for both B and H; physicists usually do not. In 1930 the International Electrotechnical Commission, of which the U.S. National Committee is the electrical standards committee of the American Standards Association, adopted the following names for units in the c.g.s. electromagnetic system: Magnetomotive force, gilbert; magnetizing force, oersted; magnetic flux, maxwell; magnetic flux density, gauss. The name oersted has been used for a unit of reluctance in the U.S.

TABLE XI

Magnetic Potential and Magnetomotive Force

Multiply Number of → by ↘ to Obtain ↓	Abampere-turns	Ampere-turns	Gilberts
Abampere-turns	1	0·1	$7·958 \times 10^{-2}$
Ampere-turns	10	1	0·7958
Gilberts	12·57	1·257	1

Table XII

General Conversion Factors

	To Convert	*Into*	*Multiply By*
Length	Centimetres	Inches	0·3937
	Centimetres	Feet	0·03281
	Feet	Centimetres	30·48
	Feet	Metres	0·3048
	Inches	Centimetres	2·54
	Inches	Metres	0·0254
	Kilometres	Miles	0·6214
	Kilometres	Feet	3,281
	Metres	Yards	1·094
	Metres	Feet	3·281
	Metres	Inches	39·37
	Miles	Kilometres	1·6093
	Miles	Feet	5,280
Mass	Grammes	Grains	15·432
	Grammes	Ounces	0·03527
	Kilogrammes	Lbs.	2·2046
	Kilogrammes	Ounces	35·274
	Ounces	Grammes	28·35
	Lbs.	Kilogrammes	0·4536
	Tons	Lbs.	2,240
	Tons	Tonnes	1·016
	Tonnes	Tons	0·9842
Area	Sq. Centimetres	Sq. Inches	0·155
	Sq. Centimetres	Sq. Feet	0·001076
	Sq. Feet	Sq. Metres	0·0929
	Sq. Inches	Sq. Centimetres	6·4516
	Sq. Kilometres	Sq. Miles	0·3862
	Sq. Metres	Sq. Feet	10·76
	Sq. Metres	Sq. Yards	1·197
	Sq. Miles	Sq. Kilometres	2·589
	Sq. Millimetres	Sq. Inches	0·00155
	Sq. Yards	Sq. Metres	0·8361

	To Convert	*Into*	*Multiply By*
VOLUME	Cubic Centimetres	Cubic Inches	0·06102
	Cubic Inches	Cubic Centimetres	16·39
	Cubic Inches	Litres	0·0164
	Cubic Feet	Cubic Metres	0·0283
	Cubic Feet	Litres	28·317
	Cubic Feet	Imperial Gallons	6·23
	Imperial Gallons	Cubic Feet	0·161
	Imperial Gallons	Litres	4·546
	Imperial Gallons	U.S. Gallons	1·205
	Litres	Cubic Centimetres	1,000
	Litres	Cubic Inches	61·0
	Litres	Cubic Feet	0·03531
	Litres	Gallons	0·22
	Litres	Pints	1·7598
	Cubic Metres	Cubic Feet	35·31
	Cubic Metres	Cubic Yards	1·308
	Pints	Litres	0·5682
	Cubic Yards	Cubic Metres	0·7646
FORCE	Dynes	Poundals	0·00007233
	Dynes	Grammes	0·0010194
	Poundals	Pounds	0·03107
ENERGY	B.O.T. Units	kW-Hours	1
	B.O.T. Units	B.T.U.s	3,411
	B.O.T. Units	Foot-lbs.	$2{\cdot}654 \times 10^6$
	B.O.T. Units	Joules	$3{\cdot}6 \times 10^6$
	B.T.U.s (1 lb. 1° F.)	Gramme Calories	252·0
	B.T.U.s	Foot-lbs.	777·4
	B.T.U.s	Watt Hours	0·29316
	B.T.U.s	B.O.T. Units	0·00029316
	Gramme Calories	Joules (Watt-seconds)	4·188
	Gramme Calories	B.T.U.s	0·00397
	Ergs	Joules	10^{-7}
	Foot-lbs.	Joules	1·356
	Foot-lbs.	Kilogramme-metres	0·1384
	Joules	Gramme Calories	0·239
	Joules	Foot-lbs.	0·7373
	Joules	Ergs	10^{-7}
	kW-Hours (see B.O.T. Units)		
	Kilogramme-metres	Foot-lbs.	7·233
	Therms	B.T.U.s	100,000

	To Convert	Into	Multiply By
POWER	B.T.U. per minute	Horse-power	0·0236
	Gramme Calories per second	Watts	4·188
	Cheval-vapeur	Horse-power	0·9863
	Cheval-vapeur	Kilowatts	0·7358
	Foot-lbs. per second	Watts	1·356
	Horse-power	Foot-lbs. per second	550·0
	Horse-power	Kilowatts	0·746
	Horse-power	Watts	746·0
	Horse-power	B.T.U. per minute	42·41
	Horse-power	Cheval-vapeur	1·014
	Horse-power	Kilogramme-metres per second	76·04
	Kilowatts	Horse-power	1·3406
	Kilowatts	B.T.U.s per hour	3,411
	Kilowatts	Gramme Calories per second	239·16
	Watts	Gramme Calories per second	0·23916
PRESSURE	Atmospheres	Lbs./sq. in.	14·7
	Lbs./sq. in.	Atmospheres	0·068
	Lbs./sq. in.	Kg./sq. cm.	0·0703
	Lbs./sq. ft.	Kg./sq. m.	4·88
	Lbs./sq. yard	Kg./sq. m.	0·543
	Inches of Mercury	Lbs./sq. in.	0·4902
	Feet of Water	Lbs./sq. in.	0·43
VELOCITY	Feet/min.	Miles/hour	0·01137
	Feet/sec.	Miles/hour	0·6818
	Miles/hour	Feet/min.	88·0
	Miles/hour	Feet/sec.	1·467
	Miles/hour	Knots	0·868
	Miles/hour	Metres/sec.	0·447

TEMPERATURE

$$T^\circ F. = \frac{9}{5} T^\circ C. + 32^\circ$$

$$T^\circ C. = \frac{5}{9} (T^\circ F. - 32^\circ)$$

T° F. = Temperature in degrees Fahrenheit

T° C. = Temperature in degrees Centigrade

31. Voltage Breakdown in Air.

When a potential difference exists between two conductors in air, as the voltage between the conductors is increased a number

of phenomena are observed which depend on the conditions of observation. In the case of open wire feeders, the first manifestation is a dark corona which may be observed as a distortion of objects behind the corona viewed through it. At a higher voltage a brush discharge occurs from any sharp points on the feeders. At a still higher voltage a luminous corona may be seen round the conductors at night. Finally, ionization of the air reaches a point where a spark passes, and if the conditions are suitable, an arc is maintained between the conductors.

The luminous corona is rare. Brush discharge from any irregularities in the conductors is however common, and for this reason *all conductors which are to carry high voltage should as far as possible have all excrescences rounded off so that the radius is not less than about an inch*. Conductors should not be operated under conditions where either brush discharge or corona is visible, and the presence of dark corona is an indication that a feeder is being operated above its safe working value.

The working conditions of feeders in common use are given in XVI:6.

The breakdown voltage of an air condenser depends on the frequency, the spacing of the vanes, their thickness and the way in which the edges of the vanes are finished. The edges should have a section which is semicircular. The surface of the vanes should be smooth and free from dirt and dust. The following gives the important dimensions of a number of condensers in practical use.

Typical Dimensions of Practical Condensers Working on Voltage and Wavelengths Shown

No.	Peak Voltage		Lowest Operating Wavelength	Thickness of Plates		Average Spacing between Plates
	No mod.	100% p.p.m.		Stator	Rotor	
	Volts	*Volts*	*Metres*	*Inches*	*Inches*	*Inches*
1	1,000	2,000	200	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{3}{32}$
2	2,500	5,000	200	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{3}{16}$
3	3,000	6,000	200	0·094	0·094	0·282
4	9,500	19,000	48·5	$\frac{3}{16}$	$\frac{1}{16}$	1
5	11,000	22,000	13	1	1	1

For information on the behaviour of air condensers at radio frequency, reference should be made to Bibliography V:3.2.

31.1. Use of Spark Gaps as Voltage Limiters. When a circuit is subject to surges of voltage above the value of voltage for which the circuit is designed it is customary to bridge a spark gap across the circuit which is set to such a value that when the limiting safe voltage is reached, the spark gap breaks down, and an arc is formed which takes sufficient current to lower the voltage to a safe value. Such a spark gap is always constituted by a pair of metal spheres because the sparking voltage between two spheres is independent of frequency and of the wave form of the voltage surge. This independency of frequency extends up to quite high frequencies: the performance of a spark gap at 13 Mc/s is indistinguishable from its performance at zero frequency.

The spark gap is normally provided with a pair of horns extending above the gap and arranged so that the separation between the horns at the top is several times the separation at the bottom. With such a form of horn gap the arc which is formed is carried upwards by the flow of hot air generated by the arc itself, and if, as is usually the case, the high voltage surge is of short duration, the arc eventually reaches a point where the gap is so large that the arc is extinguished. Some horn gaps are fitted with a "magnetic blow-out" which merely consists of an electromagnet so arranged that the interaction between the current in the arc and the magnetic field causes the arc to travel towards the wide end of the horn gap.

Figures for sparking distances with sphere gaps are given in *Tables of Physical and Chemical Constants* by G. W. C. Kaye and T. H. Laby.

For needle points and all forms of gap except sphere gap the sparking voltages vary widely with frequency, and it is impossible to formulate any generally applicable rule to give the sparking distance for a given voltage and frequency. Reference should, however, be made to the Bibliography, V:3.

Spark gaps are placed across high-voltage modulation and audio-frequency drive transformers and chokes. On medium and long waves spark gaps are sometimes connected between the anodes of valves, delivering high radio-frequency voltages, and earth.

32. Other Physical Effects.

In engineering practice a large number of physical effects are involved in addition to purely electrical effects. A brief description of some of the more important relevant effects is given below with details of the corresponding physical constants which determine the magnitudes of these effects for substances of interest.

33. Mass and Weight.

The *mass* of a body may be determined by its conditions of motion when acted upon by a single force: it may be determined by the ratio of the force divided by the acceleration. Alternatively the mass of a moving body may be determined by the momentum divided by the velocity or by twice the kinetic energy divided by the square of the velocity.

For normal engineering purposes the mass of a body is a constant. The ultra-high frequencies in use in radio technique are, however, approaching the value at which the mass of an electron can no longer be considered to be a constant. So far, however, this is only likely to affect valve design, as its effect on circuit constants is not likely to be revealed with the degree of accuracy of measurements used by the practical engineer.

The *weight* of a body is the force with which it is attracted towards the earth at the surface of the earth. This force increases as measurements are made at higher and higher latitudes, so that the weight of a body at the poles is about 5% greater than its weight at the equator. The only serious engineering importance of this is that a spring balance for measuring weights, or a weight-controlled electrical meter would have a calibration which varied with latitude.

For practical engineering purposes the weight of a body is assumed to be constant and equal to its mass. The mass of a body is therefore equal to its weight.

34. Density and Specific Gravity.

The *density* of a material is the amount of mass contained in unit volume: e.g. grammes per cubic centimetre.

The *specific gravity* of a material is the ratio of its density to the density of water = the weight of a cubic centimetre of the material divided by the weight of a cubic centimetre of water.

The densities of a number of elements and common substances are given in *Tables of Physical and Chemical Constants* by G. W. C. Kaye and T. H. Laby.

Comparison of the Merits of Aluminium and Copper as Conductors.

Since the resistivity of a metal is defined as the resistance per cubic centimetre, it follows that if a limitation of size is imposed on the conductor the use of copper will give the lowest possible resistance.

If the limitation is one of weight, however, an aluminium conductor will provide the lowest resistance. For a given weight the conductivity of a conductor is inversely proportional to the product of the resistivity and the density of the material used. Taking resistivities at 20° C., the product of the resistivity and the density is :

Copper . .	$1{\cdot}726 \times 8{\cdot}93 = 15{\cdot}4$
Aluminium .	$3{\cdot}23 \times 2{\cdot}7 = 8{\cdot}73$

For a given conductivity or resistance the weight of a copper conductor is therefore $15{\cdot}74/8{\cdot}73 = 1{\cdot}765$ times as great as the aluminium conductor of the same conductivity or resistance.

Aluminium should therefore be used where weight reduction is more important than volume reduction.

35. Relations between Different Kinds of Energy.

Energy is present wherever there is any kind of activity or any kind of stress. Each kind of activity and each kind of stress may be measured by the amount of energy it contains. Conversely, each kind of activity and each kind of stress provides an independent measure of energy, and may be said to constitute a different form of energy.

Energy of one kind may be converted into energy of another kind. Sometimes this conversion occurs without loss, that is to say, all the energy in one form is converted to only one other form. Sometimes loss occurs during the conversion ; that is, the energy is not all converted to one other form only, but some of it is dissipated in still other forms. When allowance is made for such losses it is found that a certain amount of energy described in the measure appropriate to one form always corresponds to the same amount of energy described in the measure appropriate to any other form.

When energy in one form contained in one system is converted to energy in another form contained in another system, work is said to be done in the second system by the first system. Work is therefore another name for energy.

The rate at which work is done is called *power* : power is the rate of doing work. In the example just given, the first system supplies power to the second system of value equal to the rate of increase of energy in the second system. The amount of energy supplied in a given time is therefore equal to the power multiplied by the time.

Depending on the kind of energy and the associated conditions, it is sometimes most simple to measure power and to deduce energy conditions by multiplying by appropriate amounts of time; sometimes it is most convenient to measure energy direct and to deduce the power by measuring the energy content of a system at two different times and so determine the rate of increase of energy, i.e. the power.

Energy corresponding to activity is called kinetic energy, while energy corresponding to stress is called potential energy. Kinetic energy exists in a moving electron, atom or molecule and in a travelling bus or a train. Potential energy exists in a toboggan and its crew at the beginning of their run, and is converted first to kinetic energy and finally to heat during the course of the run. Since heat consists of vibration of the particles of the substance containing the heat, heat is really a form of kinetic energy. Just as potential energy exists in the interaction of the gravitational fields of the toboggan and the earth, so potential energy exists in electric fields and in magnetic fields.

Here only three kinds of energy are involved; mechanical work, heat, and electrical energy as represented by electrical power multiplied by time. The units in which these are measured will now be described.

Mechanical Work. If a man pushes a barrow up a hill he does work on the system represented by the barrow and the hill.

The amount of this work is most simply measured by the mean force applied to the barrow in its direction of movement and the distance travelled. In mechanics, the product of force and distance through which the force acts is used as the measure of work. The most common units used for measuring force and distance for this purpose are feet and pounds. If a force of ten pounds is applied continuously to a body in its direction of motion while it is translated through a thousand feet, the amount of work done is said to be ten thousand foot-pounds. The unit of mechanical work is therefore the foot-pound. The unit of mechanical power is the foot-pound per second. A horse-power is arbitrarily defined as being equal to 550 foot-pounds per second = 33,000 foot-pounds per minute.

Heat. Several units for measuring heat are in use:

The Mean Gramme Calorie is one-hundredth of the amount of heat required to raise the temperature of one gramme of water

from 0° to 100° Centigrade. (For all practical purposes the mean gramme calorie may be assumed to be equal to the amount of heat required to raise the temperature of one gramme of water by one degree Centigrade.)

The Large Calorie = 1,000 mean gramme calories.

The British Thermal Unit is specified as the amount of heat required to raise the temperature of 1 lb. of water by one degree Fahrenheit.

The Therm = 100,000 B.T.U.s.

Power in the form of heat is measured in Calories per second or B.T.U.s per second.

Electrical Power supplied to a circuit is obtained in watts by multiplying together the R.M.S. volts across the circuit, the R.M.S. amperes flowing in the circuit, and the power factor.

The Kilowatt (kW) = 1,000 watts

The Milliwatt = 0·001 watt

Electrical Energy supplied to a circuit is measured as the product of the power supplied to the circuit and the time during which it is supplied.

Electrical Energy is normally measured in watt-seconds (joules), and kilowatt-hours.

The Joule = 1 watt-second.

The B.O.T. Unit = 1 kilowatt-hour.

The relations between the above units are given in Table XII.

For instance, 1 mean gramme calorie = 4·188 joules. This means that by the expenditure of this amount of heat in a conversion device which was 100% efficient it would be possible to supply a power of 4·188 watts for one second. Conversely, an expenditure of power of 1 watt would supply heat at the rate of 1/4·188 = 0·239 mean gramme calories per second.

36. Current-carrying Capacity of Conductors.

At D.C. the current-carrying capacity of an insulated copper conductor in amperes may safely be taken as 1,000 times the area of cross-section of the conductor in square inches. For instance, a No. 16 S.W.G. copper wire (see Table I of this chapter) will carry 3·22 amps., while a No. 8 S.W.G. copper wire will carry 20·1 amps.

At R.F. the current-carrying capacity is proportional to the diameter instead of the area and in amperes is equal to 76·2 times the diameter of the conductor in inches. Since, in practice, the requirements of mechanical strength result in the wall of any tubular conductor being considerably more than three times the depth of penetration (see II:13.1), the current-carrying capacity of a tubular conductor is equal to that of a solid round conductor of the same diameter. For instance, the current-carrying capacity at R.F. of a copper tube 1 in. in diameter is 76·2 amps., while that of a tube half an inch in diameter is 38·1 amps. The figure of 76·2 is not critical, but is given to conform to the original and alternative rule of thumb which was three amperes for each millimetre of diameter.

99. NUMERICAL EXAMPLES

99. Numerical Examples.

1. A conductor 4 centimetres long perpendicular to a magnetic field of 10,000 lines per square centimetre moves at right angles to itself and the field with a velocity of 100 centimetres per second. What is the magnitude of the e.m.f. generated, in volts?

[A. $4 \times 10{,}000 \times 100 \times 10^{-8} = 4 \times 10^{-2} = 0{\cdot}04$ volts.]

2. A battery of 12 volts supplies 10 amperes for 10 hours. State (*a*) Quantity of electricity supplied.

[A. 100 ampere-hours = $100 \times 3{,}600$ ampere-seconds = 360,000 coulombs.]

(*b*) Power supplied.

[A. 120 watts.]

(*c*) Energy supplied.

[A. 1,200 watt-hours = 1·2 kW. hours = 1·2 B.O.T. units.]

3. (*a*) Voltages of 1, 2, 10 and 100 volts are placed across a resistance of 1Ω; what is the current in each case?

[A. Respectively 1, 2, 10 and 100 amps.]

(*b*) 1,000 volts is placed across resistances of 1, 10, 100 and 1,000 ohms. What is the current in each case?

[A. Respectively, 1,000, 100, 10 and 1 amps.]

(*c*) 1,200 volts is placed across 300 Ω. What is the current?

[A. 4 amperes.]

(*d*) 1 volt across each of three unknown resistances gives the following currents: 0·5 amperes, 10 milliamps., 100 microamps. What are the resistances?

[A. 2 Ω, 100 Ω, 10,000 Ω.]

4. Given resistances of 100, 1,000 and 10,000 ohms and a milliammeter reading 0–10 milliamps., how would you measure a voltage believed to be about 50 volts?

5. What is the resistance of a copper wire 1 kilometre long and of area of cross-section 1 square millimetre?

Given $\rho = 1{\cdot}7 \times 10^{-6}$ ohms.

[A. $\dfrac{100{,}000 \times 1{\cdot}78 \times 10^{-6}}{0{\cdot}01} = 17{\cdot}8\ \Omega$.]

6. If a length of 25 S.W.G. wire (0·02 in. dia.) 1 mile long has a resistance of 134·4 Ω, what is the loop resistance of a pair of 33 S.W.G. wires (0·01 in. dia.) $1\frac{1}{2}$ miles long?

[A. $2 \times 1{\cdot}5 \times 134{\cdot}4 \times \left(\dfrac{0{\cdot}02}{0{\cdot}01}\right)^2 = 1{,}602{\cdot}8\ \Omega$.]

7. What is the inductance of a coil constructed by winding a former 2 inches in diameter, with $1\frac{1}{2}$ inches length of winding space available along its length, when wound full with 26 S.W.G. enamel-covered wire (0·02 in. dia.)?

[A. No. of turns $= \dfrac{1{\cdot}5}{0{\cdot}02} = 75 \quad L = \dfrac{2^2 \times 1{\cdot}5^2 \times 75^2}{(18 \times 2) + (40 \times 1{\cdot}5)} = 527\ \mu\text{H}$.]

8. With what number of turns should the former of Question 7 be wound full, in order to provide an inductance of 200 μH?

[A. $75 \times \sqrt{\dfrac{200}{527}} = 46{\cdot}2$, say 46 turns.]

9. What is the A.C. resistance per 1,000 feet of No. 6 S.W.G. copper wire at 10 and 100 kc/s, and at 1, 10 and 100 Mc/s?

[A. $r_0 = \frac{1}{2} \times 0{\cdot}192 \times 2{\cdot}54$ cm. $= 0{\cdot}244$ cm. From Table 1 $R_0 = 0{\cdot}2764$ ohms per 1,000 ft. The value of d for copper, as found from Fig. 2 for each frequency, is tabulated below. For

frequencies outside the range of Fig. 2, the value of d and R_s is found by using the fact that d is inversely proportional to the square root of the frequency and R_s is proportional to the square root of the frequency. The value of r_0/d is also tabulated. R_3 is found from equation (2), and then R_2 is found from R_0 via Fig. 3, or from R_3 via Fig. 4.

The table below gives the steps in the working :

Frequency	10 kc/s	100 kc/s	1 Mc/s	10 Mc/s	100 Mc/s
d	$6{\cdot}65 \times 10^{-2}$	$2{\cdot}1 \times 10^{-2}$	$6{\cdot}65 \times 10^{-3}$	$2{\cdot}1 \times 10^{-3}$	$6{\cdot}65 \times 10^{-4}$
r_0/d	3·67	11·6	36·7	116	367
R_s	$2{\cdot}53 \times 10^{-5}$	8×10^{-5}	$2{\cdot}53 \times 10^{-4}$	8×10^{-4}	$2{\cdot}53 \times 10^{-3}$
R_3	0·504	1·59	5·04	15·9	50·4
R_2/R_0 from Fig. 3	2·1				
R_2 from R_0 via Fig. 3	0·582				
R_2/R_3 from Fig. 4	1·15	1·04	$\simeq 1{\cdot}0$	$\simeq 1{\cdot}0$	$\simeq 1{\cdot}0$
R_2 from R_3 via Fig. 4	0·58	1·65	5·04	15·9	50·4

R_2 is then the A.C. resistance.]

10. It is required to make up an air condenser of 20 $\mu\mu$F with an inch separation between the plates ; what area of plate is required ?

[A. $C = 0{\cdot}0884\dfrac{A\varepsilon}{d}\ \mu\mu\text{F} = 20\ \mu\mu\text{F}$

$\therefore A = \dfrac{20d}{0{\cdot}0884\varepsilon}$ where $d = 2{\cdot}54$ and $\varepsilon =$ unity

$$\therefore A = \frac{20 \times 2\cdot54}{0\cdot0884} = 575 \text{ square centimetres}$$

$$= \frac{575}{2\cdot54^2} = 89 \text{ square inches}$$

The condenser may therefore be constituted by two square plates of just under $9\frac{1}{2}$ ins. side 1 ins. apart.]

11. What is the approximate maximum capacity of a variable air condenser of normal type in which a stack of 21 semicircular plates of diameter 6 cms. meshes with a fixed stack of 20 semicircular plates of 8 cms. diameter? The length of the stack of moving plates, measured along the spindle, is 6·2 cms., and the average spacing between each fixed plate and the moving plate each side of it is a half the thickness of a plate. All plates are of equal thickness.

[A. The number of air spaces is equal to twice the number of plates in the stack having the most plates minus $2 = 2 \times 21 - 2 = 40$. If s is the average spacing between plates, $2s$ is the thickness of each plate and $40s+(20+21) \times 2s = 6\cdot2$ cms., so that

$$s = 6\cdot2/122 = 0\cdot051 \text{ cm.}$$

The capacity of the whole condenser is 40 times the capacity of the condenser formed between the effective plate area each side of each air space. The effective area is the area of the smaller plate $= \frac{1}{2}\pi r^2 = \frac{1}{2}\pi \times 3^2 = 14\cdot1$ square centimetres.

The approximate capacity is therefore

$$40 \times 0\cdot0884 \times \frac{14\cdot1}{0\cdot051} = 975\ \mu\mu\text{F}.$$

This estimate of capacity neglects the reduction of capacity due to the part of the fixed plate which is cut away to make room for the spindle, and the increases of capacity due to the spindle and the "end effects" occurring at the circumference of each moving plate. For practical purposes the condenser capacity would be taken as 0·001 μF (1,000 $\mu\mu$F).]

12. In a test on the efficiency of a high-power amplifier, thermometers record the rise in temperature of cooling water, due to its passage through the amplifier, while flow-meters record the rate of flow of water in gallons per minute. Derive an expression giving the number of kilowatts supplied to the water in terms of the temperature rise R in degrees centigrade and the rate of water flow F in gallons per minute.

[A. One gallon = 4·546 litres = 4,546 c.c.

Hence one gallon of water weighs 4,546 grammes, since the specific gravity of water is unity.

Hence one gallon per minute raised in temperature 1° C. corresponds to 4,546 gramme calories per minute $= \frac{4{,}546}{60} = 75{\cdot}8$ gramme calories per second $= 75{\cdot}8 \times 4{\cdot}188 = 318$ watts $= 0{\cdot}318$ kilowatts.

Hence F gallons per minute with a temperature rise of R° C. corresponds to $0{\cdot}318FR$ kW.]

CHAPTER III

THE SINE WAVE AND VECTORS

A SINE wave has already been shown in Fig. 1/I:4 at *A*. This kind of wave is of great importance in all calculations dealing with varying quantities such as are involved in the transmission of speech and music, since problems involving sine waves enable comparatively simple solutions to be obtained, and from a practical point of view, it is permissible to regard all wave forms as being composed of a number of sine waves of appropriate frequency, amplitude and *phase*. The meaning of phase is given later in this chapter.

1. Trigonometrical Definitions.

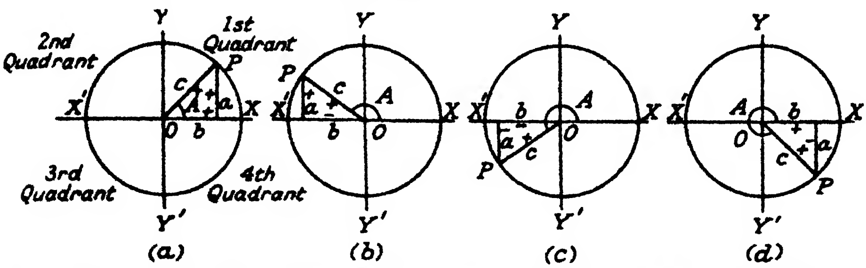


FIG. 1/III:1.—Definition of Values of Trigonometrical Functions in each Quadrant.

In the right-angled triangles in Fig. 1, *a*, *b* and *c* are the lengths of the sides, while *A* is the angle through which the arm *OP* has rotated from an assumed initial position coincident with *OX*. The following definitions completely describe the *functions*: sin, cos, tan, cosec, sec and cot ; $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$. It should be understood that sin, tan, cos, etc., are merely names arbitrarily assigned to the different ratios by which the magnitude of an angle can be determined.

Definitions.

$\sin A = \frac{a}{c}$ $\qquad \sin^{-1}\frac{a}{c} = A =$ the angle whose sine is $\frac{a}{c}$

$\cos A = \frac{b}{c}$ $\qquad \cos^{-1}\frac{b}{c} = A =$,, ,, ,, cosine is $\frac{b}{c}$

$\tan A = \frac{a}{b}$ $\qquad \tan^{-1}\frac{a}{b} = A =$ the angle whose tangent is $\frac{a}{b}$

Similarly $\sin B = \frac{b}{c}$, $\cos B = \frac{a}{c}$, etc.

Also

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{c}{a}$$

$$\sec A = \frac{1}{\cos A} = \frac{c}{b}$$

$$\cot A = \frac{1}{\tan A} = \frac{b}{a}$$

Full Names of Functions.

sin = sine $\qquad$ cosec = cosecant

cos = cosine $\qquad$ sec = secant

tan = tangent $\qquad$ cot = cotangent

Signs of the Trigonometrical Functions.

Consistent with the usual conventions relating to the measurement of quantities expressing the distance of a point from the origin (crossing point) of axes crossing at right angles, signs are allocated to distances measured along the axes $X'OX$ and $Y'OY$, as follows:

Distances of a point, to the right of $Y'OY$, from $Y'OY$ are positive.
" " " " " " left " $Y'OY$, " $Y'OY$ are negative.
" " " " above $X'OX$, from $X'OX$, are positive.
" " " " below $X'OX$, " $X'OX$, " negative.

As the result of these conventions, the lengths a and b in Fig. 1 are sometimes positive and sometimes negative, with the result that the trigonometrical functions change sign as the arm OP rotates. *Distances measured along OP are always considered positive:* this is of course an arbitrary assumption.

The plane in which OP rotates is divided into four quadrants by the two axes $X'OX$ and $Y'OY$, and these are conveniently numbered from 1 to 4 as shown in Fig. 1 (a). The signs of a, b, and c are indicated in Fig. 1. The signs of the trigonometrical functions in each quadrant are therefore as follows:

Quadrant	1st	2nd	3rd	4th
Sine . .	+	+	−	−
Cosine . .	+	−	−	+
Tangent . .	+	−	+	−
Cosecant . .	+	+	−	−
Secant . .	+	−	−	+
Cotagent . .	+	−	+	−

These signs are very important and must be observed rigidly in all formulae. For instance, the sine of 135° is $\frac{1}{\sqrt{2}}$, the cosine of 135° is $-\frac{1}{\sqrt{2}}$, while the tangent of 135° is -1. The angle whose tangent is 1 is either 45° or 225°. The symbolic way of stating this is $\tan^{-1} 1 = 45°$, and $\tan^{-1} 1 = 225°$. Similarly $\sin^{-1} A$ means "the angle whose sine is A": $\cos^{-1} A$ means "the angle whose cos is A", etc.

A certain amount of ambiguity appears to exist because, if the value of the sine or cosine, etc., of an angle is known, there are two angles which correspond. In practice this never causes any ambiguity, although it may cause some head-scratching, because other indications are always present to give a clear indication of which angle should be taken. Very often, for instance, the mathematical answer is indeed that both angles give the required result, but the requirements of economy usually say in unmistaken terms

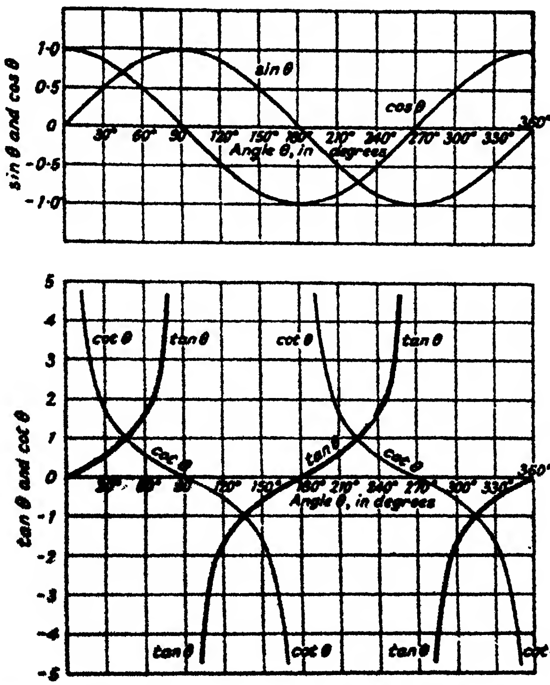


FIG. 2/III:1.—Values of cos θ, sin θ, tan θ and cot θ.

which angle is the one to use. Fig. 2 shows the values of $\sin\theta$, $\cos\theta$, $\tan\theta$ and $\cot\theta$ plotted against θ.

Trigonometrical functions are related to one another by simple formulae. The more important of these are given in CI:2, and a number of these formulae are used in deriving useful relations between electrical quantities. It is quite unnecessary for engineers to be able to establish these formulae from first principles; but, if the method of establishing them is required, reference should be made to any standard work on trigonometry, such as *A New Trigonometry for Schools* by Borehardt and Perrott (pub. G. Bell & Sons).

2. Form of a Sine Wave.

A sine wave may be generated in the way illustrated in Fig. 1/III:2. This method of generation constitutes a definition and a description of a sine curve or wave which is consistent with the sine curve drawn in Fig. 2/III:1. A point O, about which turns a rotating arm OP, moves from left to right along the axis $T'T''$ with relative velocities of rotation and translation such that the arm travels through distance T while it makes one revolution. A point A moves along the vertical axis $A'A'$ (which moves with O) so that AO is always equal to PR where R is the projection of P on the axis TT'. The trace of the point A then describes a sine wave as indicated by the full line curve.

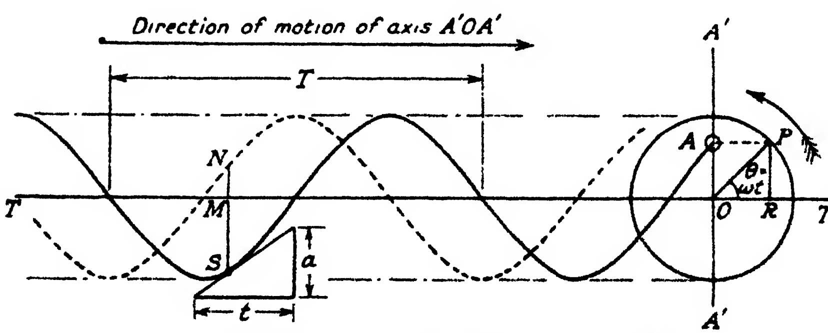


FIG. 1/III:2.—Method of Generating a Sine Wave.

If the arm OP makes f revolutions per second, its angular velocity, which is usually called ω is $2\pi f$ radians per second. (This is because the fundamental measure of angle is the radian. The number of radians in an angle is equal to the arc of circle subtended by the angle divided by the radius of the circle. Hence,

in one revolution OP transverses $\frac{2\pi r}{r} = 2\pi$ radians, r being the radius of any circle drawn with the apex of the angle as centre.)

$$\omega = 2\pi f \text{ radians per second} \quad . \quad . \quad . \quad (1)$$

f is then said to be the frequency of the sine wave and ω the angular frequency or angular velocity. The frequency of a wave is therefore the number of waves or complete cycles per second.

It is evident that the sine wave is a trace of the magnitude $PR = OP \sin \theta$ (by definition of a sine, $\frac{PR}{OP} = \sin \theta$, hence $PR = OP \sin \theta$), and since θ is equal to the angular velocity ω multiplied by time, $\theta = \omega t$ and

$$PR = OP \sin \omega t \quad . \quad . \quad . \quad . \quad (2)$$

where t is time in seconds.

If PR represents i the instantaneous magnitude of a current, then OP represents the maximum value of the current which may be written as $\hat{\imath}$ so that

$$i = \hat{\imath} \sin \omega t \quad . \quad . \quad . \quad . \quad (3)$$

and $T'T''$ is now an axis of time to a scale such that the time $T = \frac{1}{f}$ seconds. Similarly, such a wave can be used to indicate a voltage $v = \hat{v} \sin \omega t$ where v and $\hat{v}$ are respectively the instantaneous and maximum values of an *alternating* sinusoidal voltage.

The use of the term alternating will now be clear, since an alternating voltage or current alternately flows in each of two directions; starting from zero, it rises to a maximum in one sense and falls to zero again, after which it rises to a maximum in the reverse sense and falls to zero again.

$\hat{v}$ and $\hat{\imath}$ are called the peak value of volts and current, respectively, and are sometimes written as $V_{max.}$ and $I_{max.}$. v and i are the instantaneous values of voltage and current.

An important property of the sine wave is connected with its slope. Draw a tangent to the curve at any point such as S, and draw a right-angled triangle with part of this tangent as hypotenuse and the other two sides respectively parallel to the axis of time and the axis of amplitude. Let the lengths of these two sides be a and t as shown. Then evidently a/t is equal to the ratio $\frac{\text{change of amplitude}}{\text{change of time}}$ at point S, in other words ***a/t* is the rate of change of amplitude of the full-line sine wave.**

Further, the magnitude of a/t is given at each point on the

full-line sine wave, by the ordinate at that point of a second sine wave drawn $\frac{1}{4}$ of a period or cycle (i.e. $\frac{1}{4}T$) ahead of the full-line sine wave, and a maximum amplitude ω times the amplitude of the full-line curve. Such a curve is shown dotted in Fig. 1 to a relative scale of $1/\omega$. As an example the value of a/t at S is given by $MN \times \omega$. The dotted line curve is said to ***lead*** on the full-line curve by quarter of a period or cycle (i.e. $\frac{1}{4}T$), and since in that time the arm $\dot{O}P$ has made quarter of a revolution, the ***lead*** of the dotted curve is also said to be 90° or $\pi/2$ ahead of the full-line curve. The characteristic which determines the relative positions of two sine waves of the same frequency, along the time axis, is called ***phase***. These two curves are said to differ in phase by 90°.

Hence if the full-line curve is said to be described by $I = \hat{\imath} \sin \omega t$ or $v = \hat{v} \sin \omega t$, for instance, the dotted curve is described by

$$i = \hat{\imath}\omega \sin\left(\omega t+\frac{\pi}{2}\right) = \hat{\imath}\omega \sin(\omega t+90^\circ)$$

or

$$\dot{v} = \hat{v}\omega\left(\sin \omega t+\frac{\pi}{2}\right) = \hat{v}\omega (\sin \omega t+90^\circ)$$

Summary.

The rate of change of a quantity varying sinusoidally with time is also a sinusoidal quantity of the same frequency and of ω times the amplitude and having a lead of 90°, where ω is the angular frequency.

Inspection of the curves of Fig. 1 shows that the rate of change is positive when the quantity in question is increasing, and negative when it is decreasing.

The frequency of a sine wave is the number of periods, T, per second, usually referred to as cycles per second.

The convention for cycles per second adopted in this book is c/s. Other conventions are also used, such as p:s, c.p.s., p.p.s., ~ and Herz (German).

e.g. 1,000 cycles per second = 1,000 c/s.
= 1,000 p:s.
= 1,000 c.p.s.
= 1,000 p.p.s.
= 1,000 ~
= 1,000 Herz = 1 kH = 1 kiloherz.
= 1 kilocycle (per second) = 1 kc/s.

1,000,000 c/s = 1 megacycle per second = 1 Mc/s.

Note. The above statement relating to the rate of change of

a sine wave must be accepted without proof. In any particular case it can be verified by drawing the sine wave and a tangent at any required point, and determining the slope of this tangent. See example **10** in III:99. The proof of the general case may be demonstrated by elementary calculus, but is quite unimportant provided the meaning of the above statement is understood.

3: Representation of Sine Waves by Vectors.

A cook usually describes a trifle by the recipe for making it.

In the same way a sine wave can be described by the way it is drawn; either on real paper or on an entirely fictitious plane; such as the plane represented by the drawing in Fig. 1/III:2. This drawing really contains two planes in relative motion: the plane on which the sine wave is drawn; and the plane carrying the moving axis $A'OA'$ and the rotating arm OP.

It is evident that if the axis TT' is an axis of time the "rate" of travel of the axis $A'OA'$, etc., is fixed since it moves one second every second. In order to describe the full-line sine wave it is then only necessary to describe the angular movement of the arm OP and to say how big it is. The angular movement of OP is completely described by specifying its rate of angular movement and the time at which it started rotating: the latter may or may not be important. The rate of angular movement of OP may be described completely by specifying the number of revolutions it makes per second: the frequency f of the sine wave; or by specifying the number of radians the arm OP traverses per second: the angular velocity ω of the sine wave.

Just as position has no meaning except with regard to some frame of reference, so angular position has no meaning without reference to some angular direction of reference. If the question is asked, "Where is Timbuctoo?", the only satisfactory answer is to show a map of Africa with Timbuctoo marked on it. The map is then the frame of reference. If the question is asked, "What is the angular position of OP?", the only satisfactory answer is to specify the angle it makes with some other direction. This other direction may be the direction of OP at some particular time, or may be the simultaneous direction of some other sine-wave-generating arm. Most commonly it is useful to specify the angle between two generating arms which rotate at the same rate, in which case this angle is equal to the phase difference between the two generated sine waves. One of the two generating arms is then called the reference arm, and for purpose of representation

on paper may be drawn in any direction whatever; the other is then drawn to the same scale of magnitude, making the appropriate angle with the reference arm. The sense of the angle between the two arms is determined according to the convention for the direction of rotation shown in Fig. 1/III:2. If the second arm is generating a sine wave leading in phase on the reference sine wave the arm generating the second wave is drawn in advance of the first arm: leading the first arm in the direction of rotation. If the second sine lags on the first the generating arm follows the first arm in the direction of rotation. *It should be noticed that the direction of rotation is always anti-clockwise.*

Quantities which have direction and magnitude are called *vectors*. The representations of the generating arms of sine waves, whether drawn on paper, pictured in the imagination, or portrayed by letter symbols, are therefore vectors.

A sinusoidal voltage wave or a sinusoidal current wave can therefore be represented by a vector. This vector, when drawn on paper, can only represent either the position of the generating arm relative to its position at some other time, or its position relative to some other generating arm.

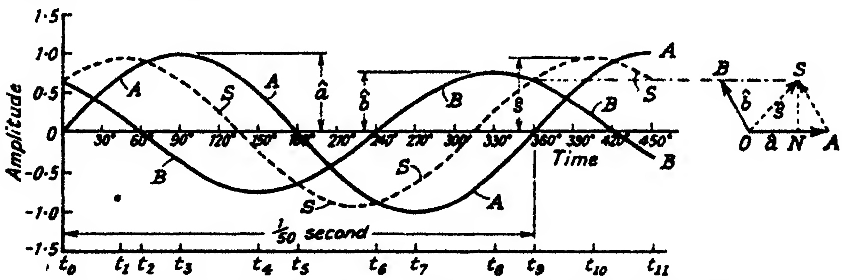


FIG. 1/III:3.—Sine Waves and Equivalent Vector Representation.

The two full-line curves on Fig. 1 are respectively a plot of the two sine waves:

$$a = \hat{a} \sin \omega t \quad . \quad . \quad . \quad . \quad \text{curve } A \quad . \quad . \quad (1)$$

$$b = \hat{b} \sin (\omega t + 120^\circ) \quad . \quad . \quad \text{curve } B \quad . \quad . \quad (2)$$

where $\omega = 2\pi f = 360^\circ \times f$, $f = 50$ c/s, $\hat{a} = 1{\cdot}0$, and $\hat{b} = 0{\cdot}75$. $\hat{a} \sin \omega t$ therefore represents a sine wave having zero amplitude and positive rate of increase of amplitude at time $t = 0$ which is indicated on the diagram as t_0. Its peak amplitude is 1·0.

$\hat{b} \sin (\omega t + 120^\circ)$ represents a sine wave leading on the first sine wave by 120°. Its peak amplitude is 0·75.

The frequency of both waves is the same: 50 c/s; so that the

quantity $2\pi ft = 360° \times ft = 360° \times 50t$ increases by 360° every $1/f$ seconds, that is to say, every $1/50 = 0{\cdot}02$ second. The quantity $\sin \omega t$ therefore passes through a complete cycle of values every 0·02 second: each cycle lasts $0{\cdot}02 = 1/50$ second, so that there are 50 cycles per second.

Curve A is plotted by drawing a time scale (in Fig. 1 this is just over 1/50 second long) and a scale of amplitude at right angles, and then plotting the values of a and b from equations (1) and (2) by giving t all values from zero up to 0·02 second (the curves in Fig. 1 have been plotted up to a slightly higher value of time). For instance, if $t = 1/150$ second, $\omega t = 360° \times 50/150 = 120°$. Hence $a = 1{\cdot}0 \sin 120° = 0{\cdot}866$ and $b = 0{\cdot}75 \sin 240° = -0{\cdot}75{\cdot}0{\cdot}866 = -0{\cdot}65$. These values of a and b are then plotted as amplitudes against the value of $t = 1/150$ second.

The diagram on the right of Fig. 1 shows the vectorial representation of these two frequencies (i.e. sine waves). For this purpose curve A has been taken to be the reference curve. The vector OA representing frequency A has therefore been drawn horizontal and pointing to the right. This evidently corresponds to the position of the generating arm at time t_0, and incidentally at time t_8, which is 1/50 second later, and at all times later which are separated from t_0 by an integral multiple of 1/50 second.

The vector OB representing frequency B, has been drawn leading OA by 120°, simply because if the curves A and B are generated by arms in relative positions corresponding to those of OA and OB they will appear in their relative positions as shown in Fig. 1.

The length of $OA = \hat{a} = 1{\cdot}0$, while the length of $OB = \hat{b} = 0{\cdot}75$.

3.1. Addition of Vectors. Apart from the fact that it is much simpler to draw a straight line than it is to draw a sine wave, the drawing of the two lines OA and OB appears rather aimless. It nevertheless can be made to serve a useful purpose. This is that it gives immediately the vector (generating arm) of the sine wave which results from adding the ordinates of the two waves A and B. The resultant of two sine waves *of the same frequency* is always a sine wave. The resultant of the two waves A and B has been drawn in Fig. 1 simply by adding the ordinates of the two curves A and B. It is shown as the dotted curve of amplitude $\hat{s}$. This curve is of different amplitude and different phase from either of the curves A and B.

The magnitude and direction of OS, the vector corresponding to curve S, is obtained by adding vector OB to vector OA as shown

on the right of Fig. 1. Vectors are added by drawing them tip to tail, and the resultant is given by drawing a line from the tail of the first vector to the tip of the last vector. AS is a line drawn parallel to OB and of the same length as OB. The resultant vector is OS, which defines the resultant curve S both in amplitude and phase. It should be noted that as OA and OB rotate with constant angular separation, OS also rotates at the same angular velocity and always keeps the same length. By applying the construction used in Fig. 1/III:2 it can be made to draw the curve S. This can be demonstrated by carrying out the geometrical constructions described with reference to Figs. 1/III:2 and 1/III:3, and anyone with a reasonable degree of intuition should be convinced by the account given above. The rigid analytical proof is, however, given below because it not only establishes the formula for the addition of two sine waves of the same frequency and of different amplitude and relative phase, but also establishes the formula for the addition of two vectors with an angle between them. Although desirable, it is not essential to follow this proof at the present stage, but the meanings of equations (4) and (10), and the relation between them, should be understood.

3.2. Derivation of Formulae for the Addition of Sine Waves and Vectors. Addition of Sine Waves. Consider the general case where the phase difference between the waves A and B is ϕ instead of 120°.

The sum of the two sine waves A and B is

$$\begin{aligned} a+b &= \hat{a}\sin\omega t+\hat{b}\sin(\omega t+\phi) \\ &= \hat{a}\sin\omega t+\hat{b}\cos\phi\sin\omega t+\hat{b}\sin\phi\cos\omega t \\ &= (\hat{a}+\hat{b}\cos\phi)\sin\omega t+\hat{b}\sin\phi\cos\omega t \\ &= \sqrt{(\hat{a}+\hat{b}\cos\phi)^2+\hat{b}^2\sin^2\phi}\,\sin\left(\omega t+\tan^{-1}\frac{\hat{b}\sin\phi}{\hat{a}+\hat{b}\cos\phi}\right) \quad . \quad (3) \\ &= \sqrt{\hat{a}^2+\hat{b}^2+2\hat{a}\hat{b}\cos\phi}\,\sin\left(\omega t+\tan^{-1}\frac{\hat{b}\sin\phi}{\hat{a}+\hat{b}\cos\phi}\right) \quad . \quad (4) \end{aligned}$$

This is a sine wave of amplitude equal to the square root of the quantity under the root and leading on curve A by an angle

$$\tan^{-1}\frac{\hat{b}\sin\phi}{\hat{a}+\hat{b}\cos\phi}$$

The derivation of (3) from the preceding line depends on the following relation which is proved immediately below.

$$A\sin\omega t+B\cos\omega t = \sqrt{A^2+B^2}\sin\left(\omega t+\tan^{-1}\frac{B}{A}\right) \quad . \quad (5)$$

Proof.

$A \sin \omega t + B \cos \omega t$

$$= -j\frac{A}{2}e^{j\omega t} + j\frac{A}{2}e^{-j\omega t} + \frac{B}{2}e^{j\omega t} + \frac{B}{2}e^{-j\omega t}$$

$$= -j\tfrac{1}{2}[Ae^{j\omega t} - Ae^{-j\omega t} + jBe^{j\omega t} + jBe^{-j\omega t}]$$ (See equations (4) and (5)/III:5.)

$$= -j\tfrac{1}{2}\left[\sqrt{A^2+B^2}\,e^{j\omega t}\big/\tan^{-1}\frac{B}{A} - \sqrt{A^2+B^2}\,e^{-j\omega t}\big\backslash\tan^{-1}\frac{B}{A}\right]$$

Putting $\tan^{-1}\dfrac{B}{A} = \theta$ the expression becomes

$$-j\tfrac{1}{2}\sqrt{A^2+B^2}[e^{j\omega t}.e^{j\theta} - e^{-j\omega t}.e^{-j\theta}]$$

$$= \sqrt{A^2+B^2} \times -j\tfrac{1}{2}[e^{j(\omega t+\theta)} - e^{-j(\omega t+\theta)}]$$

$$= \sqrt{A^2+B^2}\sin\left(\omega t + \tan^{-1}\frac{B}{A}\right)$$

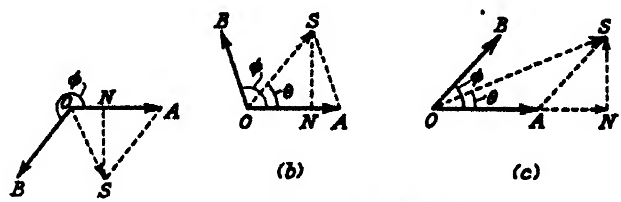


FIG. 2/III:3.—Addition of Two Vectors.

Addition of Vectors. Fig. 2 shows three configurations of two vectors OA and OB with their resultant OS and the construction line AS parallel to OB which determines the position of the point S. SN is the normal from S on to OA or OA produced. OA is the reference vector and is drawn horizontal; the angle ϕ between OA and OB is defined as the angle through which OB must be rotated in a positive (counter-clockwise) direction in order to reach its direction as shown from an initial position of coincidence with OA. In Fig. 1 (*a*) the angle ϕ is between 180° and 270°; in Fig. 1 (*b*) ϕ is between 90° and 180°; in Fig. 1 (*c*) ϕ is between 0° and 90°. SN is the normal from S on to OA or OA produced. Then in each configuration, from the definition of sine and cosine:

$$NS = AS\sin\phi = OB\sin\phi \quad . \quad . \quad . \quad (6)$$

$$AN = AS\cos\phi = OB\cos\phi \quad . \quad . \quad . \quad (7)$$

Then, considering the triangle OSN, and remembering the signs of the sine and cosine functions in each quadrant it is evident that:

$$OS = \sqrt{ON^2+NS^2} = \sqrt{(OA+AN)^2+NS^2}$$
$$= \sqrt{(OA+OB\ \cos\phi)^2+OB^2\ \sin^2\phi}$$
$$= \sqrt{OA^2+2OA.OB\ \cos\phi+OB^2\ \cos\phi+OB^2\ \sin^2\phi}$$

and, since $\cos^2\phi+\sin^2\phi = 1$

$$OS = \sqrt{OA^2+OB^2+2OA.AB\cos\phi} \quad . \quad . \quad (8)$$

Also

$$\tan\theta = \frac{NS}{ON} = \frac{NS}{OA+AN}$$

$$\therefore\quad \theta = \tan^{-1}\frac{NS}{OA+AN} = \tan^{-1}\frac{OB\sin\phi}{OA+OB\ \cos\phi}. \quad (9)$$

It is customary to use a single symbol to indicate the complete vector of magnitude OS and angle θ with regard to the reference vector OA. This symbol is sometimes written $\overrightarrow{OS}$, but in the discussion below single symbols will be written for such vectors: in the present case S will be used.

In this case

$$S = \sqrt{OA^2+OB^2+2OA.OB.\cos\phi}\ \underline{/\tan^{-1}\frac{OB\sin\phi}{OA+OB\cos\phi}} \quad . \quad (10)$$

This is a vector of magnitude OS, leading the reference vector by an angle θ.

Since $OA = \hat{a}$ and $OB = \hat{b}$

$$S = \sqrt{\hat{a}^2+\hat{b}^2+2\hat{a}\hat{b}\cos\phi}\ \underline{/\tan^{-1}\frac{\hat{b}\sin\phi}{\hat{a}+\hat{b}\cos\phi}} \quad . \quad (10a)$$
$$= OS\underline{/\theta}.$$

This vector corresponds to a sine wave of amplitude OS leading curve A by an angle θ. *It therefore describes the same sine wave as does equation* (4).

4. The Complex Plane.

This is a very simple device for representing vectors which express the magnitudes of impedance in A.C. circuits. In such circuits it is found that the current is not always in phase with the alternating voltage effective across the circuit. The quantity describing the ratio: alternating voltage/resulting current: is called the ***impedance***, see V:4, and is represented by a vector of magnitude equal to the voltage magnitude divided by the current magnitude, and having an angle equal to the amount by which the voltage leads on the current. An angle of negative lead is the same as an angle of lag;

an angle of negative lag is the same as an angle of lead. (For method of division of and by vectors see V:7.)

As explained, it is customary to use single letters to denote vectors, and when it is required to indicate the magnitude a vertical line is drawn each side of the letter. As an example $|Z|$ is the magnitude of the vector Z. The angle of the vector is shown separately. The symbol for a leading angle is $\angle$; the symbol for a lagging angle is $\overline{\diagdown}$.

It is therefore customary to write $Z = |Z|\underline{/\phi}$, which means that Z is a vector of magnitude $|Z|$ and angle ϕ.

If V is the vector describing a voltage applied across a circuit, and I is the vector describing the resultant current, V is the reference vector of magnitude $|V|$ and therefore of zero angle, while the angle of the current vector I is equal to the angle by which the current lags on the vector V, which is always drawn horizontally. It is then customary to write

$$Z = |Z|\underline{/\theta} \text{ say} = \frac{V}{I} = \frac{|V|\underline{/O}}{|I|\underline{/\phi}} = \left|\frac{V}{I}\right|\underline{/\phi} \quad . \quad . \quad (1)$$

where ϕ is the angle by which the current lags on the voltage. In this case $|Z| = \left|\frac{V}{I}\right|$ and $\theta = \phi$.

It is not usual to write $V\underline{/O}$, but merely to write V. If no angle sign appears it is understood that the angle of the vector is zero. As a matter of convention, note that

$$\underline{/\phi} = \overline{\diagdown -\phi} = \frac{1}{\overline{\diagdown\phi}} = \frac{1}{\underline{/-\phi}} \quad . \quad . \quad . \quad (2)$$

(See V:7 and III:6.)

In equation (1) the angle of the impedance $= \phi =$ the angle by which the current lags on the voltage. In other words, an impedance with a leading angle means that the current lags behind the voltage: the justification for the use of a leading angle is that the voltage leads on the current. In equation (1) the magnitude of the impedance $Z = |V|/|I|$.

Probably the simplest way of regarding an impedance is that it is the voltage per unit current, in which case, the voltage is given by

$$V = IZ = |IZ|\underline{/\phi} \quad . \quad . \quad . \quad . \quad (3)$$

where I is now the reference vector.

The main object of the above is to show that a necessity exists for having a method of representing impedances which are vectors

having an apparent absolute direction. This is done in the most obvious way.

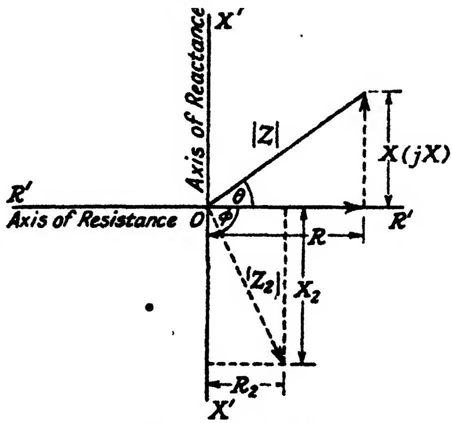


FIG. 1/III:4.—Impedance Vectors in Complex Plane.

Two axes are drawn at right angles as in Fig. 1, a horizontal axis $R'OR'$ and a vertical axis $X'OX'$. Zero angle vectors are then drawn along OR' and the direction of increase of angle is counter-clockwise. A vector having any angle is then drawn from the origin O, of length proportional to its magnitude on the scale of the diagram, making an angle with OR' equal to the angle of the vector.

In accordance with standard conventions, directions normal to $X'OX'$ are positive on the right of $X'OX'$ and negative on the left. Similarly directions normal to $R'OR'$ are positive, when above $R'OR'$, and negative when below.

Two vectors have been drawn on Fig. 1, which are intended to represent two impedances: $Z = |Z|\underline{/\theta}$ and $Z_2 = |Z_2|\overline{\diagdown}\phi$, one having a positive angle and the other having a negative angle.

It will be noted that the vector Z is the sum of two other vectors: a vector of magnitude R and zero angle, and a vector of magnitude X which leads on the vector R by 90°, and is therefore parallel to the axis $X'OX'$. Since, as is shown in V:1 to 4, impedances are usually constituted by circuit elements which contribute component impedances respectively parallel to OR' and $X'OX'$ (i.e. of the form of R and X respectively) it is convenient to have simple means of representing and classifying these component impedances. Impedances similar to R are called resistances, and are often, but not always, represented by the symbol R; impedances similar to X are called reactances, and are usually represented by the

symbol X with the letter j in front to indicate that the impedance is to be drawn with a lead of 90°. In other words, $X/\underline{90°} = jX$. It is therefore possible to write the equation

$$Z = R+jX = Z\underline{/\theta} \quad . \quad . \quad . \quad . \quad (4)$$

Similarly

$$Z_2 = R_2 - jX_2 = Z\overline{\setminus\phi} \quad . \quad . \quad . \quad (4a)$$

The impedance Z is then said to have a component of positive reactance, while the impedance Z_2 is said to have a component of negative reactance. A reactance may be either positive or negative. A positive reactance corresponds to a leading angle (and a lagging angle of current behind the voltage across the impedance); a negative reactance corresponds to a lagging angle (and a leading angle of current relative to the voltage across the impedance).

Note that $R = Z\cos\theta$, $X = Z\sin\theta$, $\theta = \tan^{-1}\frac{X}{R}$. (5)

$$R_2 = Z_2\cos\phi,\ X_2 = Z_2\sin\phi,\ \phi = \tan^{-1}\frac{X_2}{R_2} \qquad (5a)$$

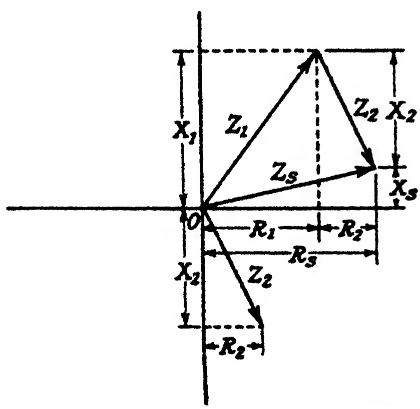


FIG. 2/III:4.—Addition of Impedances.

Impedances may be added in the complex plane by the ordinary rule for the addition of vectors: any number may be drawn in tip to tail and the line from the tail of the first vector to the tip of the last is the resultant vector.

It follows that in Fig. 2 $Z_s = Z_1+Z_2 = |Z_1|\underline{/\theta_1}+Z_2\underline{/\theta_2}$

$$\therefore \quad Z_s = |Z_1|\cos\theta_1+|Z_2|\cos\theta_2+j[|Z_1|\sin\theta_1+|Z_2|\sin\theta_2]$$

$$= R_s+jX_s \text{ say.}$$

$$= \sqrt{R_s^2+X_s^2}\ \underline{/\tan^{-1}\frac{X_s}{R_s}}$$

5. Representation of the Angle of a Vector by $e^{j\theta}$

For reasons which are explained below, but which may be ignored for most practical purposes, it is sometimes customary to represent a vector of unit length and angle θ by $e^{j\theta}$. A vector $Z = |Z|\underline{/\theta}$ may therefore be written $|Z|e^{j\theta}$. In other words:

$$|Z|\underline{/\theta} = |Z|e^{j\theta}$$

and

$$1\underline{/\theta} = e^{j\theta} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

It follows that whenever $e^{j\theta}$ appears, it may be replaced by $1\underline{/\theta}$.

(It is important to notice that in practice it is impossible at all times to preserve the convention that Z means $|Z|\underline{/\theta}$. Sometimes a vector is written $Z\underline{/\theta}$ in which case Z has the same meaning as $|Z|$ in the rigid convention.)

Consider a vector $Z = |Z|\underline{/\theta} = |Z|e^{j\theta} = R+jX$

Then $\quad R = |Z| \cos\theta \quad$ and $\quad X = |Z| \sin\theta$

$\therefore \quad |Z|e^{j\theta} = |Z| \cos\theta + j|Z| \sin\theta$

$\therefore \quad e^{j\theta} = \cos\theta + j \sin\theta \quad . \quad . \quad . \quad . \quad (2)$

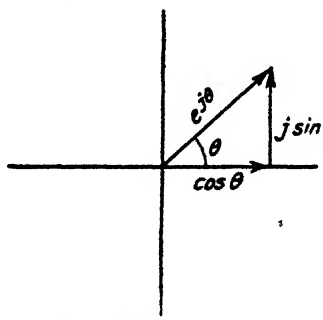


FIG. 1/III:5.—Unit Vector $e^{j\theta}$ as the Sum of $\cos\theta$ plus $j \sin\theta$.

If it is accepted that $e^{j\theta}$ represents unit vector at angle θ, equation (2) becomes a truism as can be seen by reference to Fig. 1 where the vector $e^{j\theta}$ is evidently the sum of the vector $\cos\theta$ and $j \sin\theta$.

Justification of Use of $e^{j\theta}$. The conventional justification given below is rather unconvincing to a beginner because it makes use of expansions which are derived by means of calculus, and with which a beginner cannot be expected to be familiar. It consists in establishing the truth of equation (2) without assuming that $e^{j\theta} = 1\underline{/\theta}$, but assuming the validity of the expansions used; apparently a far greater assumption.

It is known that

$$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}+\frac{x^6}{6!}+\ldots$$

$$\therefore e^{j\theta} = 1+j\theta-\frac{\theta^2}{2!}-j\frac{\theta^3}{3!}+\frac{\theta^4}{4!}+j\frac{\theta^5}{5!}-\frac{\theta^6}{6!}-$$

It is also known that

$$\cos\theta = 1-\frac{\theta^2}{2!}+\frac{\theta^4}{4!}-\frac{\theta^6}{6!}+\ldots$$

$$\sin\theta \quad \theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}-$$

$$\therefore \quad \cos\theta+j\sin\theta = 1+j\theta-\frac{\theta^2}{2!}-j\frac{\theta^3}{3!}+\frac{\theta^4}{4!}+j\frac{\theta^5}{5!}-\frac{\theta^6}{6!}\ldots$$

$$= e^{j\theta} \text{ which is equation (2).}$$

Similarly

$$\cos\theta - j\sin\theta = e^{-j\theta} \quad . \quad . \quad . \quad (3)$$

Adding (2) and (3) and dividing by 2:

$$\cos\theta = \tfrac{1}{2}(e^{j\theta}+e^{-j\theta}) \quad . \quad . \quad . \quad . \quad (4)$$

Subtracting (3) from (2) and dividing by 2:

$$\sin\theta = \frac{1}{j2}(e^{j\theta}-e^{-j\theta}) = -j\frac{1}{2}(e^{j\theta}-{}^{-j\theta}) \quad . \quad . \quad (5)$$

(For meaning and use of j, see V:2.)

6. Negative Vectors and Negative Angles.

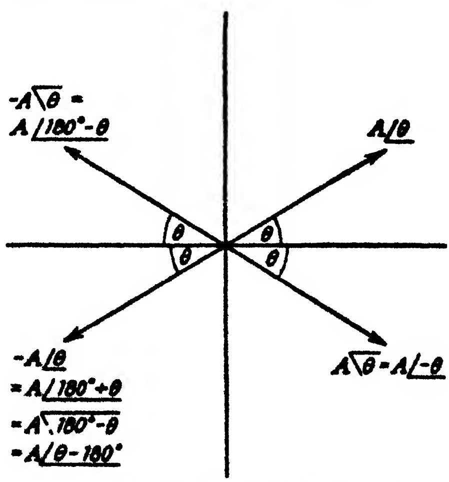


FIG. 1/III:6.—Distinction between Vectors with Negative Angles and Negative Vectors.

A minus sign before an angle changes it from a leading angle to a lagging angle and vice versa.

$$A\underline{/-\theta} = A\overline{\diagdown\theta}$$
$$A\overline{\diagdown-\theta} = A\underline{/\theta}$$

A minus sign before a vector reverses its direction. The direction of a vector may also be changed by adding or subtracting 180° from its angle.

Reference to Fig. 1 will make these conventions clear.

From this it will be evident that $A\underline{/-\theta}$ is *not* equal to $-A\underline{/\theta}$.

7. Cosine Waves.

It requires no proof to demonstrate that all the arguments in this chapter might equally well have been developed in terms of cosine waves instead of in terms of sine waves.

In practice cosine waves are often used and there is no fundamental reason why sine waves should be given preference. Sine waves may be preferred because they pass through the zero of the time scale in a direction of increasing positive amplitude; but this is normally of less practical value than the fact that sine is shorter to say than cosine.

Speaking loosely, the only difference between a sine and a cosine is that the cosine wave leads the sine wave by 90°. In specific terms the cosine of a quantity varying linearly with time leads 90° on the sine of the same quantity.

8. Soltution of Triangles.

Since addition of vectors often involves the solution of triangles, the main formula for solution of triangles are given below.

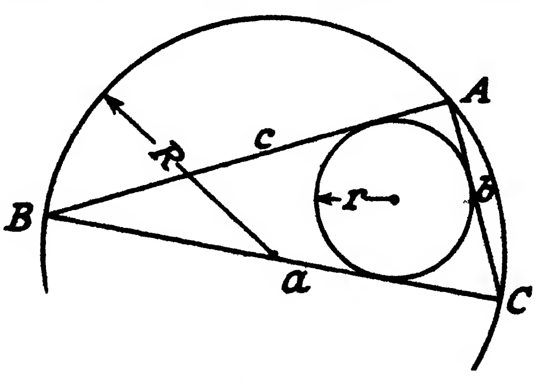


FIG. 1/III:8.—Triangle with Inscribed and Escribed Circles.

$$s = \tfrac{1}{2}(a+b+c) \qquad (1)$$

$$\text{Area} = \tfrac{1}{2}ab \sin C = \sqrt{s(s-a)(s-b)(s-c)} \qquad (2)$$

$$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \qquad (3)$$

$$\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}} \qquad (4)$$

$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}} \qquad (5)$$

$$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2} \qquad (6)$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R \qquad (7)$$

$$c^2 = a^2+b^2 - 2ab\cos C \qquad (8)$$

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}} \qquad (9)$$

99. Numerical Examples.

1. Look up in tables of trigonometrical functions the values of sin 0°, sin 30°, sin 60°, sin 90°, cos 0°, cos 30°, cos 60°, cos 90°, tan 0°, tan 45°, tan 90°. These values should be memorized.

2. What is the value of $\sin^{-1} 0$, $\sin^{-1} 0{\cdot}5$, $\sin^{-1} 0{\cdot}866$, $\sin^{-1} 1{\cdot}0$, $\cos^{-1} 0$, $\cos^{-1} 0{\cdot}5$, $\cos^{-1} 0{\cdot}866$, $\cos^{-1} 1{\cdot}0$, $\tan^{-1} 0$, $\tan^{-1} 1{\cdot}0$, $\tan^{-1}\infty$?

3. What is the value of sin 120°, sin 150°, sin 180°, sin 210°, sin 240°, sin 270°, sin 300°, sin 330°, sin 360° ?

[A. 0·866, 0·5, 0, − 0·5, − 0·866, − 1, − 0·866, − 0·5, 0.]

4. What is the value of cos 120°, cos 150°, cos 180°, cos 210°, cos 240°, cos 270°, cos 300°, cos 330°, cos 360° ?

[A. − 0·5, − 0·866, − 1, − 0·866, − 0·5, 0, 0·5, 0·866, 1·0.]

5. What is the value of tan 135°, tan 180°, tan 225°, tan 270°, tan 315°, tan 360° ?

[A. 1, 0, − 1, − infinity, − 1, 0.]

6. What is the value θ of $\sin^{-1} 0{\cdot}5$; $\sin^{-1} - 0{\cdot}5$; $\cos^{-1} 0{\cdot}5$; $\cos^{-1} - 0{\cdot}5$; $\tan^{-1} 1{\cdot}0$; $\tan^{-1} - 1{\cdot}0$?

[A. 30° or 150°; 210° or 330° or − 30°; 60° or 300°, 120° or 240°, 45° or 225°, 135° or 315° or − 45°; in addition, corresponding to each of the above angles θ there are an infinite number of angles equal to the above inverse functions given by $\theta \pm n360°$ where n is any integer.]

7. What is the value of sin ($\tan^{-1}$ 1·0), tan ($\sin^{-1}$ 0·5), cos ($\sin^{-1}$ 0·866), $\sin^{-1}$ (cos 30°), $\cos^{-1}$ (sin 60°), $\tan^{-1}$ (cos 0°) ?

[A. 0·707, 0·577, 0·5, 60°, 30°, 45°.]

8. Plot one cycle of the sine wave $a = A \sin \omega t$ when $A = 10$ and $f = 1{,}000$, choosing appropriate scales of amplitude and time. (Note: Plot against degrees and then convert scale of degrees to time.)

9. Plot on the diagram of example **8** the cosine wave $b = B \sin \omega t$ where $B = 10$ and $f = 1{,}000$. What is the relation of this curve to the curve of example **8** ?

[A. The cosine curve is a sine curve leading the sine curve of example **8** by 90°.]

10. Draw a sine wave of unit amplitude and frequency one cycle per second. Draw tangents to the curve at the points where it crosses the axis and measure the tangent of the angle of slope as the ratio of the vertical intercept measured to the scale of amplitude (of the sine wave) used, divided by the horizontal intercept measured in seconds. This ratio will be found to be equal to 2π: the rate of change of amplitude is 2π. For 2 cycles per second the rate of change is $2\pi \times 2$ and for f cycles per second the rate of change is $2\pi f = \omega$. For an amplitude of A, instead of unity, the rate of change is $A\omega$.

Repeat for any other point on the curve and show that the rate of change of amplitude is always equal to $A\omega \cos \omega t$.

11. Plot the curve $a = \cos 2\pi t$ and the curve $a = \sin (2\pi t + 90°)$ on the graph of example **10.** (*a*) What is the relation between the two curves in this example ? (*b*) What is their relation to the curve in example **8** ?

[A. (*a*) The two curves are identical. (*b*) The curves are sine waves of unit amplitude and frequency one cycle per second and lead 90° on the wave of example **8.**]

12. What is the difference between a sine wave and a cosine wave of the same amplitude and frequency ?

[A. The cosine wave is identical in form with the sine wave but leads on it in phase by 90°.]

13. Is there any advantage in using sine waves rather than cosine waves ?

[A. None.]

14. Express the following cosine waves as sine waves :
(*a*) $A \cos \omega t$; (*b*) $-A \cos \omega t$; (*c*) $A \cos (\omega t + 90°)$; (*d*) $A \cos (\omega t - 90°)$; (*e*) $-A \cos (\omega t + 90°)$; (*f*) $-A \cos (\omega t - 90°)$; (*g*) $A \cos (\omega t + \phi)$; (*h*) $A \cos (\omega t + 90° - \phi)$; (*j*) $-A \cos (\omega t - 90° + \phi)$.

[A. Draw the corresponding vectors, when the following answers appear : (*a*) $A \sin (\omega t + 90°)$; (*b*) $A \sin (\omega t - 90°)$; (*c*) $-A \sin \omega t$; (*d*) $A \sin \omega t$; (*e*) $A \sin \omega t$; (*f*) $-A \sin \omega t$; (*g*) $A \sin (\omega t + 90° + \phi)$; (*h*) $-A \sin (\omega t - \phi)$; (*j*) $-A \sin (\omega t + \phi)$.]

15. Express the following sine waves as cosine waves: (*a*) $A \sin \omega t$; (*b*) $-A \sin \omega t$; (*c*) $A \sin (\omega t+90^\circ)$; (*d*) $A \sin (\omega t - 90^\circ)$; (*e*) $-A \sin (\omega t+90^\circ)$; (*f*) $-A \sin (\omega t - 90^\circ)$; (*g*) $A \sin (\omega t+\phi)$; (*h*) $A \sin (\omega t+90^\circ - \phi)$; (*j*) $-A \sin (\omega t - 90^\circ+\phi)$.

[A. (*a*) $A \cos (\omega t - 90^\circ)$; (*b*) $A \cos (\omega t+90^\circ)$; (*c*) $A \cos \omega t$; (*d*) $-A \cos \omega t$; (*e*) $-A \cos \omega t$; (*f*) $A \cos \omega t$; (*g*) $A \cos (\omega t - 90^\circ+\phi)$; (*h*) $A \cos (\omega t - \phi)$; (*j*) $A \cos (\omega t - \phi)$.]

16. Express the sum of the two vectors $|Z_1|\underline{/\phi}$ and $|Z_2|\underline{/\theta}$ in the form $A+jB$.

[A. $|Z_1| \cos \phi+|Z_2| \cos \theta+j|Z_1| \sin \phi+j|Z_2| \sin \theta$.]

17. Express the sum of the two vectors R_1+jX_1 and R_2+jX_2 in the form $Z\underline{/\phi}$.

$$\left[\text{A. } \sqrt{(R_1+R_2)^2+(X_1+X_2)^2}\ \underline{\Big/\tan^{-1}\frac{X_1+X_2}{R_1+R_2}}.\right]$$

18. Add together the two vectors

$10\underline{/30^\circ}$	and	$10\underline{/30^\circ}$
$10\underline{/30^\circ}$	and	$10\overline{\backslash 30^\circ}$
$10\underline{/30^\circ}$	and	$-10\underline{/30^\circ}$
$10\underline{/30^\circ}$	and	$-10\overline{\backslash 30^\circ}$
$10\underline{/120^\circ}$	and	$10\overline{\backslash 30^\circ}$
$10\underline{/210^\circ}$	and	$-10\underline{/30^\circ}$

[A. $20\underline{/30^\circ}$; $17{\cdot}32\underline{/0^\circ}$; 0; $10\underline{/90^\circ}$; $20 \sin 15^\circ\underline{/45^\circ} = 5{\cdot}18\underline{/45^\circ}$; $20\underline{/210^\circ} = -20\underline{/30^\circ}$.]

19. Find the resultant of the two sine waves:

$A \sin \omega t$ and $A \sin (\omega t+90^\circ)$
$A \sin \omega t$ and $A \sin (\omega t - 120^\circ)$.

[A. $\sqrt{2} \times A \sin (\omega t+45^\circ)$; $A \sin (\omega t - 60^\circ)$.]

20. Add together the two vectors $10\underline{/30^\circ}+20\underline{/60^\circ}$ (*a*) to obtain the answer in the form $Z\underline{/\theta}$, (*b*) to obtain the answer in the form $R+jX$.

[A. (*a*) $Z = \sqrt{10^2+20^2+2 \times 10 \times 20 \cos (60 - 30^\circ)} = 29{\cdot}1$

$$\theta = 30^\circ+\tan^{-1}\frac{20 \sin (60^\circ - 30^\circ)}{10+20 \cos (60^\circ - 30^\circ)}$$

$$= 30^\circ+\tan^{-1}\frac{10}{27{\cdot}32} = 30^\circ+20^\circ\ 8' = 50^\circ\ 8'.$$

(*b*) $R = 10 \cos 30^\circ+20 \cos 60^\circ = 8{\cdot}66+10 = 18{\cdot}66$
$X = 10 \sin 30^\circ+20 \sin 60^\circ = 5+17{\cdot}32 = 22{\cdot}32$.

Check: $\text{Tan}^{-1}\dfrac{22{\cdot}32}{18{\cdot}66} = 50^\circ\ 7'$.

$\sqrt{R^2+X^2} = \sqrt{18{\cdot}66^2+22{\cdot}32^2} = 29{\cdot}05$ (Slide rule).]

CHAPTER IV

RELATIONS BETWEEN STEADY VOLTAGES AND DIRECT CURRENTS

1. Resistance and Conductivity.

IF a steady voltage E is applied across a straight piece of wire (i.e. if a potential difference E volts is applied between its ends), a *direct current* flows of magnitude depending on R the *resistance* of the wire and the magnitude of the voltage applied.

Resistance is measured in *ohms*, the value of the ohm being so chosen that in *linear* circuits *current in amperes* = *voltage* (*in volts*) divided by *resistance in ohms*.

$$\text{or } I = \frac{E}{R} \left(\text{and } R = \frac{E}{I}\right) \quad . \quad . \quad . \quad (1)$$

where I is the symbol for current.

A linear circuit is one in which the magnitude of the current flowing is directly proportional to the voltage applied; this is the normal type of circuit.

Conductance (G) is measured in *mhos*, the value of the mho being chosen so that in linear circuits

Current in amperes = *voltage* (*in volts*) × conductance (in mhos)

$$\text{or} \qquad I = EG \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Note that from equations (1) and (2) $G = \frac{I}{E} = \frac{1}{R}$. Hence conductivity is the reciprocal of resistance, and vice versa.

2. Internal Electromotive Force in Generators.

It will be evident that since voltage causes an electric current to flow round a circuit it may be called an electromotive force. This term, while used generally in this way, has particular reference to the electromotive force *internal* to generators. When the generator is on open circuit, that is when no circuit is connected to its output terminals, the voltage appearing across its output terminals is equal to its internal e.m.f. As soon as the generator delivers current to an external circuit, part of the e.m.f. is expended in driving the current through the generator itself. If, therefore, a circuit element containing an internal e.m.f. can be open circuited, its e.m.f. can be measured as the potential difference across its terminals.

3. Resistances in Series.

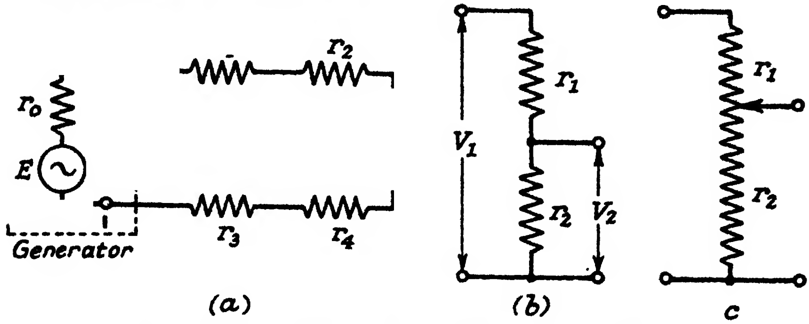


FIG. 1/IV:3.—Arrangements of Resistances in Series.

Fig. 1 (*a*) shows four elements of resistance having respective resistance values r_1, r_2, r_3, r_4 ohms connected to the terminals 1, 1 of a direct-current generator. In this arrangement the resistances are said to be connected in series. r_0 is the internal resistance of the generator and E is its internal e.m.f.

The current in the circuit is

$$I = \frac{E}{r_0+r_1+r_2+r_3+r_4} \qquad (1)$$

which is equivalent to saying that the total resistance constituted by a number of resistances in series is the sum of these resistances.

The voltage required to drive a current I through a resistance of magnitude r_0 ohms is

$$V = Ir_0$$

so that the voltage appearing across terminals 1, 1 is

$$V_g = E - V = E - Ir_0 \quad . \quad . \quad . \quad (2)$$

V_g is then said to be the terminal voltage of the generator while E is its internal e.m.f.

It is to be noted that if $r_1+r_2+r_3+r_4$ = infinity, then $V = E$; in other words the terminal voltage of a generator on open circuit is equal to its internal e.m.f.

The voltage $V = Ir_0$ is called the internal drop (of voltage) in the generator. The voltage across any resistance such as r_2 is

$$Ir_2 = \frac{r_2}{r_0+r_1+r_2+r_3+r_4}\,E = \frac{r_2}{r_1+r_2+r_3+r_4}\,V_g$$

This follows from equations (1) and (2).

It is evidently permissible to consider the voltage relations in

any part of such a circuit regardless of the remainder. Referring to Fig. 1 (*b*), if V_1 is the voltage across r_1 and r_2 in series, and V_2 is the voltage across r_2, it may be shown as above that

$$V_2 = \frac{r_2}{r_1+r_2}V_1 \quad . \quad . \quad . \quad . \quad (3)$$

This solution holds when two resistances are in series in a circuit, whatever their relation to the remainder of the circuit, provided that the resistances are linear, and that no internal e.m.f.s are induced in the resistances from sources not shown. It is evident, therefore, that two resistances connected in this way provide a means of producing any required voltage from a source of voltage which is too high. Such an arrangement of resistances is called a *potentiometer*. Potentiometers may be fixed as shown in Fig. 1 (*b*) or variable as shown in Fig. 1 (*c*), where a contact slides along a single resistance, the two halves of which then constitute variable resistances.

4. Resistances in Parallel.

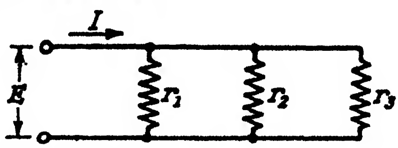


FIG. 1/IV:4.—Resistances in Parallel.

Fig. 1 shows a voltage E applied across a number of resistances r_1, r_2, r_3 in parallel. Assume the conductivities of these resistances to be respectively g_1, g_2, g_3, etc.

Then from (2)/IV:1 the total current flowing into terminals 1, 1 is

$$I = E(g_1+g_2+g_3+ \ . \ . \ .) = E\left(\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}+ \ . \ . \ .\right)$$

$$\therefore \quad \frac{E}{I} = \frac{1}{\dfrac{1}{r_1}+\dfrac{1}{r_2}+\dfrac{1}{r_3} \ . \ . \ .}.$$

But from (1)/IV:1 $\dfrac{E}{I}$ is equal to the resistance appearing at terminals 1, 1, so that the effective resistance of a number of resistances in parallel is

$$R = \frac{1}{\dfrac{1}{r_1}+\dfrac{1}{r_2}+\dfrac{1}{r_3}+} \quad . \quad . \quad . \quad (1)$$

Note that if there are only two resistances in parallel

$$R = \frac{1}{\frac{1}{r_1}+\frac{1}{r_2}} = \frac{r_1 r_2}{r_1+r_2} \quad . \quad . \qquad (2)$$

5. Back Electromotive Force.

In the circuit of Fig. 1(*a*)/IV:3 each of the voltages created across the resistances r_0, r_1, r_2, r_3, r_4, is said to constitute a back e.m.f. since it opposes the flow of current. This is a definition which appears to have little justification, if beginners regard back e.m.f.s as back voltages while remembering they are called back e.m.f.s they may avoid some confusion. The total of the back e.m.f.s is equal to the applied e.m.f. If these back e.m.f.s are e_0, e_1, e_2, e_3, e_4, then

$$e_0+e_1+e_2+e_3+e_4 = E$$

It is, however, more usual to consider that when the numerical values of e_0, e_1, e_2, etc., are inserted in this equation, they will be prefixed with a minus sign, indicating that they are back e.m.f.s, in which case the equation must be modified to read

$$E+e_0+e_1+e_2+e_3+e_4 = 0 \quad . \quad . \quad . \qquad (1)$$

Hence the general rule :

The sum of the e.m.f.s round any circuit is zero.

6. Power in Direct-Current Circuits.

The *power* supplied to a circuit is measured in *watts*. One watt is supplied to a circuit when a steady voltage of 1 volt drives a direct current of 1 ampere through the circuit. Hence the power supplied is given by

$$P = EI \text{ watts} \quad . \quad . \quad . \quad . \qquad (1)$$

where E is in volts and I in amperes.

Since when a voltage is applied across a resistance R, the current $= I = \frac{E}{R}$, $\therefore E = IR$.

Substituting these relations in (1)

$$P = I^2R = \frac{E^2}{R} \text{ watts} \quad . \quad . \quad . \qquad (2)$$

6.1. Energy stored in a Condenser. Since the definition of a Farad is the capacity which will be charged with one coulomb when a p.d. of one volt is maintained across the condenser, the quantity of electricity stored in a condenser is given by

$$Q = EC \text{ coulombs} \qquad . \quad . \quad . \quad (3)$$

where E is the voltage across the condenser and C is the capacity of the condenser in Farads.

If a condenser is charged by a constant current I amps. flowing into the condenser, starting from zero charge, the charge on the condenser is

$$Q = It = EC \text{ coulombs},$$

where t is the time in seconds during which the condenser is charged and E is the final voltage to which the condenser is charged.

Hence $E = \dfrac{It}{C}$ or $It = EC$ and the mean voltage averaged over the charging time is $E = \frac{1}{2}E$. The total energy supplied to the condenser is therefore

$$W_c = EIt = \tfrac{1}{2}EEC = \tfrac{1}{2}E^2C \text{ watt seconds or joules} \quad . \quad (4)$$

6.2. Energy stored in an Inductance. A definition of a Henry is that it is the inductance in which, when one volt is maintained across it, the current increases at one ampere per second. The rate of increase of current in an inductance of L Henrys with a steady voltage E maintained across it is therefore $\dfrac{E}{L}$ and the instantaneous value of current at any time t seconds after the application of the voltage is

$$i = \frac{E}{L}t \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Hence if at this time the current has risen to a value I, the mean current is $I = \frac{1}{2}I$ where $I = \dfrac{E}{L}t$ so that $Et = LI$.

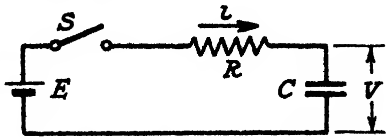
The rate at which energy is flowing into the inductance is Ei, and assuming the initial current to be zero, the energy stored in the inductance is given by the total energy which has flowed into the inductance which is

$$W_L = IEt = \tfrac{1}{2}ILI = \tfrac{1}{2}LI^2 \text{ watt seconds or joules} \quad . \quad (6)$$

where L is in Henrys and I is in amperes.

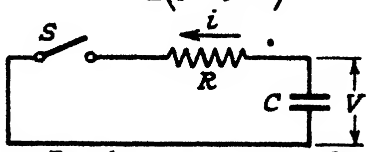
7. Time Constants

7.1. Resistance Capacity Circuits.

(a)


$$i = \frac{E}{R}\epsilon^{-\frac{1}{RC}t}$$

$$V = E\left(1 - \epsilon^{-\frac{1}{RC}t}\right)$$

(b)


$$i = \frac{E_0}{R}\epsilon^{-\frac{1}{RC}t}$$ where $E_0 = \frac{Q_0}{C}$ and

Q_0 = change on condenser at time $t = 0$.

$$V = E_0\epsilon^{-\frac{1}{RC}t}$$

FIG. 1/IV:7.—Charge and Discharge Currents and Voltages in Condenser-Resistance Circuit when Switch S is closed at time $t = 0$.

In the equations below C is in Farads, L in Henrys, R in ohms, and t is time in seconds.

If a battery of internal e.m.f. E is connected so as to charge a condenser of capacity C Farads through a resistance R ohms, as shown in Fig. 1 (*a*), the switch S being closed at time $t = 0$, the instantaneous value of charging current at any time t seconds after the closing of the switch S is given by

$$i = \frac{E}{R}\epsilon^{-\frac{1}{RC}t} \qquad (1)$$

The instantaneous voltage across the condenser at any time t is

$$v = E\left(1 - \epsilon^{-\frac{1}{RC}t}\right) \quad . \quad . \quad . \quad (2)$$

In these relations and the relations below, ϵ is the base of Napierian logarithms = 2·71828 . . .

Similarly, with reference to Fig. 1 (*b*), if at time $t = 0$ a resistance R is shunted across a condenser C carrying a charge Q_0 coulombs, and hence charged to a voltage $E_0 = \frac{Q_0}{C}$, the instantaneous dis-

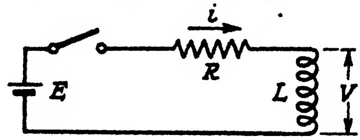
charging current and condenser voltage are given respectively by

$$i = \frac{E_0}{R}e^{-\frac{1}{RC}t} \quad \text{and} \quad v = E_0e^{-\frac{1}{RC}t} \qquad . \quad . \quad (3)$$

where t is the time after the closing of the switch in seconds.

7.2. Resistance Inductance Circuits.

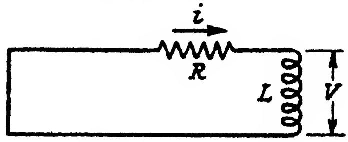
(a)



$$i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$V = Ee^{-\frac{R}{L}t}$$

(b)



$$i = I_0e^{-\frac{R}{L}t}$$ where I_0 = current at time $t = 0$.

$$V = I_0Re^{-\frac{R}{L}t}$$

FIG. 2/IV:7.—Charge and Discharge Currents and Voltages in Inductance-Resistance Circuit.

The instantaneous current through and voltage across the inductance in circuit in Fig. 2 (*a*) at time t seconds after the closing of the switch are

$$i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) \quad \text{and} \quad v = Ee^{-\frac{R}{L}t} \qquad . \quad . \quad (4)$$

Similarly if a current I_0 exists in the circuit of Fig. 2 (*b*) at time $t = 0$, the currents and voltages at times t seconds later are

$$i = I_0e^{-\frac{R}{L}t} \quad \text{and} \quad v = I_0Re^{-\frac{R}{L}t} \qquad . \quad . \quad (5)$$

Remember. In equations (1) to (5) L is in Henrys, C in Farads, R in ohms and t is time in seconds.

In this case there is a voltage v across the inductance and a voltage $-iR$ across the resistance (depending on the circuits taking off the voltage v it may, however, be convenient to reverse the sign conventions).

The quantities RC and L/R are called the *time constants* of the circuits of Figs. 1 and 2 because they determine the time required for the quantities $e^{-\frac{1}{RC}t}$ and $e^{-\frac{1}{RL}t}$ to reach the value $1/e$. For

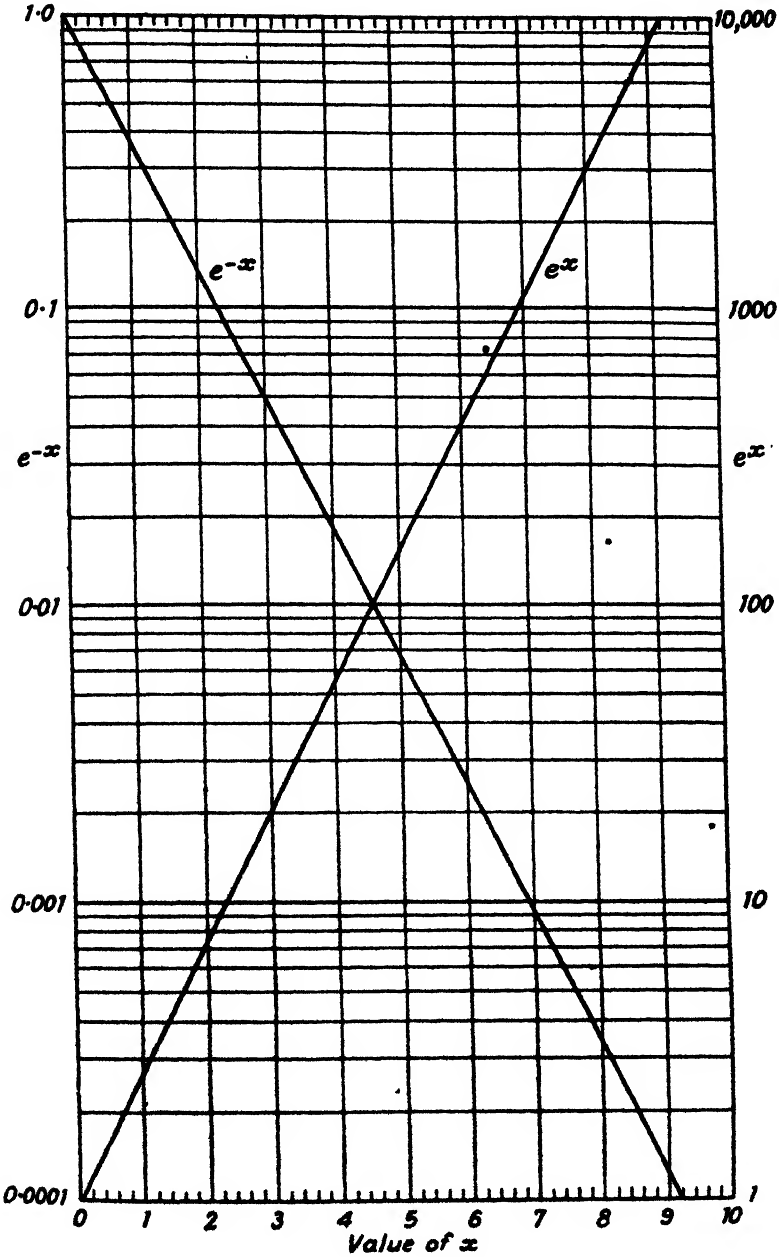


FIG. 3/IV:7.—Value of ϵ^{-x} and ϵ^{x}.

instance, when $t = RC$, $e^{-\frac{1}{RC}t} = e^{-\frac{1}{RC}RC} = e^{-1} = \frac{1}{e}$. The time constants of Fig. 1 are the times required for the charging currents and discharging currents respectively to fall to $\frac{1}{e}$ of their initial values. The time constants of the circuits of Fig. 2 are the times required for the current through and the voltages across the inductance and resistance to fall to $\frac{1}{e}$ of their initial values.

Example 1. If in the circuits of Fig. 1 $R = 1\ \text{M}\Omega$ and $C = 1\ \mu\text{F}$,
$RC = 10^6 \times 10^{-6} = 1$ second.

Example 2. If in the circuits of Fig. 2 $L = 10$ Henrys and $L = 1{,}000\ \Omega$,

$$\frac{L}{R} = \frac{10}{1{,}000} = 0{\cdot}01 \text{ second.}$$

If the time constant of a circuit is T $\left(\text{i.e. if } T = RC \text{ or } \frac{L}{R}\right)$ the value of $e^{-\frac{t}{T}} = e^{-\frac{1}{RC}t}$ or $e^{-\frac{R}{L}t}$ may be read off from Fig. 3 by entering the value of $\frac{t}{T}$ on the axis of x.

99. Numerical Examples.

1. What are the respective conductivities of resistances 1, 10, 1,000 ohms; 1, 10, 100 megohms?

[A. 1, 0·1, 0·001 mhos; 1, 0·1, 0·01 micromhos.]

2. A generator of internal resistance 100 ohms and internal e.m.f. 12,000 volts is connected to a load of 500 ohms. What is the current and what is the voltage across the load?

[A. 20 amperes, 10,000 volts.]

3. A generator with an open circuit voltage of 26 volts delivers only 24 volts when connected to a load of 12 ohms. What is the internal resistance of the generator?

[A. Voltage across load $= \frac{24}{26} \times$ internal e.m.f.

$\therefore$ external resistance $= \frac{24}{26} \times$ total resistance

$\therefore$ total resistance $= \frac{26}{24} \times 12 = 13$.

Internal resistance $= 13 - 12 = 1$ ohm.]

4. A generator is supplying 10 amps. at 25 volts to a load

through a regulating resistance, and it is observed that an increase of 2 amperes consequent on the adding of a second load in parallel drops the volts to 24 volts. How much resistance must be taken out of the regulating resistance to restore the voltage to 25?

[A. The original load resistance is $\frac{25}{10} = 2\cdot5$ ohms; the new total load resistance is $\frac{24}{12} = 2$ ohms. The currents are inversely proportional to the total resistance; hence if R be the resistance of regulating resistance plus generator

$$\frac{10}{12} = \frac{R+2}{R+2\cdot5} \quad \therefore R = 0\cdot5 \text{ ohms.}$$

The internal e.m.f. of the generator $= 10(0\cdot5+2\cdot5) = 30$ volts. If R_2 is the new value of resistance provided by generator plus regulating resistance in order to give 25 volts, then

$$\frac{2}{R_2+2} \times 30 = 25. \quad R_2 = 0\cdot4 \text{ ohms.}$$

Hence $0\cdot5 - 0\cdot4 = 0\cdot1$ ohms must be removed from the regulating resistance.]

5. What is the resistance presented by three resistances in parallel of value 1,000, 250 and 200 ohms?

[A. 100 ohms.]

6. What is the resistance of 1 MΩ and 1 Ω in parallel correct to four significant figures?

[A. 1·000 Ω.]

7. Design a fixed potentiometer to produce a voltage of 100 volts from a battery of 300 volts and to take a current drain of 10 mA.

[A. Total resistance of potentiometer $= \frac{300}{10 \times 10^{-3}} = 30{,}000\ \Omega$ and should consist of a 20,000 Ω resistance in series with a 10,000 Ω resistance. 100 volts can then be tapped off across the 10,000 Ω resistance.]

8. A battery of 100 volts is connected through one megohm to a condenser of one microfarad as in Fig. 1(*a*)/IV:7. What will be the voltage across the condenser after 0·5, 1·0, 2·0 and 5·0 seconds?

[A. $RC = 1$ second $= T$

$$\therefore \qquad \frac{t}{T} = 0\cdot5,\ 1\cdot0,\ 2\cdot0 \text{ and } 5\cdot0.$$

From Fig. 3/IV:7, $e^{-\frac{t}{T}} = 0\cdot6,\ 0\cdot37,\ 0\cdot13,\ 0\cdot0066.$

Hence $v = 100\left(1 - e^{-\frac{t}{T}}\right) = 40,\ 63,\ 87$ and $99\cdot34$ volts.]

CHAPTER V

RELATIONS BETWEEN ALTERNATING VOLTAGES AND CURRENTS

THESE are formulated in terms of sinusoidal voltages and currents.

Conventions and Symbols.

Quantity	*Symbol*	*Unit*
Instantaneous voltage	e	Volt
Peak voltage	$\hat{e}$	"
R.M.S. voltage and vector voltage	E	"
Instantaneous current	i	Ampere
Peak current	$\hat{\imath}$	"
R.M.S. current and vector current	I	"
Resistance	R	Ohm
Inductance	L	Henry
Capacity	C	Farad

1. Relation between Voltage across a Resistance, and Current.

If a voltage $\hat{e}\sin\omega t$ is applied across a resistance of magnitude R ohms a current flows of magnitude

$$i = \frac{\hat{e}}{R}\sin\omega t$$

The instantaneous values of the current through the resistance and the voltage across the resistance rise and fall simultaneously, in other words, the current and voltage are in phase.

2. Relation between Voltage across an Inductance, and Current.

It has already been stated that the Henry, the unit of inductance, is so chosen that, when a steady potential difference of 1 volt is maintained across a resistanceless coil with an inductance of 1 Henry, the current increases at the rate of 1 ampere per second. Conversely, if a current through an inductance is caused to change at the rate of 1 ampere per second, a back e.m.f. of 1 volt will be created across the inductance. If the applied voltage is not held steady but varies so that the current varies, when the current is increasing the "back" e.m.f. will oppose the voltage giving rise to the current and when the current is decreasing the "back"

e.m.f. will aid the voltage giving rise to the current. In other words, the voltage across an inductance of L Henrys is

$$e = L \times \text{rate of change of current}$$

If a current $\hat{\imath} \sin \omega t$ flows through an inductance L, since the rate of change of current is $\hat{\imath}\omega \sin (\omega t + 90°)$ the voltage across the inductance is therefore

$$e = \hat{\imath}L\omega \sin (\omega t + 90°) \quad . \quad . \quad . \quad (1)$$

Hence the voltage is sinusoidal with a peak value $\hat{e} = \hat{\imath}L\omega$ and leads in phase on the current by 90°. Conversely, the current lags on the voltage by 90°.

For brevity in calculations, voltages and currents are often not written in full as $\hat{e} \sin \omega t$ and $\hat{\imath} \sin \omega t$, but are merely indicated by V and I, the sin ωt being omitted. V and I may then be spoken of as vector voltage and vector current. Where a phase difference is to be indicated there are two alternative methods of representation :

A lead of 90° is indicated by $\underline{/90°}$ or the prefix j.

A lag of 90° is indicated by $\overline{\backslash 90°}$ or the prefix $-j$.

Equation (1) is then written

$$E = jIL\omega \text{ or } E = IL\omega\underline{/90°} \quad . \quad . \quad . \quad . \quad (2)$$

So that $\dfrac{E}{I} = jL\omega$, using the more common notation (3)

The quantities E and I are then called vector voltage and vector current respectively.

Since the quantity $jL\omega$ relates the voltage across the circuit and the current through the circuit it plays an analogous part to resistance, but evidently does not behave like a resistance, and it is given the name ***impedance*** ; the quantity $L\omega$ is called the ***reactance*** of the inductance.

Since the sign before the reactance is positive, an inductance is said to have a positive reactance.

Reactance, like Resistance, is measured in ohms.

Equation (1) may therefore be written

$$I = \frac{E}{jL\omega} = \frac{E}{L\omega\underline{/90°}} = \frac{E\overline{\backslash 90°}}{L\omega} = \frac{E\underline{/90°}}{L\omega} = -j\frac{E}{L\omega} \text{ (see V:7)} \quad . \quad (4)$$

Each of the five expressions to which I is equated express the same thing in different ways ; that the magnitude of I is $\dfrac{V}{L\omega}$ and that the phase of I lags 90° behind the applied voltage. It is hardly

necessary to point out that if the current lags behind the voltage by 90° the voltage leads on the current by 90°.

From (4) four conventions emerge:

$$\frac{1}{j} = -j\,;\ \therefore j^2 = -1 \text{ and } \frac{}{\angle 90^\circ} = \overline{\diagdown 90^\circ}, \text{ similarly } \frac{}{\overline{\diagdown 90^\circ}} = \angle 90^\circ \quad . \qquad (5)$$

Also $$j = \angle 90^\circ \text{ and } -j = \overline{\diagdown 90^\circ} \quad . \quad . \quad . \qquad (5a)$$

It is to be noted that an alternating current, flowing through a resistanceless inductance, on the average supplies no power to it. During the first quarter of each current cycle the inductance absorbs energy which is stored in the magnetic field associated with the coil of the inductance. This storage of energy terminates when the peak current is reached and during the remainder of the half-cycle the inductance returns the whole of the energy to the circuit. The same is repeated during the succeeding half-cycle, the only difference being that the sense of the magnetic field is reversed. The inductance therefore behaves towards the voltage in an analogous manner to a weight held in the hand and shaken, voltage replacing force, and current replacing velocity.

3. Relation between Voltage across a Capacity, and Current.

If a steady voltage is applied across a condenser, a current flows in the normal direction it would be expected to flow under the influence of the applied voltage if a resistance were substituted for the condenser. The current does not continue flowing indefinitely as in the case of a resistance, but only until the back e.m.f. of the condenser is equal to the applied voltage. The condenser is then said to be charged to the voltage of the applied voltage source. Theoretically the condenser charges instantaneously: in practice, of course, the theoretical condition is not realized owing to internal impedance in the supply source and the resistances and inductances of connecting leads.

The charge held by a condenser of capacity C Farads when charged to E volts is

$$Q = EC \text{ coulombs} \quad . \quad . \quad . \quad . \qquad (1)$$

Since in this equation C is a constant, if current flows into the condenser owing to E changing, the rate of change of $Q = C \times$ the rate of change of E. But the rate of change of Q is equal to the current flowing into the condenser since 1 ampere = 1 coulomb per second.

Hence the current flowing into the condenser

$$i = C \times \text{rate of change of } E$$

If a voltage $e = \hat{e} \sin \omega t$ is applied across the condenser the rate of change of e is $\hat{e}\omega \sin (\omega t + 90°)$ and the current flowing into the condenser is

$$i = C\omega\hat{e}(\sin \omega t + 90°) \quad . \quad . \quad . \quad . \quad (2)$$

Hence the current is sinusoidal with a peak value $\hat{i} = C\omega\hat{e}$ and leads in phase on the voltage by 90°.

In terms of the conventions already established therefore

$$I = EjC\omega = EC\omega/90° \qquad (3)$$

so that

$$\frac{E}{I} = \frac{1}{jC\omega} = -j\frac{1}{C\omega} \quad . \qquad (4)$$

A condenser is therefore said to have a negative reactance of $\frac{1}{C\omega}$ ohms, and an impedance of $-j\frac{1}{C\omega} = \frac{1}{jC\omega}$ ohms.

Note that from equation (4)

$$E = \frac{I}{jC\omega} = -j\frac{I}{C\omega} \quad . \quad . \quad . \quad (5)$$

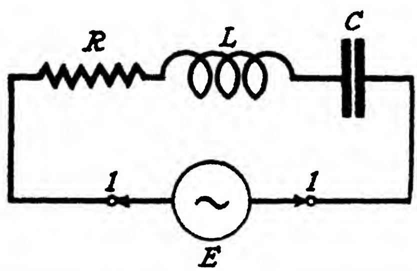


FIG. 1/V:4.—Circuit containing Resistance, Inductance, and Capacity.

4. Impedance and Complex Quantities.

It is now necessary to consider what happens in a circuit having more than one type of element in it. Consider the circuit of Fig. 1 in which a sinusoidal voltage E is applied across a circuit containing an inductance L, a capacity C and a resistance R in series. It is understood that the voltage E is of the form $\hat{e} \sin \omega t$, but the only reference to this necessary is the insertion of the quantity ω in the terms of reactance.

Let I be the current flowing in the circuit. Then since the e.m.f.s round the circuit must add up to zero (this rule applies in A.C. circuits as well as in D.C. circuits) the total back e.m.f.s in the circuit must add up to E. For simplicity the normal convention is abandoned here and the back e.m.f.s are assumed to be positive.

Hence from equation (2) of V:2 and (5) of V:3

$$IjL\omega + \frac{I}{jC\omega} + IR = E \quad . \quad . \qquad (1)$$

$$\therefore \quad \frac{E}{I} = jL\omega + \frac{1}{jC\omega} + R = R + j\left(L\omega - \frac{1}{C\omega}\right) = Z \text{ say} \quad . \qquad (2)$$

The quantity Z representing the relation between E and I in a complex circuit containing both reactance and resistance is called the ***impedance*** of the circuit, and is measured in ***ohms***. An impedance expressed in this form consists of a ***real*** or ***resistive*** part plus an ***imaginary*** or ***reactive*** part, both measured in ohms.

Now write equation (1) in the full fundamental form

$$E = \hat{\imath}R \sin \omega t + \hat{\imath}L\omega \sin (\omega t + 90°) + \frac{\hat{\imath}}{C\omega} \sin (\omega t - 90°) \quad . \qquad (3)$$

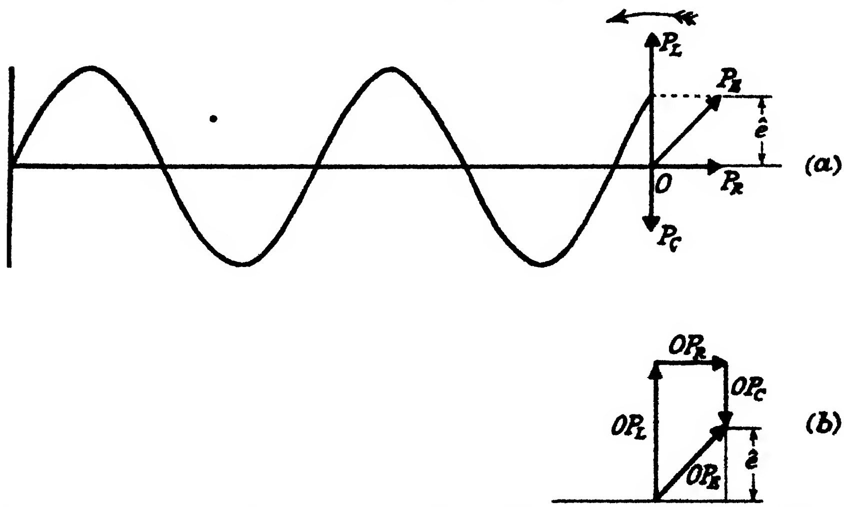


FIG. 2/V:4.—Illustrating the Representation of Voltages and E.M.F.s by Vectors.

Since the phase is purely a relative quantity it is permissible to choose any of these quantities as a reference. Choose the back e.m.f. $\hat{\imath}R \sin \omega t$ and represent this as a sine wave in the manner shown in Fig. 1/III:2 together with its generating radius arm $OP_R = \hat{\imath}R$. This is shown in Fig. 2 (*a*). Then the generating arm corresponding to $\hat{\imath}L\omega \sin (\omega t + 90°)$ must be drawn leading 90° on OP_R; let it be of magnitude $OP_L = \hat{\imath}L\omega$ as shown. Similarly $OP_C = \hat{\imath}/C\omega$, the generating arm corresponding to the quantity $\frac{\hat{\imath}}{C\omega} \sin (\omega t - 90°)$ is drawn lagging 90° on OP_R. The resulting generating arm which represents E, the summation of the three

quantities concerned, is obtained by adding together the lengths OP_R, OP_L, OP_C. This is done as in Fig. 2 (*b*), giving the resultant $OP_E = \hat{e}$.

The most simple justification of this method of addition is that the instantaneous value of the e.m.f. corresponding to OP_E, that is $\hat{e}$ in Fig. 2, is then always equal to the sum of the instantaneous e.m.f.s corresponding OP_R, OP_L and OP_C. The reader should be able to verify this for himself by visualizing or drawing the e.m.f.s corresponding to OP_R, OP_L and OP_C, as sine waves generated by OP_R, OP_L and OP_C in their proper mutual angular relations.

The quantities OP_R, OP_L and OP_C have both magnitude and direction (the direction of OP_R was shown arbitrarily, and the directions of OP_L and OP_C were defined by the angles they make with OP_R). Such quantities are called *vectors*. OP_R, OP_L and OP_C are therefore said to represent the vector voltages across the circuit elements in question. Similarly OP_E is the vector voltage applied to the circuit, and for purposes of calculation is equated to E. Similarly, I is the vector current in the circuit. It should be noted that while E represents the same quantity that is represented by $e = \hat{e} \sin \omega t$, it is doing so according to another convention, and while it is sometimes useful to replace E by $\hat{e} \sin \omega t$, it is not permissible to *equate* E to $\hat{e} \sin \omega t$, since one is a vector quantity and the other is not. The same remarks, of course, apply to I and $i = \hat{\imath} \sin \omega t$. The magnitudes assigned to E and I are written $|E|$ and $|I|$ and again vary in practice. It is evident from the above argument that strictly $|E| = \hat{e}$ and $|I| = \hat{\imath}$, but since in all calculations we are concerned with a ratio between E and I or between E at the input to a circuit and the voltage at the output of a circuit, it will be evident that if for instance $|E|$ is for convenience considered to be equal to the R.M.S. value of voltage (see section on Power in A.C. Circuits below), then $|E| = \dfrac{\hat{e}}{\sqrt{2}}$ and $|I| \dfrac{\hat{\imath}}{\sqrt{2}}$. It is therefore unnecessary to specify which value has been assigned to E and I provided it is understood that they represent vector voltages and can be designated either as peak or R.M.S. values when making practical use of any formulae derived.

It is evident that OP_E leads on OP_R by an angle

$$\phi = \tan^{-1}\frac{OP_L - OP_C}{OP_R} = \tan^{-1}\frac{\hat{\imath}L\omega - \dfrac{\hat{\imath}}{C\omega}}{\hat{\imath}R} = \tan^{-1}\frac{L\omega - \dfrac{1}{C\omega}}{R}. \qquad (4)$$

This will be evident from the sense of the arrow in Fig. 2 (*a*) indicating the direction of rotation of OP_R, OP_E, etc.

Further

$$OP_E = \hat{e} = \sqrt{OP_R^2+(OP_L - OP_C)^2} = \sqrt{(\hat{\imath}R)^2+\left(\hat{\imath}L\omega - \frac{\hat{\imath}}{C\omega}\right)^2}$$

$$= \hat{\imath}\sqrt{R^2+\left(L\omega - \frac{1}{C\omega}\right)^2}$$

$$\therefore \qquad \frac{\hat{e}}{\hat{\imath}} = \left|\frac{E}{I}\right| = \sqrt{R^2+\left(L\omega - \frac{1}{C\omega}\right)^2} \quad . \quad . \quad . \quad . \quad (5)$$

The angle ϕ is the angle by which the phase of E leads on the phase of the voltage across R and therefore is the angle by which E leads on the current I; the current I has the same instantaneous values at every point in the circuit; in other words the current is of the same magnitude and in the same phase all along the circuit.

The relation between the vector quantities E and I is defined by the ratio of their magnitudes and the angle between their directions:

$$\frac{E}{I} = \frac{\hat{e}}{\hat{\imath}}\underline{/\phi} = \sqrt{R^2+\left(L\omega - \frac{1}{C\omega}\right)^2}\ \underline{/\tan^{-1}\left(\frac{L\omega - \frac{1}{C\omega}}{R}\right)} \quad . \quad (6)$$

But from equation (2) the relation between E and I is defined by

$$\frac{E}{I} = R+j\left(L\omega - \frac{1}{C\omega}\right) = Z, \text{ say.}$$

Hence

$$Z = R+j\left(L\omega - \frac{1}{C\omega}\right) = \sqrt{R^2+\left(L\omega - \frac{1}{C\omega}\right)^2}\ \underline{/\tan^{-1}\frac{L\omega - \frac{1}{C\omega}}{R}} \quad (7)$$

It will now be evident that the quantities R and $j\left(L\omega - \frac{1}{C\omega}\right)$ are also vector quantities since they are mutually at right angles.

The impedance of a circuit can therefore be expressed either as the sum of a resistance and a reactance added at right angles, or as a magnitude and an angle. The left and right hand sides of equation (7) respectively represent these two forms of expression.

In equation (7), putting $R = A$ and $\left(L\omega - \frac{1}{C\omega}\right) = B$, a general equation can be obtained constituting a law of addition of resistance and reactance.

$$Z = A+jB = \sqrt{A^2+B^2}\ \underline{/\tan^{-1}\frac{B}{A}} \quad . \quad . \quad (8)$$

Quantities such as A are called "real" quantities and quantities such as B are called "imaginary" quantities. This nomenclature is not ideal and it is usually preferable to call quantities such as A *resistance* and quantities such as B *reactance*. The whole quantity $A+jB$ is called a complex quantity. When expressed in the form on the right hand side of equation (8) an impedance is said to consist of a *magnitude* or *modulus*, and an *angle* or *argument*.

The magnitude of an impedance Z is represented by $|Z|$. In equation (8) $|Z| = \sqrt{A^2+B^2}$. The angle of the impedance is $\phi = \tan^{-1}\frac{B}{A}$ (see Fig. 3).

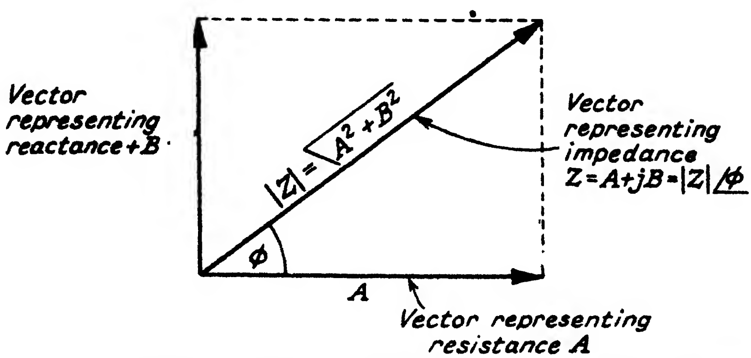


FIG. 3/V:4.—Addition of Vectors of Resistance and Reactance to constitute Impedance.

5. Addition and Subtraction of Complex Quantities.

The rules for the addition and subtraction of quantities in the form $A+jB$ are summarized in the equation

$$(A_1+jB_1)\pm(A_2+jB_2)+\text{ etc.} = A_1\pm A_2\pm\text{ etc.}+j(B_1\pm B_2\pm\text{ etc.}) \quad (1)$$

It is evident that equation (8) of the last section can be written

$$Z = A+jB = |Z|\underline{/\phi} \text{ where } |Z| = \sqrt{A^2+B^2} \text{ and } \phi = \tan^{-1}\frac{B}{A}$$

Also reference to Fig. 1/III:1 shows that

$$A = |Z|\cos\phi \text{ and } B = |Z|\sin\phi \quad . \quad . \quad . \quad (2)$$

$$\therefore \quad |Z|\underline{/\phi} = |Z|\cos\phi+j|Z|\sin\phi. \quad . \quad . \quad (3)$$

Hence the rule for addition and subtraction of quantities in the form $|Z|\underline{/\phi}$ is given by

$$|Z_1|\underline{/\phi_1}\pm|Z_2|\underline{/\phi_2}\pm\text{ etc.} = |Z_1|\cos\phi_1\pm|Z_2|\cos\phi_2\pm\text{ etc.}$$
$$+j|Z_1|\sin\phi_1\pm j|Z_2|\sin\phi_2\pm\text{ etc.} \quad . \quad (4)$$

6. Multiplication of Complex Quantities.

The rule for multiplying together two complex quantities defined by magnitude and angle is given by the formula

$$P\underline{/\theta} \times Q\underline{/\phi} = PQ\underline{/\theta+\phi} \quad . \quad . \quad . \quad (1)$$

Proof. Let $P\underline{/\theta} = A+jB$ and $Q\underline{/\phi} = C+jD$

then
$$\left.\begin{aligned} P &= \sqrt{A^2+B^2}, \quad \theta = \tan^{-1}\frac{B}{A} \\ Q &= \sqrt{C^2+D^2}, \quad \phi = \tan^{-1}\frac{D}{C} \end{aligned}\right\} \quad (2)$$

Substituting from (2) into (1)

$$P\underline{/\theta} \times Q\underline{/\phi} = \sqrt{(A^2+B^2)(C^2+D^2)} \underline{/\tan^{-1}\frac{B}{A}+\tan^{-1}\frac{D}{C}}$$

Since $\tan^{-1}x+\tan^{-1}y = \tan^{-1}\dfrac{x+y}{1-xy}$ (This must be accepted)

$\therefore$ $P\underline{/\theta} \times Q\underline{/\phi}$.

$$= \sqrt{A^2C^2+B^2D^2+A^2D^2+B^2C^2} \underline{/\tan^{-1}\frac{AD+BC}{AC-BD}}. \quad (3)$$

Now multiply together $A+jB$ and $C+jD$ by elementary algebra

$$(A+jB)(C+jD) = AC-BD+j(AD+BC)$$

$$= \sqrt{(AC-BD)^2+(AD+BC)^2} \underline{/\tan^{-1}\frac{AD+BC}{AC-BD}}$$

$$= \sqrt{A^2C^2+B^2D^2+A^2D^2+B^2C^2} \underline{/\tan^{-1}\frac{AD+BC}{AC-BD}}$$

which is the same as the result obtained in equation (3).

7. Division of Complex Quantities.

The rule for division of complex quantities defined by magnitude and angle is given in the formula:

$$\frac{P\underline{/\theta}}{Q\underline{/\phi}} = \frac{P}{Q}\underline{/\theta-\phi} \quad . \quad . \quad . \quad . \quad (1)$$

Proof. Again assume the conditions of equation (2) of the last section.

Substituting from (2) of the last section into (1) above

$$\frac{P\underline{/\theta}}{Q\underline{/\phi}} = \frac{\sqrt{A^2+B^2}}{\sqrt{C^2+D^2}} \underline{/\tan^{-1}\frac{B}{A}-\tan^{-1}\frac{D}{C}}$$

Since $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \dfrac{x - y}{1 + xy}$

$$\frac{P\angle\theta}{Q\angle\phi} = \sqrt{\frac{A^2+B^2}{C^2+D^2}} \Big/ \tan^{-1} \frac{BC - AD}{AC+BD} \quad . \quad . \quad (2)$$

Now divide $A+jB$ by $C+jD$ and incidentally *note the method of eliminating imaginary terms from the denominator.*

$$\frac{A+jB}{C+jD} = \frac{A+jB}{C+jD} \times \frac{C-jD}{C-jD} = \frac{AC+BD+j(BC-AD)}{C^2+D^2}$$

$$= \sqrt{\frac{A^2C^2+B^2D^2+B^2C^2+A^2D^2}{C^2+D^2}} \Big/ \tan^{-1} \frac{BC-AD}{AC+BD}$$

$$= \sqrt{\frac{A^2+B^2}{C^2+D^2}} \Big/ \tan^{-1} \frac{BC-AD}{AC+BD}$$

which is the same as the result obtained in (2).

8. Impedances in Series.

If a number of impedances

$$z_1 = R_1 + L_1 j\omega + \frac{1}{C_1 j\omega}$$

$$z_2 = R_2 + L_2 j\omega + \frac{1}{C_2 j\omega}$$

$$z_3 = R_3 + L_3 j\omega + \frac{1}{C_3 j\omega}$$

are placed in series, the total impedance is given by the sum of all the series elements;

$$Z = z_1 + z_2 + z_3 = R_1 + R_2 + R_3 + (L_1 + L_2 + L_3) j\omega$$
$$+ \frac{1}{j\omega}\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \quad . \quad (1)$$

9. Condensers in Series.

Consistent with equation (1) of the last section, if a number of condensers C_1, C_2, C_3, etc. are placed in series, the total impedance is that of a negative reactance

$$X = \frac{1}{j\omega}\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \text{etc.}\right)$$

that is the condensers are equivalent to a single capacity of value C such that

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \text{etc.}$$

or
$$C = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\text{etc.}} \qquad (2)$$

Note that if there are only two condensers the effective capacity of the two in series is given by

$$C = \frac{C_1C_2}{C_1+C_2} \qquad (3)$$

10. Impedances in Parallel.

When a voltage E is applied across a number of impedances z_1, z_2, z_3, etc., in parallel, the current taken by each impedance is independent of the current taken by the other impedance and the total current I is given by

$$I = \frac{E}{z_1}+\frac{E}{z_2}+\frac{E}{z_3}+\text{etc.}$$

$$\therefore \quad \frac{E}{I} = Z = \frac{1}{\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}+\text{etc.}} \quad . \quad . \quad . \quad (1)$$

Evidently in practice z_1, z_2, z_3 are complex quantities and are expressed in the form $A+jB$ or $P/\underline{\theta}$. Further, the effective impedance Z is also required in the form $A+jB$ or in the form $P/\underline{\theta}$.

Rationalization. Before considering the method of effecting the necessary transformation of equation (1), it is necessary to explain a simple process known as *rationalization*. Rationalization consists in the elimination of reactive terms from part of an expression, the part concerned usually being the denominator of fractions.

The denominator of a fraction is rationalized by multiplying the fraction above and below by the denominator with the sign of the reactive term reversed.

For instance :

$$\frac{A+jB}{C+jD} = \frac{A+jB}{C+jD} \times \frac{C-jD}{C-jD} = \frac{AC+BD+j(BC-DA)}{C^2+D^2} \quad . \quad (2)$$

Also

$$\frac{1}{A+jB} = \frac{A-jB}{A^2+B^2} = \frac{A}{A^2+B^2} - j\frac{B}{A^2+B^2} \quad . \quad . \quad (3)$$

The following conversions are not really rationalization but are very useful:

$$\frac{A+jB}{C+jD}=\frac{\sqrt{A^2+B^2}\,\big/\tan^{-1}\frac{B}{A}}{\sqrt{C^2+D^2}\,\big/\tan^{-1}\frac{D}{C}}=\sqrt{\frac{A^2+B^2}{C^2+D^2}}\,\Big/\tan^{-1}\frac{B}{A}-\tan^{-1}\frac{D}{C}. \quad (4)$$

$$\frac{1}{A+jB}=\frac{1}{\sqrt{A^2+B^2}}\Big\backslash\tan^{-1}\frac{B}{A}. \quad . \quad . \quad (5)$$

Evaluation of Equation (1) *when the Individual Impedances are expressed in the form* $A+jB$.

Assuming that

$$z_1=A_1+jB_1,\ z_2=A_2+jB_2,\ z_3=A_3+jB_3,\ \text{etc.}$$

Then

$$Z=\frac{1}{\frac{1}{A_1+jB_1}+\frac{1}{A_2+jB_2}+\frac{1}{A_3+jB_3}\text{ etc.}}$$

$$=\frac{1}{\frac{A_1}{A_1^2+B_1^2}+\frac{A_2}{A_2^2+B_2^2}+\frac{A_3}{A_3^2+B_3^2}+\text{etc.}-j\left[\frac{B_1}{A_1^2+B_1^2}+\frac{B_2}{A_2^2+B_2^2}+\text{etc.}\right]} \quad (6)$$

Equation (6) is of the same form as the left-hand sides of equations (3) and (5) and may be converted to the form $A+jB$ by means of equation (3) or to the form P/θ by means of equation (5).

Evaluation of Equation (1) *when the Individual Impedances are expressed in the Form* P/θ.

Assuming that $z_1=P_1/\theta_1$, $z_2=P_2/\theta_2$, $z_3=P_3/\theta_3$, etc.

Then

$$Z=\frac{1}{\frac{1}{P_1/\theta_1}+\frac{1}{P_2/\theta_2}+\frac{1}{P_3/\theta_3}+\text{etc.}}$$

$$=\frac{1}{\frac{1}{P_1}\backslash\theta_1+\frac{1}{P_2}\backslash\theta_2+\frac{1}{P_3}\backslash\theta_3+\text{etc.}}$$

$$=\frac{1}{\frac{1}{P_1}\cos\theta_1+\frac{1}{P_2}\cos\theta_2+\frac{1}{P_3}\cos\theta_3+\text{etc.}-j\left[\frac{1}{P_1}\sin\theta_1+\frac{1}{P_2}\sin\theta_2+\frac{1}{P_3}\sin\theta_3+\text{etc.}\right]} \quad . \quad (7)$$

Equation (7) is of the same form as the left-hand sides of equa-

tions (3) and (5) and may be converted to the form $A+jB$ by means of equation (3) or to the form $P\angle\theta$ by means of equation (5).

The whole of the analysis in this section is of great importance because it illustrates the methods of handling complex quantities. In the remaining chapters many formulae will be derived, of forms differing from the right-hand side of equation (1), but which may be reduced to a useful simple form by means of conversions using the same principles.

The exact stage at which numerical values are inserted in any computation depends on the purpose of the computation and the degree of difficulty in reducing the expression obtained to the required form by purely algebraic methods. Where a numerical answer is required immediately, it is sometimes easier to introduce numerical values at a comparatively early stage in the calculation. Reference should be made to XXIV:2.2, Method of Evaluation of Expressions Containing Complex Quantities.

Numerical Example. Find the impedance at 1,000 c/s constituted by three impedances in parallel consisting respectively of a resistance of 1,000 ohms, a condenser of 0·3 μF in series with 500 ohms, and an impedance which at 1,000 c/s is equal to $500\angle 60°$.

The reactance of 0·3 μF at 1,000 c/s is $\dfrac{159}{0\cdot 3} = 530\ \Omega$, so that the impedance containing the condenser is $500 - j530$.

Hence the effective impedance of the three in parallel is

$$Z = \frac{1}{\dfrac{1}{1{,}000}+\dfrac{1}{500-j530}+\dfrac{1}{500\angle 60°}}$$

$$= \frac{1}{0\cdot 001+\dfrac{500}{500^2+530^2}+j\dfrac{530}{500^2+530^2}+\dfrac{1}{500}\cos 60° - j\dfrac{1}{500}\sin 60°}$$

$$= \frac{1}{0\cdot 001+0\cdot 000945+j0\cdot 001+0\cdot 002\times 0\cdot 5-j0\cdot 002\times 0\cdot 866}$$

$$= \frac{1}{0\cdot 00295-j0\cdot 000732}$$

$$= \frac{0\cdot 00295}{0\cdot 00295^2+0\cdot 000732^2}+j\frac{0\cdot 000732}{0\cdot 00295^2+0\cdot 000732^2}$$

To avoid confusion in working with decimals, multiply above and below by 10^6,

then $$Z = \frac{2{,}950}{2{\cdot}95^2+0{\cdot}732^2}+j\frac{732}{2{\cdot}95^2+0{\cdot}732^2}$$
$$= 320+j79 \text{ ohms.}$$

If this is required in the form P/θ

$$Z = \sqrt{320^2+79^2}\,\Big/\underline{\tan^{-1}\frac{79}{320}}$$
$$= 329/\underline{13^\circ\ 53''} \text{ ohms}$$

The angle is here expressed to a greater degree of accuracy than is warranted either by the probable accuracy of the calculation (done on a slide rule) or the normal required accuracy. The answer is therefore $Z = 329/\underline{14^\circ}$ ohms.

11. Admittance.

Admittance (Y) is the reciprocal of impedance and is measured in mhos.

$$Y = \frac{1}{Z} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Admittance in A.C. circuits is, therefore, the analogue of conductance in D.C. circuits.

12. Vector and Scalar Notation and Mixed Notation.

It has been explained that a voltage or e.m.f. $e = \hat{e}\sin\omega t$ may be represented by a vector E. The quantity $e = \hat{e}\sin\omega t$ is then said to be a scalar representation of the voltage: a quantity defined by its magnitude at each instant of time. Consistent with this notation the first two columns of the table below give corresponding scalar and vector notations defining identical quantities. In this a, $\hat{a}$ and A may be replaced *respectively* by e, $\hat{e}$ and E, or by i, $\hat{i}$, and I, etc.

Scalar Notation		*Vector Notation on Basis that A Represents â sin ωt*	*Mixed Notation in Terms of sin ωt*
$\hat{a}\sin(\omega t+90^\circ)$	$= \hat{a}\cos\omega t$	jA or $A/\underline{90^\circ}$	$j\hat{a}\sin\omega t$
$\hat{a}\sin(\omega t-90^\circ)$	$-\hat{a}\cos\omega t$	$-jA$ or $A\overline{\backslash 90^\circ}$	$-j\hat{a}\sin\omega t$
$\hat{a}\cos(\omega t+90^\circ)$		$-A$ or $A/\underline{180^\circ}$	$-\hat{a}\sin\omega t$
$\hat{a}\cos(\omega t-90^\circ)$	$\hat{a}\sin\omega t$	A	$\hat{a}\sin\omega t$
$\hat{a}\sin(\omega t+\theta)$		$A/\underline{\theta}$	$\hat{a}/\underline{\theta}\sin\omega t$
$\hat{a}\sin(\omega t-\theta)$		$A\overline{\backslash\theta}$	$\hat{a}\overline{\backslash\theta}\sin\omega t$
$\hat{a}\cos(\omega t+\theta)$	$(\sin\omega t+\theta+90^\circ)$	$A/\underline{90^\circ+\theta}$	$\hat{a}/\underline{90^\circ+\theta}\sin\omega t$
$\hat{a}\cos(\omega t-\theta)$		$A/\underline{90^\circ-\theta}$	$\hat{a}/\underline{90^\circ-\theta}\sin\omega t$

It must be continually remembered that from a practical point of view, while vectors defining impedance have an absolute meaning, vectors describing voltage and current only have an absolute meaning as far as their magnitude is concerned. The angle of a current or voltage vector only requires meaning when related to another vector.

For instance, if an impedance $Z = A+jB = P\underline{/\phi}$, A, B, P and ϕ all have absolute meanings independent of all other quantities which may exist simultaneously.

If a voltage is defined by $E\underline{/\phi}$, the $\underline{/\phi}$ remains meaningless until some other quantity is defined or determined. Any voltage or e.m.f. which acts alone and is the " prime mover " in a circuit is therefore usually defined by a letter without an angle, and all other symbols, which are either entered in the problem or emerge from the problem with angles attached to them, have their phase defined with regard to the " prime mover ". For instance, if a sinusoidal e.m.f. E is applied across an impedance $P\underline{/\phi}$, the current which flows is $I = \dfrac{E}{P\underline{/\phi}} = \dfrac{E}{P}\overline{\backslash\phi}$: the positive angle in the impedance causes the current I to have a negative or lagging angle ϕ : the current I lags behind the voltage E by an angle ϕ.

The quantities in the first column of scalar notation above are represented in the second column of vector notation (and the column of mixed notation discussed below) on the basis that they describe co-existent quantities in the same time-scale. For instance, if in a problem it was found as the result of calculation that the input voltage to a part of a network was $\hat{e} \sin(\omega t+90°)$ and the input current was $\hat{\imath} \cos(\omega t - \theta)$, the input impedance would be given by

$$\frac{\hat{e}\sin(\omega t+90°)}{\hat{\imath}\cos(\omega t-\theta)} \equiv \frac{jE}{I\underline{/90°-\theta}} = \frac{E\underline{/90°}}{I\underline{/90°-\theta}}$$

$$= \frac{E}{I\underline{/-\theta}} = \frac{E}{I\overline{\backslash\theta}} = \frac{E}{I}\underline{/\theta} \quad . \quad . \quad . \quad (1)$$

In this case, if E is the R.M.S. value of the voltage, I is the R.M.S. value of the current ; or if $E = \hat{e}$, $I = \hat{\imath}$.

Mixed Notation. The above process is simple and convenient, but involves the introduction and definition of extra symbols E and I and the complete rewriting of the equations or formulae. The mixed notation in the third column enables this extra work to be avoided. The method of derivation of the mixed notation is sufficiently obvious to require no explanation. Just as a quantity in scalar notation cannot be *equated* to a quantity in vector notation, so a

quantity in mixed notation cannot be equated to quantities in either of the other notations. For this reason, all the quantities in any expression or equation must be written in terms of the same notation.

Example. Two e.m.f.s, $\hat{e}_1 \sin \omega t$ and $\hat{e}_2 \sin \omega t$, acting simultaneously at different points of a network drive currents i_1 and i_2 through an impedance element in the network. If

$$i_1 = \hat{i}_1 \sin(\omega t - 90^\circ) \quad \text{and} \quad i_2 = \hat{i}_2 \sin(\omega t + \theta),$$

what is the magnitude of the total current and its phase angle with regard to the applied e.m.f.s?

In mixed notation the total current

$$\begin{aligned} i = i_1 + i_2 &= -j\hat{i}_1 \sin \omega t + \hat{i}_2 \underline{/\theta} \sin \omega t \\ &= (-j\hat{i}_1 + \hat{i}_2 \underline{/\theta}) \sin \omega t \\ &= (-j\hat{i}_1 + j\hat{i}_2 \sin\theta + \hat{i}_2 \cos\theta) \sin \omega t \end{aligned}$$

$$\sqrt{\hat{i}_2^2 \cos^2\theta + (\hat{i}_2 \sin\theta - \hat{i}_1)^2} \; \underline{/ \tan^{-1} \frac{\hat{i}_2 \sin\theta}{\hat{i}_2 \cos\theta}} \qquad (2)$$

Sin ωt is omitted from (2), which therefore defines i as a vector, i.e. in vector notation.

13. The Principle of Superposition.

This applies to circuits containing linear elements of resistance, inductance and capacity, that is, elements in which the magnitude of the current flowing through them is directly proportional to the magnitude of the voltage across them. The elements considered above are all of this type.

The principle of superposition states that the current in a linear circuit containing a number of sources of e.m.f. is equal to the sum of the currents which would be caused by each e.m.f. acting alone.

This principle may conveniently be treated as an axiom. Proofs are, however, given in *Transmission Networks and Filters* by T. E. Shea, and in *Elementary Differential Equations* by T. C. Fry.

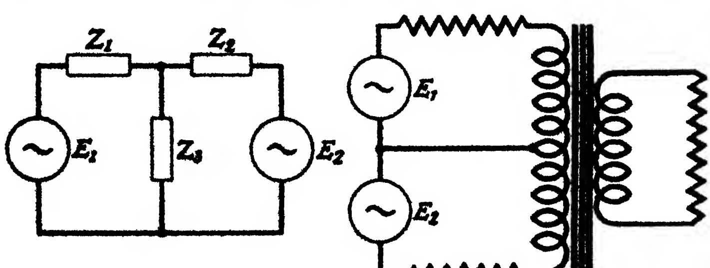


FIG. 1/V:13.—Typical Circuits containing more than one Source of E.M.F.

Fig. 1/V:13 shows typical circuits which occur with more than one source of e.m.f. z_1, z_2 and z_3 represent impedances.

14. Reciprocity Theorem.

This states that:

If any e.m.f. of any wave form whatever, located in a conductor at one point of a circuit, produces a current in a conductor at any other point in the circuit, the same e.m.f. acting at the second point produces the same current at the first point. For proof see *Transmission Networks and Filters* by T. E. Shea.

15. Thévenin's Theorem.

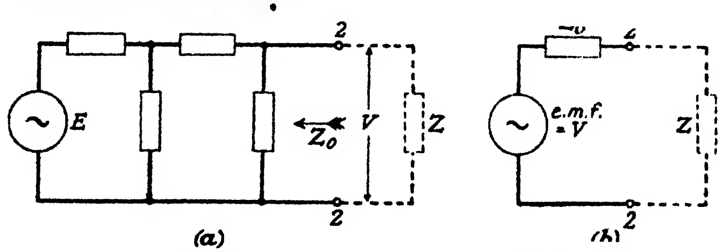


Fig. 1/V:15.—Replacement of a Circuit at (*a*) by Circuit at (*b*) for Calculation of Output Current when connected to a load Z.

This states that if the open circuit voltage at the output of any network containing a source or sources of e.m.f. is V, and the impedance looking back into its output terminals (with the internal sources of e.m.f. inoperative) is Z_0, the current which flows in any load Z connected to its output terminals is

$$I = \frac{V}{Z_0 + Z}$$

In other words, for the purpose of calculating the load current, the circuit can be represented by a generator of internal impedance Z_0 and of internal e.m.f. V, e.g. in Fig.1/V:15 the circuit at (*a*) can be replaced by the circuit at (*b*) to calculate the current through the load Z.

Proof. If in the circuit at (*a*), before connecting Z, an e.m.f. $-V$ is inserted in series with Z, no current will flow. The e.m.f. V may, by the principle of superposition, be regarded as having caused a current to flow through Z of such character as to annul the current which would have flowed due to E.

But the current to which e.m.f. $-V$, if acting alone, would give rise, is $-V/(Z_0+Z)$. Consequently, the current to which E would give rise, when acting alone, is $V/(Z_0+Z)$.

16. Parallel Representation of Impedances.

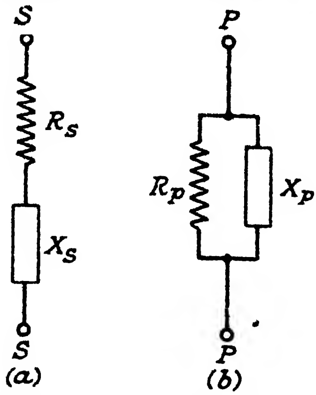


FIG. 1/V:16.—Equivalent Series and Parallel Representations of Impedance.

Consider the impedances Z_s looking into terminals s, s, in Fig. 1 (*a*) and Z_p looking into terminals p, p, in Fig. 1 (*b*).

Evidently $Z_s = R_s + jX_s$

while
$$Z_p = \frac{jX_pR_p}{R_p + jX_p} = \frac{jX_pR_p(R_p - jX_p)}{R_p^2 + X_p^2}$$
$$= \frac{X_p^2R_p + jX_pR_p^2}{R_p^2 + X_p^2}$$
$$= \frac{R_p}{1 + \frac{R_p^2}{X_p^2}} + j\frac{X_p}{1 + \frac{X_p^2}{R_p^2}}$$

At the frequency of measurement the two impedances are evidently identical, i.e. $Z_s = Z_p$, if:

$$R_s = \frac{R_p}{1 + \frac{R_p^2}{X_p^2}} \text{ and } X_s = \frac{X_p}{1 + \frac{X_p^2}{R_p^2}} \quad . \quad . \quad . \quad (1)$$

Solving (1) for R_p and X_p:

$$R_p = R_s\left(1 + \frac{X_s^2}{R_s^2}\right) \text{ and } X_p = X_s\left(1 + \frac{R_s^2}{X_s^2}\right) \quad . \quad . \quad (2)$$

Equation (2) is merely another way of expressing the relations of equation (1).

It follows that a measurement made on an impedance at one frequency cannot determine whether the elements of the impedance are in series or in parallel. There is nothing new in this since it is

implicit in the previous analysis that, regardless of whether the elements are in series or in parallel, any network of linear circuit elements of resistance, inductance and capacity will present an impedance in the form $R+jX$, or R_s+jX_s, if we care to use different symbols.

It now follows that, by making the transformations expressed by the above equations, we can express an impedance in the form R_p in parallel with jX_p. This may be written $R_p//jX_p$ so that:

$$Z_p = R_p//jX_p \quad . \quad . \quad . \quad (3)$$

It often happens in measurements made on a bridge that the impedance elements on the known side of the bridge are in parallel, so that the answer is presented in the form that the unknown impedance is equivalent to two known impedances R_p and jX_p in parallel; that is, it presents the same impedance as these two impedances in parallel. The problem then is: given R_p and X_p what is the equivalent series impedance of the network under measurement? The answer is then given by $Z_s = R_s+jX_s$ where R_s and jX_s have the values determined by equation (1).

The inverse problem is solved by means of equation (2).

Equation (2) provides the justification for the classical method of tuning the anode circuits of class C amplifiers. In most cases these consist of a parallel tuned circuit consisting of an inductance having a resistance effectively in series with it tuned by a parallel condenser. It is required that this combination, which is indicated in Fig. 2, should be adjusted, by varying the condenser C, so that the impedance looking into terminals 1, 1, is a pure resistance, i.e. an impedance with zero reactance component. The method of tuning is to vary the condenser until the anode current is a minimum, that is until the impedance 1, 1, is a maximum. It can easily be seen that this is the condition for zero reactance.

Put X_L = the reactance of the inductance L.

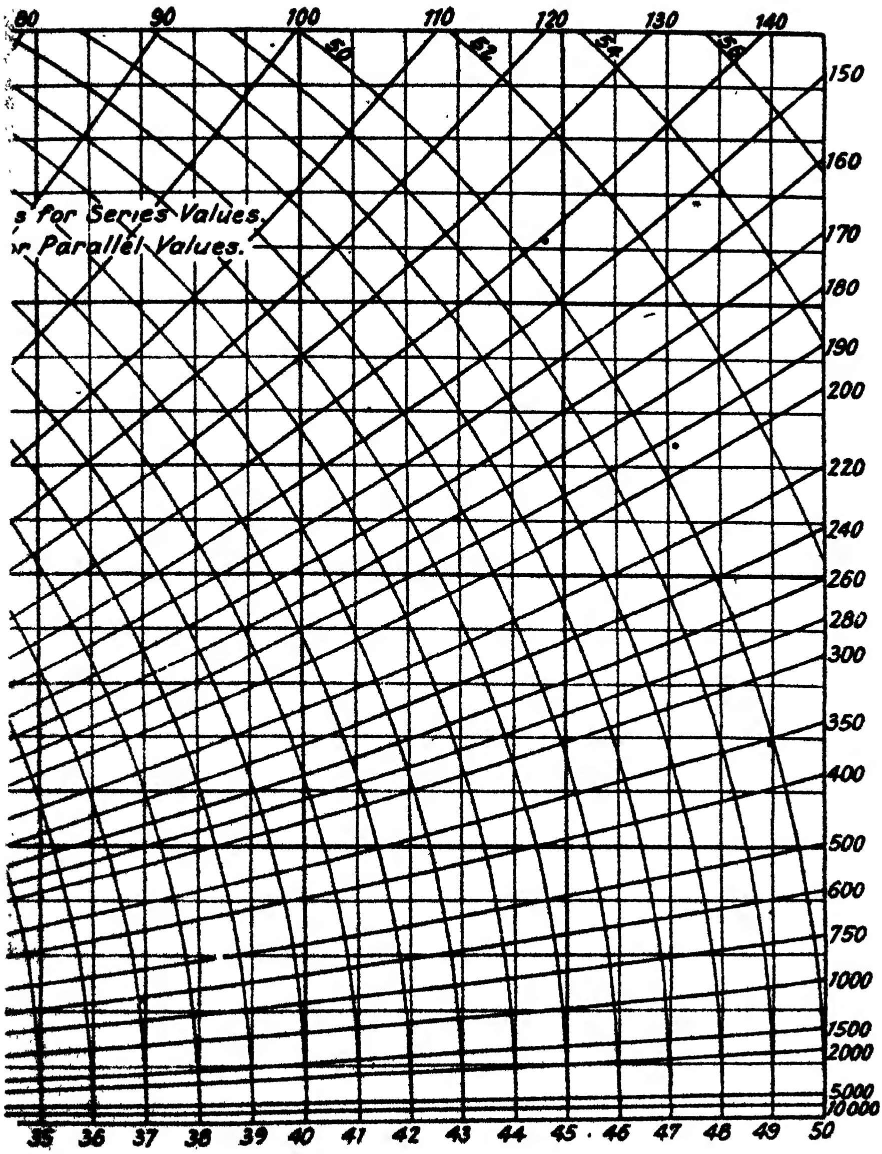
X_C = the reactance of the capacity C.

Z_{22} = the impedance looking into terminals 2, 2, with condenser C removed

$= R_p//jX_p$

Z_{11} the impedance looking into terminals 1, 1

$$\text{Then} \quad R_p = R\left(1+\frac{X_L^2}{R^2}\right) \text{ and } X_p = X_L\left(1+\frac{R^2}{X_L^2}\right) \quad . \quad (4)$$



Circuits.

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It follows that if X_C is made equal to $-X_p$ as defined by equation (4) the parallel reactance will be cancelled (see below) which means that $Z_{11} = R_p$ which is evidently the maximum possible value of Z_{11}, since the impedance of a resistance (e.g. R_p) shunted by any value of reactance is always less than the impedance of the resistance alone. Since the reactance has been cancelled it is a tautology to say that Z_{11} is free from reactance : this is explicit in the statement $Z_{11} = R_p$.

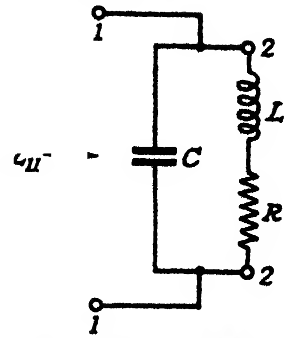


FIG. 2/V:16.—Parallel Resonant Circuit.

In case there is any doubt that the effect of shunting a reactance X_p by a reactance X_C where $X_C = -X_p$, is to cancel the reactance, apply the formula for impedances in parallel developed in V:10.

The resultant impedance is given by :

$$Z = \frac{1}{\frac{1}{jX_p} + \frac{1}{jX_C}} = \frac{1}{\frac{1}{jX_p} - \frac{1}{jX_p}} = \text{infinity.}$$

Equations (1) and (2) are fundamental equations of great importance and to save time in calculation they have been embodied in the chart shown in Fig. 3/V:16. This chart provides means for converting impedances expressed in the form $R_s + jX_s$ into the form $R_p//jX_p$ (R_p in parallel with jX_p), and vice versa.

The chart consists of a system of rectangular co-ordinates with one set of circles described from centres on the horizontal axis and one set of circles described from centres on the vertical axis.

The series values of the impedances are determined by the rectangular co-ordinates and the parallel values of the impedances by the circular co-ordinates.

For each conversion from series to parallel or from parallel to series there are two ways of using the chart.

Conversion from Series to Parallel. Insert R_s on horizontal axis and X_s on vertical axis. At point of intersection of lines through R_s and X_s on rectangular co-ordinates read off R_p on the circle described with centre on the horizontal axis and X_p on the circle described with centre on the vertical axis; the circles in question being those which pass through the plotted point.

Alternatively, enter R_s on the vertical axis and X_s on the horizontal axis and read off R_p from the circle described with

sloping line downwards to the right of the figure; where the value of capacity is read off as 500 $\mu\mu$F.

Example 4. What value of inductance is required in order to produce a reactance of 100,000 ohms at 30 c/s?

Enter the 30 c/s at the bottom of the figure and follow the vertical line through 30 c/s upwards until it meets the horizontal line through 100,000 ohms; from the point of intersection follow the diagonal inductance line upwards towards the right where the value of inductance is read off (in this case at the top of the figure) at 500 Henrys.

Example 5. What value of capacity must be provided to produce a reactance of 60 ohms at 90 c/s?

Enter 90 c/s at the bottom of the figure and follow the vertical line through 90 c/s upwards until it meets the horizontal line through 60 ohms; from the point of intersection follow the capacity diagonal line downwards towards the right of the figure where it is read off (in this case at the bottom of the figure) as 30 μF.

Example 6. What is the resonant frequency of 100 μH and 0·001 μF?

Enter the 100 μH and the 0·001 μF on the right of the figure and follow both lines along until they meet. From the point of intersection follow the vertical line to the bottom of the chart where the frequency is read off as 500 kc/s.

18. Kirchhoff's Laws.

These are two.

(1) In any network of conductors and/or impedances the algebraic sum of all the currents flowing towards any junction of conductors (and/or impedances) is zero. It is hardly necessary to point out that, if the convention is adopted that currents flowing towards the junction are positive, then all currents flowing away from the junction are negative, and vice versa.

(2) The algebraic sum of the e.m.f.s around any closed part of a network is zero. For this purpose, the e.m.f. developed in any part of the circuit between any two neighbouring junction points in the circuit is equal to the product of the impedance of the direct path between those two junction points multiplied by the algebraic sum of the currents flowing through the said direct path.

The method of applying these laws for solving circuit problems will now be illustrated by considering two examples.

Example 1. Fig. 1 shows two generators of internal impedances A and B and having internal e.m.f.s e_1 and e_2, both of the same frequency, feeding a common load impedance C. This constitutes a circuit with two meshes, and it is required to find the current through C.

In this case it is simplest to assume that i_1 is the current flowing round one mesh, as shown and in the direction shown in Fig. 1,

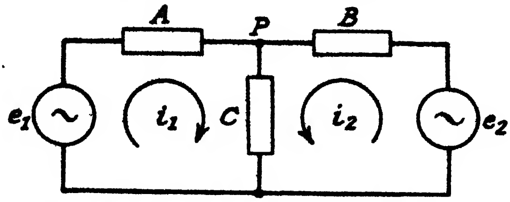


FIG. 1/V:18.—Two Generators supplying a Common Load.

and i_2 is the current flowing round the other mesh, as shown and in the direction shown. It is then obvious without more argument that the current through C is equal to i_1+i_2. (A rather more complicated way of regarding the problem, which is quite unnecessary, is to say that i_1 is the current flowing from left to right through A and that i_2 is the current flowing from right to left through B. Since both these currents flow towards the point P, which is the common junction point of the paths A, B and C (and of those paths only), the current through C must be of such magnitude and such direction that the total current flowing towards P is zero. This means that the current through C is of magnitude i_1+i_2, and of direction away from P. (This, of course, illustrates the application of Kirchhoff's first law.)

The e.m.f. round each mesh is now equated to zero, which results in the two equations :

$$Ai_1+C(i_1+i_2) - e_1 = 0 \qquad (1)$$

$$Bi_2+C(i_1+i_2) - e_2 = 0 \qquad (2)$$

From (1) : $$(A+C)i_1+Ci_2= e_1 \qquad (3)$$

From (2) : $$Ci_1+(B+C)i_2 = e_2 \qquad (4)$$

Multiplying (3) by $(B+C)$ and (4) by C

$$\therefore \quad (B+C)(A+C)i_1+(B+C)Ci_2 = (B+C)e_1 \qquad (5)$$

$$C^2i_1+(B+C)Ci_2 = Ce_2 \qquad (6)$$

Subtracting (6) from (5)

$$\therefore \quad (AB+BC+CA)i_1 = (B+C)e_1 - Ce_2$$

$$\therefore \; i_1 = \frac{(B+C)e_1 - Ce_2}{AB+BC+CA} \qquad (7)$$

Similarly $$i_2 = \frac{(C+A)e_2 - Ce_1}{AB+BC+CA} \qquad (8)$$

Adding (7) and (8) gives the current through C which is

$$i_1+i_2 = \frac{Be_1+Ae_2}{AB+BC+CA} \qquad (9)$$

See also Example 18 in V:99.

Example 2. Fig. 2 shows the general case of a bridge circuit.

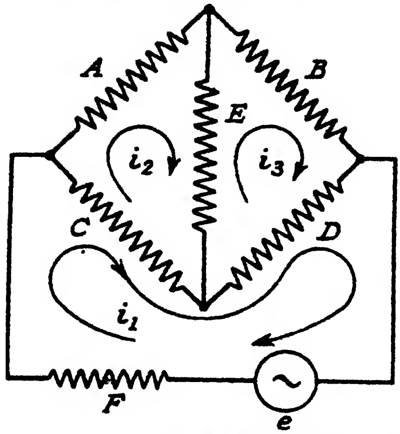


FIG. 2/V:18.—General Case of Bridge Circuit.

It has three empty meshes, i.e. meshes which do not embrace circuit elements. *A third law may now be introduced, which is that the number of circulating currents necessary to provide a solution is equal to the number of empty meshes.*

Unless expediency indicates (as it did in the first example) the circulating currents are all introduced with the same sense of rotation, though this is by no means essential. The three currents to be found, i_1, i_2 and i_3, have been drawn in Fig. 2, all flowing clockwise. Then by inspection

$$i_A = i_2, i_B = i_3, i_C = i_1 - i_2, i_D = i_1 - i_3, i_E = i_2 - i_3, i_F = i_1 \qquad (10)$$

Equating the e.m.f.s round each mesh to zero

$$i_1F + (i_1 - i_2)C + (i_1 - i_3)D - e = 0 \quad . \quad . \quad (11)$$

$$i_2A + (i_2 - i_3)E + (i_2 - i_1)C \quad = 0 \quad . \quad . \quad (12)$$

$$i_3B + (i_3 - i_1)D = (i_3 - i_2)E \quad = 0 \quad . \quad . \quad (13)$$

Regrouping

$$(C+D+F)i_1 - Ci_2 - Di_3 = e \quad . \quad . \quad . \quad (14)$$

$$-Ci_1 + (A+C+E)i_2 - Ei_3 = 0 \quad . \quad . \quad . \quad (15)$$

$$-Di_1 - Ei_2 + (B+D+E)i_3 = 0 \quad . \quad . \quad . \quad (16)$$

As is often the case where Kirchhoff equations are involved, the solution of these equations is laborious although straight-forward.

Solving for i_1, i_2 and i_3

$$i_1 = \frac{[E(C+A) + (B+D)(A+C+E)]e}{H} \quad . \quad . \quad (17)$$

$$i_2 = \frac{[E(C+D) + C(B+D)]e}{H} \quad . \quad . \quad . \quad . \quad (18)$$

$$i_3 = \frac{[E(C+D) + D(A+C)]e}{H} \quad . \quad . \quad . \quad . \quad (19)$$

Where

$$H = (A+B)(CD+EF)+(C+D)(AB+EF) \\ +(E+F)(AD+BC)+E(AC+BD)+F(AB+CD) \quad . \quad (20)$$

The currents through each member of the network, i_A, i_B, etc., are then given by equation (10).

From the above it will be seen that a simple advantage results from making the sense of all mesh currents the same (i.e. all clockwise or all anti-clockwise), which is that the e.m.f. developed in any impedance between two neighbouring junctions is equal to the impedance multiplied by the arithmetical difference of the currents. The sense of the e.m.f. is always determined by making the circulating current appropriate to the mesh under consideration, positive; the same current will of course be negative when other meshes are being considered.

19. Impedance Potentiometers.

Just as in the case of direct current two resistances may be placed in series with one another to tap off a fraction of the potential difference across any voltage source, so in the case of alternating currents two impedances may be connected in series for the same purpose.

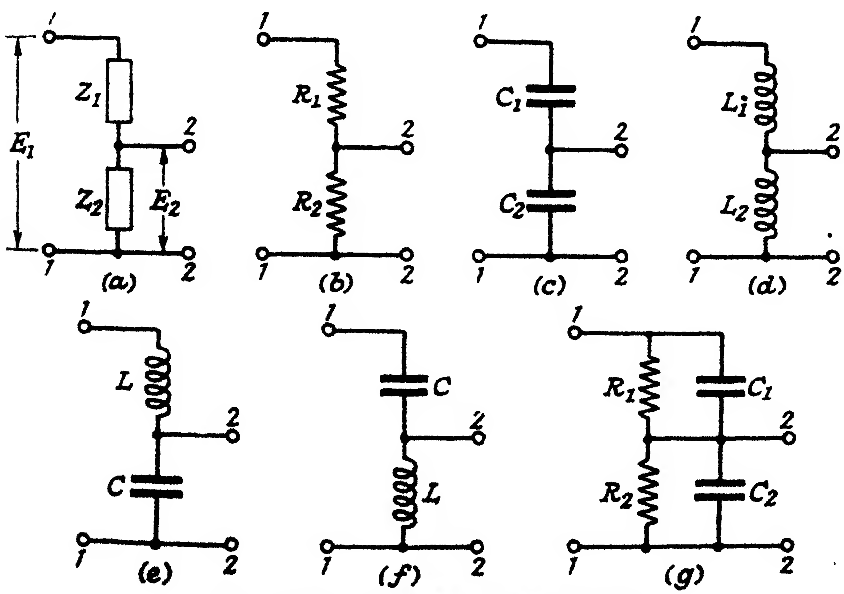


FIG. 1/V:19.—Impedance Potentiometers.

Fig. 1 shows a number of typical impedance potentiometers, the general case being shown at (*a*). In every case, if E_1 is the A.C. potential difference applied at terminals 1, 1, and E_2 is the resultant p.d. at 2, 2, then :

$$E_2 = \frac{Z_2}{Z_1 - Z_2} \times E_1$$

In case (*b*) $Z_1 = R_1$ and $Z_2 = R_2$

In case (*c*) $Z_1 = \frac{-j}{C_1\omega}$ and $Z_2 = \frac{-j}{C_2\omega}$, and so on.

Cases (*e*) and (*f*) are of some interest because they are the only ones in which E_2 may be, but will not necessarily be, greater than E_1 and they can therefore be used to obtain a voltage step-up when required. For instance, suppose that in case (*e*) the reactance of L is 100 ohms and the reactance of C is -75 ohms, then :

$$. \ E_2 = \frac{-j75}{j100 - j75} \times E_1 = -3E_1.$$

The minus sign indicates that E_2 is 180° out of phase with E_1, and this may be seen to be a case of lag. Since L has a greater reactance than C, the current entering 1, 1 lags 90° behind the voltage at 1, 1, and the resultant voltage across C lags 90° on the current.

In general, if the reactance of Z_2 is greater than the algebraic sum of the reactances of Z_1 and Z_2 (this is evidently possible only if the réactances of Z_1 and Z_2 are of opposite sign) E_2 is greater than E_1.

The rules governing the phase of E_2 with reference to E_1, in the case where Z_1 and Z_2 are pure reactances, are quite simple :

If Z_1 is less than Z_2 or if the reactances of Z_1 and Z_2 are of the same sign, E_2 is always in phase with E_1. If Z_1 is greater than Z_2 and of positive reactance, Z_2 having a negative reactance, E_2 lags 180° behind E_1. If Z_1 is greater than Z_2 and of negative reactance, Z_2 having a positive reactance, E_2 leads E_1 by 180°.

Case (*g*) is important because if $\frac{X_1}{X_2} = \frac{R_1}{R}$ (where X_1 is the reactance of C_1 and X_2 is the reactance of C_2) E_2 is in phase with E_1 at all frequencies, and the ratio of $\frac{E_2}{E_1}$ is independent of frequency. Practical applications occur where incidental capacities are introduced across one arm of a potentiometer. Phase shift and variation

of the potentiometer ratio with frequency can be eliminated by connecting an appropriate capacity across the other arm.

99. Numerical Examples.

Values of reactance in the examples below should be checked by referring to the reactance chart on Fig. 1/V:17.

1. What is the reactance X and the impedance Z of:
(a) 1 Henry at 1,000 c/s.
(b) 1 μH at 10^6 c/s = 1 Mc/s.
(c) 1 μF at 1,000 c/s.
(d) 1 $\mu\mu$F at 10^6 c/s = 1 Mc/s.

[A. (a) $X = L\omega = 2\pi \times 1{,}000 = 6{,}280\ \Omega$; $Z = j6{,}280\ \Omega$.
(b) $X = 10^{-6} \times 2\pi \times 10^6 = 6{\cdot}28\ \Omega$; $Z = j6{\cdot}28\ \Omega$.

(c) $Z = \dfrac{1}{jC\omega} = -j\dfrac{1}{C\omega} \therefore X = -\dfrac{1}{10^{-6} \times 2\pi \times 1{,}000} = -159\ \Omega$

$Z = -j159\ \Omega$.

(d) $X = \dfrac{-1}{10^{-12} \times 2\pi \times 10^6} = -159{,}000\ \Omega$.]

2. What is the reactance X of 5 Henrys at 1,000 c/s, 100 c/s, 10 kc/s?

(Refer to answer 1 (a) above.)

[A. At 1,000 c/s $X = 5 \times 6{,}280 = 31{,}400\ \Omega$.

At 100 c/s $X = 5 \times 6{,}280 \times \dfrac{100}{1{,}000}$ $3{,}140\ \Omega$.

At 10 kc/s $X = 5 \times 6{,}280 \times \dfrac{10{,}000}{1{,}000} = 314{,}000\ \Omega$.]

3. What is the reactance X of 300 μH at 1 Mc/s, 500 kc/s, 100 kc/s?

(Refer to answer 1 (b) above.)

[A. At 1 Mc/s $X = +300 \times 6{\cdot}28 = 1884\ \Omega$.

At 500 kc/s $X = +300 \times 6{\cdot}28 \times \dfrac{500{,}000}{1{,}000{,}000} = 942\ \Omega$.

At 100 kc/s $X = +300 \times 6{\cdot}28 \times \dfrac{100{,}000}{1{,}000{,}000} = 188{\cdot}4\ \Omega$.]

4. What is the reactance X of 0·5 μF at 1,000 c/s, 30 c/s and 10 kc/s?

(Refer to answer 1 (c) above.)

[A. At 1,000 c/s $X = -\dfrac{159}{0{\cdot}5} = -318\ \Omega$.

At 30 c/s $X = -\frac{159}{0{\cdot}5} \times \frac{1{,}000}{30} = -10{,}600\ \Omega.$

At 10 kc/s $X = \frac{159}{0{\cdot}5} \quad \frac{1{,}000}{10{,}000} = -31{\cdot}8\ \Omega.]$

5. What is the reactance X of 200 $\mu\mu$F at 1 Mc/s, 500 kc/s, 100 kc/s?

(Refer to answer 1 (*d*) above.)

[A. At 1 Mc/s $X = -\frac{159{,}000}{200} = -795\ \Omega.$

At 500 kc/s $X = -\frac{159{,}000}{200} \times \frac{1{,}000{,}000}{500{,}000} = -1{,}590\ \Omega.$

At 100 kc/s $X = -\frac{159{,}000}{200} \times \frac{1{,}000{,}000}{100{,}000} = 7{,}950\ \Omega.]$

6. What is the equivalent inductance of four inductances in parallel of value 1,000, 500, 250, 200 mH (millihenrys)?

[A. 100 mH.]

7. What is the equivalent capacity of four capacities in series of value 1,000, 500, 250 and 200 $\mu\mu$F?

[A. 100 $\mu\mu$F.]

8. What is the equivalent capacity of two condensers in series, each of 1 μF?

[A. 0·5 μF.]

9. What is the reactance X of 100 μH in series with 300 $\mu\mu$F at 1 Mc, 2 Mc, 900 kc/s and 500 kc/s?

[A. At 1 Mc $X = +628 - 530 = +98\ \Omega.$

At 2 Mc $X = +1{,}256 - 265 = +991\ \Omega.$

At 900 kc/s $X = +565 - 590 = -25\ \Omega.$

At 500 kc/s $X = +314 - 1{,}060 = -746\ \Omega.]$

10. What is the reactance X of 100 μH in parallel with 300 $\mu\mu$F at 1 Mc, 2 Mc, 900 kc/s and 500 kc/s? (Refer to Example 9.)

[A. At 1 Mc $X = \frac{1}{\frac{1}{628} - \frac{1}{530}} = \frac{628 \times -530}{628 - 530} = -3{,}400\ \Omega.$

At 2 Mc $X = \frac{1{,}256 \times -265}{1{,}256 - 265} = -336\ \Omega.$

At 900 kc/s $X = \frac{565 \times -590}{565 - 590} = +13{,}330\ \Omega.$

At 500 kc/s $X = \frac{314 \times -1{,}060}{314 - 1{,}060} = +445\ \Omega.]$

11. Express 100 $\Omega+j200\ \Omega$ as a resistance and reactance in parallel.

[A. $R_p = 100\left(1+\frac{200^2}{100^2}\right) = 500\ \Omega.$

$X_p = 200\left(1+\frac{100^2}{200^2}\right) = 250\ \Omega.$

$Z = 500\ \Omega//j250\ \Omega.$]

12. Express 4,000 $\Omega - j2{,}000\ \Omega$ in parallel representation.

[A. $R_p = 4{,}000\left(1+\frac{2{,}000^2}{4{,}000^2}\right) = 5{,}000\ \Omega.$

$X_p = 2{,}000\left(1+\frac{4{,}000^2}{2{,}000^2}\right) = 10{,}000\ \Omega.$

$Z = 5{,}000\ \Omega//-j10{,}000\ \Omega.$]

13. What is the parallel representation of 10 Henrys in series with 62,800 ohms at 1,000 c/s ?

[A. The reactance of 10 Henrys at 1,000 c/s is $10 \times 2\pi \times 1{,}000 = 62{,}800\ \Omega.$

$R_p = 62{,}800\ (1+1) = 125{,}600\ \Omega.$
$X_p = 62{,}800\ (1+1) = 125{,}600\ \Omega.$
$Z = 125{,}600\ \Omega//j125{,}600\ \Omega.$]

14. An A.C. generator with an internal impedance of 1,000 ohms which is a pure resistance and an open circuit voltage of 100 volts R.M.S. at a frequency of one Mc/s, is connected to a circuit consisting of an inductance of 1 mH in series with 100 $\mu\mu$F and a resistance of 500 Ω. What is the R.M.S. value of the current which flows ; and what is its phase relation to the e.m.f. in the generator ? What is the terminal voltage of the generator ?

[A. The reactance of 1 mH = 1 millihenry, at 1 Mc/s is 6,280 Ω ; the reactance of 100 $\mu\mu$F at 1 Mc/s is − 1,590 Ω ; the total series reactance is 6,280 − 1,590 = 4,090 Ω. The total series resistance is 1,000+500 = 1,500 Ω.

$$\text{The current} = \frac{100}{\sqrt{1{,}500^2+4{,}690^2}} = \frac{100}{4{,}920}$$
$$= 0{\cdot}0204 \text{ amps.} = 20{\cdot}4 \text{ mA.}$$

$$\text{The current phase angle} = \tan^{-1}\frac{4{,}690}{1{,}500} = 72^\circ\ 14'\ \text{lag.}$$

The terminal voltage of the generator is

$$100 \times \frac{500+j4{,}690}{1{,}500+j4{,}690} = 100\frac{4{,}720\angle 83^\circ\ 55'}{4{,}920\angle 72^\circ\ 14'}$$

i.e. the terminal voltage of the generator leads on the generator e.m.f. by 11° 41′.]

15. What impedance is presented by the following impedance combinations (impedances in ohms)?

(*a*) $1{,}000+j1{,}000$ in series with $2{,}000//-j2{,}000$.

(*b*) $1{,}000-j1{,}000$ in parallel with $2{,}000//j2{,}000$.

(*c*) $500+j500$ in series with $500-j500$.

(*d*) $500+j500$ in parallel with $500-j500$.

(*e*) $1{,}000//j1{,}000$ in parallel with $2{,}000//-j1{,}000$.

[A. (*a*) $2{,}000\ \Omega$. (*b*) $1{,}000\ \Omega$. (*c*) $1{,}000\ \Omega$. (*d*) $500\ \Omega$. (*e*) $666{\cdot}6\ \Omega$.]

16. What impedance is presented by the following impedance combinations (impedances in ohms)?

(*a*) $1{,}000\underline{/45^\circ}$ in parallel with $1{,}000\overline{\backslash 45^\circ}$.

(*b*) $1{,}414{\cdot}2\overline{\backslash 45^\circ}$ in series with $1{,}000-j1{,}000$.

(*c*) $1{,}414{\cdot}2\underline{/45^\circ}$ in parallel with $2{,}000//-j2{,}000$.

(*d*) $3{,}451\underline{/72^\circ\ 39'}$ in series with $7{,}022\underline{/72^\circ\ 39'}$.

(*e*) $1{,}000\underline{/60^\circ}$ in series with $1{,}000\underline{/30^\circ}$.

[A. (*a*) $707\ \Omega$. (*b*) $2{,}828{\cdot}2\overline{\backslash 45^\circ} = 2{,}000-j2{,}000$. (*c*) $1{,}000\ \Omega$. (*d*) $10{,}473\underline{/72^\circ\ 39'}$. (*e*) $(500+866)\ \Omega+j(866+500)\ \Omega = 1{,}930\underline{/45^\circ}$.

17. What impedance is presented by the following impedance combinations (impedances in ohms)?

(*a*) $1{,}570\underline{/47^\circ}$ in series with $800\overline{\backslash 60^\circ}$.

(*b*) $4{,}750\underline{/67^\circ}$ in parallel with $2{,}000\underline{/20^\circ}$.

(*c*) $500+j780$ in parallel with $700-j400$.

(*d*) $500+j780$ in parallel with $1{,}500\overline{\backslash 40^\circ}$.

(*e*) $465//j875$ in series with $785//-j480$.

[A. (*a*) $1{,}570\cos 47^\circ+800\cos 60^\circ+j(1{,}570\sin 47^\circ-800\sin 60^\circ)$

$$= 1{,}070+400+j(1{,}148-693) = 1{,}470\ \Omega+j455\ \Omega.$$

(*b*) $$\frac{1}{\dfrac{1}{4{,}750\underline{/67^\circ}}+\dfrac{1}{2{,}000\underline{/20^\circ}}} = \frac{1}{\dfrac{}{4}\overline{\backslash 67^\circ}+\dfrac{}{2{,}000}\overline{\backslash 20^\circ}}$$

$$= \frac{1}{0{\cdot}00021\cos 67^\circ+0{\cdot}0005\cos 20^\circ-j(0{\cdot}00021\sin 67^\circ+0{\cdot}0005\sin 20^\circ)}$$

$$= \frac{1}{0{\cdot}000082+0{\cdot}00047-j(0{\cdot}000193+0{\cdot}000179)}$$

$$\frac{1}{0{\cdot}000552 - j0{\cdot}000372} = \frac{10^6}{552 - j372}$$

$$= 10^6\left[\frac{552}{552^2+372^2} + j\frac{372}{552^2+372^2}\right]$$

$$= 1{,}250 + j843.$$

This calculation may be checked by an alternative method making use of equation (10)/III:3:

$$\frac{1}{\dfrac{1}{4{,}750\underline{/67^\circ}} + \dfrac{1}{2{,}000\underline{/20^\circ}}} = \frac{10^4}{2{\cdot}1\overline{\backslash 67^\circ} + 5\overline{\backslash 20^\circ}}$$

$$= \frac{10^4}{\sqrt{2{\cdot}1^2+5^2+2\times 5\times 2{\cdot}1\cos 47^\circ}\;\overline{\Big\backslash\, 20^\circ + \tan^{-1}\dfrac{2{\cdot}1\sin 47^\circ}{5+2{\cdot}1\cos 47^\circ}}}$$

$$= 1{,}515\underline{/33^\circ\ 27'} = 1{,}263 + j835,$$

which represents as good agreement as can be expected by slide-rule. The insertion of 20° under the angle sign is necessary because the vector chosen as a reference vector has an angle of 20° lag.

$$(c)\quad \frac{(500+j780)(700-j400)}{500+j780+700-j400}$$

$$= \frac{350{,}000+312{,}000+j(546{,}000-200{,}000)}{1{,}200+j380}$$

$$= \frac{662{,}000+j346{,}000}{1{,}200+j380} = \frac{747{,}000}{1{,}260}\underline{/27^\circ\ 37' - 17^\circ\ 35'}$$

$= 593\underline{/10^\circ}$ neglecting the 2′ which is not warranted by slide-rule accuracy.

$$(d)\quad 1{,}500\overline{\backslash 40^\circ} = 1{,}500\cos 40^\circ - j1{,}500\sin 40^\circ$$

$$= 1{,}150 - j965 = 1{,}960 /\!/ -j2{,}335$$

$$500+j780 = 1{,}710 /\!/ j1{,}100$$

The parallel representation of the two impedances in parallel is then:

$$\frac{1{,}960\times 1{,}710}{1{,}960+1{,}710} /\!/ \frac{-j2{,}335\times j1{,}100}{-j2{,}335+j1{,}100} = 913 /\!/ j2{,}075$$

The series representation is therefore: $765+j337$. This example is of particular interest since it introduces a new method of calculating the value of impedances in parallel: the impedances are represented in their parallel form, the values of the resistances in parallel and the reactances in parallel are calculated, and the resultant parallel representation is then converted to the series form

of representation. Compare the calculations in this example with those in example (*b*) above.

(*e*) $465/\!/j875 + 785/\!/-j480 = 362 + j193\cdot5 + 214 - j349$
$= 576 - j155\cdot5.$]

18. Two generators are connected to a common load of impedance $L/\underline{\phi}$. The open circuit voltage of generator 1 is e_1 and its internal impedance is $A/\underline{\alpha}$. The open circuit voltage of generator 2 is $e_2/\underline{\theta}$ at the same frequency and its internal impedance is $B/\underline{\beta}$. What is the magnitude of the current through the load, and its phase with regard to e_1.

[A. *Method* 1. By the principle of superposition each generator drives a current through the load of amplitude independent of the other generator. The impedance seen by generator 1 acting alone is $\dfrac{BL/\underline{\beta+\phi}}{B/\underline{\beta}+L/\underline{\phi}}$ and its terminal voltage on load is

$$E_1 = \frac{\dfrac{BL/\underline{\beta+\phi}}{B/\underline{\beta}+L/\underline{\phi}}}{A/\alpha + \dfrac{BL/\underline{\beta+\phi}}{B/\underline{\beta}+L/\underline{\phi}}}e_1$$

$$= \frac{BL/\underline{\beta+\phi}}{AB/\underline{\alpha+\beta}+AL/\underline{\alpha+\phi}+BL/\underline{\beta+\phi}}e_1$$

Similarly, the terminal voltage of generator 2 on load, when acting alone, is

$$E_2 = \frac{AL/\underline{\alpha+\phi}}{AB/\underline{\alpha+\beta}+AL/\underline{\alpha+\phi}+BL/\underline{\alpha+\phi}}e_2/\theta$$

The load current is then given by

$$\frac{E_1+E_2}{L/\underline{\phi}} = \frac{Ae_2/\underline{\alpha+\theta}+Be_1/\beta}{AB/\underline{\alpha+\beta}+AL/\underline{\alpha+\phi}+BL/\underline{\beta+\phi}}$$

Method 2. Apply Kirchhoff's laws and assume that generator 1 delivers current i_1 and generator 2 delivers current i_2 in such sense that the load current $= i_1 + i_2$. Equating the e.m.f.s round the two meshes to zero:

$$i_1A/\underline{\alpha} + (i_1+i_2)L/\underline{\phi} - e_1 = 0 \qquad . \quad . \quad (1)$$
$$i_2B/\underline{\beta} + (i_1+i_2)L/\underline{\phi} - e_2/\underline{\theta} = 0 \qquad . \quad . \quad (2)$$

Subtracting (2) from (1)

$$\therefore \; i_1A/\underline{\alpha} - i_2B/\underline{\beta} = e_1 - e_2/\underline{\theta}$$

$$\therefore \; i_1 = \frac{e_1 - e_2/\underline{\theta} + i_2B/\underline{\beta}}{A/\alpha} \qquad . \quad . \quad . \quad (3)$$

Substituting (3) in (1)

$$\therefore\ e_1 - e_2\underline{/\theta} - i_2B\underline{/\beta} + i_2L\underline{/\phi} + \frac{L}{A}\underline{/\phi - \alpha}(e_1 - e_2\underline{/\theta})$$

$$+\frac{BLi_2}{A}\underline{/\phi+\beta-\alpha} - e_1 = 0$$

$$\therefore\ i_2 = \frac{e_2\underline{/\theta} + \frac{L}{A}\underline{/\phi-\alpha}(e_1 - e_2\underline{/\theta})}{L\underline{/\phi} - B\underline{/\beta} + \frac{BL}{A}\underline{/\phi+\beta-\alpha}}$$

$$= \frac{Ae_2\underline{/\alpha+\theta} + L\underline{/\phi}(e_1 - e_2\underline{/\theta})}{AB\underline{/\alpha+\beta} + AL\underline{/\alpha+\phi} + BL\underline{/\beta+\phi}}$$

Similarly

$$i_1 = \frac{Be_1\underline{/\beta} + L\underline{/\phi}(e_2\underline{/\theta} - e_1)}{AB\underline{/\alpha+\beta} + AL\underline{/\alpha+\phi} + BL\underline{/\beta+\phi}}$$

The load current is then given by

$$i_1 + i_2 = \frac{Ae_2\underline{/\alpha+\theta} + Be_1\underline{/\beta}}{AB\underline{/\alpha+\beta} + AL\underline{/\alpha+\phi} + BL\underline{/\beta+\phi}}.\Big]$$

19. Two A.C. generators supply sinusoidal current at the same frequency to a common load through resistances or attenuators which mask the impedances of each generator so that disconnecting either generator does not change the load facing the other generator. Each generator is adjusted to supply equal current to the load, and then both generators are connected to the load. If the ratio: voltage magnitude across the load with both generators operating divided by the voltage magnitude with either generator driving the load alone: is r, what is the phase difference between the individual load voltages supplied by each generator?

[A. Let the load voltage magnitude with one generator be e, and with both generators be E, and let ϕ be the phase difference between the individual voltages supplied by each generator.

Then $$E = 2e\cos\frac{\phi}{2}$$ (Prove by drawing vector diagram)

$$\therefore\ \phi = 2\cos^{-1}\frac{E}{2e}$$

$$= 2\cos^{-1}\frac{r}{2}.\Big]$$

CHAPTER VI

SELECTIVE CIRCUITS: RESONANCE AND TUNING

Conventions.

E = e.m.f. in volts.
I = Current in amperes.
L = Inductance in Henrys.
C = Capacity in Farads.
R = Resistance in Ohms.
Z = Impedance.
Z_r = Impedance of parallel resonant circuit at resonant frequency.
f_s = Resonant frequency of series resonant circuit.
f_r = Resonant frequency of parallel resonant circuit.
f_n = Natural frequency of resonant circuit.
Q_s = Value of Q at series resonant frequency.
Q_r = Value of Q at parallel resonant frequency.
Q_n = Value of Q at natural frequency.
$\omega_s = 2\pi f_s$; $\omega_r = 2\pi f_r$; $\omega_n = 2\pi f_n$.

The discussion below distinguishes three frequencies associated with resonant circuits: f_s, f_r and f_n. Of these, f_s and f_n apply to every series resonant circuit as described in section 1 immediately following; f_r applies to every parallel resonant circuit as described in section 2, while, when this circuit is oscillating as a closed resonant circuit, so that it behaves as a series resonant circuit, f_n and f_s also apply and have the values obtained by considering the series path.

While the distinction between these three frequencies is important, in practice, for any given set of values of L, C and R, they are usually so nearly equal that, for most computations, the distinction can be neglected.

The relation between these three frequencies depends on the variation of Q with frequency, but they are usually so close together that $Q_s = Q_r = Q_n = Q$, say.

It is shown below that under this condition

$$f_s = f_r\sqrt{1+\frac{1}{Q^2}} = \frac{f_n}{\sqrt{1-\frac{1}{4Q^2}}}$$

If $Q = 5$, for instance,

$$f_0 = 1{\cdot}0198\, f_r = 0{\cdot}994987\, f_n$$

and as Q is increased the three frequencies rapidly approach one another still more closely. Since values of Q below 5 seldom occur, the error involved in assuming the three frequencies to be the same is normally negligible.

In general, any network of reactances and resistances is said to be in resonance at any pair of terminals constituting a driving point, when the impedance at those terminals is a pure resistance. The resonant frequency of a two-terminal network is therefore the frequency at which the driving-point impedance is free from reactance. A network containing only one inductance and one capacity has only one resonant frequency. In general, as the number of reactances are increased indefinitely, the number of resonant frequencies increases without limit. We are here concerned only with networks containing one inductance and one capacity, and having only one resonant frequency.

The natural frequency of a network is the frequency at which it oscillates when subjected to any impulse such as may be supplied by connecting a battery to the driving-point terminals. We are here concerned only with networks which have one natural frequency.

1. Series Resonance.

Consider a simple series circuit consisting of an inductance L, a capacity C and a resistance R in series with an alternating e.m.f. E of frequency $f = \frac{\omega}{2\pi}$.

From equation (1) of V:4, the current through the circuit is:

$$\frac{E}{R + j\left(L\omega - \frac{1}{C\omega}\right)} \qquad (1)$$

from which it is evident that the current is zero at zero frequency and at infinite frequency, and rises to a maximum at the frequency at which $L\omega = \frac{1}{C\omega}$ *when the reactance term disappears.*

The current which flows therefore varies with frequency in the way shown in Fig. 1.

The frequency at which maximum current occurs is called the *resonant* frequency, the condition when the frequency is such that the maximum current flows is called *resonance*, and the curve Fig. 1

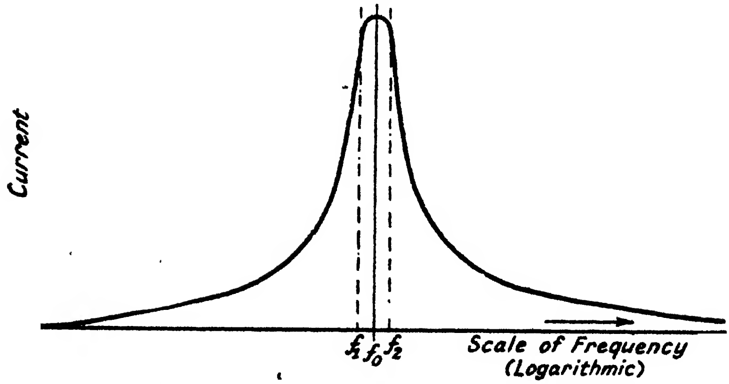


FIG. 1/VI:1.—Resonance Curve : Response Curve of a Resonant Circuit.

is called a *resonance* curve. The process of adjusting a circuit to resonance at any required frequency is called *tuning* ; this may be done by adjusting the magnitude of either L or C or both, until $L\omega = \frac{1}{C\omega}$ as shown by the fact that the current in the circuit is at maximum.

It is evident that, while the response of the circuit is a maximum at the resonant frequency, the response at frequencies in the immediate neighbourhood of resonance falls away slowly so that it would, for instance, be possible to pass a band of frequencies through the circuit extending from f_1 to f_2, see Fig. 1, with a small percentage difference in amplitude. Such a circuit can be used to select a band of frequencies such, for instance, as is constituted by a modulated carrier-wave carrying programme. As will appear later, however, this form of circuit is only one of many which are used. See XXV.

Resonant Frequency of Inductance L and Capacity C. The value of ω at the resonant frequency (which is called f_0) is called ω_0. So that

$$L\omega_0 = \frac{1}{C\omega_0}$$

$$\omega_0 = \frac{}{\sqrt{LC}}$$

and

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad (2)$$

The sharpness of resonance usually depends on the ratio of L to C and the value of $\frac{L\omega_0}{R}$ where R is the resistance of the coil ;

this ratio is usually called Q, i.e.

$$Q = \frac{L\omega_0}{R}$$

In a series resonant circuit the greater the value of $\frac{L}{C}$ and Q the sharper the resonance curve and the greater the selectivity; but as selectivity is increased in this way the ends of a fixed band of frequencies located symmetrically in the resonance curve suffer increased loss of response. (See VI:2.5.) If this band of frequencies is carrying intelligence, loss of response in this way degrades the fidelity with which the intelligence is transmitted.

When such a circuit is used to select a band of frequencies, a compromise has to be effected between the requirements of selectivity and fidelity.

2. Parallel Resonance.

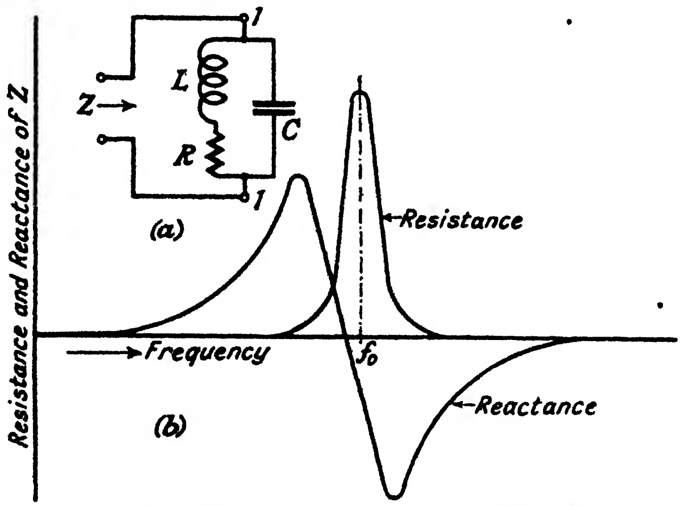


Fig. 1/VI:2.—Variation with Frequency of the Resistance and Reactance Components of the Impedance of a Parallel Resonant Circuit.

Fig. 1 shows at (*a*) another type of resonance circuit sometimes called a parallel resonance circuit and sometimes called an anti-resonant circuit because at resonance the impedance looking into terminals 1,1 goes to a maximum. The resistance represents the resistance of the coil constituting the inductance L. Strictly, a resistance should also be shown in series with the condenser, but usually this resistance is small enough to be neglected.

The impedance looking into terminals 1,1 is given by

$$Z = \frac{1}{\dfrac{1}{R+jL\omega}+jC\omega} = \frac{R+jL\omega}{1 - LC\omega^2+jRC\omega} \quad . \quad . \quad . \quad (1)$$

$$= \frac{(R+jL\omega)(1 - LC\omega^2 - jRC\omega)}{(1 - LC\omega^2)^2+R^2C^2\omega^2}$$

$$= \frac{R(1 - LC\omega^2)+RLC\omega^2+j[L\omega(1 - LC\omega^2) - R^2C\omega]}{(1 - LC\omega^2)^2+R^2C^2\omega^2}$$

$$= \frac{R+j\omega[L - C(R^2+L^2\omega^2)]}{(1 - LC\omega^2)^2+R^2C^2\omega^2} = A+jB, \text{ say} \quad . \quad . \quad (2)$$

The curves labelled Resistance and Reactance in Fig. 1 (*b*) show the way in which the quantities A and B vary with frequency.

Designate by ω_0 the value of ω at which the magnitude of the reactance $L\omega$ of the inductance is equal to the magnitude $\dfrac{1}{C\omega}$ of the capacity.

Then

$$L\omega_0 = \frac{1}{C\omega_0}, \ \omega_0 = \frac{1}{\sqrt{LC}} \text{ and } C = \frac{1}{L\omega_0^2} \quad . \quad . \quad (3)$$

Denote the ratio $\dfrac{L\omega_0}{R}$ by Q_0, that is

$$Q_0 = \frac{L\omega_0}{R} \text{ and } R = \frac{L\omega_0}{Q_0} \quad . \quad . \quad . \quad (4)$$

In XXIV:2.23 it is shown that by substituting the value of C from (3) and of R from (4) in (2)

$$Z = L\omega_0\left[\frac{\dfrac{1}{Q_0}}{\left(1 - \dfrac{\omega^2}{\omega_0^2}\right)^2+\dfrac{1}{Q_0^2}\dfrac{\omega^2}{\omega_0^2}} + j\frac{\left(1 - \dfrac{1}{Q_0^2} - \dfrac{\omega^2}{\omega_0^2}\right)\dfrac{\omega}{\omega_0}}{\left(1 - \dfrac{\omega^2}{\omega_0^2}\right)^2+\dfrac{1}{Q_0^2}\dfrac{\omega^2}{\omega_0^2}}\right] \quad . \quad (5)$$

This expression for Z should always be used for evaluating Z as it is very much simpler for numerical calculation. Note that $\dfrac{\omega}{\omega_0} = \dfrac{f}{f_0}$. See also XXI:5.

When $\omega = \omega_0$, the impedance obtained by substituting the value of ω in (5) is

$$Z = QL\omega_0 - jL\omega_0 \quad . \quad . \quad . \quad (6)$$

2.1. Condition at Parallel Resonance. Parallel resonance occurs at the frequency at which the reactance of the condenser C is equal to the equivalent parallel reactance (see V:16) of R and L

in series. The parallel tuned circuits in transmitters and receivers are normally adjusted, by the process of tuning, so that the circuit impedance is a maximum: this produces the same condition (see VI:2.2).

The impedance looking into the circuit at the resonant frequency is evidently a pure resistance of magnitude equal to the equivalent parallel resistance of L and R in series.

Conventions.

f_r = resonant frequency.

$\omega_r = 2\pi f_r$.

$Q_r = \dfrac{L\omega_r}{R}$.

Z_r = impedance at resonant frequency.

Then

$$Z_r = R\left(1+\frac{L^2\omega_r^2}{R^2}\right) = R+\frac{L^2\omega_r^2}{R}$$

$$= \frac{L\omega_r}{Q_r}+Q_rL\omega_r$$

$$= L\omega_r\left(Q_r+\frac{1}{Q_r}\right) \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Note that

$$\frac{R}{2\pi L} = \frac{f_r}{Q_r} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

An important quantity in a high-voltage circuit is the kVA to kW ratio. This is the product of the kilovolts across the condenser $C \times$ by the R.M.S. current through C and divided by the power supplied to the circuit. See VII:14.1.

Hence the kVA/kW ratio

$$= \frac{Z_r}{\dfrac{1}{C\omega_r}} = \frac{L\omega_r\left(Q_r+\dfrac{1}{Q_r}\right)}{L\omega_r\left(1+\dfrac{1}{Q_r^2}\right)} = Q_r \quad . \quad . \quad . \quad (8a)$$

It is usual to assume that the impedance at resonance is $QL\omega_r$, and the error consequent on this assumption is therefore

$$\frac{\dfrac{1}{Q_r}}{Q_r+\dfrac{1}{Q_r}} \times 100 = \frac{100}{Q_r^2+1} \text{ per cent} \quad . \quad . \quad (9)$$

e.g. if $Q = 5$, the answer is 3·84% low. As Q increases the error reduces approximately as Q^2.

2.11. Relation between f_r and f_0. f_0 and f_r are here considered to apply respectively to a series resonant circuit and a parallel resonant circuit with the configuration of Fig. 1/VI:2, both circuits having the same value of L, C and R. Equating the reactance of the condenser to the equivalent parallel reactance of L and R:

$$\frac{1}{C\omega_r} = L\omega_r\left(1+\frac{R^2}{L^2\omega_r^2}\right)$$

$$\therefore \quad C = \frac{1}{L\omega_r^2+\dfrac{R^2}{L}}$$

Then
$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi}\sqrt{\omega_r^2+\frac{R^2}{L^2}}$$

$$= \sqrt{f_r^2+\frac{R^2}{4\pi^2L^2}} = \sqrt{f_r^2+\frac{f_r^2}{Q_r^2}} \text{ : (from (8))}$$

$$\therefore \quad f_0 = f_r\sqrt{1+\frac{1}{Q_r^2}} \quad . \quad . \quad . \quad (10)$$

2.2. Conditions for Maximum Impedance of a Parallel Resonant Circuit. Four different conditions for the maximum value of impedance exist, depending on which of the quantities L, C, R and f is made variable. Normally only one of these is of interest: that in which C is varied. This has been considered under VI:2.1 above. It is evident that maximum impedance will occur when the reactance of C is equal to the equivalent parallel reactance of L and R. This is because the parallel reactance will then be infinity, and the impedance of any resistance shunted by any finite reactance is always less than that of the resistance alone. The value of C for maximum impedance is

$$C = \frac{1}{L\omega^2\left(1+\dfrac{R^2}{L^2\omega^2}\right)} \quad . \quad . \quad . \quad (11)$$

Similarly it may be shown that if the frequency f is varied, L, C and R being constant, maximum impedance will occur at a frequency

$$f = \sqrt{\frac{\sqrt{1+\dfrac{2CR^2}{L}}}{LC}-\frac{R^2}{L^2}} \quad . \quad . \quad . \quad (12)$$

The conditions for the variation of L and R are more complicated, but only one is normally of interest: the value of R to give maximum impedance at the resonant frequency f_r. Since the magnitude of this impedance is $QL\omega_r = \frac{L\omega_r^2}{R}$, and changing R does not change ω_r, it is evident that, for maximum impedance, R should be as small as possible.

2.3. Natural Frequency. The natural frequency of a closed inductance capacity circuit is the frequency at which it oscillates freely after being excited by a short impulse. A detailed consideration of the natural frequency is given in VI:5, while the mechanism of oscillation is described in XI:1.

The natural frequency of a resonant circuit is given by

$$= \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = \sqrt{\frac{1}{4\pi^2 LC} - \frac{R^2}{16\pi^2 L^2}} \qquad (13)$$

$$= f_0\sqrt{1 - \frac{1}{4Q_n^2}} \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

where $Q_n = \frac{L\omega_n}{R}$, $\omega_n = 2\pi f_n$ so that $\frac{R}{2\pi L}$ $\frac{f_n}{Q_n}$.

Hence, when $Q_n = 5$, $f_n = 0{\cdot}995\, f_0$.

As Q_n is increased the natural frequency progressively approaches the resonant frequency.

2.4. Practical Application of Parallel Resonant Circuit.

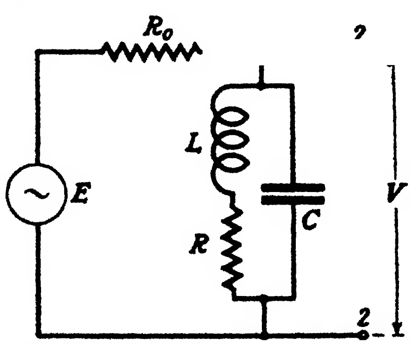


FIG. 2/VI:2.—Common Method of Use of a Parallel Resonant Circuit.

The circuit of Fig. 1 (*a*) is usually employed as a selective circuit in the form shown in Fig. 2, where it is used to supply at 2,2, a band of frequencies in the neighbourhood of the resonance frequency of the parallel resonant circuit.

Evidently $$V \doteq \frac{Z}{Z+R_0} E \qquad (15)$$

where Z is defined by equations (1), (2) or (5).

If, as is always the case in practice, at frequencies away from resonance the value of Z falls to values much lower than R_0, it is evident that the response of such a circuit is similar to that of Fig. 1/VI:1, though not of exactly the same form. R_0 here may be considered to represent the internal impedance of a generator such as a valve.

The arrangement of Fig. 2 is one of the most common circuits in use.

The practical use of tuned and coupled tuned circuits is described in VII:13 and CII to which reference should be made. See also X:22 and following sections for behaviour of class C amplifiers with tuned anode circuits.

2.5. Selectivity and Response. Two conflicting requirements determine the design requirements of simple tuned circuits.

Selectivity is the term used to describe the amount of discrimination afforded by a tuned circuit between a wanted frequency and an unwanted frequency. See XIX:19.4.

Response is the term used to describe the amount by which the tuned circuit introduces differences in amplitude of a received band of wanted frequencies, usually constituted by a carrier and its sidebands. Evidently such differences are undesirable. See VIII:3.

2.51. Selectivity. A number of attempts have been made to express the selectivity of a selective circuit in general terms which can serve as a basis of comparison between different selective circuits. For purpose of general comparison the simplest criterion to adopt is to calculate the output voltage or current at the frequency to which the circuit is tuned and at twice this frequency for equal applied voltages at each frequency. The ratio of the voltages or currents at the two frequencies then gives a useful measure of selectivity.

To obtain a measure of performance in a given practical case, since the selective circuit is usually tuned to a wanted carrier frequency, the most useful thing to do is to take the ratio of output voltage at the frequency to which the circuit is tuned to the output voltage at the nearest interfering frequency, assuming equal input voltages at each frequency.

The ratio of voltages at wanted and unwanted frequencies may be expressed in decibels if required. See VII:5.

Series Tuned Circuit. This is normally of little importance. Its selectivity is given as a ratio by $\frac{|Z_u|}{R}$ and in decibels by $20 \log_{10} \frac{|Z_u|}{R}$, where Z_u is the impedance of the circuit at the unwanted frequency and R is the series resistance in the circuit. It follows that to increase the selectivity of a series circuit the ratio $\frac{L\omega}{R}$ must be increased.

Parallel Tuned Circuit. This is normally used in the circuit of Fig. 2 and the selectivity is given by the ratio of $\frac{V}{E}$ at the wanted frequency to $\frac{V}{E}$ at the unwanted frequency.

Examination of equations (5) and (15) shows that the selectivity will be a maximum when the Q of the circuit at the frequency to which the circuit is tuned is as large as possible, and when the value of L is as small as possible.

The selectivity ratio is:

$$\frac{\dfrac{V}{E}\text{ at wanted } f}{\dfrac{V}{E}\text{ at unwanted } f} = \frac{QL\omega}{R_0+QL\omega} \times \frac{R_0+Z_u}{Z_u}$$

where Z_u is the impedance of the parallel tuned circuit at the unwanted frequency as given by equation (5).

It is evident that the complete selectivity of a tuned circuit may be represented by plotting the selectivity ratio at all frequencies in a frequency range extending each side of the frequency to which the circuit is tuned.

2.52. Response. The response ratio of a tuned circuit at any chosen frequency is the ratio of the output current or voltage of the tuned circuit at the frequency to which the circuit is tuned to the output voltage or current at the chosen frequency. The complete response curve of a tuned circuit may therefore be plotted as the reciprocal of the ordinates of the selectivity curve.

It follows that a high selectivity ratio at any frequency corresponds to a small response ratio at that frequency. Further, as the selectivity ratio at any frequency, remote from the tuning frequency, is increased, the response ratios at frequencies near the tuning frequencies are reduced. Since the ideal response ratio is normally unity it follows that *increasing the selectivity of a tuned circuit degrades the response.*

2.6. Choice of Values of Elements in Tuned Circuits. This refers to parallel tuned circuits used in the anodes of valves in the way indicated in Fig. 2.

In radio transmitters and receivers the values of the condensers are determined at low frequencies by the largest variable condensers which can be economically constructed or are commercially available (about 1,000 $\mu\mu$F) and at high frequencies by the valve capacities and stray circuit capacities. At high frequencies the tuning condenser is made as small as possible consistent with a reasonable tuning range being obtained. This is to obtain an adequate impedance looking into the tuned circuit. At frequencies near 20 Mc/s the variable tuning condensers may have maximum values of about 50 $\mu\mu$F, and lower still at higher frequencies. The inductances are then made of values suitable to tune with the capacities over chosen bands of frequency.

The Q of the coils used in radio receivers is made as high as possible.

In radio transmitters the overall Q of the circuit is equal to the kVA to kW ratio which is usually adjusted to about 5 (see VII:14) in medium- and long-wave transmitters, but may be as high as 15 in short-wave transmitters. Higher values of Q in medium- and long-wave transmitters would degrade the response too much, and give rise to too large values of (circulating) current through the inductance and capacity. Lower values of Q would mean that, in order to obtain a high enough impedance at the driving (carrier) frequency, the reactance of L and C would have to be increased to such a value that the reactance at twice carrier frequency would be appreciable compared to the reactance at the carrier frequency. This is undesirable, see X:22.

It is evident that if the value of C is fixed from considerations of valve capacities and stray capacities, the impedance of the tuned circuit depends on Q. It follows that in short-wave transmitters a value of Q or kVA/kW ratio higher than 5 may be required. Fortunately this does not matter since, for a given value of Q and L to C ratio, the fall in response of any frequency depends on its distance from the carrier expressed as a percentage of the carrier frequency. The consequence is that for a given Q and L to C ratio the fall in response of a frequency situated a *fixed* distance from the carrier frequency becomes less as the carrier frequency is increased.

It will be clear that in radio transmitters in the medium-wave range, where no other limitation on condenser size operates, the condenser and inductance should have such a value that, with a

circuit Q of 5, the correct impedance is offered to the valve anode.

In this discussion the resistance which determines the circuit Q is substantially constituted by *referred* resistance introduced into the circuit by inductive coupling (see VII:13.5), and is therefore in effect the ultimate load resistance to which power is supplied by the valve. The immediate load resistance is, of course, the impedance facing the valve anode: this is, the impedance which determines the valve performance.

3. Rejector and Acceptor Circuits.

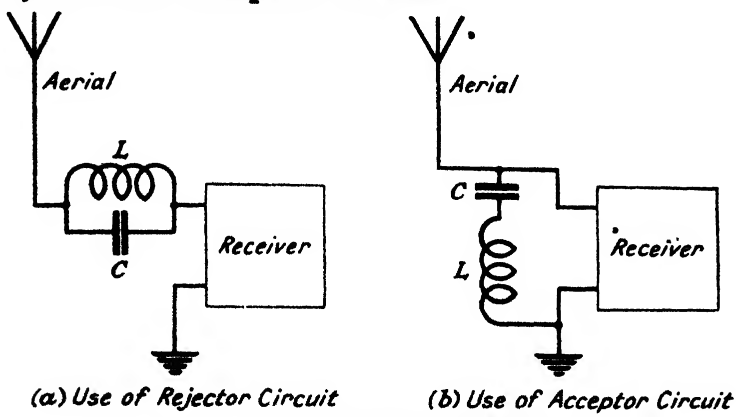


FIG. 1/VI:3.—Use of Rejector and Acceptor Circuits.

In Fig. 1 at (*a*) is shown a parallel arrangement of inductance and condenser in series between an aerial and a wireless receiving set. If the circuit is tuned to any unwanted frequency (i.e. if $L\omega = \frac{1}{C\omega}$ where $f = \frac{\omega}{2\pi}$ is the unwanted frequency) it is evident that the resonant circuit presents a high impedance to the unwanted frequency and a much lower impedance to frequencies away from resonance. In the medium-wave band, by using a small value of inductance, e.g. a few microhenrys, tuned by its appropriate condenser, an unwanted frequency within 100 kilocycles of the wanted frequency can be cut out if the ratio of field strength of wanted and unwanted frequencies is not too great.

In Fig. 1 at (*b*) is shown a series resonant circuit in shunt across the input to a receiver. This is called an acceptor circuit and in the arrangement shown performs the same function as the rejector circuit of Fig. 1 (*a*), in that at the unwanted frequency it constitutes a low impedance bypass or shunt across the input to the receiver.

Unwanted frequencies therefore go through the shunt instead of into the receiver.

For design of rejector and acceptor circuits, see XVIII:5.22.14 and XXV:6 and examples at the end of Chapter VI.

4. Other Types of Selective Circuit.

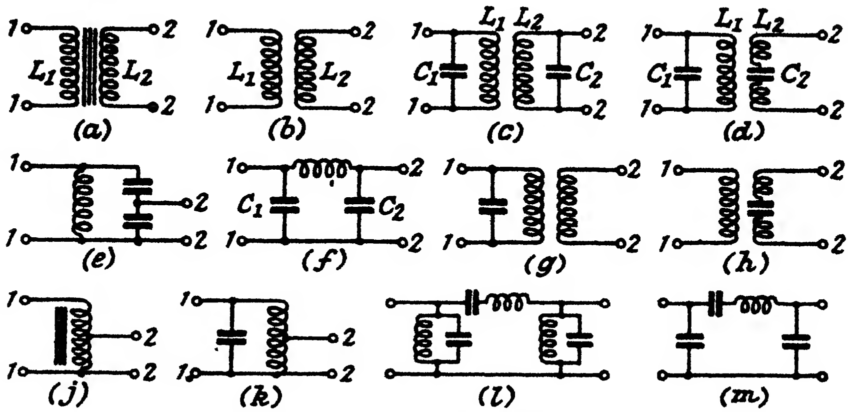


FIG. 1/VI:4.—Band Pass Circuits.

Fig. 1 shows a number of other types of selective circuit.

These have the property of passing bands of frequency, and are therefore called band-pass circuits. The circuits (*a*) and (*b*) represent respectively an iron-cored and an air-cored transformer, while that at (*j*) represents an iron-cored auto-transformer; see VII:11.

The remainder of the circuits are members of a class of circuit called Filters. The general design of Filters is described in XXV, while the design of the circuits shown at *C*, *D*, *E* and *F* is described in VII:14.

5. Decay Factor, Decay Index, Decrement, *Q*, and Natural Frequency of a Circuit.

If in the series resonant circuit of Fig. 1 switch *S* is closed, the resultant current which flows depends on the resistance and the *L* to *C* ratio of the circuit.

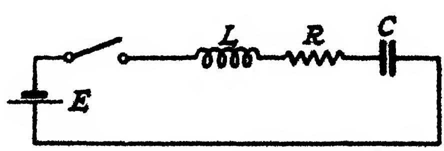


FIG. 1/VI:5.—Circuit to Illustrate Decrement.

5.1. Case 1. When $\frac{1}{LC} > \frac{R^2}{4L^2}$ that is when $Q_0 > 0{\cdot}5$.

In this case the current is defined by

$$i = -\frac{E}{2\pi f L} e^{-\frac{R}{2L}t} \sin 2\pi f_n t \qquad (1)$$

where e is the base of Naperian logarithms and

$$f_n = -\frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \quad . \qquad (2)$$

This is a sine wave of initial amplitude $\frac{E}{2\pi f L}$ and frequency f, which diminishes in amplitude with time in accordance with the magnitude of the factor $e^{-\frac{R}{2L}t}$.

The frequency f_n is called the *Natural Frequency* of the circuit and evidently differs from the resonant frequency which is given by

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \qquad (2a)$$

The quantity $e^{-\frac{R}{2L}t}$ may conveniently be termed the *Decay Factor* while the quantity $\frac{R}{2L}$ may be called the *Decay Index*.

5.11. Decrement. Since the time displacement t between two successive cycles of the decaying wave is $1/f_n$, the ratio between the amplitude of two successive waves is

$$r = e^{-\frac{R}{2L}t} \quad \epsilon^{-d} \quad . \quad . \quad . \quad . \qquad (3)$$

d is then called the ***decrement*** of the circuit so that

$$d = \frac{R}{2f_n L} \qquad (4)$$

Unless Q is less than 5 it is safe to assume that the natural frequency is equal to the resonant frequency, in which case the decrement is

$$d = \frac{\pi}{Q} \quad . \quad . \quad . \quad . \qquad (5)$$

See equations (10) and (14)/VI:2.

5.2. Measurement of Q, Decrement, Decay Factor and Decay Index of a Coil. Assuming that the resonant frequency is sufficiently near the natural frequency, it is only necessary to measure one of these quantities in order for the other three to be determined.

Method 1. Applicable to circuits containing condensers of negligible loss where currents of sufficient magnitude to be measured can be passed through circuit.

An e.m.f. of the required frequency is introduced in series with the coil and a variable condenser which must either be calibrated or be marked with graduations so that the capacity at any setting can be measured.

The circuit is tuned to resonance by adjustment of the condenser. Call the value of the capacity at resonance C_r. The current I_r at resonance is noted.

The condenser is then varied above and below C_r to the values at which the current is $0{\cdot}707 I_r$. Let C_1 be the lower value of capacity so obtained and C_2 the higher value of capacity.

Then, if L is the inductance of the coil,

$$1/\omega C_1 - \omega L = R \quad \text{and} \quad \omega L - 1/\omega C_2 = R$$

Hence $$R = 1/2\omega C_1 \quad 1/2\omega C_2 = \frac{C_2 - C_1}{2\omega C_1 C_2}$$

further, when the Q of the coil is large, C_r is very nearly equal to $\sqrt{C_1 C_2}$, so that

$$R = \frac{C_2 - C_1}{2\omega C_r^2} \tag{6}$$

and $$\frac{C_2 - C_1}{2C_r} = R\omega C_r = \frac{R}{\omega L} = \frac{1}{Q} \tag{7}$$

It is important to note that the means for inducing the e.m.f. and the means for measuring the current flowing must together introduce into the circuit a total resistance which is negligible in comparison with the resistance R to be measured. In practice it is difficult to be certain that such a condition has been realized and the circuit of Fig. 2 is so designed that the errors due to these causes are eliminated.

Method 2. *Bridge Measurement.* This consists in measuring separately on an R.F. bridge the inductance and resistance of the circuit. Such a measurement gives directly the Q of the circuit at the frequency of measurement. If this measurement is made at the resonant frequency of the circuit, and if the natural frequency, calculated from the values of R and L so obtained and the value of C for resonance, is near to the resonant frequency, R and L can also be used for the calculation of decrement, etc., as above. If the natural frequency so obtained differs appreciably from the resonant frequency (i.e. if $Q << 5$), then the true natural frequency must be obtained by a series of approximations. Measurement of

L and R is made at the natural frequency so determined and a new value of natural frequency is determined. Measurement of R and L is made at the new natural frequency and a third value of natural frequency is found. This process is continued until the difference between successive values of natural frequency is sufficiently small. The final values of R and L are then used to calculate the value of the decrement, decay index and decay factor per second. Such a procedure is, however, not likely to have much practical application.

Each measurement of L and R is made with the circuit constants set at the values which they are to have when the circuit is in use. Under this condition the circuit will have zero reactance at the resonant frequency, and the resistance R will, in general, be too small to be measured directly on a normal R.F. bridge.

To produce an impedance which can be conveniently measured on an R.F. bridge a lossless condenser is placed in series with the circuit to introduce an extra reactance, or if a coil alone is being measured, to cancel its inductance partially. In measurements made at frequencies below resonance in the neighbourhood of the natural frequency there will be an inherent circuit reactance given by the difference between the reactances of the circuit inductance and capacity. The reactance will be capacitative and will therefore add to the extra reactance, giving a total reactance X.

The impedance measured by the bridge is therefore $R - jX$ and will be expressed by the bridge in terms of the equivalent parallel circuit; a resistance R_p in parallel with a reactance X_p. The value of R is then given by

$$R = \frac{R_p}{1+R_p^2/X_p^2} \tag{8}$$

As a guide to the value of external reactance to add in order to bring R_p and X_p within the range of the bridge assuming the value of R to have been guessed at R_g, the expected values of R_p and X_p are given by

$$R_p = R_g(1+X^2/R_g^2) \quad X_p = X(1+R_g^2/X^2) \quad . \quad . \tag{9}$$

The value of L can be measured directly at each frequency.

Method 3. Fig. 2 shows a circuit which operates on somewhat the same principle as a device put on the market by some manu-

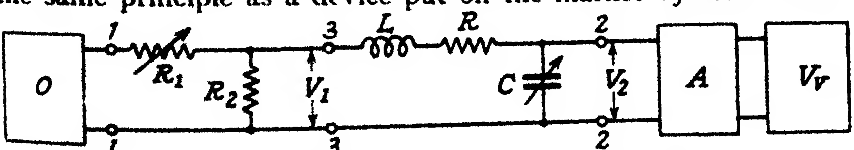


FIG. 2/VI:5.—Circuit for Measurement of Q.

facturers, and called by them a Q meter. O is an oscillator supplying current through resistance R_2 which has a value of about 0·1 ohm. L,R is the coil under measurement and C is a variable condenser. A is an amplifier having an input circuit of which the impedance is very high compared with the reactance of L, and therefore compared with the reactance of C, when tuned to resonance with L.

With the circuit in the configuration shown in Fig. 2, C is adjusted until V_2 is a maximum. The value of V_2 is then observed. By connecting the input of the amplifier across terminals 3,3, the value of V_1 is then measured. The Q of the coil is then equal to V_2/V_1. This should be sufficiently obvious for a proof to be unnecessary.

In practice, since absolute methods of measuring voltage are not always available, and since signal generators with adjustable outputs of known level are available, a slight modification of the above procedure is often adopted. The oscillator is replaced by a signal generator, resistance R_1 is omitted, and the amplifier and valve voltmeter are replaced by a radio receiver fed through a very small capacity direct to the grid of the first signal frequency valve. A micro-ammeter is connected in the diode load of the receiver to serve as an indicating device.

Signal is then applied and the receiver tuned, condenser C is adjusted to give maximum deflection on the micro-ammeter and the deflection noted. The output of the signal generator is also noted ; call this V_1. The input of the receiver is then transferred to point 3,3, the series coupling condenser being of course retained, and the output from the signal generator is adjusted until the same deflection is obtained on the micro-ammeter as before. The output of the signal generator is then observed ; call this V_2. The value of Q is then given by V_2/V_1. In case the absolute output level of the signal generator is not given, but the number of decibels difference in level between V_1 and V_2 is known, the value of Q is evidently equal to the voltage ratio corresponding to this decibel difference.

Q Meter Circuit. For comparison with the above method, a Q meter circuit is shown in Fig. 3/VI:5.

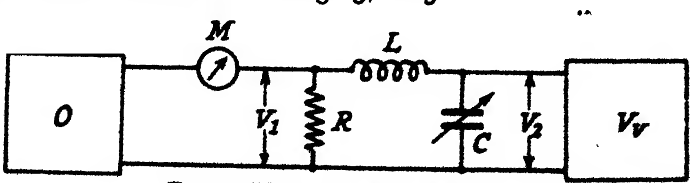


FIG. 3/VI:5.—Q Meter Circuit.

O is an oscillator which passes a known current through resistance R. At resonance $Q = V_2/V_1$. If, for instance, the current is a quarter of an ampere, and $R = 0{\cdot}04$ ohms, then $V_1 = 0{\cdot}01$ volts and $Q = 100V_2$.

This method has the advantage of simplicity and direct reading, and provided the voltmeter is calibrated directly from reading of the voltage developed across R by the current through the milliammeter M, no serious error is likely to be introduced by the current measurement.

For low-inductance coils of high Q a correction must be made to allow for the shunt introduced across R by the tuned circuit.

Case 2. When $\frac{1}{LC} < \frac{R^2}{4L^2}$ that is when $Q_0 < 0{\cdot}5$.

In this case the current is defined by

$$i = \frac{-Ee^{-\frac{R}{2L}t}}{2Lx}(e^{xt} + e^{-xt})$$

where

$$x = \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} \quad . \quad . \quad . \quad . \quad (10)$$

No oscillation takes place : the current first increases and then decays to zero without oscillation.

99. NUMERICAL EXAMPLES

99. Numerical Examples.

1. What is the resonant frequency of 200 μH and 100 $\mu\mu$F ?

[A. $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{200 \times 10^{-6} \times 100 \times 10^{-12}}} = 1{\cdot}125$ Mc/s]

2. What is the impedance of an inductance of 200 μH in parallel with a capacity of 100 $\mu\mu$F, at the resonant frequency, if at that frequency the resistance of the inductance is 10 ohms?

[A. $f_0 = 1{\cdot}125$ Mc/s. $L\omega_0 = 200 \times 6{\cdot}28 \times 1{\cdot}125 = 1{,}410\ \Omega$

$$Q = \frac{L\omega_0}{R} = \frac{1{,}410}{10} = 141$$

Hence Z at resonance $= QL\omega_0 = 141 \times 1{,}410 = 200{,}000\ \Omega$.]

3. What value of inductance will resonate with a capacity of 50 $\mu\mu$F at 10 Mc/s?

[A. $$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore\ L = \frac{1}{4\pi^2 f_0^2 C^2} = \frac{1}{4\pi^2 \times 10^{14} \times 50 \times 10^{-12}}\,\text{H}$$

$$= \frac{10^6}{4\pi^2 \times 5{,}000}\,\mu\text{H} = 5{\cdot}07\ \mu\text{H.}]$$

4. If the ratio of the maximum capacity of a variable condenser to its minimum value is 10 to 1, what is the ratio between the highest and lowest wave-length to which this condenser is capable of tuning a given inductance, and why?

[A. $\sqrt{10} = 3{\cdot}16$ because the resonant frequency is inversely proportional to the square root of the tuning capacity.]

5. Given an interfering signal on 900 kc/s, a receiver with an input impedance of 10,000 ohms (resistive), an aerial with an effective capacity of 100 $\mu\mu$F, a 100-μH coil having a Q of 50 and a variable condenser with a maximum capacity of 300 $\mu\mu$F, which will be more selective: a series rejector circuit or a shunt acceptor circuit? And which will introduce most attenuation at the unwanted frequency?

[A. The reactance of 100 $\mu\mu$F in the neighbourhood of 900 kc/s is about 1,800 Ω. The total series impedance when a rejector is used is therefore $-j1{,}800\ \Omega+$rejector impedance $Z_r+10{,}000\ \Omega$. When an acceptor is used the aerial is a generator of internal impedance $-j1{,}800\ \Omega$ working into a load constituted by the acceptor impedance Z_a in parallel with 10,000 ohms. For approximate purposes 10,000 Ω in shunt across the acceptor may be neglected. If e is the e.m.f. in the aerial (assumed constant at all frequencies) the voltage across the receiver input when the rejector is used is:

$$E_r = \frac{10{,}000e}{10{,}000+Z_r-j1{,}800} \qquad . \quad . \quad . \quad (1)$$

and when the acceptor is used this voltage is:

$$E_a = \frac{Z_a e}{Z_a-j1{,}800} \qquad . \quad . \quad . \quad . \quad (2)$$

At resonance (i.e. at $f_0 = 900$ kc/s, the frequency to which acceptor and rejector are tuned: $\omega_0 = 2\pi f_0$).

$$Z_a = \frac{L\omega_0}{Q} = \frac{6{\cdot}28 \times 0{\cdot}9 \times 100}{50} = \frac{565}{50}$$
$$= 11{\cdot}3 \text{ ohms resistance.}$$

At a frequency $f = 1$ *Mc/s*, which may represent a wanted frequency,

$$Z_a = 11{\cdot}3+j\left[565\frac{f}{f_0} - 565\frac{f_0}{f}\right] = 11{\cdot}3+j565\left[1{\cdot}11 - \frac{1}{1{\cdot}11}\right]$$
$$= 11{\cdot}3+j119 = j119, \text{ say.}$$

At resonance $Z_r = QL\omega_0 = 50 \times 565 = 28{,}250$ ohms resistance.

At frequency $f = 1$ *Mc/s*, from equation (5)/VI:2 the resistance

component of the impedance is negligible in comparison with the reactive component, and the value of the reactive component is substantially equal to its value when Q is infinity :

$$\therefore \quad Z_r = jL\omega_0 \frac{\frac{f}{f_0}}{1 - \frac{f^2}{f_0^2}} = \frac{jL\omega_0}{\frac{f_0}{f} - \frac{f}{f_0}}$$

Note that this is an approximation, and does not hold when $\frac{\omega}{\omega_0}$ approaches unity very closely ; in this case the full formula must be used.

$$\therefore \quad Z_r = \frac{j565}{\frac{1}{1{\cdot}11} - 1{\cdot}11} = \frac{-j565}{0{\cdot}21} = -j2{,}690$$

Then at $f_0 = 900\ kc/s$, from equation (1)

$$E_r = \frac{10{,}000e}{10{,}000 + 28{,}250 - j1{,}800}$$

$$= \frac{e}{3{\cdot}8} \text{ neglecting the } -j1{,}800 \text{ ohms.}$$

and $E_a = \frac{11{\cdot}3e}{11{\cdot}3 - j1{,}800} = \frac{je}{160}$ neglecting the 11·3 ohms in the denominator.

Hence the voltage suppression ratio of the acceptor is

$$\frac{160}{3{\cdot}8} = 42 \text{ times that of the rejector.}$$

Selectivity.

This is best expressed for each wanted frequency as the ratio of the voltage suppression ratio at the unwanted frequency to that at the wanted frequency.

At 1 Mc/s $\quad E_r = \frac{10{,}000e}{10{,}000 - j2{,}690 - j1{,}800}$.

$$\therefore \quad |E_r| = \frac{e}{1{\cdot}1}$$

The selectivity ratio is therefore $\frac{3{\cdot}8}{1{\cdot}1} = 3{\cdot}45$.

At 1 Mc/s $\quad E_a = \frac{j119e}{j119 - j1{,}800} = -\frac{e}{14{\cdot}1}$

The selectivity ratio is therefore $\frac{160}{14{\cdot}1} = 11{\cdot}35$.

The acceptor circuit therefore has the highest suppression ratio and the highest selectivity. The acceptor circuit is, however, not necessarily the best circuit to use because, if the voltage suppression ratio of the rejector is adequate, this circuit has the advantage that less attenuation is introduced to frequencies near the disturbing frequency.]

6. In the case above another receiver with an input impedance of 1,000 ohms is used, and it is required to receive a signal at 1 Mc/s with a field strength of 1 millivolt per metre in the face of an interfering field on 900 kc/s of 100 millivolts per metre. The required speech to interference ratio is 60 db., and the receiver selectivity ratio between 900 kc/s and 1 Mc/s is 80 db. Should a rejector or an acceptor circuit be used?

[A. The unwanted signal field is 40 db. above the wanted field, and the unwanted signal is therefore reduced to 80 − 40 = 40 db. below the wanted signal. The rejector acceptor circuit must therefore have a selectivity ratio of 10 corresponding to the 20 db. difference between 60 and 40 db.

Rejector.

At 900 kc/s $E_r = \dfrac{1{,}000e}{1{,}000+28{,}250-j1{,}800} = \dfrac{e}{29\cdot3}$

neglecting $-j1{,}800$.

At 1 Mc/s $|E_r| = \left|\dfrac{1{,}000e}{1{,}000-j2{,}690-j1{,}800}\right| = \dfrac{e}{4\cdot6}$.

The selectivity ratio is therefore $\dfrac{29\cdot3}{4\cdot6} = 6\cdot4$.

Acceptor.

The selectivity ratio of the acceptor is 11·35, which is calculated as in example 5 and is therefore just over 20 db. The acceptor circuit should therefore be used since it has an adequate selectivity ratio, whereas the rejector has not. It should be noted that the acceptor is chosen in spite of the fact that its suppression ratio at 1 Mc/s, the wanted frequency, is $\dfrac{1}{14}$, while that of the rejector is only $\dfrac{1}{4\cdot6}$. This is related to the fact that 1 mV/m is a strong signal and the reduction of $\dfrac{1}{14}$ will still leave a signal loud enough to ride over the noise inherent in the receiver.]

7. In general, what value of condenser gives the greatest selec-

tivity, (*a*) in a rejector circuit, (*b*) in an acceptor circuit, and why ? (*c*) What are the practical limitations, apart from the sizes of components available, which prevent the maximum selectivity being realized ? (*d*) How would you word a general specification for the design of rejector and acceptor circuits ? (*e*) Is a rejector circuit better than an acceptor circuit ?

[A. (*a*) The smallest condenser possible because the rejector is an impedance in a series circuit and the change in impedance of the rejector is least masked by other elements in the circuit when the rejector impedance is as large as possible. (*b*) The largest condenser possible because an acceptor circuit is in shunt across the receiver input impedance and the change of impedance of the acceptor is least masked by the receiver input impedance when the acceptor impedance is as low as possible. (*c*) The practical limitations which prevent the maximum selectivity being realized are the field strength of the wanted signal, the sensitivity of the receiver and the noise level inherent in the receiver. (*d*) Rejector and acceptor circuits should be constructed from coils of the highest possible Q, and should be designed for the highest selectivity permitted by the limitations under (*c*) or by the values of variable condenser available, whichever limitation is the more stringent. (*e*) Fundamentally there is no difference between a rejector circuit and an acceptor circuit : either may be designed to give the same performance with coils of a given Q. In practice, owing to the limitations on the size of variable condenser which are available, one or other will be found to give a better performance, depending on the receiver input impedance, sensitivity, selectivity and inherent noise, and also on the absolute strengths of wanted and unwanted signals and their frequency separation.]

CHAPTER VII

POWER IN ALTERNATING CURRENT CIRCUITS

1. Power delivered to a Resistance by a Sinusoidal Alternating Current.

If a current $i = \hat{i} \sin \omega t$ flows through a resistance R the instantaneous power is given by

$$p = R(\hat{i} \sin \omega t)^2 \quad . \quad . \quad . \quad . \quad (1)$$

$(\hat{i} \sin \omega t)^2$ is a curve obtained by plotting for each value of time the square of the ordinate of $\hat{i} \sin \omega t$ at that time. As can be determined by plotting, the mean height of such a curve is $\frac{\hat{i}^2}{2}$ so that the average power delivered over a cycle is

$$P = R\frac{\hat{i}^2}{2} \quad . \quad . \quad . \quad . \quad (2)$$

The quantity $\frac{\hat{i}^2}{2}$ is evidently the mean effective (i.e. effective in contributing power) value of the current squared and is called the mean square current. The root of the mean square current evidently corresponds to the magnitude of a direct current supplying the same power to the resistance as the alternating current. This quantity which is called the *Root Mean Square* value of the current is evidently equal to $\frac{\hat{i}}{\sqrt{2}}$;

$$\text{R.M.S. current} = \frac{\text{Peak Current}}{\sqrt{2}} = I_{\text{R.M.S.}} \quad . \quad . \quad (3)$$

Similarly

$$\text{R.M.S. voltage} = \frac{\text{Peak Voltage}}{\sqrt{2}} = I_{\text{R.M.S.}} \quad . \quad . \quad (4)$$

The power supplied by an alternating current to a resistance R is therefore given by

$$P = RI^2_{\text{R.M.S.}} = \frac{V^2_{\text{R.M.S.}}}{R} = V_{\text{R.M.S.}} \times I_{\text{R.M.S.}} \quad . \quad . \quad (5)$$

$$= R\frac{\hat{i}^2}{2} = \frac{\hat{e}^2}{2R} = \frac{1}{2}\hat{e}\hat{i} \quad . \quad . \quad (6)$$

2. R.M.S. Sum of a Number of Alternating Currents or Voltages.

If a number of alternating currents *of any wave form whatever*

having R.M.S. values I_1, I_2, I_3, etc., flow through a resistance R, each current delivers the same power to the resistance as if the other currents were absent.

The single current of R.M.S. value $I_{\text{R.M.S.}}$ which would supply the same power to the resistance as the sum of the powers supplied by the individual currents is called the *R.M.S. sum of the currents* and must evidently satisfy the relation

$$I^2_{\text{R.M.S.}}\,R = I_1^2R + I_2^2R + I_3^2R + \ldots$$

$$\therefore\ I_{\text{R.M.S.}} = \sqrt{I_1^2+I_2^2+I_3^2+\ldots} \qquad . \qquad . \qquad (1)$$

Similarly for R.M.S. voltages

$$V_{\text{R.M.S.}} = \sqrt{V_1^2+V_2^2+V_3^2+\ldots} \qquad . \qquad . \qquad (2)$$

3. Power in Circuits containing both Reactance and Resistance.

When a current flows through a circuit containing resistance as well as reactance the power supplied to the circuit depends only on the magnitude of the current and the value of the effective series resistance presented at the terminals of the circuit.

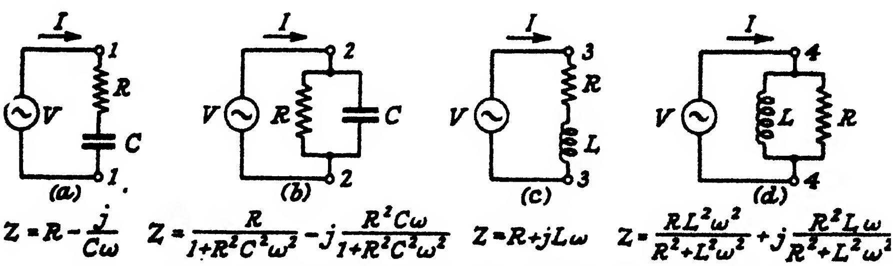


FIG. 1/VII:3.—Circuits Illustrating Power Dissipated in A.C. Circuits.

Referring to Fig. 1, for instance, the power supplied in each case corresponds to the power supplied by the current I flowing through a resistance equal to the real part of the impedances presented respectively at terminals 1,1, 2,2, 3,3, 4,4, i.e. through

R in case (*a*)

$\dfrac{R}{1+R^2C^2\omega^2}$ in case (*b*)

R in case (*c*)

$\dfrac{RL^2\omega^2}{R^2+L^2\omega^2}$ in case (*d*)

Consider the general case where an R.M.S. voltage V is applied across an impedance $A+jB$.

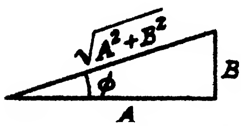
The magnitude of the R.M.S. current is

$$I = \frac{V}{|A+jB|} = \frac{V}{\sqrt{A^2+B^2}} \quad . \quad . \quad . \quad (1)$$

the power

$$P = I^2A = \frac{V^2A}{A^2+B^2} = V \times \frac{V}{\sqrt{A^2+B^2}} \times \frac{A}{\sqrt{A^2+B^2}}$$

$$= VI\cos\phi \text{ where } \phi = \tan^{-1}\frac{B}{A} \quad . \quad . \quad . \quad . \quad (2)$$



$\tan\phi = \frac{B}{A}$ $\sin\phi = \frac{B}{\sqrt{A^2+B^2}}$

$\cos\phi = \frac{A}{\sqrt{A^2+B^2}}$

FIG. 2/VII:3.—Triangle Illustrating Relation between Resistive and Reactive Components of Impedance and Power Factor.

A glance at Fig. 2 will make this clear. ϕ is evidently the angle by which the current lags (or leads if B is negative) on the voltage.

4. Power Factor.

Cos ϕ is called the *Power Factor*, and since it is always less than unity, the effect is to reduce the power supplied by a given value of VI. Hence, as the power factor becomes lower, in constant voltage systems, more current has to be supplied to provide any given amount of power. This introduces greater energy loss due to resistance of the power mains and hence a low power factor is undesirable. Corresponding disadvantages occur in other cases which will be discussed.

Owing to the nature of the loads offered by rotating machinery, and other plant such as mercury arc rectifiers, the power factor usually corresponds to a lag of the current behind the voltage.

Condensers are sometimes connected across the circuit to correct the power factor (i.e. bring it back to approximately unity) by introducing a leading current. Over-excited alternators are sometimes used for the same purpose and are then called synchronous condensers.

5. Power Ratios : The Decibel.

The nature of the mechanism of hearing is approximately such

that equal percentage increases of sound power entering the ear cause equal increases of loudness.

This means that the loudness of a sound is proportional to the logarithm of the sound power.*

Partly for this reason, but largely for convenience in calculation, power ratios which express amplification, attenuation or change of level in audio frequency, and related radio-frequency circuits are usually expressed by a unit which is proportional to the logarithm of the power ratio. *This unit is called the decibel* (usual abbreviation = *db.*).

The usefulness of the decibel is much overrated, but it has its uses, and because it is employed extensively it is essential to become familiar with it in all its aspects.

The *decibel* is a tenth of a *bel*, the number of bels by which two power levels are said to *differ* being given by the power to which 10 must be raised to be equal to the corresponding power ratio $\frac{P_1}{P_2}$, i.e. if B is the number of bels then N the number of decibels $= 10B$,

also
$$10^B = \frac{P_1}{P_2} \qquad . \quad . \quad . \qquad (1)$$

and taking logarithms to base 10

$$B \log_{10}(10) = \log_{10}\frac{P_1}{P_2}. \quad \text{But } \log_{10}(10) = 1$$

$$\therefore\ B = \log_{10}\frac{P_1}{P_2}$$

$\therefore\ N = 10 \log_{10}\frac{P_1}{P_2}$ = *number of decibels difference in level between the two powers* P_1 *and* P_2 . (2)

If both P_1 and P_2 are in unity power-factor circuits (i.e. circuits without reactance, or in which the reactance has been cancelled) of the same resistance R:

$$P_1 = I_1^2 R = \frac{V_1^2}{R} \qquad P_2 = I_2^2 R = \frac{V_2^2}{R}$$

and
$$\frac{P_1}{P_2} = \frac{I_1^2}{I_2^2} = \frac{V_1^2}{V_2^2}$$

Hence
$$N = 10 \log_{10}\left(\frac{I_1}{I_2}\right)^2 = 20 \log_{10}\frac{I_1}{I_2} \qquad . \quad . \qquad (3)$$

* If L is loudness and P is the sound power, then the nature of hearing is approximately such that $dL = K_1\frac{dP}{P}$ when K_1 is a constant. Integrating, $L = K_1 \log P + K_2$ where K_2 is the constant of integration.

Similarly
$$N = 20 \log_{10} \frac{V_1}{V_2} \tag{4}$$

Amplification when expressed in decibels is usually called "*gain*", and *attenuation* expressed in decibels is usually called "*loss*".

Originally the term "gain" was applied only to values of amplification expressed in decibels; recently, however, it has become common practice to use "gain" to mean "power amplification factor", i.e. the ratio: output power (e.g. of an amplifier) divided by input power.

Similarly, the gain of an aerial array in any direction is now most commonly employed to mean the ratio of the power density radiated in that direction divided by the power density radiated in all directions by a hypothetical isotropic (radiating equal power density in all directions) array emitting the same total power as the aerial array for which the gain is defined (instead of 10 times the logarithm of that ratio = the gain in decibels).

Table I gives the relation between decibels, current ratios and power ratios and also gives the corresponding attenuation in Nepers, a unit in use on the Continent.

TABLE I

Decibels	Nepers	Power Ratio	Current or Voltage Ratio (for Equal Impedances)
1	0·115	1·259	1·122
2	0·230	1·585	1·259
3	0·345	1·995	1·413
4	0·461	2·512	1·585
5	0·576	3·162	1·778
6	0·691	3·981	1·995
7	0·806	5·012	2·239
8	0·921	6·310	2·512
9	1·036	7·943	2·818
10	1·151	10	3·162
15	1·727	31·623	5·623
20	2·303	10^2	10
30	3·454	10^3	31·623
40	4·605	10^4	10^2
50	5·756	10^5	316·23
100	11·513	10^{10}	10^5

Fig. 1 shows the relation between power ratio and current or voltage ratio and number of decibels, in graphical form.

The Neper is so defined that two powers P_1 and P_2 differ in

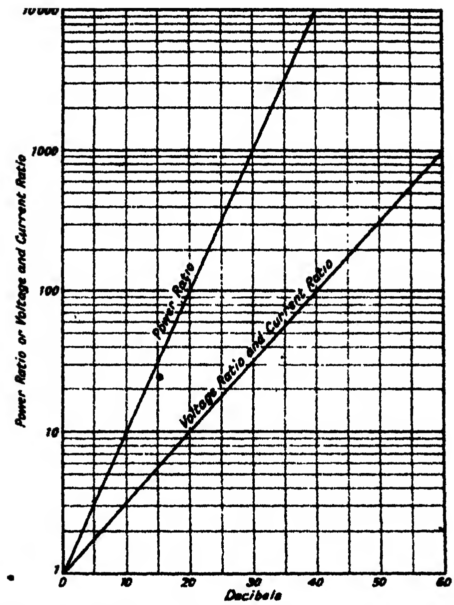


FIG. 1/VII:5.—Relations between Power Ratio, Current Ratio = Voltage Ratio and Number of Decibels in Circuits of Equal Impedance.

level by N Nepers when

$$e^{2N} = \frac{P_2}{P_1}$$

that is when

$$N = \frac{1}{2} \log_e \frac{P_2}{P_1}$$

6. Level and Volume.

6.1. Level of Tone. In low-power audio-frequency circuits the power due to steady sinusoidal currents (tone) flowing in the circuits is expressed in terms of milliwatts, or in terms of *level.* For this purpose a power of one milliwatt is called reference level or zero level. Other powers are then specified by their ratio to zero level (one milliwatt) expressed in decibels. Powers higher than one milliwatt then correspond to positive levels and powers below one milliwatt to negative levels.

$$\textit{Zero power level} = 1 \textit{ milliwatt}$$

If P is the power in milliwatts,

$$\textit{Power Level} = 10 \log_{10} P \textit{ db.}$$

The standard impedance, for which instruments measuring level

are often designed, is a non-reactive impedance equal to 600 ohms. If an R.M.S. voltage V across a resistance of 600 ohms gives rise to a power of 1 milliwatt supplied to the resistance, the value of the R.M.S. voltage is given by

$$\frac{V^2}{600} = 10^{-3} \text{ so that } V = 0{\cdot}775 \text{ volts R.M.S.}$$

This gives rise to the convention that

$$\begin{aligned} \textit{Zero Voltage Level} &= 0{\cdot}775 \textit{ volts R.M.S.} \\ &= 1{\cdot}095 \textit{ volts peak} \end{aligned}$$

Consistent with the derivation of power level

$$\begin{aligned} \textit{Voltage Level} &= 20 \log_{10} \frac{\textit{R.M.S. Voltage}}{0{\cdot}775} \textit{ db.} \\ &= 20 \log_{10} \frac{\textit{Peak Voltage}}{1{\cdot}095} \textit{ db.} \end{aligned}$$

In a circuit with a zero angle impedance of 600 ohms the voltage level is always equal to the power level, but when the circuit impedance Z is other than 600 ohms the relation between voltage level and power level is given by

$$\textit{Voltage Level} = \textit{Power Level} + 10 \log_{10} \frac{Z}{600} \textit{ db.}$$

The voltage level is therefore not a true measure of the power in the circuit, although it may be a very useful quantity for a particular purpose, as will appear below.

6.2. The Nature of Programme and Noise. In a circuit carrying programme the voltage wave form consists of a succession of peaks with an envelope varying over a wide range of amplitude. It is found that if the tops of the highest peaks are suppressed by a limiting device with zero time constant, provided the duration of the suppressed portion is less than about 5 milliseconds, no noticeable distortion occurs.

When limitation has been carried out in this way the residual peak amplitude will be called the *essential amplitude.*

Essential amplitude is therefore the residual peak voltage observed if voltage peaks of duration less than 5 milliseconds are suppressed or disregarded.

The *instantaneous peak voltage* is defined as the essential amplitude of the voltage peak nearest in time to the chosen instant at which the peak voltage is specified.

$$\textit{Instantaneous Voltage Level} = 20 \log_{10} \frac{\textit{Instantaneous Peak Voltage}}{1{\cdot}095} \textit{db.}$$

Instantaneous Power Level

$$= \textit{Instantaneous Voltage Level} - 10 \log_{10} \frac{Z}{600} \text{ db.}$$

In any given period of time the *Peak* (Voltage or Power) *Level* is the maximum value of the Instantaneous (Voltage or Power) Level during the period.

The *Minimum Level* is the lowest value of the instantaneous level which carries useful intelligence or entertainment value.

The *Level Range* is the difference in decibels between the Peak Level and the Minimum Level.

For the present purpose Noise and Interference may be considered to be of the same nature as Programme, and the above definitions apply also to these.

6.3. Volume of Programme and Noise. For the purpose of adjusting the levels in a programme circuit, so that the transmitter will be modulated to the required degree, 1,000 c/s tone is transmitted over the programme circuit and the gains of all amplifiers are adjusted so that the level of the tone at a number of selected points is at the required level. When this has been done the input gain of each transmitter supplied by the programme chain is adjusted so that the modulation is 40%, which is 8 db. below 100% modulation. The choice of this figure of 8 db. was originally due to the characteristics of the level measuring device used, see XV:4.1, but it may for the moment be considered to be chosen arbitrarily, because it is not always convenient or possible to modulate transmitters 100% with tone.

When programme is applied, the control operator at the input to the programme chain continually adjusts the input level so that during loud passages the peak level at the input to the chain reaches peak values 8 db. above the initial value of line-up tone. As a result, the programme peak levels just modulate the transmitters 100%. If a transmitter is subject to noticeable distortion at modulations below 100% this is allowed for by adjusting the modulation on line-up tone to a value below 40%.

Corresponding to each of the definitions of level given above there is a definition of *Volume* which exceeds the corresponding definition of level by 8 db. At a point in a distortionless circuit where zero level of line-up tone occurs, zero volume also occurs, but whereas the zero level of line-up tone has a peak voltage of 1·095 watts, the programme power peaks up to an instantaneous level 8 db. higher, i.e. to 1·095 × 2·512 volts. Line-up tone is then expressed in terms of

Level, and programme is expressed in terms of Peak Volume, which for brevity is normally referred to as Volume.

Since, owing to the variation of circuit impedances, voltage is easier to measure than power, level and peak volume are respectively measured in terms of voltage-level and peak voltage-volume (not peak-voltage volume) : the hyphens are inserted in this case only to avoid confusion. *Peak Voltage-Volume is then normally referred to as Volume.*

The use of volume and level has a parallel in the specification of the heights of mountains and buildings. The height of a building is specified with regard to ground level, and the height of a mountain with regard to sea level, and it is evident that the two kinds of height could conveniently be distinguished by the use of two different words.

It will be evident that the Volume Range of a programme is equal to the Level Range.

6.31. Measurement of Volume. Volume is measured on a type of peak voltmeter called a peak programme meter which has such a time constant that, while on steady tone it gives a steady deflection proportional to the voltage level, on programme it swings up to a reading proportional to the peak level and peak volume. The characteristics of the peak programme meter in use in the B.B.C. conform substantially with the recommendations of the C.C.I. and Volume only has any meaning when referred to such an instrument. This instrument is calibrated on steady tone so that when bridged across a 600-ohm resistance dissipating one milliwatt (i.e. when driven with 0·775 volts R.M.S.) it gives a deflection up to division 5. There are divisions from 1 to 7 on the meter, the interval between any two divisions corresponding to a change of 4 db. in input level. Division 7 therefore corresponds to an input level 8 db. higher than division 5.

A programme is said to be at zero volume if it causes a peak programme meter, calibrated to read 5 *on tone of* 1,000 *c/s at zero voltage level, to peak up to* 7 *during loud passages.*

As indicated above, consonant with the definition of power level there is a definition of Power Volume as follows :

$$\textit{Power Volume} = \textit{Peak Power Level} - 8 \textit{ db.}$$

The term " Power Volume " means the same thing as Programme Power, a term which is now abandoned.

A programme is said to be at zero power volume in a circuit of impedance Z, when the (voltage) volume is zero and Z = 600 *ohms, and is non-reactive.*

When Z is unequal to 600 ohms but is non-reactive:

$$\textit{Power Volume} = \textit{Volume} - 10 \log_{1}\frac{\quad}{600}$$

Hence, if the circuit impedance is known the power volume can be found by measuring the volume on a programme meter and making the calculation above.

Remember that when the term " Volume " is used alone, voltage volume is always implied, and this is the quantity measured on a peak programme meter, and the quantity used invariably when referring to the direct measurements of programme meters, which are normally the only measurements of level made on programme.

6.4. Applications.

When the term " Level " is used with reference to line-up tone, voltage level is always implied. At other times the term " Level " generally, but not always, means power level. The context and a normal sense of physical requirements generally indicate which meaning applies.

For the comparison of two quantities respectively in circuits of different impedance, either power, power level or power volume must be used.

In the B.B.C., for the comparison of speech and noise in the same circuit and at the same point in it, Volume is always used to measure both the speech and the noise. The programme-to-noise ratio expressed in decibels is equal to the programme volume minus the noise volume. In other parts of this book, reference will, however, be made to speech and noise levels because the term level is in current use outside the B.B.C. and also because it is likely to have a more permanent standing since at any time it may be decided to change the definition of volume, for instance, to enable transmitters to be lined up initially at a value of modulation other than 40%.

Usually these levels may be regarded as peak power levels, but it will be recognized that this measure is extremely limited since the distribution of instantaneous levels varies from programme to programme and depends on the amount of level range compression introduced by the control operator, and by any physical device in the circuit. Hence, two programmes which had the same peak power level (or the same volume) might have very different effects when appearing as cross-talk in a neighbouring circuit. The same remark applies to noise. Without entering into very involved discussions it is however impossible to clarify the situation further, and the measures described are adequate for many purposes.

An exception to the use of Volume for the expression of speech-to-noise ratio occurs in XIX:19.71, where receiver noise is measured at the output of a weighting network by means of a meter indicating R.M.S. values. This is to conform with the Specification for Testing Radio Receivers issued by the Radio Manufacturers' Association of Great Britain.

7. Levels in Speech and Music Circuits.

The level of the sound in a studio fluctuates over a level range which varies with the nature of the programme, but may be as large as 100 db. although the effective range for broadcast purposes is appreciably less.

If this range of levels were sent out from a radio transmitter without overloading the transmitter on the highest levels, the lower levels when received would be lost in background noise due to atmospherics and other noise. For this reason, the control operators, by adjustment of their volume control potentiometers, endeavour to compress the Volume range into a compass of about 20 db. by keeping the deflection of a peak programme meter between division 1 and division 6. (Owing to the decay time constant of the programme meter the resulting level range is appreciably greater than 20 db.) In addition, at the input to certain transmitters, a further compression of 4 to 8 db. is introduced automatically by means of a device called a limiter. See XXII.

8. Level of Line-up Tone and Use of Line-up Tone.

Line-up Tone is an audio-frequency sinusoidal current used for the purpose of adjusting all levels in the chain between the volume control operators and the transmitter.

Before transmission over a circuit takes place, line-up tone is transmitted from an oscillator through the control operator's position in the originating studio building, and through the outgoing lines to the transmitters, where it is radiated. Before the tone is passed on from each centre to the next, it is adjusted to its correct outgoing level, and all programme meters across the line have their sensitivity adjusted so that the line tone gives a deflection to division 4, i.e. 2 divisions = 8 db. below full deflection. The transmitters (with certain exceptions) are then adjusted to give 40% modulation, i.e. 8 db. below 100% modulation. See XV:4.1 for procedure.

When programme is transmitted through the chain, the volume control operator in the studio building where the programme is

produced, continually adjusts the volume, so that the high level passages peak up to 6 and so modulate the transmitters 100%, and low-level passages peak up to 1, so that the volume range is that corresponding to 5 divisions of 4 db. each, total 20 db.

40% modulation has not always been used for the initial line up of transmitters, since on some transmitters appreciable distortion is introduced below 100% modulation, while on other transmitters where distortion is not so important higher modulation has been used for line up. In the B.B.C. at the time of writing (May 1952) all R.M. transmitter line up at 40% modulation with one exception which lines up at 64%.

It will be evident that the absolute level to which line-up tone is adjusted at any other point than in the transmitter does not affect the percentage modulation of the transmitter, but only the level of programme at that point. It is only possible to give approximate levels for line-up tone at different points. The level in control rooms is usually between +10 db. and − 10 db. The level sent to line is usually about +4 db. (which, to prevent overloading of repeaters, should not be exceeded), while the level received at transmitters naturally varies with the line length, but is seldom below − 40 db., although in extreme cases it may fall to − 50 db.

9. Impedance Matching.

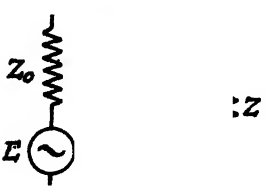


FIG. 1/VII:9.—Generator of Internal Impedance Z_0 working into a Load Z.

Fig. 1 shows a generator of internal impedance Z_0 working into a load Z.

If Z_0 and Z are both reactanceless resistances equal to R_0 and R respectively, then the maximum power will be supplied to the load R when $R = R_0$.

The maximum power is transferred from a generator to a load when the impedance of the load is equal to the impedance of the generator (reactance being absent from both impedances).

Conjugate Impedances. If $Z_0 = R_0+jX_0$ and $Z = R+jX$, the conditions for maximum power transference are

$$R_0 = R \quad \text{and} \quad X_0 = -X$$

When these conditions are satisfied the impedances Z_0 and Z are said to be conjugate impedances.

Conditions to be Observed in Impedance Matching. From the above it will be evident that the maximum power is supplied

to a load by a generator when the load impedance is conjugate to the generator impedance.

Normally, however, an attempt is made to make the impedance of all loads and generators of zero angle and therefore of zero reactance. Such an impedance is said to be a resistive impedance, or a pure resistance. Needless to say, these attempts are not always successful.

It has not been proved above, but it is a fact, that when a reactive load is connected to a generator having a reactive impedance of the same angle as that of the load, maximum power is supplied to the load when the *magnitude* of the load impedance is equal to the *magnitude* of the generator impedance.

If a resistive load is connected to a generator with a fixed impedance which has a reactive component and the resistance of the load is varied, maximum power will be supplied to the load when the resistance of the load is equal to the magnitude of the impedance of the generator.

If a reactive load which contains a resistive component is connected to a generator with a fixed resistance impedance, and the magnitude of the load impedance is varied without varying its angle, maximum power will be supplied to the load when the magnitude of the load impedance is equal to the resistance of the generator.

It is hardly necessary to point out that if the impedance of the load is a pure reactance, no power is supplied to it.

Standardization of impedance magnitudes for different types of apparatus has, unfortunately, not progressed very far, but where such standards exist, the design procedure is to make all generators with resistive impedances equal to the standard impedance applicable to the type of equipment in question, and to make all loads with resistive impedances of the same magnitude, as far as these ideal conditions can be realized.

Although the characteristic impedances of the circuits in underground cables, used for telephonic communication and for programme circuits in broadcasting, are highly reactive at low frequencies, it is customary to terminate them in resistive impedances.

Aerial feeders are often terminated in aerial arrays which possess considerable reactance, and, since the characteristic impedance of the feeder is substantially resistive, it is desirable to insert networks which offer towards the array an impedance equal to its conjugate (when connected to the feeder). As a corollary, when this is done, the impedance offered by the network (when connected to the aerial array) towards the feeder is equal to the characteristic impedance of the feeder.

10. Reflection Loss.

The reflection loss is the loss of power in decibels consequent on impedance mismatch between load and generator. In a particular case, therefore, it is the number of decibels expressing the ratio between the power transmitted to the load, and the power which would have been transmitted if the load impedance had been equal to the generator impedance.

Reflection Loss with Zero Angle Impedances. Referring to Fig. 1/VII:9, and considering the case where $Z_0 = R_0$ and $Z = R$ (R and R_0 being zero angle impedances), when $R = R_0$ the power delivered to the load is

$$P_m = I_m^2 R = \left(\frac{E}{2R_0}\right)^2 R_0 = \frac{E^2}{4R_0}$$

where I_m is the current flowing in the circuit.

When R is not equal to R_0 the power delivered to the load is

$$P_n = I_n^2 R = \left(\frac{E}{R_0+R}\right)^2 R \text{ where } I_n \text{ is the current}$$

Hence the reflection loss

$$\text{R.L.} = 10 \log_{10} \frac{P_m}{P_n} = 10 \log_{10} \frac{E^2}{4R_0} \times \frac{(R_0+R)^2}{E^2 R}$$

$$= 10 \log_{10} \frac{(R_0+R)^2}{4RR_0} \text{ db.} \tag{1}$$

$$= 20 \log_{10} \frac{R_0+R}{2\sqrt{RR_0}} \text{ db.} = 20 \log_{10} \frac{1+r}{2\sqrt{r}}$$

where
$$r = \frac{R}{R_0} \quad . \quad . \quad . \tag{2}$$

Fig. 1 shows the relation between the reflection loss R.L. and r, plotted from (2). It is to be noted that if $1/r$ is substituted for r in equation (2), the value of R.L. is unchanged, hence to use Fig. 1 enter $r = \dfrac{R}{R_0}$ if $R > R_0$ and $r = \dfrac{R_0}{R}$ if $R_0 < R$.

Reflection Loss with Complex Impedances. In this case let $Z_0 = |Z_0|\underline{/\phi}$ and $Z = |Z|\underline{/\theta}$.

Then

$$P_m = I_m^2 |Z_0| \cos\phi = \frac{E^2}{4|Z_0|^2} \times Z_0 \cos\phi$$

$$= \frac{E^2}{4|Z_0|} \cos\phi$$

$$P_n = \frac{E^2}{|Z_0+Z|^2} |Z| \cos\theta$$

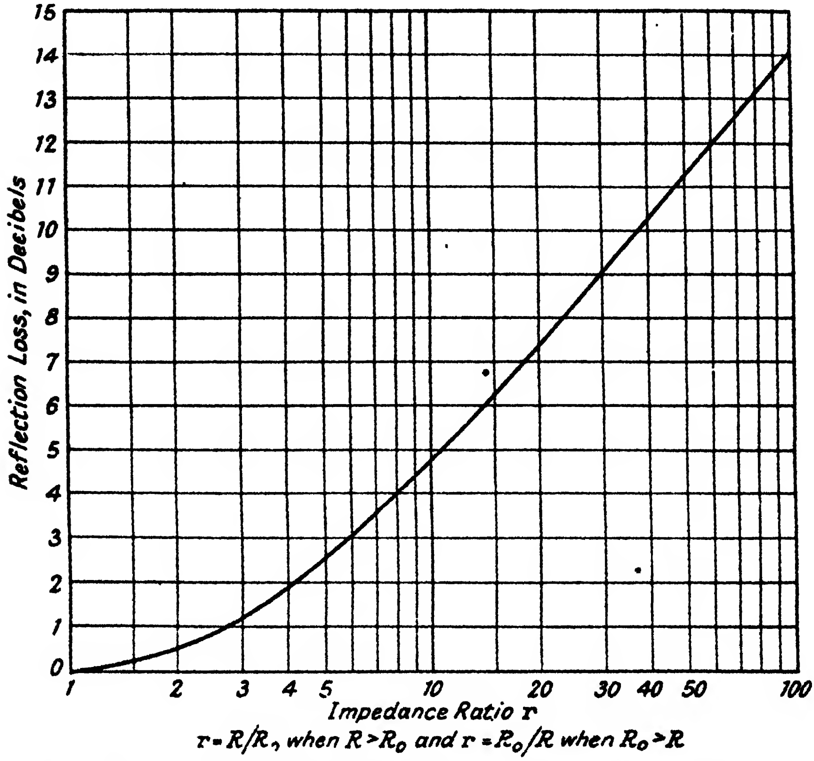


FIG. 1/VII:10.—Variation of Reflection Loss with Impedance Ratio for Zero Angle Impedances.

$$\therefore \ \text{R.L.} = 10 \log_{10} \frac{P_m}{P_n} = 10 \log_{10} \left[\frac{E^2}{4|Z_0|} \cos\phi \times \frac{|Z_0 + Z|^2}{E^2|Z| \cos\theta} \right]$$

$$= 10 \log_{10} \left[\frac{|Z|^2 + |Z_0|^2 + 2|ZZ_0| \cos(\phi - \theta)}{4|ZZ_0|} \times \frac{\cos\phi}{\cos\theta} \right]. \quad (2a)$$

For method of expansion of $|Z_0 + Z|^2$, see expansion of denominator of equations (4) to (5) immediately below.

If the reactance of the load is opposite in sign to that of the generator, the reflection loss may be, but is not always, negative: this is a "reflection gain".

10.1. Transition Loss. This is only of interest in the case of complex impedances.

The *transition loss* is the number of decibels expressing the ratio between the power transmitted to the load and the power which would have been transmitted if the load impedance was the conjugate of the generator impedance.

It will be evident, therefore, that the transition loss can be reduced (but not necessarily to zero) by inserting a transformer of *appropriate impedance ratio: see VII:11 below. If, in addition,* reactances are added on one side of the transformer, or both, so as to cancel the algebraic sum of the reactances of load and generator, the transition loss will be reduced to zero.

In the case where Z_0 is an impedance equal to $A+jB = |Z_0|\underline{/\phi}$ and Z is an impedance $C+jD = |Z|\underline{/\theta}$, *the transition loss must express the difference between the received power level and the power received when $C = A$ and $D = -B$ which is equivalent to the conditions $|Z| = |Z_0|$ and $\theta = -\phi$, that is under the conditions of conjugate impedance match.*

With a conjugate impedance match the power received is

$$P_m = \frac{E^2}{4A} = \frac{E^2}{4|Z_0| \cos \phi} \quad . \quad . \quad . \quad (3)$$

Under any other condition the power received is

$$P_n = I_n^2 C = \frac{E^2C}{|A+C+j(B+D)|^2} = \frac{E^2C}{(A+C)^2+(B+D)^2} \quad . \quad . \quad (4)$$

$$= \frac{E^2C}{(|Z| \cos \phi + |Z_0| \cos \theta)^2 + (|Z| \sin \phi + |Z_0| \sin \theta)^2}$$

$$= \frac{E^2|Z| \cos \theta}{|Z|^2 + |Z_0|^2 + 2|Z|\,|Z_0|\,(\cos \phi \cos \theta + \sin \phi \sin \theta)}$$

But since $\cos \phi \cos \theta + \sin \phi \sin \theta = \cos (\phi - \theta)$

$$P_n = \frac{E^2|Z| \cos \theta}{|Z|^2+|Z_0|^2+2|Z|\,|Z_0| \cos (\phi - \theta)} \quad . \quad . \quad (5)$$

Hence from (3) and (4) the transition loss

$$\text{T.L.} = 10 \log_{10} \frac{P_m}{P_n} = 10 \log_{10} \frac{(A+C)^2+(B+D)^2}{4AC} \quad . \quad (6)$$

While from (3) and (5)

$$\text{T.L.} = 10 \log_{10} \frac{|Z|^2+|Z_0|^2+2|ZZ_0| \cos (\phi - \theta)}{4|ZZ_0| \cos \phi \cos \theta} \quad . \quad (7)$$

Either equation (6) or (7) can be used according to the form in which Z and Z_0 are expressed.

From equations (2*a*) and (7)

$$\text{Reflection Loss} = \text{Transition Loss} + \log_{10} \cos^2 \phi \quad . \quad (8)$$

11. Use of Impedance Transforming Circuits to avoid Reflection Loss.

An impedance transforming network is a four-terminal arrange-

ment of reactances which when terminated at one pair of terminals in one value of resistance presents at the other pair of terminals another value of resistance. Typical networks in use in radio circuits are shown in Fig. 1/VI:4. When a suitably designed network of this type is inserted between a generator and a load of different impedance, no reflection loss takes place (generator and load having zero angle impedances).

Transformers. The most common impedance matching devices in use are those shown at (*a*) and (*b*) which each consists of two inductances wound on the same core. The core at (*a*) is iron, iron alloy in the form of stampings of sheet metal, or iron alloy dust pressed into an aggregate with some form of cementing material, while the core at (*b*) is of air.

Such devices are called *Transformers* and over the band of frequencies for which they are designed, possess, *in common with all the other structures shown in Fig.* 1/VI:4, the property of transforming the impedance magnitude as observed at terminals 1,1 and 2,2, and hence of transforming the magnitudes of voltages and currents.

In the case of structures (*a*), (*b*), (*c*) and (*j*), when both windings are on the same core, the voltage transformation is equal to the turns ratio, and the impedance transformation is equal to the square of the turns ratio. This may be explained more clearly in symbols.

Let T_1 = the number of turns on L_1.
Let T_2 = the number of turns on L_2
V_1 = the voltage applied to terminals 1,1
V_2 = the voltage observed across terminals 2,2
Z_2,R_2 = the impedance or resistance connected across 2,2
Z_1,R_1 = the impedance or resistance observed at 1,1
I_2 = the current flowing in R_2
I_1 = the current flowing into terminals 1,1.

Then

$$Z_1 = \sqrt{\frac{L_1}{L_2}}Z_2 \quad . \quad . \quad . \quad . \quad (1)$$

and

$$\frac{V_2}{V_1} = \frac{T_2}{T_1} = \frac{I_1}{I_2} = \sqrt{\frac{R_2}{R_1}} = \sqrt{\frac{L_2}{L_1}} \quad . \quad . \quad (2)$$

so

$$V_2I_2 = V_1I_1 = I_1^2R_1 = I_2^2R_2 = \frac{V_1^2}{R_1} = \frac{V_2^2}{R_2} = \text{power flowing in the circuit} \; . \quad (3)$$

Evidently relation (2) can be established from the fact that the power entering the transformer equals the power leaving it. In practice this is only approximately true, on account of power dis-

sipated in the windings and core of transformers, and transformers approximate to the idea condition expressed by (1), (2) and (3), only over a limited frequency range.

The structure at J, which consists of a single tapped inductance, is called an *auto-transformer*, and is nearly always an iron-cored structure except when used as part of a structure as at K.

In general, iron-cored transformers are used only in 50-cycle power circuits and in audio-frequency circuits, while dust cores are used in audio-frequency circuits and low-power radio-frequency circuits. Air core transformers are used in both low-power and high-power radio-frequency circuits. *The same remarks apply to the use of cores in inductances.*

A transformer which is satisfactory for working between impedances of 60 and 600 ohms would not be suitable for working between 60,000 and 600,000 ohms, the design requirements in the two cases being different. For this reason it is not customary to talk about an impedance ratio of 1 : 10, but of 60 : 600 ohms, or 60,000 : 600,000 ohms as the case may be. When ordering an audio-*frequency transformer it is necessary to specify:*

1. The impedance ratio.
2. The frequency band to be transmitted: specify the frequencies at the ends of the band.
3. The permissible midband transmission loss and the permissible variation of loss over the working band of frequencies. (In practice the inductances are not free from resistance and this results in transmission loss through the transformer.)
4. The power to be handled.
5. Whether a screen is required between windings.
6. Whether either of the windings is to be balanced, i.e. if it is to be connected to a balanced circuit.
7. The amount of direct current through each winding.
8. The breakdown voltage effective between each winding and all other windings and screen and case.

Screening between windings is introduced to eliminate interfering currents, due to neighbouring circuits. (See XVII:9.)

While, in the case of the structures at A, B, C and J, the voltage ratios and impedance ratios are determined by the turns ratio, no such simple relations exist in the case of the other structures in Fig. 1/VI:4. Once designed, however, to work between two impedances R_1 and R_2, they behave like transformers and over the frequency band for which they give satisfactory operation the current, voltage and power relations of equations (2) and (3) are

substantially valid. *It might be truer to say that the frequency range for which they are effective is that over which the relations of equations (2) and (3) hold sufficiently closely.*

The design of certain types of impedance transforming circuits or coupling circuits shown in Fig. 1/VI:4 is described in VII:14.

12. Insertion Loss and Gain.

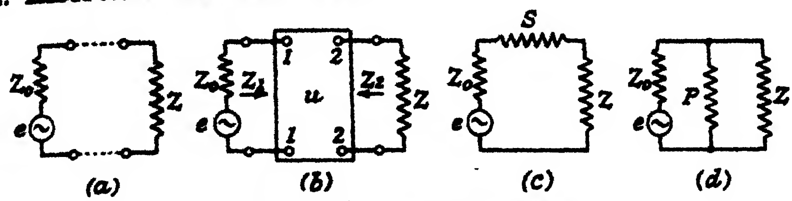


FIG. 1/VII:12. Insertion Loss.

Fig. 1 shows at (*a*) a generator of internal impedance Z_0 supplying a load of impedance Z. At (*b*) a *fourpole* or *transducer* is inserted between the same generator and the same impedance. *Fourpole* and *transducer* are respectively the German and American general terms descriptive of any type of physical arrangement such as a line, a network, or an amplifier, which when an e.m.f. is applied at its input terminals 1,1 generates an e.m.f. in its output circuit of which 2,2 are the terminals.

The insertion loss or gain of a fourpole is the difference in level (in decibels) in the load consequent on the insertion of the fourpole in the circuit.

In Fig. (1) it is the difference in level appearing in Z consequent on the insertion of the fourpole.

The magnitude of the insertion loss of a general fourpole is determined in XXIV:4.14. The method of calculating insertion loss in the case of particular structures will be illustrated by the case of a simple series and a simple shunt element.

If the e.m.f. of the generator is e, the current in the load with no inserted element is $e/(Z_0+Z)$.

With a series impedance element S inserted as in Fig. 1 (*c*), the current in the load is $e/(Z_0+S+Z)$. The insertion loss is therefore given by the number of decibels corresponding to the magnitude of the ratio of these two currents, and is:

$$\text{I.L.} = 20 \log_{10} \left| \frac{Z_0+S+Z}{Z_0+Z} \right| \text{ db.} \qquad (1)$$

With a shunt impedance element P inserted as in Fig. 1 (*d*) the current in the load is $eP/(Z_0Z+Z_0P+ZP)$. The insertion loss is

therefore given by the number of decibels corresponding to the ratio of the current, with no shunt, to this current, and is:

$$\text{I.L.} = 20 \log_{10} \left| \frac{Z_0 Z + Z_0 P + ZP}{PZ_0 + PZ} \right|$$

$$= 20 \log_{10} \left| \frac{\frac{1}{P} + \frac{1}{Z} + \frac{1}{Z_0}}{\frac{1}{Z} + \frac{1}{Z_0}} \right| \qquad (2)$$

As a mnemonic it may be noticed that equation (2) is of the same form as (1) with admittances instead of impedances.

Equation (2) is of importance because it gives the loss introduced in a circuit consequent on bridging any apparatus of impedance P across a circuit. Z_0 is then the impedance looking back towards the generator and Z is the impedance looking forward towards the receiving apparatus from the bridge point.

It will be evident that the additional loss consequent on the insertion of any network between a generator and its load is equal to the insertion loss. The specified loss of an attenuator is normally its loss when terminated in impedances equal to its image impedances. Its effective loss, when inserted in a circuit, i.e. its insertion loss, will only be equal to its specified loss if it is impedance matched to its generator and load. For insertion loss of a fourpole, see XXIV:4.14.

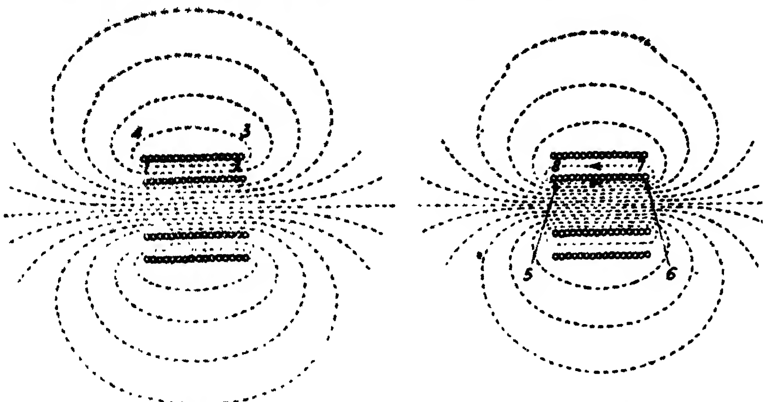
The attenuation of a filter is also normally calculated at each frequency when operating between its image impedances. As a filter is normally operated between resistive impedances equal to its geometric midband image impedances, its insertion loss will only be equal to its calculated attenuation at its geometric midband frequency: at other frequencies its insertion loss may differ appreciably from its attenuation proper. See XXV:10.

Reference should be made to XXIV:2.31 where a definition of Total Loss and its relation to Reflection Loss and Insertion Loss is given.

13. Coupled Circuits and Coupling Factor.

13.1. Leakage Flux. Consider the case of two air core coils wound one inside the other as shown in Fig. 1/VII:13. If the current flows through the outer coil, some of the resulting flux linking the outer coil will not link the inner coil. An example of this is given, for instance, by the flux following the path 1,2,3,4, in Fig. 1 (*a*). *Such flux is called leakage flux.* Flux following the path 5,6,7,8, in Fig. 1 (*b*) is also leakage flux. Evidently all the

flux doesn't link all the turns, and it is convenient to *define the equivalent flux as that flux which when linking all turns produces the same linkage as the actual flux distribution in space.*



(a) (b)

Fig. 1/VII:13.—Coupled Coils: (a) Flux Lines due to Current in Outer Coil; (b) Flux Lines due to Current in Inner Coil.

13.2. Coupling Factor.

$$\text{Let } k = \text{the ratio } \frac{\begin{array}{c}\text{Equivalent flux through inner coil}\\ \text{(due to outer coil currents)}\end{array}}{\begin{array}{c}\text{Equivalent flux through outer coil}\\ \text{(due to outer coil currents)}\end{array}}$$

The ratio k is called the coupling factor or coefficient of coupling. If Φ is the total equivalent flux through the outer coil due to the outer coil currents, then the equivalent leakage flux is evidently equal to $(1 - k)\Phi$.

If current flows through the inner coil, while the whole of the flux through the inner coil passes through the outer coil, some of the return flux of the inner coil also passes through the outer coil in reverse sense, so cancelling the effect of some of the normal flux. This will be made clear by reference to Fig. 1(b)/VII:13; see, for instance, the flux following the path 5,6,7,8.

It is not obvious, but it is a fact, that the equivalent fraction of flux through the inner coil effective in inducing currents in the outer coil is again equal to k.

If a current I_1 flows through the outer coil (of inductance L_1 say) the back e.m.f. induced in L_1 is

$$E_b = -I_1L_1j\omega$$

If a hypothetical coil L_2 were wound "occupying the same

space " as L_1 and so embracing all the equivalent flux flowing through L_1, but having t times as many turns, it would evidently have an inductance $L_2 = t^2L_1$, and would have an e.m.f. induced in it

$$E_h = tE_b = - I_1tL_1j\omega = - I_1\sqrt{L_2L_1}j\omega$$

If, now, inductance L_2 is made to shrink to a smaller diameter, the inductance being kept constant by increasing the number of turns in proper proportion, in effect it will no longer link the whole of the equivalent flux, linking L_1 due to the current through L_1, but only k times this flux and the e.m.f. induced in L_2 will now be

$$E_2 = - I_1k\sqrt{L_2L_1}j\omega. \quad . \quad . \quad . \quad (1)$$

13.3. Mutual Inductance. The quantity $k\sqrt{L_2L_1}$ has the dimensions of inductance. It is called the ***mutual inductance*** between L_1 and L_2, and has the symbol M.

13.4. Leakage Inductance. The quantities $(1 - k^2)L_1$ and $(1 - k^2)L_2$ are called the leakage inductances and are the inductances observed looking respectively into primary (L_1) and secondary (L_2) coils with the secondary and primary coils respectively short circuited. This can easily be proved from equation (5) below, by putting $Z = 0$.

The above equations and the equations which follow all hold regardless of the convention adopted for L_1 and L_2, i.e. regardless of whether L_1 and L_2 is the outer coil, and apply also whatever form of coil is used and whatever arrangement in space.

13.5. Impedance looking into Primary of Coupled Circuit. From (1)

$$E_2 = - I_1Mj\omega \quad . \quad . \quad . \quad . \quad (2)$$

This e.m.f. is induced into L_2 regardless of any current flowing in L_2.

Now suppose an impedance Z to be connected across L_2. The current flowing in the secondary circuit, i.e. through L_2 and Z is

$$I_2 = \frac{E_2}{jL_2\omega + Z} = \frac{- I_1jM\omega}{jL_2\omega + Z} \qquad (3)$$

This induces a back e.m.f. into the primary circuit, i.e. into L_1 of magnitude.

$$E_1 = - I_2Mj\omega = \frac{jI_1M\omega}{jL_2\omega + Z}jM\omega$$

$$E_1 = - I_1\frac{M^2\omega^2}{jL_2\omega + Z} \quad . \quad . \quad . \quad . \quad (4)$$

The negative sign indicates that E_1 is in the opposite direc-

tion to the voltage giving rise to I_1, and that this voltage must be increased by E_1 in order to maintain I_1.

The total voltage to be applied across the terminals 1,1 is therefore

$$V = I_1 jL_1\omega + I_1 \frac{M^2\omega^2}{jL_2\omega + Z}$$

$$\therefore \frac{V_1}{I_1} = \text{the impedance presented at terminals 1,1 ;}$$

$$= jL_1\omega + \frac{M^2\omega^2}{jL_2\omega + Z} = Z_{11} \text{ say} \quad . \quad . \quad . \quad (5)$$

In radio-frequency circuits, it is often (approximately) the case that the impedance connected to 2,2 $= Z = R - jL_2\omega$ where R is the resistance component of the load connected at 2,2 and $-jL_2\omega$ is the reactance of that load deliberately adjusted to be equal in magnitude to $L_2\omega$ and negative in sign.
In this case

$$L_1 j\omega + \frac{M^2\omega^2}{R} \qquad (6)$$

$\frac{M^2\omega^2}{R}$ is then the resistance transferred to the primary circuit from the secondary circuit ; this is sometimes called the *referred* resistance.

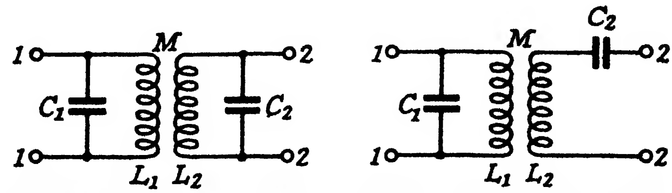
13.6. Mutual Impedance or Coupling. The quantity $M\omega$ is called the *Mutual Impedance* or *Coupling*, and, evidently, variation of this quantity changes the value of impedance transferred from secondary to primary. This variation can evidently be achieved by varying either M or ω ; but since change of transferred impedance is usually carried out at constant frequency, attention is normally directed to variation of M. *It is, however, important to note that changing the frequency changes the coupling.*

13.7. Adjustment of Coupling. Coupling can be adjusted :

(1) By changing k either by sliding either coil along its axis, or, if small enough and suitably mounted, by rotating its axis with regard to the axis of the outer coil.

(2) By changing the number of turns on either coil so as to change the value of L_1 or L_2. In practice this is usually done by changing taps, but in short-wave circuits is sometimes done by building up extra turns or by removing turns.

For a given arrangement of coils, the coupling is changed when the frequency is changed.



(a) Double Parallel Tuned Mutual Coupling

(b) Parallel-Series Tuned Mutual Coupling

FIG. 2/VII:13.

Now consider the circuits at (*a*) and (*b*) in Fig. 2 and suppose that the load across 2,2 in combination with the condenser C_2 provides an impedance Z across L_2 equal to $R - jL_2\omega$ so that the impedance thrown in series with L_1 is $\dfrac{M^2\omega^2}{R}$. With normal methods of tuning this condition obtains exactly in the case of the circuit at (*b*) and approximately in the case of the circuit at (*a*).

If C_1 is now tuned to resonate with L_1, the impedance looking into 1,1 is approximately

$$Q_eL_1\omega = \frac{L_1\omega}{\dfrac{M^2\omega^2}{R}} \times L_1\omega = \frac{L_1^2R}{M^2} \quad . \quad . \quad . \quad (7)$$

where Q_e is the apparent Q of the coil L_1 due to the resistance transferred from the secondary.

It is evident therefore that increasing the coupling while keeping the frequency constant increases the resistance thrown into the primary circuit but reduces the impedance looking into 1,1. *Reducing the coupling has the reverse effect.*

Also increasing the load resistance effective in series with L_2 decreases the resistance thrown into the primary circuit and increases the impedance seen across 2,2.

Decreasing the resistance in series with L_2 has the reverse effects.

The method of adjusting the impedance offered for instance to the anode circuit of a valve at a given frequency by terminals 1,1 of a coupling circuit, such as those shown respectively at (*a*) and (*b*) in Fig. 2, is therefore clear in the case when the coupling is changed without changing the number of turns on either winding, e.g. by sliding or rotating the coils with regard to one another.

If it is impossible to do this, then the number of turns on one of the coils must be changed.

If, for instance, the inductance of L_1 is halved, M is divided by $\sqrt{2}$, so that from (7) the primary impedance (i.e. looking into 1,1) is halved. If L_1 is doubled the impedance is doubled.

Similarly if L_2 is doubled, the primary impedance is halved, while if L_2 is halved the primary impedance is doubled.

Hence, to reduce primary impedance, remove turns from L_1 or add turns to L_2.

To increase primary impedance, add turns to L_1 or remove turns from L_2.

The decision as to whether to remove turns from one coil or to add turns to the other is made on the basis of the kVA/kW ratio (see VII:14.1) and the band width. Since these two requirements march hand in hand, the turns should always be adjusted so that as far as possible the required kVA/kW ratio is maintained.

14. Design of Impedance Matching or Coupling Circuits.

This refers to the design of any of the types of coupling circuit shown in Fig. 1/VI:4, except the transformers at (*a*), (*b*) and (*j*), but only the most commonly used of these will be considered in detail. These are shown in Figs. 1 to 4 below. For derivation of formulae in these figures, see Bibliography F7:20 and CIX.

The design of these circuits differs according to whether or not it is necessary to take into account the Kilo-Volt-Amperes in the condensers. (The kVA in a condenser is given by the product of the R.M.S. kilovolts across the condenser and the R.M.S. current through the condenser. As the cost and size of a condenser are functions of the kVA in it, the kVA is a very important quantity.)

14.1. kVA to Kilowatts Ratio. The kVA to kilowatts ratio for any condenser in a circuit is the ratio between the number of kVA in the condenser and the number of kilowatts transmitted through the circuit. For instance, if the condenser is in shunt across the circuit (as C_1 and C_2 in Fig. 1 below) at a point where the impedance of the circuit is a resistance (of value R_1 or R_2), the kVA/kW ratio is evidently equal to the ratio of current through the condenser to the current flowing into terminals 1,1, or out of 2,2, respectively, since the voltage across the condenser is the voltage across the circuit. Hence *in this case* the kVA/kW ratio $= R_1C_1\omega$ and $R_2C_2\omega$ for C_1 and C_2 respectively.

In the case of a series condenser as C_2 in Fig. 2 below, since the

current leaving 2,2 also flows through the condenser, the kVA/kW ratio is equal to the ratio of voltage across 2,2 to that across condenser C_2. Hence *in this case*

$$\text{kVA/kW} = \frac{1}{R_2 C_2 \omega} \quad \text{See also VI:2.1 and 2.6.}$$

14.2. Current per Kilowatt through Inductances. It should be remembered that the current is proportional to the square root of the number of kilowatts, so that to obtain the actual current flowing through any inductance the figures below should be multiplied by the square root of the number of kilowatts transmitted through the circuit.

Fig. 1. The resistance effectively in series with L_2 is contributed by the parallel combination of R_2 and C_2 and is given by

$$a = \frac{R_2}{1 + R_2^2 C_2^2 \omega^2}$$

But $\omega = 2\pi f$ where f is the carrier frequency $= \frac{1}{2}(f_1 + f_2)$ and

$C_2 = \dfrac{1}{2\pi(f_2 - f_1)R_2}$ where f_1 and f_2 are the limits of the pass band.

See formula for C_2 on Fig. 1.

$$\therefore\ a = \frac{R_2}{1 + \dfrac{R_2^2 4\pi^2 f^2}{4\pi^2 R_2^2 (f_2 - f_1)^2}} = \frac{R_2}{1 + \dfrac{\frac{1}{4}(f_1 + f_2)^2}{(f_2 - f_1)^2}} = \frac{R_2}{1 + \dfrac{1}{B^2}}$$

where $B \doteq \dfrac{f_2 - f_1}{\frac{1}{2}(f_1 + f_2)}$ = the band-width factor.

(Note that $100B$ = the percentage band width, by definition.)

If I_2 is the current per kilowatt in L_2, the power flowing in the secondary circuit per kilowatt

$$= 1 \text{ kilowatt} = I_2^2 a = \frac{I_2^2 R_2}{1 + \dfrac{1}{B^2}} = 1{,}000$$

$$\therefore\ I_2 = \sqrt{\frac{1{,}000\left(1 + \dfrac{1}{B^2}\right)}{R_2}} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Similarly

$$I_1 = \sqrt{\frac{1{,}000\left(1 + \dfrac{1}{B^2}\right)}{R_1}} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Fig. 2. As before

$$I_1 = \sqrt{\frac{1{,}000\left(1+\frac{1}{B^2}\right)}{R_1}} \quad . \quad . \quad . \quad (3)$$

While I_2 is given directly by

$$I_2^2R_2 = 1{,}000 \quad \therefore \; I_2 = \sqrt{\frac{1{,}000}{R_2}} \quad . \quad . \quad (4)$$

Fig. 3. The voltage per kilowatt across L is given by

$$\frac{E^2}{R_1} = 1{,}000 \qquad E = \sqrt{1{,}000R_1}\,. \quad . \quad . \quad (5)$$

The current per kilowatt in L is therefore

$$I = \frac{1}{L\omega}\sqrt{1{,}000R_1}\;. \quad . \quad . \quad . \quad (6)$$

Fig. 4. The voltage per kilowatt across L is given by

$$\left|\frac{jL\omega\sqrt{1{,}000R_1}}{jL\omega+\dfrac{R_2}{1+jR_2C_2\omega}}\right| \quad . \quad . \quad . \quad . \quad (7)$$

and the current through L is this voltage divided by $jL\omega$, hence the current per kilowatt is

$$I = \left|\frac{(1+jR_2C_2\omega)\sqrt{1{,}000R_1}}{R_2 - R_2LC_2\omega^2+jL\omega}\right| = \sqrt{\frac{1{,}000R_1(1+R_2^2C_2^2\omega^2)}{(R_2 - R_2LC_2\omega^2)^2+L^2\omega^2}}\,. \quad (8)$$

The usual procedure is to design a circuit so that the optimum compromise on kVA/kW results in the condensers ; the inductances are then designed to pass the resultant currents through them and to withstand the resultant volts across them.

In the case of straight series condensers and straight shunt condensers (as C_1 and C_2 in the two cases considered above) as the kVA to kilowatts ratio of the circuit is increased :

(1) The cost and size of the condensers increases.
(2) The selectivity of the circuit increases.
(3) The attenuation of the outer sidebands is increased in relation to those near the carrier resulting in distortion. This is evidently undesirable.
(4) The impedance presented at 1,1 to harmonics of the carrier is reduced. Where the circuit is used with class B and C amplifiers this is desirable ; see X:22.

In medium wave tuned amplifiers for modulated waves used in broadcast transmitters the optimum compromise between these four effects, in the case of straight shunt or series condensers, has

in the past been considered to be realized with a kVA/kW ratio of about 5. This figure is evidently somewhat elastic, since the overall selectivity and overall distortion is determined by the number of tuned circuits through which a modulated wave has to pass, while the cost of condensers in low-power parts of the circuit is evidently less than that in high-power parts of the circuit. Further, in drive amplifiers the question of suppression of sidebands does not arise. In a circuit such as that of Fig. 4 it may pay to go to a value of kVA/kW ratio in excess of 5 in order to realize a high step-down ratio. At the present time, therefore, it can only be said that if the kVA/kW ratio exceeds 5, some good justification should be available for the extra cost of condensers incurred. See, for instance, VI:2.6. If it is less than 5, the reactance of the circuit to harmonics of the carrier frequency must be watched.

The design of coupling circuits will first be considered for the case where consideration of the kVA/kW ratio does not arise, because the power in the circuit is so low that the normal ratings of the commercial components are well in excess of the kVA to which they are subjected. Such conditions exist in radio receivers, in the low-power drive stages of transmitters, and in broad-band amplifiers associated with receivers and certain types of drive equipment.

14.3. Design of Coupling Circuits in which the kVA/kW Ratio does not have to be considered. Design charts of four types of coupling circuit are given in Figs. 1 to 4.*

When structures are designed with the values of the elements defined by these charts they constitute band-pass impedance transforming filters of impedance ratio from R_1 to R_2, passing a band of frequencies from f_1 to f_2. R_1 and R_2 are the midband image impedances. Since the image impedances are not constant over the pass band a slight loss is introduced as the cut-off frequencies are approached from midband, which rises to one or two decibels at the cut-off frequencies, i.e. when the structures are terminated in structures equal to their midband image impedances. See XXV.

Method of using Charts on Figs. 1 *and* 2.

In using these charts note whether inductances are specified in mH or μH.

These charts show values of filter elements plotted against percentage band width for a filter structure having a value of

$$\bar{f} = \tfrac{1}{2}(f_1+f_2) = 10^6 \text{ c/s}$$

* For derivation of these charts and fundamental treatment of these coupling circuits, see CIX and Bibliography F7:20.

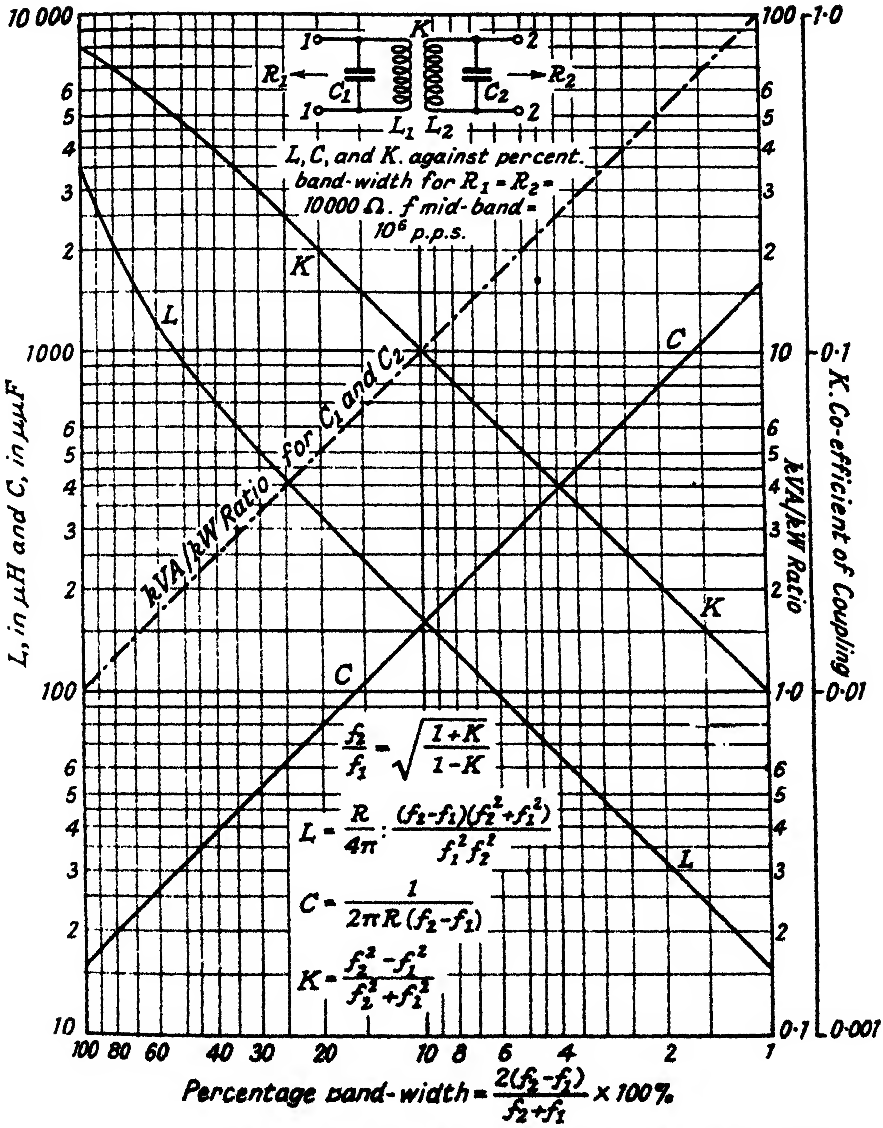


FIG. 1/VII:14.—Design Chart for Double Parallel Tuned Mutual Coupling.
(By courtesy of *The Wireless Engineer* and the B.B.C.)

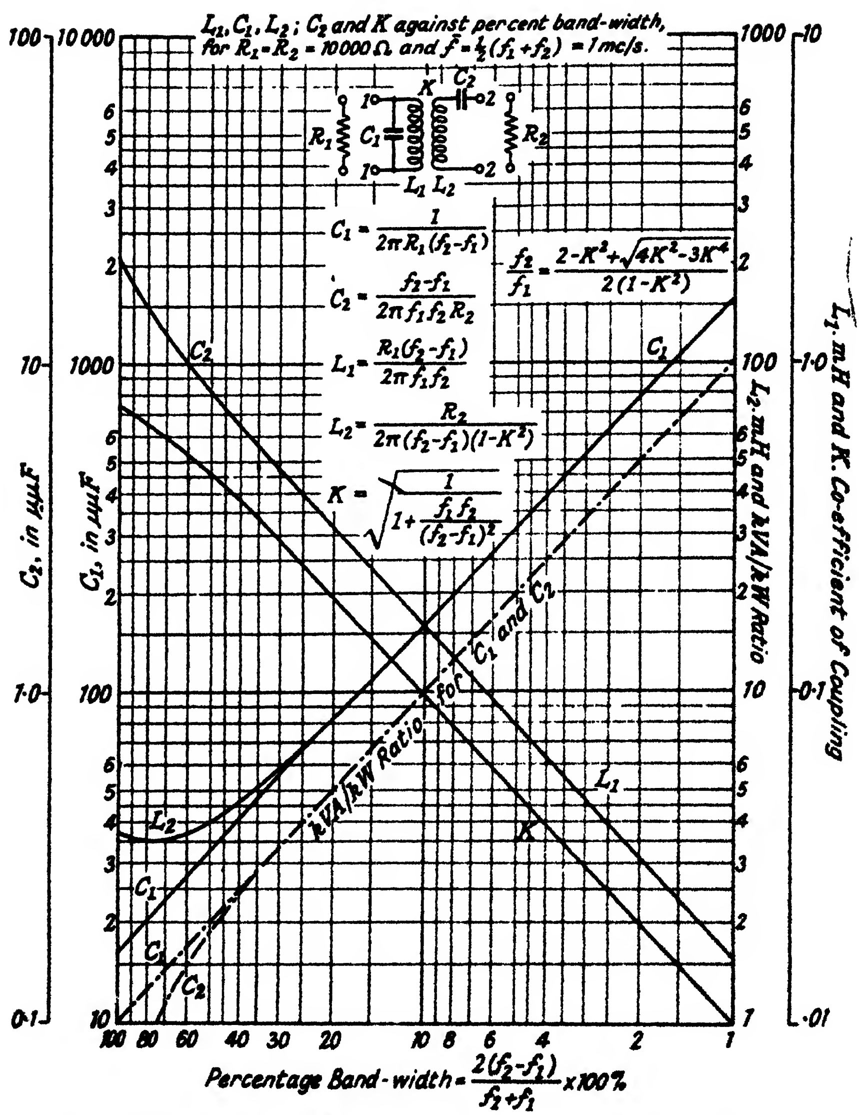


FIG. 2/VII:14.—Design Chart for Series Parallel Tuned Mutual Coupling.

(By courtesy of *The Wireless Engineer* and the B.B.C.)

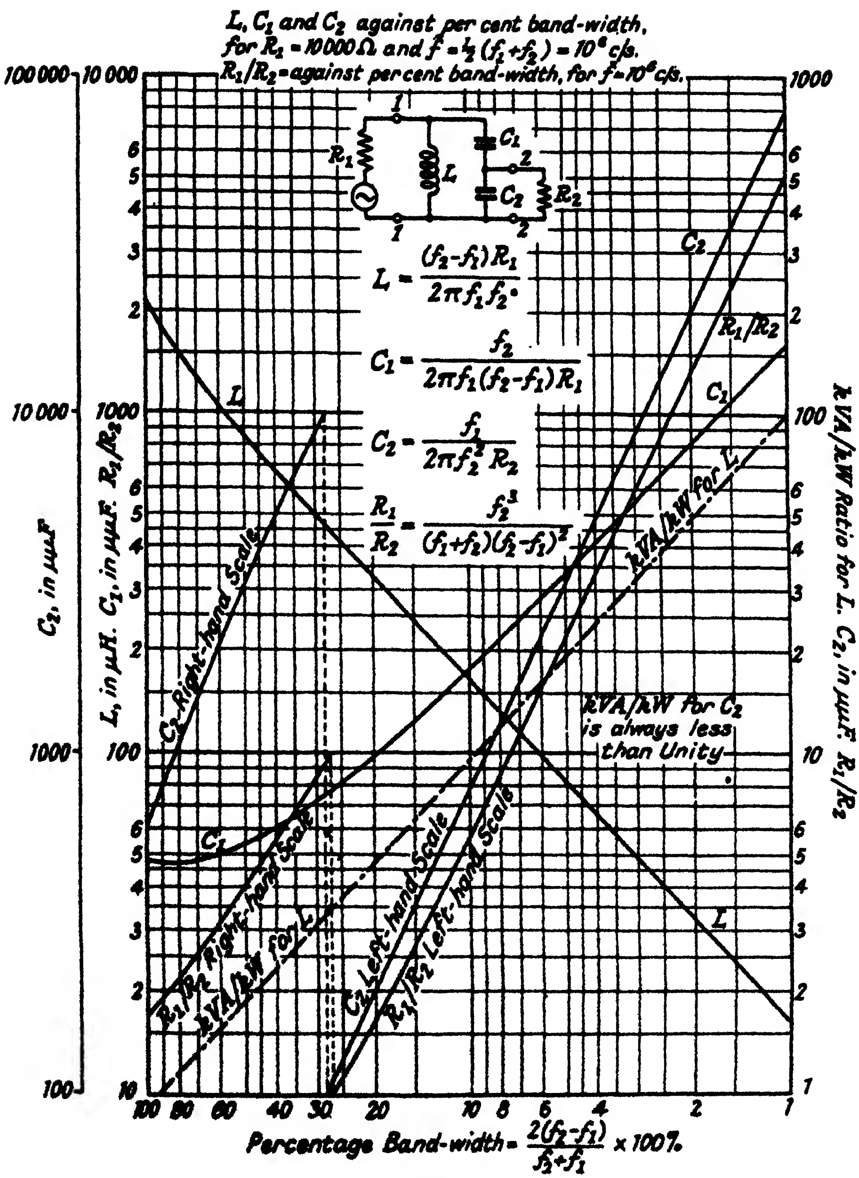


FIG. 3/VII:14.—Design Chart for Capacity Tap Coupling Circuit.

(By courtesy of *The Wireless Engineer* and the B.B.C.)

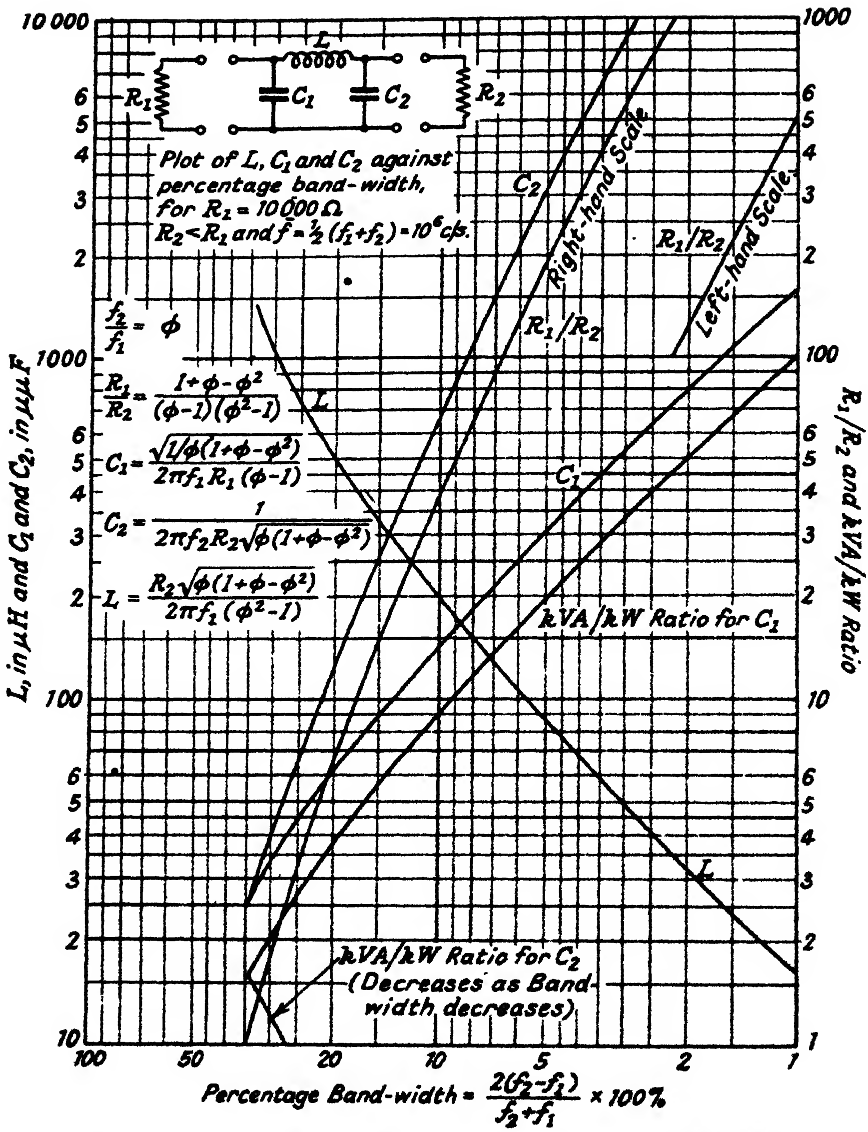


FIG. 4/VII:14.—Design Chart for Dissymmetrical π Coupling.
(By courtesy of *The Wireless Engineer* and the B.B.C.)

and designed to work between equal impedances of 10,000 ohms; f_1 and f_2 are the cut-off frequencies. The percentage band width is defined as

$$B = \frac{(f_2 - f_1)}{\frac{1}{2}(f_1 + f_2)} \times 100 = \frac{(f_2 - f_1)}{\bar{f}} \times 100$$

By means of these charts it is possible rapidly to design filters to operate between any impedances, and for any band width for which the required value of coupling is realizable.

Owing to the presence of pure mutual coupling, the band width is independent of the impedance ratio, and each side of the filter can be designed independently of the other side.

Example on Fig. 1.

Design a double parallel tuned mutual coupling circuit as shown on Fig. 1 with impedance ratio $R_1 : R_2 = 1{,}000 : 100\ \Omega$, $f_1 = 400$ kc/s, $f_2 = 600$ kc/s.

$\therefore\ \bar{f} = \frac{1}{2}(400 + 600)$ kc/s $= 500$ kc/s.

Percentage band width $= \dfrac{f_2 - f_1}{\bar{f}} = \dfrac{200}{500} = 40\%$

Design for filter on chart, reading direct from chart the values of L, C and k corresponding to 40% band width.

$$L = 700\ \mu\text{H},\ C = 38\ \mu\mu\text{F},\ \underline{k = 0{\cdot}39}$$

The value of k is then determined, but the values of L and C have to be transformed for frequency and impedance.

Transforming for Frequency.

The values of the elements are *changed so that their reactance remains constant.*

$$L = 700 \times \frac{10^6}{0{\cdot}5 \times 10^6} = 1{,}400\ \mu\text{H}$$

$$C = 38 \times \frac{10^6}{0{\cdot}5 \times 10^6} = 76\ \mu\mu\text{F}$$

Transforming to 1,000 *ohms:*

$$L_1 = 1{,}400 \times \frac{1{,}000}{10{,}000} = 140\ \mu\text{H}$$

$$C_1 = 76 \times \frac{10{,}000}{1{,}000} = 760\ \mu\mu\text{F}$$

Similarly, transforming to 100 *ohms:*

$$L_2 = 14\ \mu\text{H} \qquad C_2 = 7{,}600\ \mu\mu\text{F}$$

Example on Fig. 2.

To design a series parallel tuned mutual coupling circuit as shown on Fig. 2 with impedance ratio 1,000 : 100, $f_1 = 400$ kc/s, $f_2 = 600$ kc/s.

$\therefore \bar{f} = 500$ kc/s.

As before, percentage band width = 40%

Make the low impedance face the series condenser and the high impedance face the shunt condenser, i.e. with reference to Fig. 2 $R_1 = 1{,}000\ \Omega$, $R_2 = 100\ \Omega$.

From Fig. 2.

$L_1 = 640\ \mu$H $C_1 = 41\ \mu\mu$F (Inductances are
$L_2 = 4{,}600\ \mu$H $C_2 = 6{\cdot}4\ \mu\mu$F shown on Chart
$k = 0{\cdot}38$ in mH.)

Transforming for Frequency.

$L_1 = 640 \times 2 = 1{,}280\ \mu$H $C_1 = 41 \times 2 = 82\ \mu\mu$F
$L_2 = 4{,}600 \times 2 = 9{,}200\ \mu$H $C_2 = 6{\cdot}4 \times 2 = 12{\cdot}8\ \mu\mu$F

Transforming L_1 and C_1 to 1,000 Ω.

$$L_1 = \frac{1{,}280}{10} = 128\ \mu\text{H} \qquad C_1 = 82 \times 10 = 820\ \mu\mu\text{F}$$

Transforming L_2 and C_2 to 100 Ω.

$$L_2 = \frac{9{,}200}{100} = 92\ \mu\text{H} \qquad C_2 = 12{\cdot}8 \times 100 = 12{,}800\ \mu\mu\text{F}$$

Method of Using Charts on Figs. 3 *and* 4.

In these filters the impedance ratio and band width are interdependent, so that if the impedance ratio is chosen, the percentage band width is fixed by the curve given for the relation between R_1/R_2 and percentage band width.

The other curves give the values of the filter elements plotted against percentage band width for a structure having a value of $\bar{f} = \frac{1}{2}(f_1+f_2) = 10^6$ p:s and designed to work from one impedance of 10,000 ohms (connected on the same side as the inductance in the case of Fig. 3).

Example on Fig. 3.

(1) Given $\bar{f} = 500{,}000$ c/s impedance ratio 5,000 : 100, then $\frac{R_1}{R_2} = 50$ and from curve between R_1/R_2 and band width, the band width = 11%.

(2) Given $R_1 = 5{,}000$, $f_1 = 472{,}500$, $f_2 = 527{,}500$.
$\therefore f = 500{,}000$ c/s.

Percentage band width $= \dfrac{2(f_2 - f_1)}{f_2 + f_1} = 11\%$

From curve between R_1/R_2 and band width $\dfrac{R_1}{R_2} = 50$. Hence

$R_2 = \dfrac{5{,}000}{50} = 100.$

It will be clear that the above two examples specify the same network in different ways. The remainder of the design is then the same for both specifications.

From chart: corresponding to 11% band width

$$L = 180\ \mu\text{H} \qquad C_1 = 160\ \mu\mu\text{F} \qquad C_2 = 660\ \mu\mu\text{F}$$

Transforming for Frequency from 10^6 *c/s to* 500,000 *c/s.*

$$\therefore\ L = 180 \times 2 = 360\ \mu\text{H}$$
$$C_1 = 160 \times 2 = 320\ \mu\mu\text{F}$$
$$C_2 = 660 \times 2 = 1{,}320\ \mu\mu\text{F}$$

Transforming for Impedance from $R_1 = 10{,}000$ *to* $R_1 = 5{,}000$.

$$\therefore\ L = \frac{360}{2} = 180\ \mu\text{H}$$
$$C_1 = 320 \times 2 = 640\ \mu\mu\text{F}$$
$$C_2 = 1{,}320 \times 2 = 2{,}640\ \mu\mu\text{F}$$

The circuit of Fig. 3 *should never be used in class B and C amplifiers because of its poor harmonic suppression.*

14.31. Broad Band Amplifiers. The design charts of Figs. 1 to 4 have direct application to all circuits which are set up to final values of inductance, capacity and coupling. This procedure is adopted in broad band amplifiers, usually of low power, employed for receiving purposes, where a single coupling circuit may, for instance, be designed to pass a range of frequency with a ratio of two or three to one between the upper and the lower limiting frequency. This is only possible with class A amplifiers, however. In the case of Figs. 1 and 2, and Fig. 2 in particular, which gives a broader band for a given value of coupling factor, a ratio of $\dfrac{f_2}{f_1}$ of 3 or 4 may be realized with air-cored coils and a ratio of 100 to 1 or higher with rhometal or dust cores. A core which has yielded a coupling factor of about 0·995 is the T.C. and M. Co. R.2821 core. The corresponding value of $\dfrac{f_2}{f_1}$, taking the formula for $\dfrac{f_2}{f_1}$ on Fig. 2,

is 101. This means, for instance, that it is possible to design a single series-parallel tuned coupling circuit as shown on Fig. 2, to pass a band from 10 kc to 1 Mc or from 100 kc to 10 Mc, etc.

As already indicated, the band width on Figs. 1 and 2 is independent of impedance ratio, while the band width of the circuits on Figs. 3 and 4 is dependent on impedance ratio, and vice versa. The advantages of Figs. 1 and 2 are therefore appreciable, while the only advantage of Figs. 3 and 4 is simplicity. It is sometimes claimed for Fig. 4 that the circuit has the advantage of increased harmonic suppression in class C amplifiers, but while Fig. 3 is undoubtedly deficient in this respect and should not be used where harmonic suppression is required, there is little to choose between Figs. 1 and 4 from this point of view. Fig. 2 is intermediate in harmonic suppression between Figs. 1 and 4 bracketed together and Fig. 3.

A point of interest occurs in connection with the use of Fig. 1 in broad-band amplifiers operating at high frequency, where the valve capacities and self-capacities in the circuit impose a limit on the minimum capacity and are made to constitute the only capacity in the circuit. Since, even when this is done, the inherent circuit capacities impose a limit to the band width obtained, an extension of band width is obtained by designing the coupling with capacities equal to half the inherent circuit capacities. In such case the coupling circuit is terminated with twice the values of capacity read from Fig. 2, and while this leads to an increase in the deviation of response over the band, the deviation within any small band of frequencies, corresponding for instance to the sidebands of one carrier being received, is normally negligible. This method of procedure is sometimes considered worth while in view of the increase in overall band width which results.

A further point which has to be taken into account in connection with valves having a high internal impedance such as screen-grid valves and pentodes, is that these do not constitute a generator of impedance equal to R_1. Since the input impedance of the coupling circuit varies over the band, because the image impedance varies over the band, the use of a valve of high input impedance further increases the deviation of response over the band. This is usually tolerable, but an improvement in response can be obtained at the expense of gain by bridging a resistance equal to R_1 across the input to the circuit.

It will be evident that the design procedure is the inverse of that previously adopted, since the capacities are fixed and the values of R_1 and R_2 are adjusted until a suitable compromise is

obtained between gain and band width. Evidently as R_1 and R_2 are reduced the band width is increased and the gain is reduced.

It is evidently a simple matter to use the charts of Figs. 1, 2, 3 and 4 in reverse : starting with shunt capacity and impedance level and finishing up with band width and the values of other elements.

It will be evident that, for the purpose of coupling an anode circuit of a valve to a grid circuit of a following valve, in the case where the anode to ground capacity of the first valve is substantially equal to the effective grid to ground capacity of the following valve, any type of filter section which is terminated in shunt capacities may be used. Reference to Figs. 2, 3 and 4/XXV:5 will reveal a number of possible types. Of these the simplest is evidently to be preferred ; this is type III, in Fig. 2. Other types should, however, be inspected to find which permit the largest valves of terminating capacity to be used for a given band width.

In general, however, the structures discussed in this chapter are of more general use because they provide coupling between unequal capacities. The type III filter has still the merit of simplicity.

14.32. Tuned Amplifiers (Narrow Band Amplifiers). These are either drive amplifiers for amplifying a carrier wave before modulation or amplifiers for passing a small band of frequencies corresponding to the carrier and sidebands of a normally modulated wave, and constitute the so-called high-frequency amplifiers and power amplifiers in use in transmitters.

It is again possible to design the circuits to pass the required band of frequencies and to set up a circuit to pass the required band of frequencies.

This procedure is usually not used, however, because of the labour involved in assessing inherent circuit capacities, and it is more usual to introduce variable condensers to absorb the inherent circuit capacities by adjustment. For instance, in Figs. 1 and 2, C_1 and C_2 would be variable, while, when Fig. 3 is used, a variable condenser is shunted across the inductance, and when Fig. 4 is used, C_1 may be made variable. In Figs. 1 and 2, for use in transmitters, the coupling is usually made variable.

In such case, while Figs. 3 and 4 can best be handled by direct design from the curves, making due allowance for the effect of the capacity of the variable condenser, Figs. 1 and 2 require special treatment since *the method of adjustment of the coupling circuits in transmitters which is normally used does not result in the values of capacity being adjusted to the values given by the charts.* The result is that the circuits do not behave as band-pass circuits but have to be

considered as coupled resonant circuits, and the response over the band deviates rather more widely than when the condensers are given their chart values. Apart from this, circuits designed by taking the inductance and coupling values from the charts and having the capacities adjusted by normal tuning methods give a performance for all practical purposes identical with that of tuned coupled circuits designed by the classical method: see VII:14.5 and 14.6 below. In general, it may be stated that with classical design values a slightly higher value of coupling is required in order to pass a given band with a given deviation of response, than would be required if the band-pass values of condenser as read from the chart were used. The method of design by chart, taking into account the kVA/kW ratio is given below in VII:14.5. The classical method of design is given in VII:14.6.

14.4 Method of Adjusting and Tuning Coupled Circuits. In radio transmitters it is not customary to adjust condensers to their band-pass values, but to tune the secondary and primary circuits to resonance, i.e. to maximum *circulating* current through the tuning inductances, a radio-frequency ammeter being inserted in series with each inductance for this purpose. The current through the inductances is called the *closed circuit* current in the case of L_1 and L_2 in Fig. 1, and L_1 in Fig. 2, and the line current or feeder current, etc., in the case of L_2 in Fig. 2.

Two methods of tuning coupled circuits are in use, both of which adjust the coupled circuit so that the impedance presented towards the driving valve is resistive.

Method 1. *When the Mutual Coupling between Coils can be varied.*

(1) By varying the tuning condenser C_2, tune secondary circuit to resonance, i.e. to maximum current through L_2 or maximum voltage applied to grid of following stage.

(2) Reduce coupling until secondary current or voltage output is just readable.

(3) Repeat 1 and 2 until coupling has been reduced to the minimum at which secondary current is discernible and *secondary circuit is tuned to resonance in this condition. After this, the secondary circuit tuning must not be changed.*

(4) Adjust coupling to most probable value.

(5) By varying the tuning condenser C_1, tune primary circuit to resonance, i.e. to maximum current through L_1 or to minimum anode current in valve driving the input of the coupling circuit. These two alternative criteria of resonance give the same value of C_1.

(6) Observe impedance looking into 1,1 (in class C amplifiers this is determined from the driving valve anode current. If this is too high the impedance is too low, and vice versa.) See also X:26.31.

(7) If the impedance looking into 1,1, i.e. facing the anode of driving valve, is *too high, increase coupling*. If impedance is *too low, decrease coupling*.

(8) Repeat 5, 6 and 7 until the anode impedance reaches its correct value with the primary circuit in tune.

Method 2. *When the Mutual Coupling cannot be varied.*

I_1 = current through L_1.
I_2 = current through L_2.

Adjust secondary circuit to resonance, that is until the ratio $\frac{I_2}{I_1}$ is a maximum. Since the e.m.f. in the secondary circuit is proportional I_1, this evidently adjusts the secondary circuit to identically the same condition of resonance as the previous method.

After this, the secondary circuit tuning must not be changed.

Then proceed as in 5, 6, 7 and 8 of Method 1.

The above constitute the classical methods of tuning in universal use. They ensure that the amplifying stage which drives the primary circuit is faced with a correct resistive impedance at the carrier frequency.

The exact values to which the primary and secondary condensers C_1 and C_2 are adjusted by this method are normally of little practical importance. They are, however, determined in CII for the case of the Double Parallel Tuned Mutual Coupling, since the performance of the circuit at sideband frequencies is controlled by these values. A Parallel Series circuit as at (*b*) in Fig. 2/VII:13, when used in the anode circuit of a valve, is invariably employed with the parallel tuned side towards the valve. When tuned by the methods above, condenser C_2 is adjusted to normal resonance with L_2 so that the impedance transferred to the primary circuit is $M^2\omega^2/R$, where R is the series resistance in the secondary circuit. C_1 is adjusted to be equal to the parallel reactance offered by L_1; see V:16.

14.5. Design of Coupling Circuits in which kVA/kW Ratio has to be taken into Account. On Figs. 1 to 4, assuming the carrier frequency to be located at $f = \frac{1}{2}(f_1+f_2)$, curves of the kVA/kW ratio are plotted against percentage band width for all the important condensers except condenser C_1 and C_2 on Fig. 3. Condenser C_2 on Fig. 3 always has a kVA/kW ratio less than 1·6,

and this circuit is therefore a poor circuit from the point of view of harmonic suppression. The kVA/kW ratio of C_1 may conveniently be taken to be equal to the kVA/kW ratio for the inductance L. It is always less than this, but this assumption introduces a factor of safety.

It is now possible, if the circuit elements are adjusted individually by bridge measurements, to design a circuit either by choosing the kVA/kW ratio for one of the essential condensers and seeing whether the percentage band width is adequate, or by choosing a percentage band width and seeing whether the kVA/kW ratios in all the elements are economical. If the condensers are to be adjusted by tuning, then the charts may still be used, but in this case the kVA/kW ratio must be chosen as the basis and the performance of the circuit will not be that of a band-pass filter.

Example on Fig. 1.

Design a double parallel tuned mutual coupling, as shown on Fig. 1, with an impedance ratio 10,000:1,000 Ω and a kVA/kW ratio in both condensers equal to 5, the carrier frequency being 10^6 c/s located at $\bar{f} = \frac{1}{2}(f_1+f_2)$.

From the chart Fig. 1, the band width corresponding to a kVA/kW ratio of 5 is 20% and the values of L and C for a 20% band width, with R_1 = 10,000 ohms, are

$$\underline{L_1 = 320\ \mu\text{H}} \qquad \underline{C_1 = 80\ \mu\mu\text{F}}$$

Hence, transforming to 1,000 Ω

$$\underline{L_2 = 32\ \mu\text{H}} \qquad \underline{C_2 = 800\ \mu\mu\text{F}}$$

Corresponding to 20% band width : $\underline{k = 0{\cdot}2}$.

Example on Fig. 4.

Design a π circuit as in Fig. 4 with impedance ratio 10,000:1,000 Ω for a carrier frequency at 1 Mc/s located at $\bar{f} = \frac{1}{2}(f_1+f_2)$.

From chart Fig. 4, corresponding to $\dfrac{R_1}{R_2} = \dfrac{10{,}000}{1{,}000} = 10$ the percentage band width = 17%.

Hence $L = 400\ \mu$H, $C_1 = 75\ \mu\mu$F, $C_2 = 190\ \mu\mu$F.

The kVA/kW ratio for C_1 is 4·7 and for C_2 is 1·15.

Method of Tuning the Circuit of Fig. 3.

If for tuning purposes it is required to shunt L with a variable condenser, the admittance of this condenser should not be greater

than about 10% of that of the inductance, and the admittance of the inductance must be increased by 10% as a compensation. This means that the inductance must be divided by 1·1. Assume this inductance to be 400 μH. The variable condenser should therefore have a capacity in the middle of its range having a reactance equal to that of 4,000 μH at the carrier frequency, and the inductance used will be $\frac{400}{1\cdot 1}\mu\text{H} = 354\ \mu\text{H}$. This is equivalent to shunting the input of the circuit with a tuned circuit consisting of 4,000 μH in parallel with a condenser of the same reactance. The band-pass properties of the circuit will then be modified accordingly.

14.6. Design of Coupling Circuits without the Use of Charts. An example on this, using the classical method of design, will be considered for the circuit of Fig. 1, taking the design requirements of the example on Fig. 1 immediately above. Design is carried out at the carrier frequency.

Since the kVA/kW ratio is to be 5, the reactance of C_2 at 1 Mc/s must be a fifth of R_2, i.e. 200 ohms. *Hence* $C_2 = 800\ \mu\mu$F.

The series impedance presented by R_2 and C_2 in parallel is

$$a_2+jb_2 = \frac{R_2}{1+R_2^2C_2^2\omega^2}+j\frac{R_2^2C_2\omega}{1+R_2^2C_2^2\omega^2}$$

$$= \frac{1{,}000}{1+25}+j\frac{5{,}000}{1+25} \quad (\text{since } R_2C_2\omega = \text{the kVA/kW ratio} = 5)$$

$$= 38\cdot 5+j192\cdot 5$$

The convenient assumption is then made that the circuit is tuned so that the reactance L_2 cancels the reactance b_2. (Reference to CII and the section above on methods of tuning shows that such is not exactly the case, but the approximation is justified by its utility.)

On this basis $L_2\omega = 192\cdot 5$ so that $L_2 = 30\cdot 7\ \mu$H.

Similarly on the primary side

$$a_1+jb_1 = \frac{10{,}000}{1+25}+j\frac{50{,}000}{1+25} = 385+j1{,}925$$

Hence $L_1 = 307\ \mu$H and $C_1 = 80\ \mu\mu$F.

From equation (6)/VII:13, the resistance transferred from the secondary side to the primary side is $\frac{M^2\omega^2}{38\cdot 5}$ and thus must be equal to 385 Ω.

$$\therefore \frac{M^2\omega^2}{38\cdot 5} = \frac{k^2L_1\omega L_2\omega}{38\cdot 5} = 385$$

$$\therefore k^2 = \frac{385 \times 38\cdot 5}{192\cdot 5 \times 1925} = 0\cdot 04 \qquad \therefore k = 0\cdot 2$$

The current per kilowatt in L_1 is given by $385\,I_1^2 = 1{,}000$

$$\therefore\ I_1 = 1{\cdot}61 \text{ amps. per kilowatt}$$

Similarly $I_2 = 5{\cdot}1$ amps. per kilowatt.

In practice, the error introduced by the approximate assumption referred to above is compensated by adjustment of k during the tuning of the circuit.

Comparison of this example with the example on Fig. 1 in VII:14.5 makes it clear that the values obtained for the elements and the couplings are substantially the same by both methods. As the chart method is quicker, it is to be preferred: careful analysis shows that the classical method has no advantages.

15. Effective Inductance of Two Inductances L_1 and L_2 with Mutual Coupling M between them.

Case A. Inductances in Series. If the inductances are in series as in Fig. 1 (*a*) each inductance carries the same current I, and the back e.m.f. contributed by each inductance is of magnitude $jL\omega I$ owing to its self-inductance, and $\pm jM\omega I$ owing to the current I flowing in the other inductance.

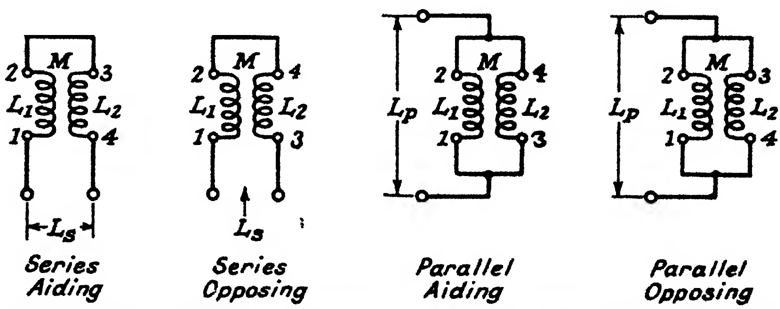


FIG. 1/VII:15.—Effective Inductance of Series and Parallel Inductance with Mutual Inductance between them.

The total back e.m.f. is therefore $(jL_1\omega + jL_2\omega \pm 2M\omega)I$, the sign of M depending on the sense of the coupling.

Hence the effective total series inductance is

$$L_s = L_1 + L_2 \pm 2M \quad . \quad . \quad . \quad . \quad (1)$$

When the sense of coupling is such as to make M positive the inductances are said to be wound in ***series aiding***, and when such as to make M negative they are said to be wound in ***series opposing***.

Case B. Inductances in Parallel. If the inductances are in parallel as in Fig. 1 (*b*)

let I_1 = the current through L_1
I_2 = ,, ,, ,, L_2
E = the e.m.f. applied across terminals 1,1
L_p = the inductance looking into terminals 1,1

Then if the coils are in parallel aiding

$$I_1L_1+I_2M = \frac{E}{j\omega}$$

$$I_1M+I_2L_2 = \frac{E}{j\omega}$$

Eliminating I_1

$$I_1ML_1+I_2M^2 = \frac{E}{j\omega}M$$

$$I_1ML_1+I_2L_1L_2 = \frac{E}{j\omega}L_1$$

Subtracting

$$\therefore\ I_2(L_1L_2 - M^2) = (L_1 - M)\frac{E}{j\omega}$$

$$\therefore\ I_2 = \frac{L_1 - M}{L_1L_2 - M^2} \times \frac{E}{j\omega}$$

Similarly

$$I_1 = \frac{L_2 - M}{L_1L_2 - M^2} \times \frac{E}{j\omega}$$

$$L_pj\omega = \frac{E}{I_1+I_2} = \frac{L_1L_2 - M^2}{L_1+L_2 - 2M}j\omega$$

Hence the equivalent inductance is

$$L_p = \frac{L_1L_2 - M^2}{L_1+L_2 - 2M} \quad . \quad . \quad . \quad . \quad (2)$$

and if $L_1 = L_2 = L$

$$L_p = \frac{L^2 - M^2}{2L - 2M} = \tfrac{1}{2}(L+M) \quad . \quad . \quad . \quad (3)$$

Similarly if M is negative

$$L_p = \frac{L_1L_2 - M^2}{L_1+L_2+2M} \quad . \quad . \quad . \quad . \quad (4)$$

and if $L_1 = L_2 = L$

$$L_p = \tfrac{1}{2}(L - M) \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The numbering convention in Fig. 1 is the standard convention for indicating the relative sense of windings on a single core. Note that connecting odd-numbered terminals to even-numbered terminals puts the windings in series aiding and in parallel opposing.

16. Measurement of Coupling Factor *k* of two Inductances.

To reduce the effect of stray capacity these measurements should be carried out at audio frequency, e.g. 1,000 c/s. Measurements made at about a fifth of the working (carrier) frequency will, however, give fairly accurate results; if radio frequency is used it should, however, be as low as possible.

Method I. The inductances L_1 and L_2 of each coil alone are measured. The effective inductances L_a and L_b of the two coils respectively in series aiding and series opposing are measured.

Then
$$L_a = L_1+L_2+2M$$
$$L_b = L_1+L_2-2M$$
$$\therefore\ 4M = L_a - L_b$$
$$\therefore\ k = \frac{L_a - L_b}{4\sqrt{L_1L_2}} \qquad (1)$$

Method II. The inductances L_1 and L_2 of each coil alone are measured.

The short circuit impedance looking into L_1 and L_2 is then measured. This is the inductance looking into one coil with the other coil short circuited.

The short circuit impedance looking into L_1 is

$$jX_1 = jL_1\omega + \frac{M^2\omega^2}{jL_2\omega} = j(L_1\omega - k^2L_1\omega).$$

See (5)/VII:13.5.

$$\therefore\ X_1 = (1 - k^2)L_1\omega$$

$$k^2 = 1 - \frac{X_1}{L_1\omega} \text{ and } k = \sqrt{1 - \frac{X_1}{L_1\omega}} \qquad (2)$$

Similarly
$$k = \sqrt{1 - \frac{X_2}{L_2\omega}} \qquad (3)$$

k should be determined by both methods and the mean value chosen.

17. Attenuators.

Any passive structure (i.e. without internal sources of e.m.f. such as would be caused by the presence of valves) which delivers less

power at its output than is supplied to its input is said to attenuate the power.

A class of attenuator of particular interest is a resistance attenuator which can be inserted in circuit without introducing reflection

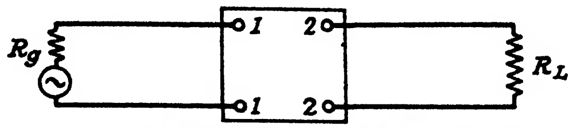


FIG. 1/VII:17.—Four Terminal Attenuators inserted between Generator and Load.

loss. Such an attenuator, when inserted between a generator and a load of impedance equal to its image impedance, introduces no reflection loss but introduces an amount of attenuation determined by the values of the resistances composing it.

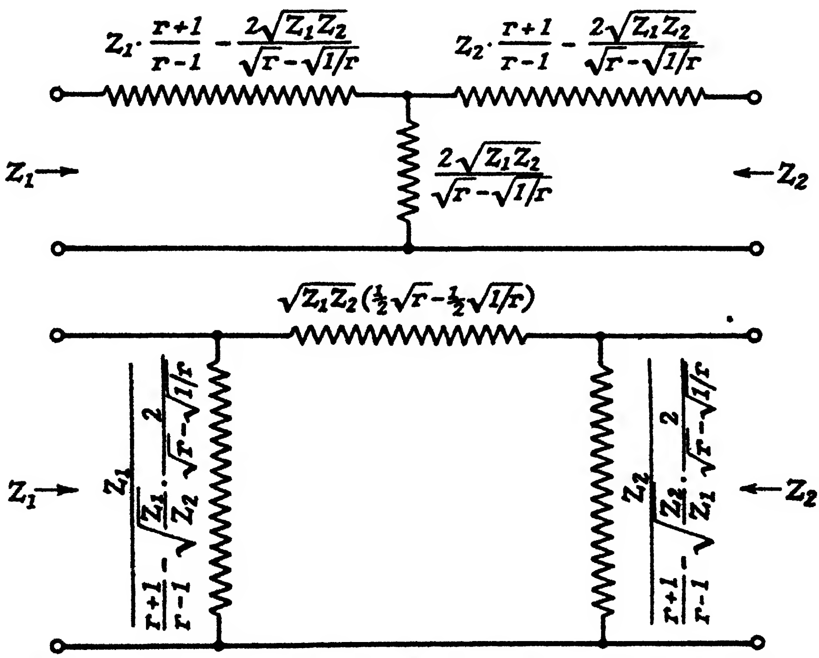
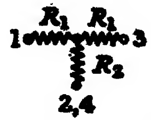
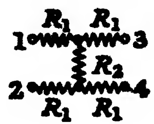
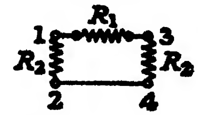
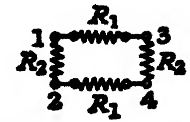
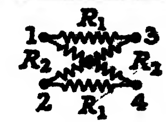


FIG. 2/VII:17.—Values of Elements of Dissymmetrical T and π Attenuators in Terms of their Image Impedances, Z_1 and Z_2, and Power Ratio : $r = \frac{P_1}{P_2}$ $(P_1 > P_2)$

This condition is represented in Fig. 1 where the impedance looking into 1,1 with the load R_L connected $= R_g$, and the impedance

TABLE I: VII: 17

										
Impedance	600 Ohms		600 Ohms		600 Ohms		600 Ohms		600 Ohms	
Loss, db.	R_1 Ohms	R_2 Ohms	R_1 Ohms	R_2 Ohms	R_1 Ohms	R_2 Ohms	R_1 Ohms	R_2 Ohms	R_1 Ohms	R_2 Ohms
0	0	∞	0	∞	0	∞	0	∞	0	∞
0·1	3·58	50,204	1·79	50,204	7·20	100,500	3·60	100,500	3·58	100,500
0·2	6·82	26,280	3·41	26,380	13·70	57,380	6·85	57,380	6·82	57,380
0·3	10·32	17,460	5·16	17,460	20·55	34,900	10·28	34,900	10·32	34,900
0·4	13·79	13,068	6·90	13,068	27·50	26,100	13·80	26,100	13·79	26,100
0·5	17·20	10,464	8·60	10,464	34·40	20,920	17·20	20,930	17·20	20,920
0·6	20·9	8,640	10·45	8,640	41·7	17,230	20·85	17,230	20·9	17,230
0·7	24·2	7,428	12·1	7,428	48·5	14,880	24·25	14,880	24·2	14,880
0·8	27·5	6,540	13·75	6,540	55·05	13,100	27·53	13,100	27·5	13,100
0·9	31·02	5,787	15·51	5,787	62·3	11,600	31·2	11,600	31·02	11,600
1·0	34·5	5,208	17·25	5,208	69·1	10,440	34·6	10,440	34·5	10,440
1·5	51·8	3,452	25·9	3,452	104·3	6,950	52·1	6,950	51·8	6,950
2·0	68·8	2,582	34·4	2,582	139·4	5,232	69·7	5,232	68·8	5,232
2·5	85·9	2,053	42·9	2,053	175·4	4,195	87·7	4,195	85·9	4,195
3·0	102·7	1,703	51·3	1,703	211·3	3,505	105·6	3,505	102·7	3,505
3·5	119·2	1,448	59·6	1,448	258·0	3,021	129·0	3,021	119·2	3,021
4·0	135·8	1,249	67·9	1,249	285·7	2,651	142·9	2,651	135·8	2,651
4·5	152·2	1,109	76·1	1,109	324·6	2,365	162·3	2,365	152·2	2,365
5·0	168·1	987·6	84·1	987·6	364·5	2,141	182·3	2,141	168·1	2,141
5·5	184·0	886·8	92·0	886·8	405·9	1,956	203·0	1,956	184·0	1,956
6·0	199·3	803·4	99·7	803·4	447·9	1,807	224·0	1,807	199·3	1,807

6·5	214·6	730·8	107·3	730·8	492·6	1,679	246·3	1,679	214·6	1,679
7·0	229·7	685·2	114·8	685·2	525·3	1,569	262·7	1,569	229·7	1,569
7·5	244·2	615·6	122·1	615·6	584·7	1,475	292·4	1,475	244·2	1,475
8·0	258·4	567·6	129·2	567·6	634·2	1,393	317·1	1,393	258·4	1,393
8·5	272·3	525·0	136·1	525·0	685·5	1,322	342·8	1,322	272·3	1,322
9·0	285·8	487·2	142·9	487·2	738·9	1,260	369·4	1,260	285·8	1,260
9·5	298·9	453·0	149·5	453·0	794·4	1,204	397·2	1,204	298·9	1,204
10·0	312·0	421·6	156·0	421·6	854·1	1,154	427·0	1,154	312·0	1,154
11·0	336·1	367·4	168·1	367·4	979·8	1,071	489·9	1,071	336·1	1·071
12·0	359·1	321·7	179·5	321·7	1,119	1,002	559·5	1,002	359·1	1,002
13·0	380·5	282·8	190·3	282·8	1,273	946·1	636·3	946·1	380·5	946·1
14·0	400·4	249·4	200·2	249·4	1,443	899·1	721·5	899·1	400·4	899·1
15·0	418·8	220·4	209·4	220·4	1,632	859·6	816·0	859·6	418·8	859·6
16·0	435·8	195·1	217·9	195·1	1,847	826·0	923·2	826·0	435·8	826·0
17·0	451·5	172·9	225·7	172·9	2,083	797·3	1,042	797·3	451·5	797·3
18·0	464·0	152·5	232·0	152·5	2,344	772·8	1,172	772·8	464·0	772·8
19·0	479·0	136·4	239·5	136·4	2,637	751·7	1,318	751·7	479·0	751·7
20·0	490·4	121·2	245·2	121·2	2,970	733·3	1,485	733·3	490·4	733·3
22·0	511·7	95·9	255·9	95·9	3,753	703·6	1,877	703·6	511·7	703·6
24·0	528·8	76·0	264·4	76·0	4,737	680·8	2,369	680·8	528·8	680·8
26·0	542·7	60·3	271·4	60·3	5,970	663·4	2,985	663·4	542·7	663·4
28·0	554·1	47·8	277·0	47·8	7,524	649·7	3,762	649·7	554·1	649·7
30·0	563·2	37·99	281·6	37·99	9,309	639·2	4,655	639·2	563·2	639·2
32·0	570·6	30·16	285·3	30·16	11,930	630·9	5,967	630·9	570·6	630·9
34·0	576·5	23·95	288·3	23·95	15,000	624·4	7,500	624·4	576·5	624·4
36·0	581·1	18·98	290·6	18·98	18,960	619·3	9,480	619·3	581·1	619·3
38·0	585·1	15·11	292·5	15·11	23,820	615·3	11,910	615·3	585·1	615·3
40·0	588·1	12·00	294·1	12·00	30,000	612·1	15,000	612·1	588·1	612·1

ATTENUATOR NETWORKS

(By courtesy of Mr. C. D. Colchester and Marconi's Wireless Telegraph Co.)

looking into 2,2 with the generator connected $= R_L$. Such a network is said to have *image impedance* Z_1 and Z_2 respectively equal to R_g and R_L.

$$\text{i.e.} \quad Z_1 = R_g, \qquad Z_2 = R_L.$$

The simplest type of attenuator is that in which $Z_1 = Z_2$, and a number of types of such attenuators are illustrated in Table I (p. 222) which gives the values of the component resistances for different values of attenuation or loss in decibels for the case when

$$Z_1 = Z_2 = 600 \ \Omega.$$

From left to right these are called respectively : *T*, *H* or balanced *T*, π, balanced π, and lattice-type attenuators. The lattice type has the advantage that self-capacity of the windings tends to cancel out but the other types are in more general use.

T and π attenuators are used in unbalanced circuits, while the other attenuators are used in balanced circuits.

Values of component resistances for the case when $Z_1 = Z_2$ is not equal to 600 Ω can be obtained by varying the component resistances proportionally.

17.1. Attenuators in which Z_1 and Z_2 are unequal : Dissymmetrical *T* and π Networks. These are sometimes useful for connecting together circuits of different impedance and are shown in Fig. 2, together with the values of their elements in terms Z_1, Z_2 and r, where Z_1 and Z_2 are the required image impedances and r is the required ratio of input power P_1 divided by output power P_2, i.e. $r = \dfrac{P_1}{P_2}$. Evidently, for each ratio of input impedance Z_1 to output impedance Z_2 there is a minimum value of attenuation which can be realized with real resistances.

99. NUMERICAL EXAMPLES.

1. 200 volts R.M.S. 50 c/s are applied across a circuit consisting of an inductance of 60 mH in series with a resistance of 40 Ω. What is the power factor and what power is applied to the circuit ?

[A. The reactance of 60 mH at 50 c/s is

$$6{,}280 \times \frac{50}{1{,}000} \times \frac{60}{1{,}000} = 18{\cdot}84\ \Omega.$$

The magnitude of the impedance of this circuit is

$$|Z| = |40+j18{\cdot}84| = \sqrt{40^2+18{\cdot}84^2} = 44{\cdot}3\ \Omega$$

Hence $$I_{\text{R.M.S.}} \quad \frac{200}{44{\cdot}2} = 4{\cdot}52 \text{ amps.}$$

The power factor $$\cos\phi = \frac{40}{|Z|} = \frac{40}{44{\cdot}3} = 0{\cdot}905$$

Hence the power $$P = EI\cos\phi$$
$$= 200 \times 4{\cdot}52 \times 0{\cdot}905 = 817 \text{ watts.]}$$

Alternatively, and preferably, the power

$$= I^2R = 4{\cdot}52^2 \times 40 = 817 \text{ watts.}$$

[A. From Table I values of elements for equivalent 600 Ω attenuator are: shunt arms 2,970 Ω, series arm 733·3 Ω. Increasing these in the ratio $\frac{1,000}{600}$ there results:

Shunt arms 4,950 Ω, series arm 1,220 Ω.]

5. Design a dissymmetrical T attenuator to work between impedances of 100 and 6,000 ohms and to have a loss of 10 db.

[A. This is impossible, as is shown by a negative answer for one of the series arms.]

6. Design a dissymmetrical T attenuator to work between impedances of 600 Ω and 200 Ω and to have a loss of 20 db.

[A. $r = 100$; put $Z_1 = 600\ \Omega$ and $Z_2 = 200\ \Omega$. Then series element on high impedance side

$$= 600\frac{101}{99} - \frac{2\sqrt{600 \times 200}}{10 - 0{\cdot}1} = 612 - 70 = 542\ \Omega$$

Low impedance side series arm

$$= 200\frac{101}{99} - 70 = 134\ \Omega. \quad \text{Shunt arm} = 70\ \Omega.]$$

7. Design a dissymmetrical π network to work between impedances of 1,000 and 2,000 ohms and to have a loss of 20 db.

[A. $r = 100$; put $Z_1 = 2,000$ and $Z_2 = 1,000$. Then shunt element on high impedance side

$$= \frac{2,000}{\frac{101}{99} - \sqrt{2}\,\frac{2}{10 - 0{\cdot}1}} = 2,730\ \Omega.$$

Similarly the shunt element on the low impedance side = 1,140 Ω. The series element $= \frac{1}{2}\sqrt{1,000 \times 2,000} \times (10 - 0{\cdot}1) = 7,000\ \Omega$.]

8. In the circuit of Fig. 2(*a*)/VII:13 $L_1 = 100\ \mu$H, $L_2 = 50\ \mu$H and $k = 0{\cdot}5$. At 1,122 kc/s the combination of the load across 2,2 and condenser C_2 provides an impedance in series with L_2 equal to $500 - jX$ where X is equal to the reactance of L_2. What is the impedance across 1,1 when the primary circuit is tuned?

[A. $M = k\sqrt{L_1L_2} = 0{\cdot}5\sqrt{100 \times 50} = 35{\cdot}35\ \mu$H

$M\omega = 35{\cdot}35 \times 10^{-6} \times 2\pi \times 1{\cdot}122 \times 10^6 = 249\ \Omega$

$M^2\omega^2 = 61,700$

Resistance transferred to primary circuit

$$\frac{61,700}{500} = 133{\cdot}4\ \Omega$$

Primary circuit reactance

$$= 100 \times 10^{-6} \times 2\pi \times 1{\cdot}122 \times 10^{6} - 705\ \Omega$$

$$Q_e = \frac{705}{133{\cdot}4} = 5{\cdot}28$$

Impedance observed across 1,1 when the primary circuit is tuned $= 5{\cdot}28 \times 705 = 3{,}725\ \Omega$.

This is the approximate answer: the exact answer is given by

$$R_r = 133{\cdot}4\left(1+\frac{705^2}{133{\cdot}4^2}\right)$$

(see equation (4)/V:16).]

9. If under the conditions of example 8 the impedance across 1,1 is too low, and it is inconvenient to change the values of inductance, how should the coupling factor be varied to lower the impedance?

[A. The coupling factor should be reduced. This will reduce the resistance transferred to the primary circuit and so increase the impedance observed across 1,1.]

10. If under the conditions of example 8 the impedance across 1,1 is too high and it is impossible to increase coupling above 0·5 but possible to vary either inductance, what should be done?

[A. The primary inductance should be reduced or the secondary inductance should be increased. As an example, if the primary inductance L_1 is halved the value of $M\omega = k\sqrt{L_1L_2}\omega$ will be divided by $\sqrt{2}$, $M^2\omega^2$ will be halved and the resistance transferred to the primary circuit will be halved. Since $L_1\omega$ has been halved (by halving L_1) the value of Q_e is unchanged, and so primary impedance $= Q_eL_1\omega$ is evidently halved. If the secondary inductance is doubled, for instance, the value of $M^2\omega^2$ is doubled, and the resistance transferred to the primary circuit is doubled, so that Q_e is halved and $Q_eL_1\omega$ the impedance across 1,1 is halved. See equation (7)/VII:13. This treatment is approximate: the exact values of primary impedance are found by using equation (4)/V:16.]

11. What single current will dissipate the same number of watts in a given resistance as the R.M.S. currents 10, 15, and 20 amps.?

[A. A current of R.M.S. value $I = \sqrt{10^2+15^2+20^2} = 25$ amps.]

12. A resistance of 1,414 Ω is shunted with a condenser having a reactance of 1,414 Ω at the frequency of the generator applied across the combination. If the generator current is 1 amp., what power is supplied to the resistance?

[A. *Method* 1. The currents in resistance and condenser are

equal and add at right angles to make up the generator current. The current through the resistance is therefore $I/\sqrt{2}$ amps. and the power dissipated in the resistance is therefore $I^2R = 707$ watts.

Method 2. The equivalent series resistance of the circuit is

$$\frac{1{,}414}{1+\frac{1{,}414^2}{1{,}414^2}} = 707 \text{ ohms.}$$

The power consequent on one amp. flowing through this is 707 watts.]

13. What are the reflection losses when a generator with a resistive internal impedance works into resistive impedances equal to twice, five and ten times, a half, a fifth and a tenth its internal impedance?

[A. Referring to Fig. 1/VII:10 the reflection losses are: twice and a half = 0·5 db.; five times and a fifth = 2·6 db.; ten times and a tenth = 4·85 db.]

14. A generator of internal impedance $1{,}000+j600$ works into a load of impedance $700 - j400$. Assuming that a series reactance and an ideal transformer are added to increase the power delivered to the load, what turns ratio should the transformer have, and what value of series reactance should be used to make the power a maximum? What is the increase in power supplied to the load expressed in decibels?

[A. The impedance ratio of the transformer should be equal to the resistance ratio = 1,000 : 600 with the high impedance side towards the generator. The transformer turns ratio is therefore $\sqrt{\frac{1{,}000}{600}} = 1{\cdot}29$. The impedance facing the generator is then: $\frac{1{,}000}{600}(600 - j400) = 1{,}000 - j667$. For maximum power in the load and therefore for maximum current supplied by the generator the reactance seen by the generator must be equal to and opposite in sign to its own internal reactance. The added reactance must therefore be $j67$ ohms. The gain in the power in the load is equal to the transition loss between the original impedances as given by equation (6) of VII:10.1, and is therefore equal to

$$10 \log_{10} \frac{(1{,}000+600)^2+(600 - 400)^2}{4 \times 1{,}000 \times 600} = 0{\cdot}46 \text{ db.}$$

It is evident that the proposed measures are not worth much.]

15. Two coils, L_1 of inductance 50 μH and L_2 of inductance 75 μH, are magnetically coupled, the coupling factor being 0·4.

A tuning condenser is connected in series with L_2 and a load resistance of 550 ohms, and the condenser adjusted to neutralize the reactance of L_2 at a frequency of 1 Mc/s. What will be the impedance looking into L_1 at this frequency? What value of condenser should be placed in parallel with L_1 to make the impedance looking into L_1 a pure resistance, and what will be the magnitude of this resistance?

[A. The mutual inductance between L_1 and L_2 is

$$M = 0{\cdot}4\sqrt{50 \times 75} = 24{\cdot}5\ \mu\text{H}$$

The magnitude of the mutual impedance is

$$M\omega = 24{\cdot}5 \times 10^{-6} \times 2\pi \times 10^{6} = 154\ \text{ohms}$$

The resistance transferred to the primary

$$L_1 = \frac{M^2\omega^2}{550} - \frac{154^2}{550} = 43{\cdot}2\ \text{ohms}$$

The impedance looking into

$$L_1 = 43{\cdot}2 + jL_1\omega = 43{\cdot}2 + j314\ \text{ohms} = 2{,}315//j320\ \text{ohms}$$

The reactance of the required tuning condenser is therefore $-$ 320 ohms which at 1 Mc/s is realized by a capacity just under 500 $\mu\mu$F. The resultant resistive impedance is of course 2,315 ohms.]

16. A single coil with a tapping has an inductance of 200 μH. The inductance looking into the coil between the tapping and one end is 100 μH, and the inductance between the tapping and the other end of the winding is 50 μH. What is the inductance measured between the tapping and both ends of the coil joined together?

[A. The inductance looking into the whole coil = 200 μH = the sum of the inductances of each section + twice the mutual inductance $M = 100 + 50 + 2M$. Hence $M = 25\ \mu$H. With the arrangement proposed the two sections are connected in parallel opposing so that the resultant inductance is:

$$\frac{100 \times 50 - 25^2}{100 + 50 + 2 \times 25} = \frac{4{,}375}{200} = 21{\cdot}9\ \mu\text{H}]$$

17. In the case of the coil of example 16 what would be the inductance measured looking into the 150 μH section with the 50 μH section shorted?

[A. The coupling factor between the two sections is

$$K = \frac{M}{\sqrt{L_1L_2}} = \frac{50}{\sqrt{150 \times 50}} = \frac{50}{86{\cdot}7} = 0{\cdot}577$$

The inductance looking into the 150 μH section with the 50 μH shorted is therefore $(1 - k^2)150 = 0{\cdot}667 \times 150 = 100\ \mu$H.]

CHAPTER VIII

HARMONIC ANALYSIS AND DISTORTION

1. Periodic Waves.

ANY wave such as that of Fig. 1 (*a*) which repeats itself at uniformly spaced intervals can be built up from a number of sine waves of different frequency, amplitude and phase ; the frequencies of the component sine waves being integral (whole number) multiples of the lowest frequency wave.

The lowest frequency wave is equal to the frequency of repetition of the wave and is called the *fundamental* (or first harmonic, but this term is not generally used) and the other waves are called the *harmonics* of the fundamental wave or fundamental frequency. The wave of frequency equal to twice that of the fundamental is called the second harmonic, the wave of frequency equal to three times that of the fundamental is called the third harmonic, and so on.

Assuming the complete wave to represent the variation of a quantity with time, the complete wave is built up from the component waves by adding at each instant in time the values of the ordinates of the component waves.

In linear circuits the currents and voltages which result from the application of an e.m.f. having any periodic wave form whatever, are identical with those which would be produced by the series of sinusoidal e.m.f.s whose instantaneous ordinates add up to give the values of the ordinates of the periodic wave in question.

For purposes of calculation it is therefore permissible to replace any periodic wave by its equivalent harmonic series. Further, if any periodic wave is applied to a selective device such as a resonant circuit, capable of being tuned to each of the frequencies of the component sine waves, it is found that the response of the device to the impressed series of waves conforms exactly to the hypothesis that the periodic wave is constituted by a series of component waves as described. In other words, a rise of current is observed in the circuit as it is tuned to resonance at each of the component frequencies.

The process of harmonic analysis consists in resolving periodic

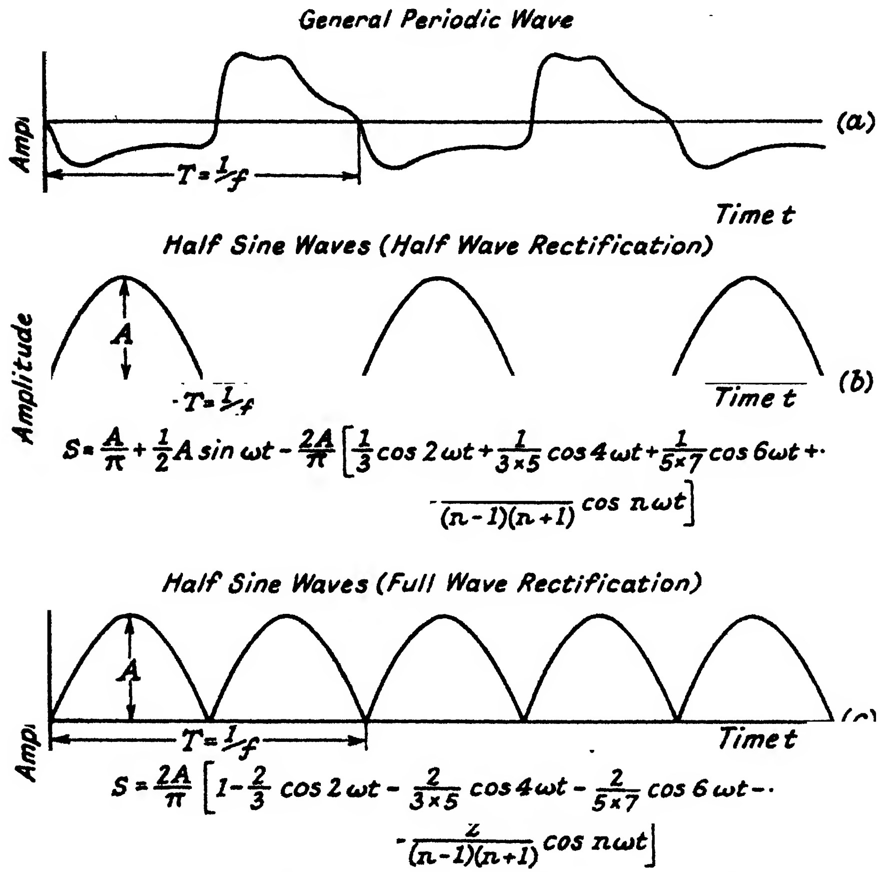


FIG. 1/VIII:1.—(*a*) Representation of General Case of a Periodic Wave. (*b*) and (*c*). Periodic Waves consisting of Half-Sine Waves with Analysis Expressions for Component Frequencies. $\omega = 2\pi f$; $f = 1/T$.

waves into their component frequencies of appropriate amplitude and phase. Two types of periodic wave are shown in Fig. 1 at (*b*) and (*c*), together with the analytical expressions which define the amplitudes and phase relations of their component harmonics.

The wave at (*b*) represents the current which would result from applying a voltage $\hat{e} \sin \omega t$ to a resistance R through a one-way device such as an ideal rectifier. It consists of series of half-sine waves of amplitude A, and frequency $f = \frac{\omega}{2\pi}$, the value of A is then $\frac{\hat{e}}{R}$.

Examination of the analytical expression shows that this current wave is equivalent to the following currents:

A direct current of amplitude	$\frac{A}{\pi}$
A fundamental frequency of amplitude	$\frac{1}{2}A$
A second harmonic frequency of amplitude	$\frac{2A}{3\pi}$
A fourth " " " "	$\frac{2A}{3 \times 5\pi}$
A sixth " " " "	$\frac{2A}{5 \times 7\pi}$

and so on, in an infinite series of progressively diminishing amplitude such that the nth harmonic is of amplitude

$$\frac{2A}{(n-1)(n+1)\pi}$$

Similarly the wave at (c) in Fig. 1 may be considered to be generated by a rectifier circuit arranged to pass each half-wave due to a sinusoidal generating e.m.f. The frequency of this wave is here called the fundamental frequency. It is equivalent to the following currents:

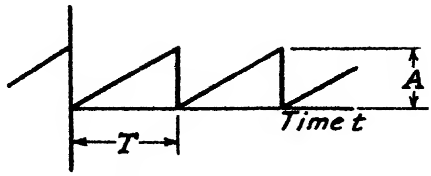
A direct current of amplitude	$\frac{2A}{\pi}$
A fundamental frequency of amplitude	zero
A second harmonic of amplitude	$\frac{4A}{3\pi}$
A fourth " " "	$\frac{4A}{3 \times 5\pi}$
A sixth " " "	$\frac{4A}{5 \times 7\pi}$

and so on, in an infinite series of progressively diminishing amplitudes such that the nth harmonic is of amplitude

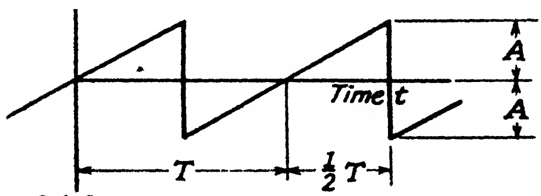
$$\frac{4A}{(n-1)(n+1)\pi}$$

It should be noted that in this case, since the fundamental frequency as defined is of zero amplitude, the second harmonic becomes the fundamental of the harmonic series.

The presentation given in which the fundamental frequency is assumed to be equal to that of the generating e.m.f. is, however, more common.

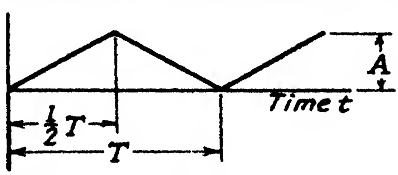


$$(a)\ S = \tfrac{1}{2}A - \frac{A}{\pi}\left[\sin\omega t + \tfrac{1}{2}\sin 2\omega t + \tfrac{1}{3}\sin 3\omega t \cdots\cdots + \frac{1}{n}\sin n\omega t + \cdots\cdots\right]$$

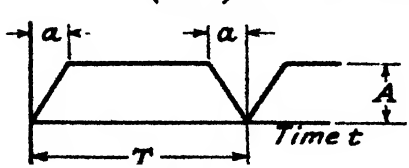


$$(b)\ S = \frac{2A}{\pi}\left[\sin\omega t - \tfrac{1}{2}\sin 2\omega t + \tfrac{1}{3}\sin 3\omega t - \cdots\cdots + \frac{(-1)^{n-1}}{n}\sin n\omega t + \cdots\cdots\right]$$

(Note change of origin between (a) and (b))

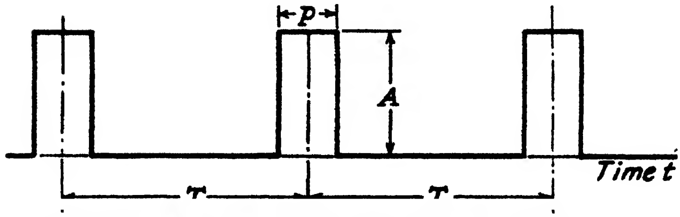


$$(c)\ S = \tfrac{1}{2}A - \frac{4A}{\pi^2}\left[\cos\omega t + \tfrac{1}{9}\cos 3\omega t + \tfrac{1}{25}\cos 5\omega t + \cdots\cdots + \frac{1}{(2n-1)^2}\cos(2n-1)\omega t + \cdots\cdots\right]$$



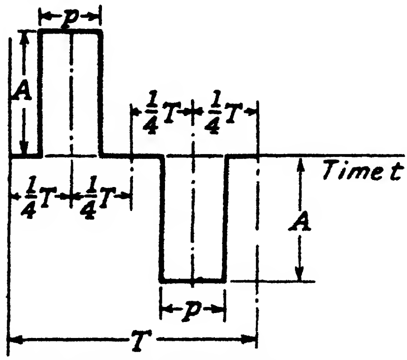
$$(d)\ S = A\left[1 - \frac{a}{2\pi} + \sum_{n=1}^{n=\infty} \frac{-1+\cos na}{n^2}\cos n\omega t\right]$$

FIG. 2/VIII:1.—Harmonic Analysis of (*a*) and (*b*) Saw-Tooth Waves, (*c*) Triangular Wave, (*d*) Trapezium Wave.



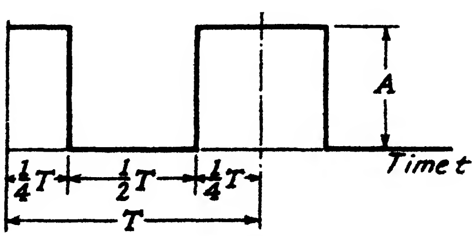
(a) Pulses of Length 'p' at Intervals 'T'

$$S = A\left[\frac{p}{T} + \sum_{n=1}^{n=\infty} \frac{2}{\pi n} \sin\frac{n\pi p}{T}\cos n\omega t\right]$$



(b) Pulses of Width 'p'

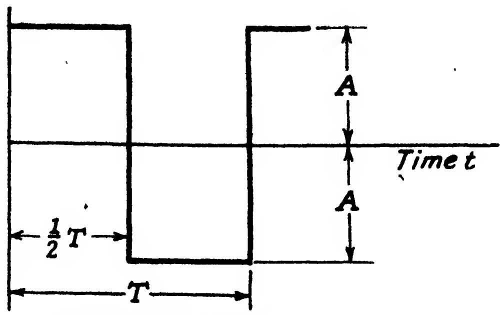
$$S = \frac{4A}{\pi}\sum_{n=1}^{n=\infty}\left(\frac{1}{2n-1}\right)\cos(2n-1)\left(1-\frac{2p}{T}\right)\frac{\pi}{2}\sin(2n-1)\omega t$$



(c) Square Wave

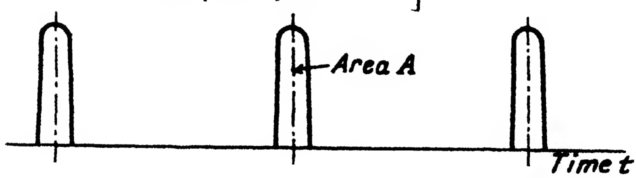
$$S = \frac{1}{2}A + \frac{2}{\pi}A\left[\cos\omega t - \frac{1}{3}\cos 3\omega t + \frac{1}{5}\cos 5\omega t - \frac{1}{7}\cos 7\omega t + \cdots\cdots + \frac{(-1)^{n-1}}{(2n-1)}\cos(2n-1)\omega t + \cdots\cdots\right]$$

Fig. 3/VIII:1.—Harmonic Analysis of Rectangular Pulses and Square Wave.



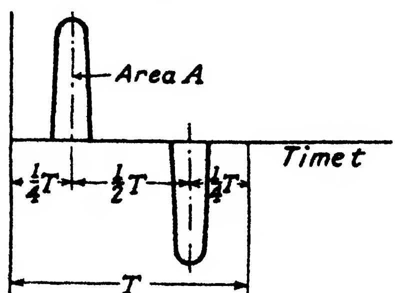
(a) Square Wave

$$S = \frac{4A}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \cdots\cdots + \frac{1}{(2n-1)}\sin(2n-1)\omega t + \cdots\cdots\right]$$



(b) Pulses of Short Duration at Intervals 'T'

$$S = \frac{A}{\pi} + \frac{2A}{\pi}\left[\sum_{n=1}^{n=\infty}\cos n\omega t\right]$$



(c) Short Pulses

$$S = \frac{2A}{\pi}\sum_{n=1}^{n=\infty}\sin(2n-1)\omega t$$

FIG. 4/VIII:1.—Harmonic Analysis of Square Wave and Constant Area Irregular Pulses of Duration short compared with their Spacing.

((b) By courtesy of R. L. Fortescue.)

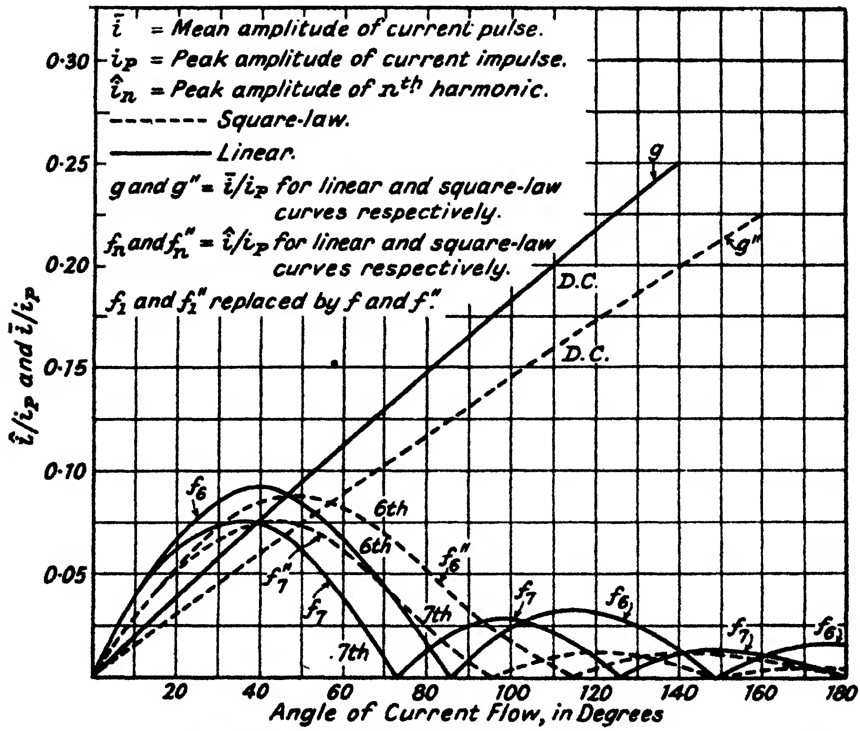


FIG. 8/VIII:1.—Analysis of Space-Current Impulses for Linear and Square-Law Tube Characteristics with Sinusoidal Input and Bias adjusted to different Angles of Current Flow.

(By courtesy of R. Calvert and B.B.C.)

terms are not of interest. The full expansion for the wave of Fig. 2 (*d*) is, for instance

$$S = A\Big[1 - \frac{a}{2\pi} - \frac{1 - \cos a}{1}\cos \omega t - \frac{1 - \cos 2a}{4}\cos 2\omega t$$

$$- \frac{1 - \cos 3a}{9}\cos 3\omega t - \frac{1 - \cos 4a}{16}\cos 3\omega t, \text{etc.}\Big]$$

The quantity a must be expressed in radians to such a scale that one period $= 2\pi$ radians. In other words, $\frac{a}{2\pi}$ = the duration of period of current rise expressed as a fraction of the whole period T.

An alternative method of expressing the series is therefore

$$S = A\left[1 - \frac{a}{T} - \frac{1 - \cos 2\pi\frac{a}{T}}{1}\cos wt - \ldots\right]$$

where $\frac{a}{T}$ is now a simple ratio, independent of the units in which a and T are measured.

The first two summations in Fig. 3, when expanded, yield a series of terms of the form:

(Prefix containing n) × (sine or cosine of a quantity not containing time) × (sine or cosine of a quantity containing time)

The sine (or cosine) of the quantity not containing time can be evaluated as a number and constitutes part of the factor determining the amplitude of each harmonic. The last term determines the frequency of each harmonic.

Fig. 5 is rather a special case. It gives the expansion of a carrier frequency $A \sin pt$ modulated 100% with a square wave. To enable an exact and comparatively simple solution to be obtained, the frequency of the carrier is made an even integral multiple ($= m$) of the fundamental frequency of the square wave. If this is not the case, a series of terms appear equal to multiples of the carrier frequency, and, in general, as such terms would be suppressed in tuning circuits, the limited solution in practice has an application extended to cases where m is not an even integral number. That is to say, the frequencies which are of such amplitude as to be important for preserving this particular wave form described are also important when m is not an even integral number, provided p and T are very near to the values inserted in the rigid solution. The remaining restrictions are inherent in the solution and are not imposed by the initial conditions. They are:

(1) Terms in which $(n+m)$ and $(n-m)$ are even disappear, i.e. they are of zero amplitude.

(2) The term in which $n = m$ disappears.

The method of expansion is to insert m in each term equal to the value chosen, and to give n all integral values which give rise to terms of important amplitude, omitting the terms which disappear.

The general method of finding the spectra of modulated waves is considered in VIII:1.1 immediately below.

1.1. Spectra of Amplitude Modulated Waves. The spectrum of the wave resulting from modulating a carrier wave with a periodic wave of any form can be obtained very easily if the spectrum of the envelope is known.

If a modulating wave contains frequency components f_1, f_2, f_3, etc., of relative amplitudes A_1, A_2, A_3, etc., the modulated wave resulting from modulating a carrier wave of frequency f_0 will contain

lated wave, measured as its distance from the zero of carrier current, is proportional to the amplitude of the modulating wave *plus* the amplitude of the unmodulated carrier.)

A particular example of this type of transmission occurs in the transmission of pulses of carrier frequency. These are produced by a process which is equivalent to modulating a carrier wave by the wave shown in Fig. 3 (*a*)/VIII:1 with no carrier transmitted in the absence of modulation. If p is the pulse duration in seconds, T is the time in seconds between the beginning of one pulse and the beginning of the next, and A is the peak amplitude of the carrier when transmitted, the envelope wave form is given by

$$S = A\left[\frac{p}{T}+\frac{2}{\pi}\sin\frac{\pi p}{T}\cos\omega t+\frac{2}{2\pi}\sin\frac{2\pi p}{T}\cos 2\omega t\right.$$
$$\left.+\frac{2}{3\pi}\sin\frac{3\pi p}{T}\cos 3\omega t+\text{etc.}\right]$$

where $\omega = \frac{2\pi}{T}$. (Incidentally $f = \frac{\omega}{2\pi} = \frac{1}{T}$, see below.)

If the carrier is of frequency f_0 and $\omega_0 = 2\pi f_0$, the modulated wave may be represented by $S \sin \omega_0 t$. This is an important deduction of general application.

In the present case

$$S\sin\omega_0 t = \frac{p}{T}A\sin\omega_0 t+\frac{2A}{\pi}\sin\frac{\pi p}{T}\cos\omega t\sin\omega_0 t$$
$$+\frac{A}{\pi}\sin\frac{2\pi p}{T}\cos 2\omega t\sin\omega_0 t$$
$$+\frac{2A}{3\pi}\sin\frac{3\pi p}{T}\cos 3\omega t\sin\omega_0 t+\text{etc.}$$
$$=\frac{p}{T}A\sin 2\pi f_0 t$$
$$+\frac{A}{\pi}\sin\frac{\pi p}{T}\cos 2\pi(f_0-f)t-\frac{A}{\pi}\sin\frac{\pi p}{T}\cos 2\pi(f_0+f)t$$
$$+\frac{A}{2\pi}\sin\frac{2\pi p}{T}\cos 2\pi(f_0-2f)t-\frac{A}{2\pi}\sin\frac{2\pi p}{T}\cos 2\pi(f_0+2f)t$$
$$+\frac{A}{3\pi}\sin\frac{3\pi p}{T}\cos 2\pi(f_0-3f)t-\frac{A}{3\pi}\sin\frac{3\pi p}{T}\cos 2\pi(f_0+3f)t+\text{etc.}$$

The frequency components in the resultant modulated wave are therefore as in the table below.

Amplitude	*Frequency*
$\frac{pA}{T}$	Carrier frequency $= f_0$
$\frac{A}{\pi}\sin\frac{\pi p}{T}$	$f_0 - \frac{1}{T}$ and $f_0 + \frac{1}{T}$
$\frac{A}{2\pi}\sin\frac{2\pi p}{T}$	$f_0 - \frac{2}{T}$ and $f_0 + \frac{2}{T}$
$\frac{A}{3\pi}\sin\frac{3\pi p}{T}$	$f_0 - \frac{3}{T}$ and $f_0 + \frac{3}{T}$, and so on

If, for instance, the pulses are of 25 micro-seconds' duration separated by intervals of 1/50 second,

$$p = 25 \text{ micro-seconds} = 1/40{,}000 \text{ second}$$
$$\text{and } T = 1/50 \text{ second} = 0{\cdot}02 \text{ second.}$$

Hence $\frac{p}{T} = \frac{1}{800}$ and $1/T = 50$.

Also $\frac{\pi p}{T} = \frac{\pi}{800}$, and since this is very small, $\sin\frac{\pi p}{T} = \frac{\pi p}{T}$, so that the amplitude of all the sidebands near the carrier frequency is $\frac{p}{T}A$, the same as the carrier frequency. As soon as $\sin\frac{n\pi p}{T}$ begins to deviate from $\frac{n\pi p}{T}$, with increase of n, the amplitude of the sidebands begins to fall.

The sidebands are evidently located at frequency intervals of 50 c/s each side of the carrier.

This section is unfortunately a good example of a case where, from the layman's standpoint, analytical treatment appears to wrap a simple subject in obscurity. The whole of the above amounts to saying that the sidebands in a modulated wave occur in relative amplitudes equal to the relative amplitudes of the modulating frequencies. The full treatment does, however, demonstrate clearly the truth of this statement, and brings out the difference between normal modulation and quiescent carrier conditions. It also establishes the phase relations of the sidebands. See also XIII:11.

2. Spectrum of Speech and Music.

The practice of regarding speech and music as being constituted by a band of frequencies may be justified from two standpoints. The first is that, regardless of the ultimate nature of speech and music, it is customary to make measurements on circuits with single frequency sinusoidal currents, and the characteristics of circuits are

described in terms of the attenuation, gain, phase, shift, etc., introduced at each frequency of measurement. Experience has established that circuits having one kind of characteristic described in such terms is good, and one having another kind of characteristic is poor. On this basis, for instance, it has been found that a circuit passing a band of frequencies from 30 to 10,000 c/s with the same efficiency gives a good, but not perfect, performance from the point of view of fidelity of reproduction, while one passing a band of frequencies from 300 to 2,000 c/s gives a poor performance.

There is, however, a more fundamental reason for regarding speech and music as constituting a band of frequencies, although the first examination of the nature of speech and music appears to lead away from this view. While it is true that during sustained vowel sounds and sibilants, or sustained musical notes, there are sometimes as many as 10 or 15 repetitions of substantially the same wave form, all these sounds are of finite duration and are not repeated indefinitely. A "*frequency*" is essentially a sine wave of steady amplitude and infinite duration. In practice, fortunately, the transient conditions consequent on the starting of a sine wave in a circuit die away so quickly that the conditions a few seconds after the application of a sinusoidal wave to a circuit are indistinguishable from those which would have existed if the wave had been applied a year before. The duration of a large percentage of the sounds of speech is, however, not sufficient for this condition to apply, and they therefore have to be considered as transients and not as periodic waves. Further, explosive sounds such as *p*, *b*, *t*, *k*, etc., are essentially non-periodic and are called transients.

It will be shown, however, that the preservation of the form of a transient is related to steady state measurements on a circuit, i.e. to measurements made with sinusoidal waves which have been applied for a sufficient time for initial transients, consequent on their starting, to die away.

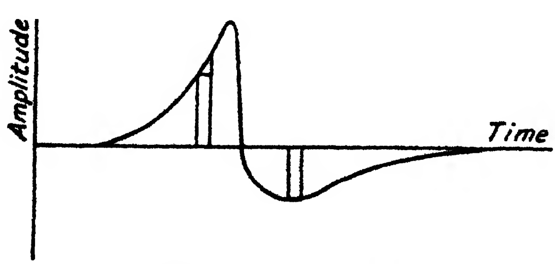


FIG. 1/VIII:2.—Typical Transient.

Fig. 1 shows a transient voltage or a current flowing in a circuit. It may be remarked in passing that owing to the presence of transformers, all transients existing in a speech or music circuit must have such a form that their arithmetic mean value is zero: the quantity of electricity flowing in a negative direction is equal to the quantity flowing in a positive direction throughout the duration of the transient.

Such a transient may be considered to be made up of an infinite number of small square-topped pulses of infinitely short duration. Two finite pulses of this form are indicated.

The effect of applying any transient voltage at the input of a circuit is to give rise at the output of a circuit to a disturbance which in general is of different shape from, and of longer duration than, the initial transient.

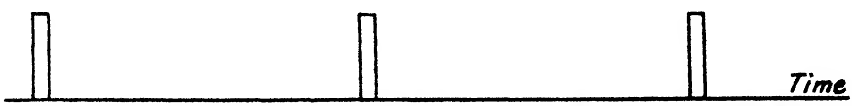


FIG. 2/VIII:2.—Periodic Pulses representing Approximations to Element of Transient.

Now consider the periodic wave of Fig. 2 which consists of a series of pulses separated by time intervals long compared with the duration of the pulses. It is evident that if the transient disturbance created by each pulse dies away substantially completely before the next pulse is applied, it is impossible by observing the circuit at any time to determine whether the last pulse was an isolated transient or part of a periodic wave. A periodic voltage wave may therefore be used to determine the effect of a transient voltage in a circuit, provided the periodic wave consists of voltages of the form of the transient separated by sufficiently large intervals.

Similarly, with the same limitations, the harmonic series for a system of periodic pulses of adequate spacing simulates the conditions produced by an isolated pulse at all times, after the occurrence of the pulse, which are less than the time interval between pulses. And since by hypothesis all voltages and currents after this period are negligible, later times are irrelevant. This point is of practical interest since it affords a simple method of determining the behaviour of circuits towards transients.

Further light on the nature of a transient can be derived from the following method of approach.

Figs. 3 to 6 show four periodic waves consisting of square pulses of duration $p = 2$ milliseconds transmitted with intervals T between

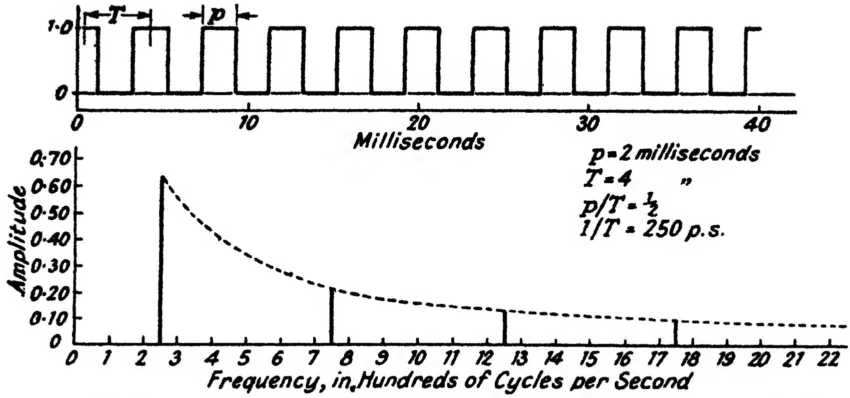


Fig. 3/VIII:2.—Unit Amplitude Square Wave of 2 Milliseconds' Duration and 4 Milliseconds' Period with Amplitude of Harmonic Frequency Components shown to an Enlarged Scale.

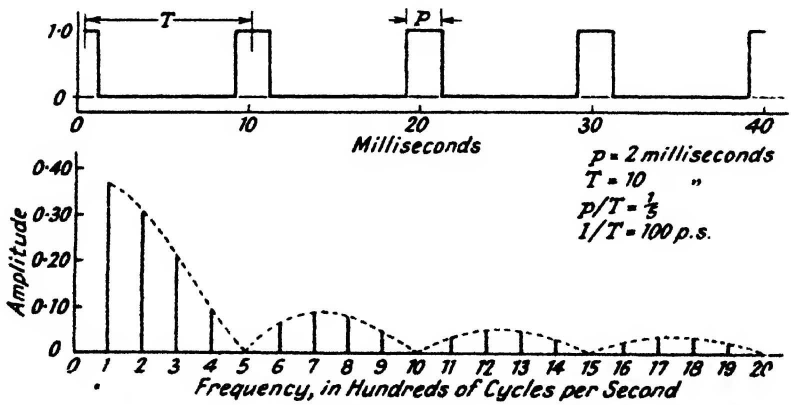


Fig. 4/VIII:2.—Unit Amplitude Square Pulse of Duration 2 Milliseconds repeated at Intervals of 10 Milliseconds with Amplitude of Harmonic Frequency Components shown to an Enlarged Scale.

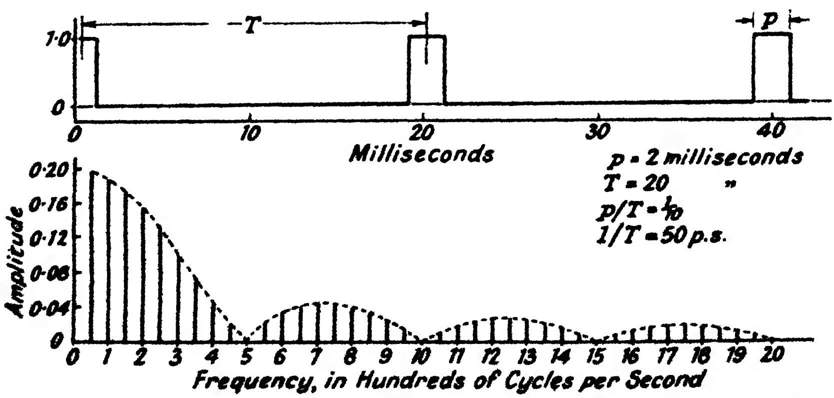


Fig. 5/VIII:2.—Unit Amplitude Pulse of 2 Milliseconds' Duration repeated at Intervals of 20 Milliseconds with Amplitude of Harmonic Frequency Components shown to an Enlarged Scale.

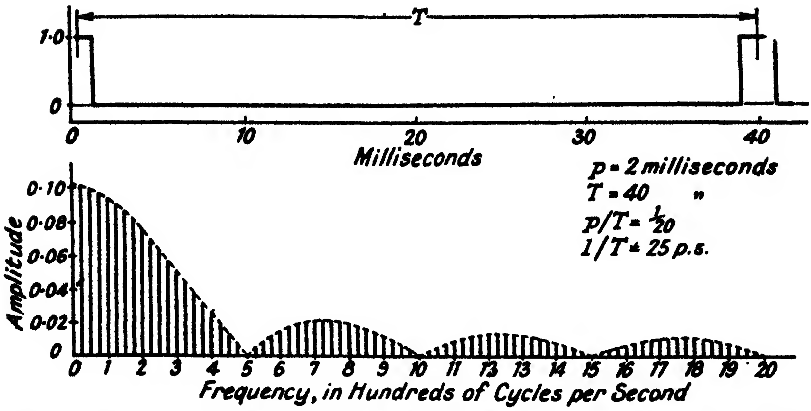


Fig. 6/VIII:2.—Unit Amplitude Square Pulse of 2 Milliseconds' Duration repeated at Intervals of 40 Milliseconds with Amplitude of Harmonic Frequency Components shown to an Enlarged Scale.

them varying from 2 to 40 milliseconds, together with the spectra of the component frequencies in each case. It will be seen that, except in the case where $p = T/2$, the frequency separation of the harmonic components is equal to the frequency of repetition of the pulse while the spectrum is modulated at frequency intervals equal to the reciprocal of p the length of the pulse in seconds. Further, as the interval between pulses is increased, the amplitude of each individual component is reduced. It follows that if the period is increased to infinity, in which case the wave is represented by a single pulse of finite duration, the wave is represented by a continuous spectrum of frequencies of zero amplitude and zero frequency spacing, the form of the spectrum being as the dotted envelope of Fig. 6. It is for these reasons that atmospherics are more severe on long waves than they are on short waves, since most atmospherics are caused by transient disturbances.

As the pulse length is gradually reduced to zero the first hump of the envelope of the spectrum gradually extends to infinity, so that the spectrum up to any finite frequency is of uniform amplitude.

The suggestion has been made that the uniform spectrum of Johnson noise (see XVIII:6.7) is caused by the transients consequent on electron impacts. If this is the case, the fact that these impacts must be of finite duration leads to the expectation that the spectrum of Johnson noise falls off at some very high frequency.

It will now be evident that the practice of considering speech and music as being composed of a band of frequencies is basically sound.

3. Distortion.

Distortion is said to occur when the component frequencies of a wave applied at the input of a circuit appear at the output of the circuit in wrong relative amplitude or phase or accompanied by additional frequencies generated by the application of the wave to the circuit.

Distortion may broadly be classed under three headings :

(*a*) *Response Distortion or Frequency-Amplitude Distortion* occurring when the different frequency components emerge from the circuit with relative amplitudes different from those in the original wave.

(*b*) *Non-Linear Distortion* when frequencies not present in the original wave appear at the output of the circuit. This is due to the presence of non-linear circuit elements.

(*c*) *Phase Distortion* is said to occur when the component frequencies emerge from the circuit with mutual phase relations different from those in the original wave.

3.1. Response Distortion : Response Characteristics. Response distortion is due to a large number of causes, among which are the variation of line attenuation with frequency, the loss characteristics of transformers and coupling circuits in audio-frequency amplifiers, and the attenuation of sideband frequencies in the tuned circuits of transmitters, and in aerial and aerial coupling circuits, the last becoming more important as the wavelength is increased. The frequency-amplitude distortion occurring in any piece of apparatus is represented by plotting the absolute loss or gain of the apparatus at each frequency, or the relative loss or gain with reference to the loss or gain at some specified frequency, e.g. 1,000 cycles. When relative gain or loss are plotted, the resultant curve is usually called a response characteristic : response characteristics are plotted so that frequencies which are most efficiently reproduced are represented by the highest ordinates. This means that to derive a response characteristic from a gain characteristic the ordinates plotted are equal to the gain at each frequency minus the gain at the *reference frequency*. When derived from a loss characteristic the ordinates of the response curve are given by the loss at the reference frequency minus the loss at each frequency plotted. Gain is then plotted as a positive quantity and loss as a negative quantity.

Fig. 1 shows a typical gain frequency characteristic and the corresponding response characteristic referred to the gain at

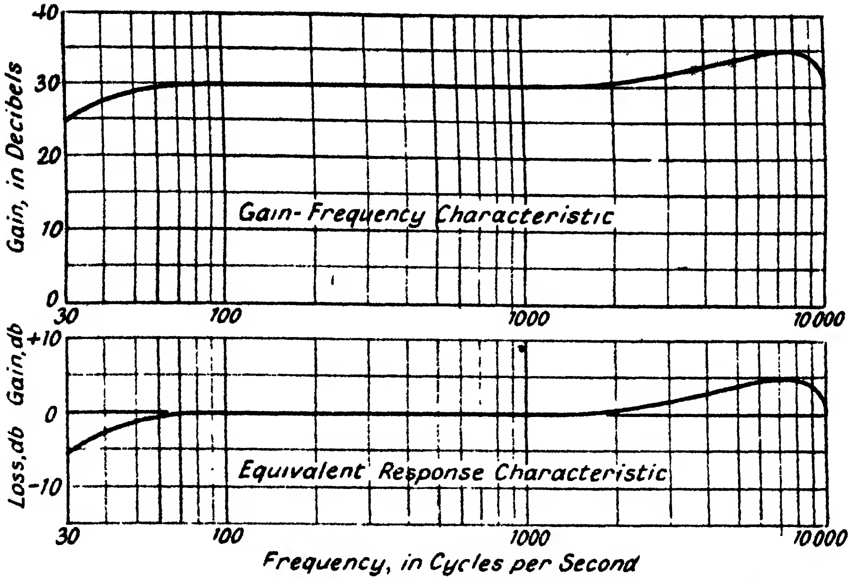


FIG. 1/VIII:3.—Typical Gain-Frequency Characteristic and Equivalent Response Characteristic. Reference frequency = 1,000 c/s.

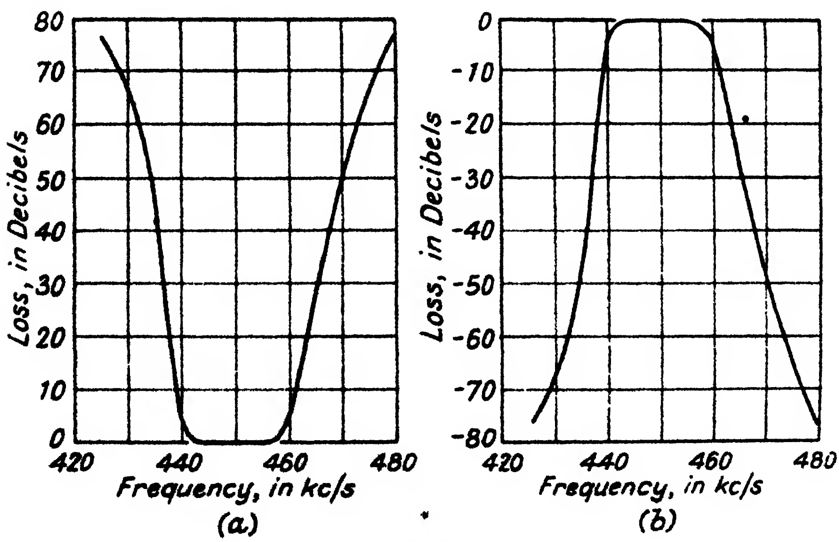


FIG. 2/VIII:3.

(a) Typical Loss Frequency Characteristic.
(b) Equivalent Response Characteristic. Reference frequency = 450 kc/s.

1,000 c/s. Fig. 2 shows a typical loss frequency characteristic and the corresponding response characteristic referred to the loss at 450 kc/s.

It is evident that the ideal gain, loss or response characteristic of an amplifier is represented by a straight line corresponding to equal reproduction at all frequencies. The loss characteristic of a filter must evidently rise in the frequency ranges where attenuation is wanted.

In general, gain characteristics are measured on amplifiers and loss characteristics over lines (e.g. underground cable circuits). It is evident, however, that for convenience a gain characteristic may be measured as a loss characteristic by inserting a suitable amount of distortionless attenuation (equal attenuation at all frequencies) such as would be provided by a resistance attenuator, in series with the amplifier.

In addition to response characteristics of lines, overall audio-frequency response characteristics are taken on radio transmitters by measuring the response from the input of the transmitter to the output of a distortionless receiving set tuned to the output wave-length of the transmitter.

Response distortion changes the timbre of speech and music and reduces the intelligibility of speech and the enjoyment of music. Its effects are exactly those to be expected from a reduction or an exaggeration of part of the audio-frequency spectrum, that is, certain notes of music and tones of speech are accentuated or diminished in intensity. Loss of high frequencies eliminates the harmonics which determine the characteristic sounds of certain instruments, e.g. a violin may sound rather like a flute; loss of low frequencies reduces the full sound of the base instruments, although owing to reconstruction of the fundamental bass notes by intermodulation in the non-linearity of the ear the loss of bass is not so pronounced as would otherwise be the case.

3.11. Permissible Deviation of Response Characteristics in Audio-Frequency Circuits. The question of permissible deviation from a straight line of the response characteristics of a piece of apparatus or a circuit is a large subject. It depends on what is possible for a given cost as well as what is desirable. It depends also on the number of pieces of apparatus occurring in tandem. From a practical point of view it is therefore most useful to specify either requirements which give good results in practice, and can be realized with reasonable expenditure, or requirements which are generally agreed, such as those published by the C.C.I. (Comité Consultatif Internationale).

Before doing this it may be generally stated that, provided the response does not change too rapidly with frequency, a variation of 3 db. in response over the range of frequencies transmitted is just detectable by the average listener. The response characteristic of a circuit determines also the range of frequencies which are transmitted. To obtain complete fidelity in audio-frequency circuits it would be necessary to transmit a frequency range from below 1 cycle per second up to about 18 or 20 kc/s. This is impracticable and it is generally agreed that the optimum compromise for high-quality circuits is reached with a frequency range extending from 30 to 10,000 c/s. This is reduced by certain land lines, while in the case of long-wave aerial circuits the inefficiency of the aerial circuits at sideband frequencies restricts the upper end of the audio-frequency range to about 6 kc/s.

It must be remembered that, in broadcasting, the receiver is an essential part of the chain, and few receivers have a good response extending above 5 kc/s even when the loud-speaker is neglected. Many loud-speakers in receivers have a poor response below 100 c/s. At the present time the receiver is the weakest point in the chain and the high quality radiated from most transmitters is spoilt by the receiver. For this reason, where important economies can be realized by so doing, the frequency range above about 6 kc/s may be neglected with some justification.

It is quite a practical proposition to build an audio-frequency amplifier with a flat response over the bulk of the range 30 to 10,000 c/s, deviating by only 0·2 db. at 10 kc/s and 0·5 db. at 30 c/s. This is a standard which should always be met in speech input and terminal amplifiers in broadcast circuits.

The C.C.I. requirements on a broadcasting transmitter call for a response flat "within 5%" (which presumably means within $\pm 0{\cdot}4$ db.) over a required frequency range from 30 to 10,000 c/s.

The C.C.I. requirements on the overall characteristics of land lines including repeaters (line amplifiers) and attenuation equalizers state that the overall frequency characteristic shall be flat within $\pm 1{\cdot}3$ db. over the range of frequencies transmitted which is to be 50 to 6,400 c/s. In apparent conflict with this, they also state that a frequency shall be considered to be effectively transmitted when the attenuation at that frequency does not exceed the 800 c/s attenuation by more than 4·3 db. What is apparently intended is that a deviation of 4·3 db. should be permitted at the ends of the frequency range, but that over the remainder of the range the tolerance is to be $\pm 1{\cdot}3$ db.

Having regard to the above, it is reasonable to expect a chain comprising studio equipment (excluding the microphone), transmitter speech input equipment, and a local transmitter, to have a deviation from the 1,000 c/s response less than 3 db. at 30 c/s and not exceeding 1·4 db. at 10 kc/s. A transmitter should be able to meet the C.C.I. requirements, and including its speech input equipment should have a deviation at 30 c/s not exceeding 1·4 db. and little in excess of 0·8 db. at 10 kc/s.

3.2. Phase Distortion in Audio-Frequency Circuits. From an apparatus point of view phase distortion in audio-frequency circuits is usually unimportant; it only occurs of serious magnitude in long land lines. A special case of phase distortion which may occur in long-wave aerial circuits is discussed later. See VIII:4.1 and 4.2.

In general, all frequencies, in traversing any circuit, suffer phase shift: there is a difference in phase at each frequency between the input voltage or current and the output voltage or current respectively.

In the case of a transmission line carrying a frequency f, if α is the phase shift per unit length at that frequency in a length of line λ such that the phase shift along it is 2π, i.e. such that $\alpha\lambda = 2\pi$, there exists one complete cycle of amplitude in length λ, so that the length of line λ contains one wavelength, or

$$\lambda = \frac{2\pi}{\alpha} = \text{the wavelength at frequency } f \qquad . \quad (1)$$

Phase Velocity. Every time the voltage or current at the sending end of the line goes through a complete cycle, which takes place f times a second, a new wavelength is launched on the line. There are therefore f wavelengths sent out per second so that each wave travels forward at a velocity

$$V = \lambda f = \frac{2\pi f}{\alpha} = \frac{\omega}{\alpha} \quad . \quad . \quad . \quad . \quad (2)$$

This velocity is called the Phase Velocity.

In order that the relative time relations of all frequencies shall be preserved at the receiving end, the velocity of all frequencies must be the same, so that V is a constant and

$$\alpha = \frac{2\pi}{V} f \quad . \quad . \quad . \quad . \quad (3)$$

Condition for Freedom from Phase Distortion. Distortionless phase-shift conditions therefore exist when the phase-shift frequency characteristic and the phase-shift angular frequency

characteristic is a straight line passing through the origin as the line A in Fig. 3.

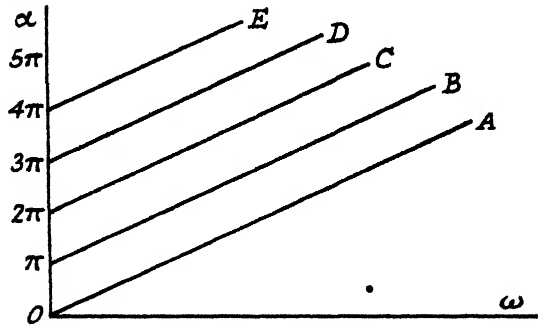


FIG. 3/VIII:3.—Distortionless Phase-Shift Characteristics.

Lines B, C, D and E, etc., also represent distortionless characteristics since they represent characteristics of the form at A in combination respectively with 1, 2, 3 and 4, etc., commutations. The cotangent of the angle of slope of this line $= \frac{\omega}{\alpha} = V$ the velocity = the distance travelled in unit time. The tangent of the angle of slope $= \frac{\alpha}{\omega} = \frac{1}{V}$ = the time taken by the wave to travel unit distance. This is called the *delay* per unit length.

If α is the total phase shift of a distortionless circuit, then $\frac{\alpha}{\omega}$ is the total delay. Although the discussion has been conducted with reference to a transmission line, it applies exactly to any four-terminal network in which, if α is the total phase shift, $\frac{\alpha}{\omega}$ is the delay. The velocity through a network of dimensions small compared with the wavelength is evidently not expressed in distance units per second, but in terms of network sections traversed per second. It can only have meaning in this respect when the network is composed of a number of identical sections in tandem. For this reason, and because, when phase distortion exists, the difference in delay for different frequencies provides a figure of merit for the circuit, it is usual to describe phase distortion in terms of delay rather than velocity.

Performance of a Circuit in which $\frac{\alpha}{\omega}$ is not a Constant. The phase velocity is the rate of travel of a wave crest which is part of an infinite sinusoidal train, whereas communication and

broadcasting engineers are concerned only with finite wave trains. The component waves of speech and music are finite wave trains. As explained above, it is possible to consider a finite wave train as being made up of an infinite number of infinite sinusoidal wave trains which for all time and all space have zero resultant except at the time and place where the finite wave exists. The wave crests of the infinite wave trains travel with the phase velocity.

Except in distortionless circuits no single velocity can be assigned to the finite wave train, since as it moves it expands or contracts along its direction of travel. (In distortionless circuits this does not occur and the wave train travels at the phase velocity.) At any instant the position of the finite wave train is defined by the envelope of the infinite wave trains composing it. The smallest number of frequencies which can possess an envelope is two, and hence the rate of travel of different parts of the whole envelope can be determined by considering the rates of travel of each component envelope constituted by two frequencies close together in each part of the frequency spectrum in turn. This consideration shows how the (component) *envelope velocity* varies with frequency. Another name for the envelope velocity is *group velocity*.

Determination of Group Velocity. Fig. 4 shows two infinite

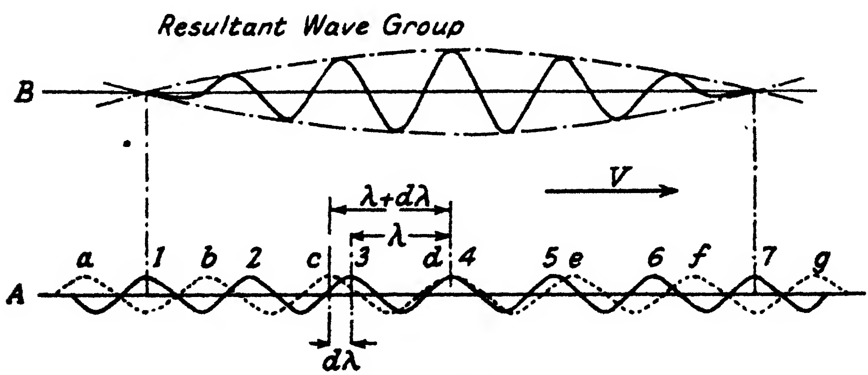


FIG. 4/VIII:3.—Wave Group.
(By courtesy of Messrs. Standard Telephones and Cables.)

sinusoidal wave trains travelling from left to right, 1, 2, 3, etc., being the crests of one train, and *a*, *b*, *c*, etc., being the crests of another infinite train contiguous in the frequency range. If the two infinite wave trains are contiguous component frequencies of a finite wave train not shown, then their frequencies, and so their wavelengths, will differ by an infinitesimal amount. If the wave train

1, 2, 3 is of wavelength λ, the wave train a, b, c, may consistently be considered to have a wavelength $\lambda+d\lambda$, and also to travel from left to right, but at a phase velocity $V+dV$, the difference in wavelength and velocity being due to the change in α consequent on the change in frequency. The resultant wave obtained by adding the ordinates of the two wave trains is shown as the full line curve at B, which is an infinite wave train with an envelope as shown by the chain dotted line. It may be regarded as an infinite train of finite *wave groups* of which one is shown in Fig. 4B.

Crests a, b, c, etc., have a velocity relative to crests 1, 2, 3, equal to dV. The time taken for crest c to overtake crest 3 is $\frac{d\lambda}{dV}$, and, when crests c and 3 become coincident, the centre of the wave group (which in Fig. 4 occurs at the point of coincidence of crests d and 4) will have travelled the distance λ backwards relative to crests 1, 2, 3, etc. The velocity of the group relative to crests 1, 2, 3, etc., is therefore

$$-\frac{\lambda}{d\lambda/dV} = \lambda\frac{dV}{d\lambda}$$

and therefore the velocity of the group is

$$= V - \lambda\frac{dV}{d\lambda} \tag{4}$$

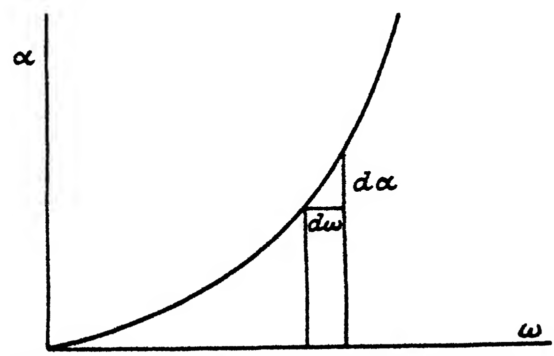


FIG. 5/VIII:3.—General Representative Phase-Shift Characteristic.

Now take the case of a transmission line in which the phase shift per unit length at any frequency $f = \frac{\omega}{2\pi}$ is defined by the curve of Fig. 5 where ω is angular velocity in radians per second.

When ω changes from ω to $\omega+d\omega$, the wavelength changes from

$$\lambda = \frac{2\pi}{} \quad . \quad . \quad . \quad . \quad (5)$$

to
$$\lambda + d\lambda = \frac{2\pi}{\alpha + d\alpha} \quad . \quad . \quad . \quad . \quad (6)$$

and the phase velocity changes from
$$V = \frac{\omega}{\alpha} \quad . \quad . \quad . \quad . \quad (7)$$

to
$$V + dV = \frac{\omega + d\omega}{\alpha + d\alpha} \quad . \quad . \quad . \quad . \quad (8)$$

Subtracting (5) from (6) and (7) from (8)
$$d\lambda = \frac{2\pi}{\alpha + d\alpha} - \frac{2\pi}{\alpha}$$
$$dV = \frac{\omega + d\omega}{\alpha + d\alpha} - \frac{\omega}{\omega}$$
$$\therefore \quad \frac{dV}{d\lambda} = \frac{\dfrac{\omega + d\omega}{\alpha + d\alpha} - \dfrac{\omega}{\alpha}}{\dfrac{2\pi}{\alpha + d\alpha} - \dfrac{2\pi}{\alpha}} = \frac{\omega}{2\pi} - \frac{d\omega}{d\alpha} \cdot \frac{\alpha}{2\pi}$$

So that the group velocity
$$g = V - \lambda \frac{dV}{d\lambda} = \frac{\omega}{\alpha} - \frac{2\pi}{\alpha}\left[\frac{\omega}{2\pi} - \frac{d\omega}{d\alpha} \cdot \frac{\alpha}{2\pi}\right]$$
$$= \frac{\omega}{\alpha} - \frac{\omega}{\alpha} + \frac{d\omega}{d\alpha} = \frac{d\omega}{d\alpha} \quad . \quad . \quad . \quad (9)$$

Delay. The delay per unit length
$$t = \frac{1}{g} = \frac{d\alpha}{d\omega} \text{ seconds} \quad . \quad . \quad . \quad (10)$$

The time taken for the group envelope to traverse l units of the line is therefore
$$T = l\frac{d\alpha}{d\omega} = \frac{d(l\alpha)}{d\omega} = \frac{da}{d\omega} \quad (11)$$

if $a = l\alpha =$ the total phase shift through the circuit.

The equation $T = \frac{da}{d\omega}$ applies to any form of network whether constituted by a line or a piece of apparatus.

Example.—A piece of apparatus has a phase shift-frequency characteristic such that the change in phase shift between 6,340 and 7,200 c/s is 2π radians, and between these frequencies the curve is substantially a straight line. What is the delay in this region?

[A. $T = \frac{da}{d\omega} = \frac{2\pi}{2\pi(7{,}200 - 6{,}340)}$ seconds

$= 1{\cdot}16$ milliseconds.]

Permissible Deviation of Delay in Audio-Frequency Circuits. It is evident that if the delay is constant over the frequency range transmitted, all parts of each finite wave train arrive simultaneously and there is no distortion. Permissible distortion can therefore conveniently be specified in terms of permissible difference in delay.

The deviation of delay in all audio-frequency apparatus, and in the envelope of modulated radio-frequency waves passing through tuned circuits including both transmitters and aerial circuits, is usually negligible, see below however. Up to the present, therefore, except in television circuits, it has only been necessary to prescribe limits for delay deviation occurring in land lines.

The C.C.I. requirements state that in broadcast circuits the difference in delay between 800 c/s and 6,400 c/s, shall not exceed 10 milliseconds, and between 800 c/s and 50 c/s, shall not exceed 70 milliseconds.

It is true that the appreciable delays which occur at low frequencies in land lines are due to the additions of many small delays introduced by the audio-frequency transformers in repeaters. But in radio transmitters there are usually not enough audio-frequency circuits to introduce a total delay of appreciable magnitude.

In reading discussions relating to phase shift through lines and networks, some confusion is likely to be caused because lag is represented as a positive angle, and lead as a negative angle.

This is in conformity with the definition of the transfer vector and image transfer constant of a fourpole, see XXIV:4 and 4.11.

The transfer vector of a symmetric fourpole is

$$u = \frac{V_1}{V_2}$$

where V_1 is the input voltage and V_2 the output voltage with the output of the fourpole connected to its image impedance. Hence, if the output voltage V_2 lags on the input voltage by an angle α,

$$u = \frac{V_1/\underline{\alpha}}{V_2\overline{\setminus\alpha}} = \left|\frac{V_1}{V_2}\right|/\underline{\alpha}$$
$$= |u|/\underline{\alpha}$$

so that the angle of u is positive.

It is this angle which is specified (i.e. the angle of u) when describing the phase shift through a fourpole.

This convention also results in the delay of a lossless network, when expressed as $\frac{d\alpha}{d\omega}$, being positive, which evidently corresponds with common-sense notions.

3.3. Non-Linear Distortion. This is due to the presence of circuit elements in which the current which flows is not directly proportional to the applied voltage, but to some other function of the voltage.

In a linear circuit of resistance R to which a voltage E is applied the current which flows is given by

$$I = \frac{1}{R}E$$

In a non-linear circuit it is in general not possible to specify a single coefficient such as $\frac{1}{R}$ to express the performance of the circuit, but to give an example, in one very common form of non-linearity the current may be expressed in terms of the voltage by a relation of the form

$$I = a+bE+cE^2+dE^3+ \text{ etc.} \qquad (12)$$

When a sinusoidal voltage $e \sin 2\pi ft$ is applied to such a non-linear circuit, the term a gives rise to direct current, the term in E gives rise to a current of frequency f; the term in E^2 to a current of frequency $2f$, and the term in E^3 to a current of frequency $3f$, and so on. When two voltages respectively of frequency f_1 and f_2 are applied simultaneously to such a non-linear circuit :

The term a gives rise to D.C.
,, ,, E ,, ,, ,, the original frequencies f_1 and f_2
,, ,, E^2 ,, ,, ,, frequencies $2f_1$, $2f_2$, f_1+f_2 and f_1-f_2
,, ,, E^3 ,, ,, ,, ,, f_1^3, f_2^3, $2f_1-f_2$, $2f_1+f_2$, $2f_2-f_1$, $2f_2+f_1$.

In both sets of cases the relative amplitudes of the different frequencies depend on the magnitudes of the coefficients in equation (12). All the products of the non-linear circuit other than those of frequency equal to the original input frequencies are called *distortion products.*

The frequencies of two and three times and other multiples of the original frequency are called *harmonics.*

The frequencies $f_2 \pm f_1$ $2f_2 \pm f_1$, etc., are sometimes called *sum and difference frequencies*, and sometimes *intermodulation products.*

In audio-frequency circuits non-linear distortion introduces frequencies which are not present in the original speech and music and, when noticeable to the same degree as frequency amplitude distortion, is much more objectionable.

Non-linear distortion is measured in several ways, of which three are worthy of notice here:

(1) By transmitting a single frequency (fundamental) through the apparatus under measurement, and measuring at the output the amplitude of second, third and fourth harmonics, etc., expressed as a percentage of the fundamental. This is done by means of harmonic analysers which are merely highly selective circuits associated with appropriate amplifiers and detectors. The input is usually varied from a small value up to full load, and a curve plotted of percentage harmonics against amplitude. Sometimes the harmonics are lumped together, the percentage harmonics being expressed as $\frac{\text{R.M.S. value of all harmonics}}{\text{R.M.S. value of fundamental}}$.

The R.M.S. value of a number of R.M.S. currents

$$i_1,\ i_2,\ i_3,\ \text{etc., is } I_{\text{R.M.S.}} = \sqrt{i_1^2+i_2^2+i_3^4},\ \text{etc.}$$

Alternately, by separating the fundamental and harmonics with low-pass and high-pass filters, and observing at the output of the filter on a square law device such as a thermocouple, the R.M.S. sum of the harmonics may be observed directly.

(2) By transmitting two equal amplitude frequencies f_1 and f_2 through the apparatus and measuring at the outputs the amplitudes of $2f_1$, $2f_2$, $3f_1$, $3f_2$, f_1+f_2, f_1-f_2, $2f_2 \pm f_1$, $2f_1 \pm f_2$.

(3) By transmitting a single frequency through the apparatus, varying its amplitude and plotting a curve of input voltage (or current) versus output voltage (or current).

The C.C.I. requirements for a transmitter state that in single frequency tests the audio-frequency harmonics shall not exceed 4% of the fundamental at 95% modulation, and this requirement is met in modern transmitters.

4. Effects of Distortion in Radio-Frequency Circuits.

The effects of distortion in radio-frequency circuits are of two kinds.

(1) Distortion of the envelope of the modulated wave which results in distortion of the audio-frequency at the receiving end.

(2) Distortion of the carrier which results in the production of harmonics of the carrier. In transmitters this may give

rise to radiation of frequencies of two, three and four, etc., times the carrier frequency, i.e. the radiation of wavelengths of half, a third, a fourth, etc., of the carrier wavelength.

Distortion of the envelope may be caused either by unequal attenuation or phase shift of the sideband frequencies due to the selective action of tuned circuits, or by any form of non-linearity.

Distortion of the carrier is a normal attribute of class C amplification and will be discussed under that heading. It gives rise to small-amplitude modulated carriers of two, three and four, etc., times the main carrier frequency, i.e. harmonics of the carrier frequency. These are suppressed by the normal circuits incorporated in the design of transmitters, but sometimes are not adequately suppressed.

4.1. Measurement of Distortion in Radio-Frequency Circuits. Distortion in radio-frequency circuits, in so far as it affects the audio-frequency envelope, is measured by measurement of the audio-frequency distortion observed on a distortionless radio receiver.

Harmonics of the carrier frequency are usually observed by making *field strength* measurements of the radiated field of these unwanted frequencies. They are evidently undesirable as they cause interference on other channels (i.e. other wavelengths).

The C.C.I. requirements state that on wavelengths above 100 metres the harmonic field strength must not exceed 300 μV/metre at a distance of 5 kilometres from the transmitting station. On wavelengths below 100 metres the power of the harmonic in the aerial must be 40 db. below the power of the fundamental and in no case must it exceed 200 milliwatts.

4.2. Effect of Attenuation and Phase Shift of Sidebands.

Let f = the envelope frequency (i.e. original modulating frequency) of a modulated wave.

f_1 and f_2 = the corresponding sideband frequencies

f_c = the carrier frequency

α = radio-frequency phase shift through circuit

α_c = the phase shift at the carrier frequency

α_1 = ,, ,, ,, ,, frequency f_1

α_2 = ,, ,, ,, ,, ,, f_2

b_1 and b_2 = the voltage ratios by which f_1 and f_2 respectively are diminished with regard to the carrier, due to attenuation.

The meanings of b_1 and b_2 may alternatively be expressed as the voltage ratios corresponding to the response of the circuit

respectively at f_1 and f_2, plotted with f_c as the reference frequency, i.e. with zero attenuation at f_c.

α_c, α_1 and α_2 are illustrated in Fig. 1, which shows a typical phase-shift frequency curve for a radio-frequency coupling circuit.

Case I. Attenuation and Phase Shift Symmetrical about Carrier Frequency: $b_1 = b_2$, $\alpha_2 - \alpha_c = \alpha_c - \alpha_1$. This is the most usual condition.

In the case of $b_1 = b_2 = b$, the only effect of the attenuation variation is to introduce an envelope response curve defined by

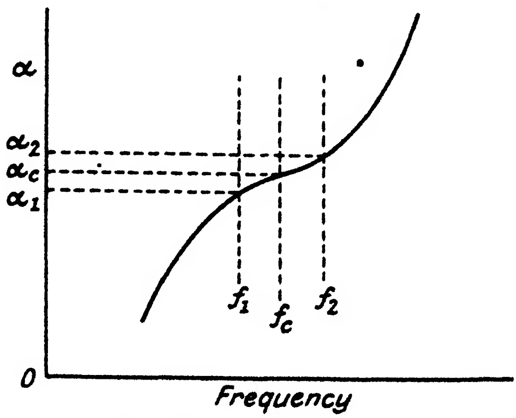


FIG. 1/VIII:4.—Phase Shift of Carrier and Sidebands.

plotting the value of b for each pair of sidebands, f_1 and f_2 corresponding to an envelope frequency f, against f.

The effect of the sideband phase shift is to introduce a phase shift in the audio-frequency envelope equal to $\alpha_2 - \alpha_c = \alpha_c - \alpha_1$ at each audio-frequency f corresponding to sideband frequencies f_1 and f_2. The resultant audio-frequency phase distortion is always negligible, although the phase shift may become important in circuits with envelope feedback. See XXIII:6 and 8.

Response distortion of appreciable magnitude occurs in the I.F. and R.F. circuits of radio receivers, and in long-wave transmitting aerial-tuning circuits.

Case II. Attenuation and/or Phase Shift Unsymmetrical about Carrier Frequency. In this case the effective audio-frequency response, as observed on a diode detector, is given by plotting against f the relative value of

$$\sqrt{(b_1+b_2)^2 \cos^2 \phi + (b_1 - b_2)^2 \sin^2 \phi} \; *$$

* See Bibliography A12:3.

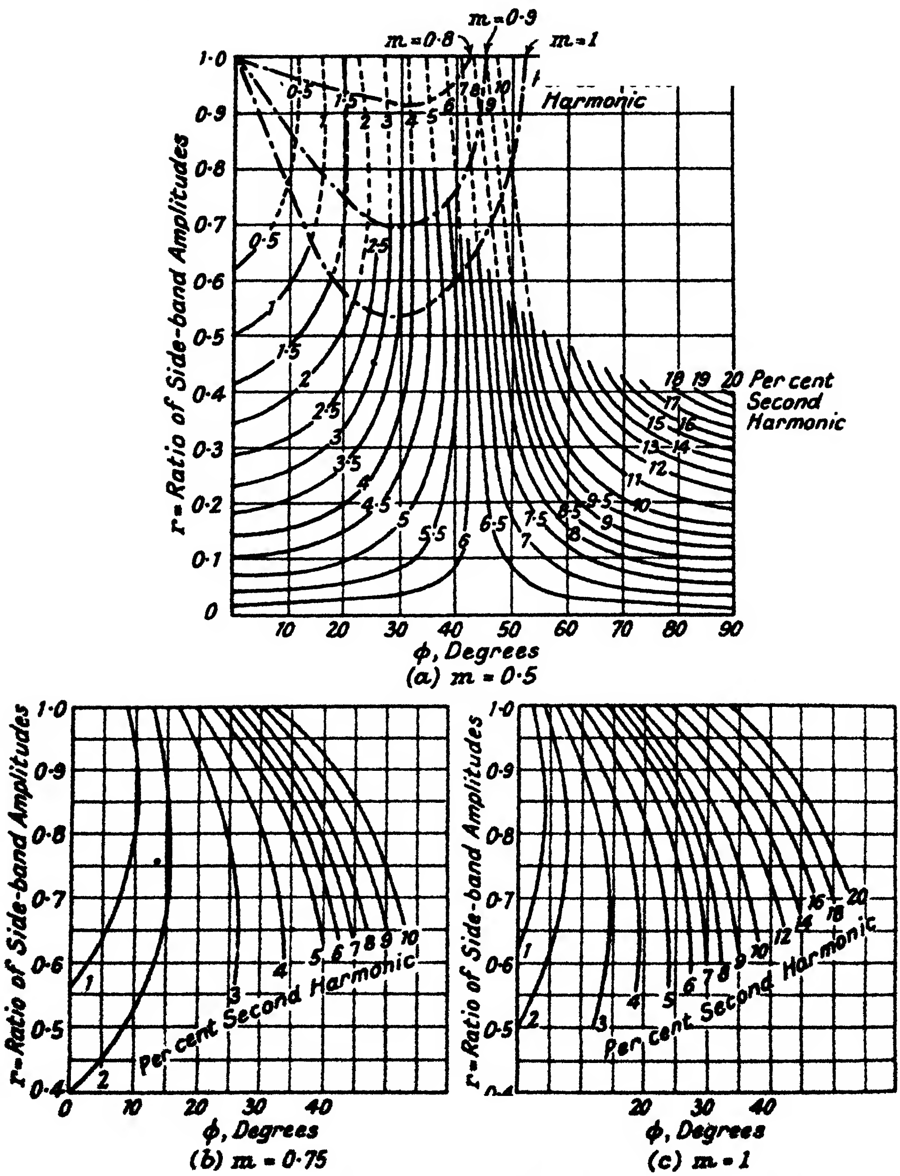


FIG. 2/VIII:4.—Curves of Constant Second Harmonic Distortion plotted against r and ϕ for three values of m. m = percentage modulation before one sideband is reduced in the ratio r. r = ratio: amplitude of reduced sideband divided by original amplitude corresponding to percentage modulation m. (The other sideband is assumed to be equal to the value corresponding to a percentage modulation m.) ϕ = phase shift of carrier from correct phase relation to sidebands for normal amplitude modulation.

(By courtesy of P. P. Eckersley and the Institution of Electrical Engineers.)

where $\quad \phi = \frac{1}{2}[\alpha_2 - \alpha_c - (\alpha_c - \alpha_1)] = \frac{1}{2}(\alpha_1+\alpha_2) - \alpha_c$

which is the amount by which the carrier phase shift α_c deviates from the phase shift $\left[\frac{\alpha_1+\alpha_2}{2}\right]$ corresponding to zero phase distortion.

The most important effect of sideband asymmetry is, however, the introduction of non-linear distortion when detection takes place on a diode detector. This is the only case of practical interest, and has been worked out by P. P. Eckersley * for the case of modulation with single-frequency sine waves.

The results are given in Figs. 2 (*a*), (*b*) and (*c*), which show lines of constant second harmonic expressed as a percentage of the fundamental frequency plotted as a function of ϕ and r, where $\phi = \frac{1}{2}(\alpha_1+\alpha_2) - \alpha_c$ and $r = \frac{b_1}{b_2}$ if $b_2 > b_1$ or $\frac{b_2}{b_1}$ if $b_1 > b_2$, i.e. r is the ratio of the amplitudes of the sideband frequencies after undergoing unequal attenuation and is always less than unity.

m is the depth of modulation of the normally modulated wave, assuming both sidebands to have the same amplitude as the large sideband. For values of modulation below 60%, i.e. below $m = 0{\cdot}6$, the distortion factor is linearly proportional to m, and for values of m below 0·6 the second harmonic after detection, expressed as a percentage of the fundamental, is obtained by multiplying the distortion percentage read off from Fig. 2 (*a*) by $2m$, e.g. the percentage second harmonic distortion for $m = 0{\cdot}25$, $\phi = 20°$ and $r = 0{\cdot}6$ is $2 \times 0{\cdot}25 \times 1{\cdot}5 = 0{\cdot}75\%$. This method of extrapolation may be extended to values of m above 0·6, using Fig. 2 (*a*), provided the parts of the chart above the chain dotted lines are not used, e.g. if $m = 1{\cdot}0$ the part of the chart above the chain dotted line marked $m = 1$ must not be used.

Above $m = 0{\cdot}6$ the distortion is no longer linearly proportional to m, and it is only possible to present a picture of the distortion by giving a set of curves for each value of m. Figs 2 (*b*) and (*c*) give the distortion directly for values of m respectively equal to 0·75 and 1·0. For instance, the value of second harmonic distortion for $m = 1$, $\phi = 25°$ and $r = 0{\cdot}9$ is 9%.

Asymmetric transmission characteristics become important in long-wave aerial circuits and in the intermediate frequency filters of superheterodyne receivers, particularly when the receiver is slightly mistuned.

* See Bibliography A12:3.

CHAPTER IX

THERMIONIC VALVES

1. The Diode.

If a potential difference is maintained between a red-hot or white-hot conductor and a cold conductor, so that the potential of the cold conductor is more positive than that of the hot conductor, the two conductors being insulated from one another and located in a gas-tight envelope evacuated to a high degree of vacuum (e.g. to a pressure below 10^{-4} millimetres of mercury), electrons flow from the hot conductor to the cold and circulate round the external circuit which is applied to maintain the potential difference.

The hot conductor is then called a *cathode*, and the cold conductor is called an *anode*.

In practical applications of this effect, the cathode is often in the form of a filament of wire and may then be called the *filament*. In such cases the filament is heated by passing a current through it from a second source of voltage. In other types of construction the cathode is *indirectly heated* by means of a heating winding, called the *heater*, in close proximity. In such cases heater and cathode are usually insulated from one another but always possess a small mutual capacity.

Valves with indirectly heated cathodes are sometimes called *equipotential*-cathode or *homopotential*-cathode valves because the cathode is all at the same potential.

The cold conductor is usually in the form of a plate or a cylinder, and for this reason is sometimes called the *plate*, but the more usual term is *anode*.

The device constituted by a cathode and anode in a sealed evacuated chamber, with the cathode and anode led out to suitable external contacts, is called a *diode*.

In Fig. 1 is shown the conventional representation of a diode with filament or heater battery and high-tension battery maintaining a p.d. between anode and cathode. The diode at (*a*) is directly heated and that at (*b*) is indirectly heated. *mA* indicates a milliammeter reading the current flowing in the external circuit which is called the *plate* current or *anode* current. *V* is a voltmeter reading the potential difference between cathode and anode.

If in either of the above circuits the applied voltage is varied

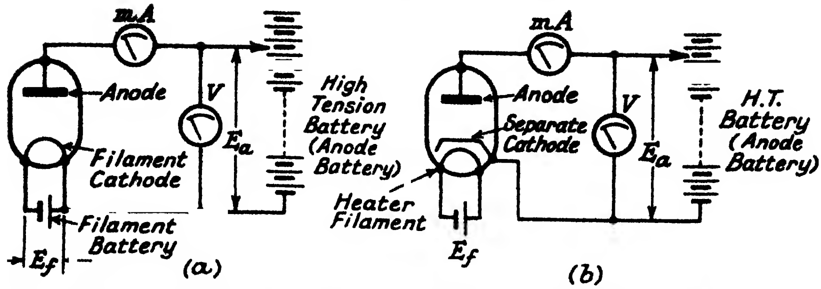


FIG. 1/IX:1.—Diodes with Filament or Heater Batteries and High Tension Anode Batteries. (*a*) Directly Heated Cathode; (*b*) Indirectly Heated Cathode.

by tapping along the H.T. battery, the current varies with applied volts in accordance with the general form of the curve of Fig. 2.

It is to be noted that if the battery were to be reversed in sense so as to make the anode negative with regard to the cathode, no current would flow; that is, the curve to the left of the axis of anode current runs along the axis of H.T. volts. The property of a diode, by virtue of which it passes current in one direction only, is called rectification. A diode is therefore one form of rectifier.

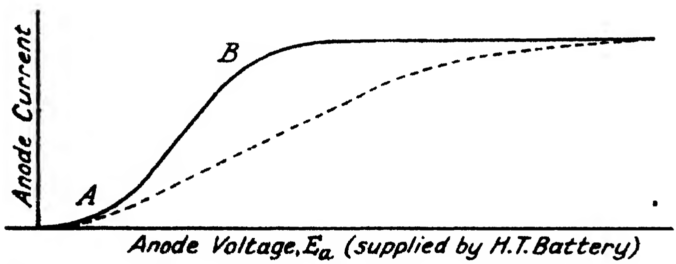


FIG. 2/IX:1.—Typical Anode-Voltage Anode-Current Characteristic of a Diode.

It is to be noted that two knees occur in the curve, one at *A* and one at *B*.

If a resistance of suitable magnitude is placed in series with the H.T. battery, and the voltage again varied, a curve of the form indicated by the dotted line can be obtained. It is important to note that this curve is much more nearly rectilinear (i.e. more nearly a straight line). In popular language it is said to be more ***linear***.

The knee at *B* is due to filament saturation; the filament can only furnish a limited amount of current, however much the H.T. voltage is increased. The same phenomena appears in triodes and in all valves depending on a heated cathode for the emission of electrons, and a valve working over the part of the anode current

anode voltage to the right of B is said to be in *filament saturation* or running into *filament limitation*. See IX:19.

Cathodes are either made of tungsten in the pure form, or, to increase the emission, the tungsten may be coated with the oxides of certain pure metals.

2. The Triode.

If a grid of fine wire is placed between the cathode and anode, the potential of the *grid* with regard to the cathode (and hence with regard to the anode) modifies the flow of electrons in the way indicated in Fig. 1/IX:2.

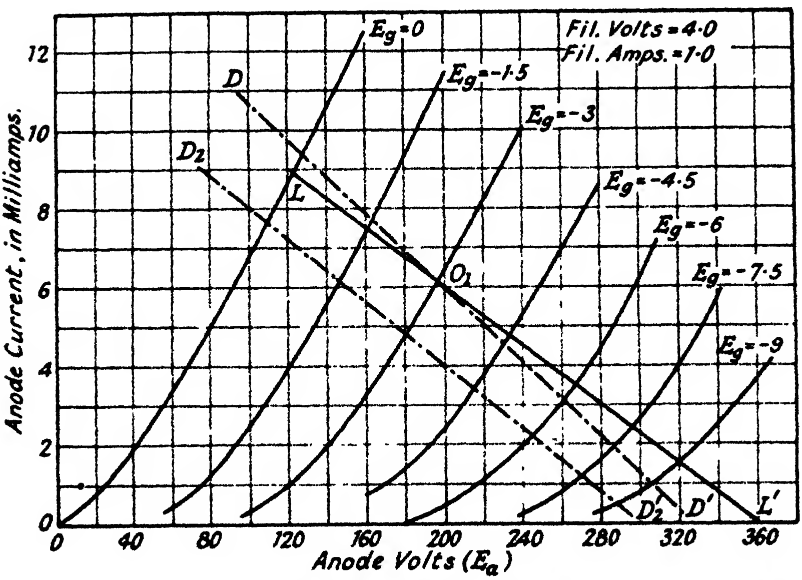


FIG. 1/IX:2.—Field of Anode Characteristics for a Mazda AC/HL Valve (Low Power Triode).

(By courtesy of the Edison Swan Electric Company.)

The combination of cathode, anode and grid in a sealed evacuated envelope is called a *valve*, since comparatively small power variations applied to the grid can cause large variation of power flowing in the anode circuit.

Fig. 1, which applies to a Mazda AC/HL valve, shows that corresponding to each of a number of steady potential differences, E_g, applied between grid and cathode there is a particular anode-voltage anode-current characteristic. Such a steady p.d. applied between grid and cathode of a valve is called the *grid bias* of the

valve. The circuit of Fig. 2 defines, E_g and E_a, the grid and anode voltages of Fig. 1/IX:2.

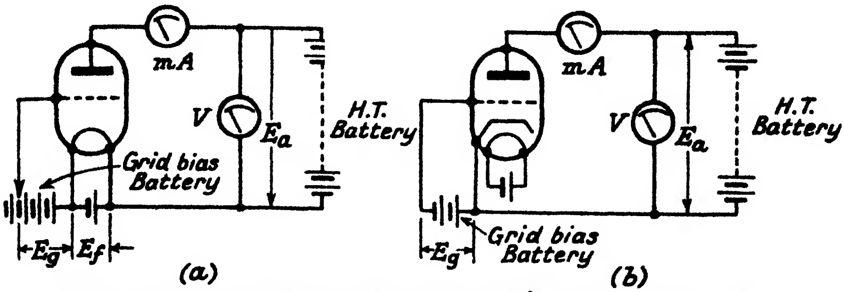


FIG. 2/IX:2.—Triodes with Applied Voltages, including Grid Bias.

The grid-bias voltages are chosen at intervals of 1·5 volts; evidently any other convenient value of interval might have been chosen.

It is evident that over a certain region the effect of changing the grid bias is merely to change the position of the anode-voltage anode-current curve, without appreciably changing its shape, and that over this region the distance that the anode characteristic is shifted along the axis of anode volts is directly proportional to the change in grid bias. The region over which these conditions hold is the region over which distortionless amplification can be obtained. This is explained below.

A set of valve characteristics plotted as in Fig. 1 will be referred to as a *valve field.*

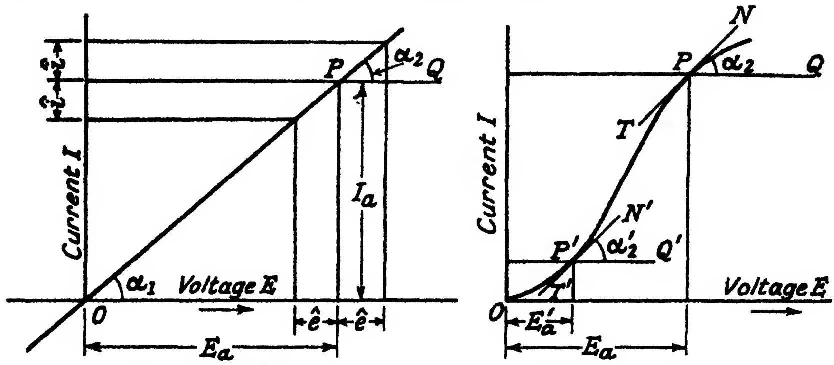
3. Anode Impedance $= R_a$.

In a direct-current circuit the value of a resistance R can be defined by the relation between E, the voltage across the resistance, and I, the current through the resistance, i.e.

$$R = \frac{E}{I} \qquad I = \frac{E}{R}$$

If this relation between E and I is plotted, it is of the form shown in Fig. 1 (*a*)/IX:3, from which it will be seen that $\frac{E}{I} = R = \cot \alpha_1 =$ the cotangent of angle of slope of the line; this is conveniently referred to as the slope.

If a steady voltage E_a is applied across the resistance, a current I_a will flow of magnitude given by $\frac{E_a}{R} = \frac{E_a}{\cot \alpha_1}$.



(a) *Linear Resistance* (b) *Non-linear Resistance*

FIG. 1/IX:3.—Relation between Voltage and Current in Linear and Non-Linear Resistances.

If, now, a voltage $\hat{e}$ sin ωt is placed in series with E_a, an alternating current $\hat{\imath}$ sin ωt will flow of magnitude defined by $\hat{\imath} = \dfrac{\hat{e}}{\cot \alpha_2}$, where α_2 is the angle between the line and the axis of voltage (and therefore also PQ, which is parallel to the axis of voltage), at point P.

Cot α_2, the slope at P, is called the *A.C. resistance*. In the case of a linear resistance $\alpha_2 = \alpha_1$, i.e. the A.C. resistance is always equal to the D.C. resistance: but in the case of a non-linear resistance (i.e. a resistance for which the relation between E and I is not a straight line) α_2 is not equal to α_1 and the A.C. resistance differs from the D.C. resistance. This is illustrated in Fig. 1 (*b*)/IX:3, where the A.C. resistance at two points, P and P', is given respectively by the cotangents of the angles of slope α_2 and α_2 at each point.

Reference to Fig. 1/IX:2 shows that a non-linear resistance is presented between the anode and cathode of a valve, and that the A.C. resistance presented to small alternating voltages varies in accordance with the values of applied anode volts and grid bias. Evidently, if the swing of the A.C. voltage is large enough, the A.C. resistance varies throughout the swing.

The A.C. resistance presented between anode and cathode of a valve to small alternating voltages is called the ***anode impedance***, and is usually symbolized by R_a. It evidently varies for different conditions of anode voltage and grid bias.

R_a = *Anode Impedance.*

The fact that in the absence of feedback (as in the case under

consideration) the anode impedance is a pure resistance, does not invalidate the use of the term impedance since an impedance may evidently be of zero angle. The instantaneous value of the anode A.C. resistance presented to voltage swings of large amplitude may evidently vary throughout each cycle, in which case non-linearity is said to be present. This is discussed under the heading of distortion. The anode impedance is not to be confused with the external impedance in the anode circuit; see discussion on load line immediately following.

4. Load Line.

A proper understanding of the use of the load line is essential to the appreciation of the operation of valves as amplifiers.

Consider the circuit of Fig. 1/IX:4, in which a resistance R is placed in series with the H.T. battery E, and consider what happens as the voltage E_g applied between grid and cathode is varied.

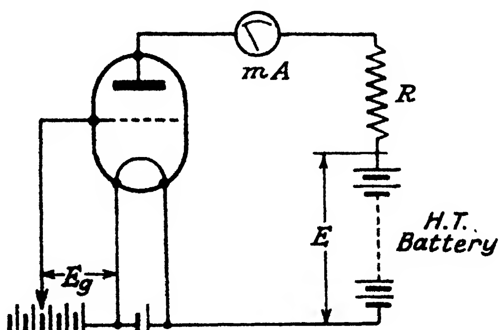


FIG. 1/IX:4.—Circuit Arrangement for which Load Line on Fig. 2/IX:4 is drawn.

Evidently the current I_a in the anode circuit varies and E_a the voltage between anode and cathode is given by

$$E_a = E - I_aR \qquad \text{(1)}$$

If $E = 10{,}000$ volts, and $R = 5{,}000$ ohms, substituting these values in (1):

$$E_a = 10{,}000 - 5{,}000 I_a$$

$$\therefore I_a = \frac{10{,}000 - E_a}{5{,}000}$$

It is important to notice that while E_a is the voltage between anode and cathode of the valve, the voltage across the load R is I_aR: this is the output voltage of the valve.

If $I_a = 0$, $E_a = 10{,}000$; if $I_a = 1{\cdot}2$ amps., $E_a = 4{,}000$, and so on, for all possible values of I_a less than 2 amps. and the saturation value of the valve filament. From these figures the value of I_a is plotted against E_a in Fig. 2/IX:4, which shows the field of anode characteristics for a Marconi ACT9 valve. This appears as the straight line LL'.

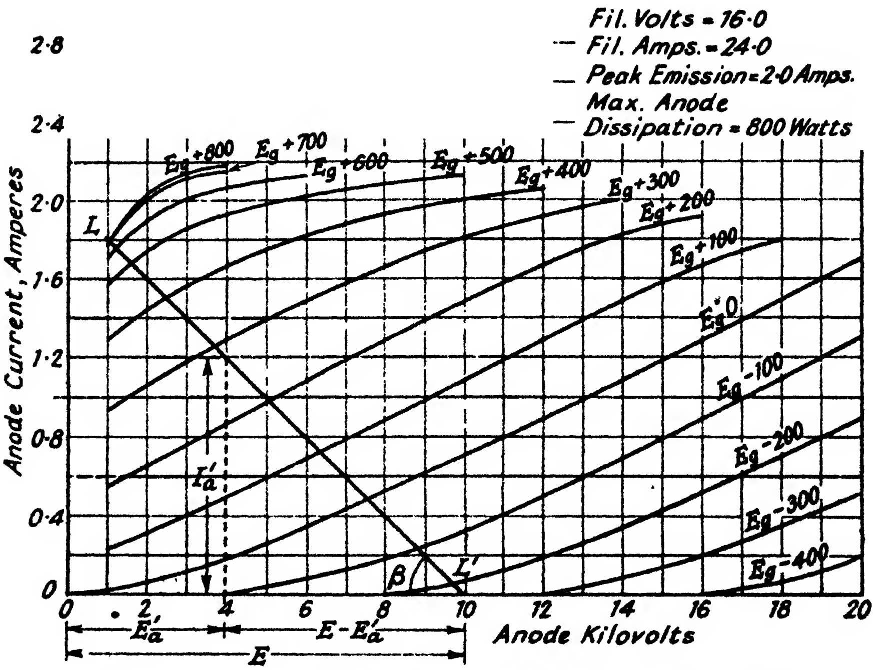


FIG. 2/IX:4.—Typical Load Line drawn on Field of Anode Characteristic of Marconi ACT9 Valve (Medium Power Triode).

(By courtesy of Marconi's Wireless Telegraph Company.)

This line is called the *load line* and determines the value of anode volts and anode current for any value of grid voltage. Once the load line has been drawn in, it is only necessary to know one of the three quantities, anode volts, anode current, or grid volts to determine the other two, e.g. if the instantaneous grid volts are known, the intersection of the corresponding anode-current anode-voltage curve with the load line gives the instantaneous *working point* of the valve, and projections from this point to the axes of anode volts and anode current give the values of instantaneous anode volts and anode current.

Considering two particular corresponding values of E_a and I_a, say E'_a and I'_a; from equation (1):

$$\frac{E - E'_a}{I'_a} = R = \cot\beta\text{; see Fig. 2/IX:4} \quad . \quad . \quad (2)$$

Hence the slope of the load line is R and the angle of slope of the load line is

$$\beta = \cot^{-1}R \quad . \quad . \quad . \quad . \quad (3)$$

The use of the load line in the important practical case where the load effectively consists of a resistance shunted by an inductance or a tuned circuit is dealt with in the section on Amplifiers.

4.1. Use of the Load Line. The load line serves the following purposes:

1. It gives by inspection:
   (*a*) The anode swing for a given grid-voltage swing.
   (*b*) The peak current for a given grid-voltage swing.
   (*c*) The power output for a given grid-voltage swing.
   (*d*) The peak current for a given anode swing.
   (*e*) The required value of grid bias.
   (*f*) The approximate amplitude of grid and anode swings which are permissible for a tolerable amount of distortion.
2. It enables the optimum load to be chosen for particular requirements of operation.

The first step in the design of a valve amplifier is to draw the load line, and while at this stage all the implications involved will not be clear, it is useful to summarize the general conditions which dictate the choice of load line.

The following requirements must be met:

1. *The peak anode current must not exceed the rated peak anode current for the valve* (see IX:19). The peak anode current is given by projecting on the axis of anode current the intersection, with the load line, of the anode-voltage anode-current characteristic corresponding to the maximum positive excursion of the grid. (This is the characteristic drawn with the grid bias which would bring the grid to the same potential that it reaches at the instant of maximum positive swing.)
2. *The rated anode dissipation of the valve* (see IX:17) *must not be exceeded.*

   In the case of class A amplifiers the *anode dissipation* is given by the product of the mean steady voltage effective

between anode and cathode and the mean anode current. This is called the *anode power input.* The mean anode current is generally called the *anode feed* and is the current measured on a D.C. ammeter or milliammeter in series with the valve anode. In class A amplifiers it is given by projecting on to the axis of anode current the point of intersection, with the load line, of the anode-voltage anode-current characteristic corresponding to the steady value of grid bias used. In the case of class C and class B amplifiers the anode feed can only be calculated when the drive is a steady sinusoidal voltage, see X:22.2. In the case of R.F. drive amplifiers, anode modulated amplifiers, and linear class B amplifiers for modulated waves, the anode dissipation may be calculated as the power input to the anode circuit, less the R.F. power output to the load in the carrier condition.

3. *The optimum compromise must be obtained between power output, efficiency and linearity* (freedom from non-linear distortion).

In class A amplifiers it is usually unnecessary to consider efficiency because class A amplifiers are generally used at low power where the power cost is so small that it can be neglected.

Linearity is ensured by seeing that the load line only traverses the region of the valve field where the anode-voltage anode-current characteristics (assumed to be drawn for equal increments of grid voltage) are equally spaced. It is, however, surprising to what extent this criterion can be violated without introducing serious distortion, and the only satisfactory method of making sure that the full power output for permissible distortion is being obtained, is to set up the circuit and make measurements of distortion.

In R.F. drive amplifiers the requirement of linearity is unimportant and can usually be neglected. The permissible grid dissipation (see X:38) must, however, not be exceeded.

In linear amplifiers for modulated waves it is necessary to limit the load line so that at 100% peak positive modulation the grid does not become more positive than the anode. If this happens, excess grid current flows, and although the permissible grid dissipation (see X:38) is not likely to be exceeded, the grid may fall to such a low impedance at the instant of maximum positive grid excursion that the tops of the grid-voltage wave are cut so that distortion occurs.

The optimum compromise is usually considered to be reached when, at 100% peak positive modulation, the anode peak volts are equal to 0·9 of the mean H.T. volts effective on the anode.

In the case of R.F. class C drive amplifiers and anode modulated amplifiers the equivalent criterion is that, in the carrier condition, the anode peak volts shall be at least 0·45 of the mean H.T. volts effective on the anode.

It will be evident from the above that in drawing the load line it is necessary to determine not only its slope, but also its length. In class A amplifiers the length of the load line is limited at each end by non-linearity, and at one end by the permissible peak-anode current, although usually the limitations of linearity and required power output prevent this limit from being approached. The permissible anode dissipation introduces a limitation on the height of the midpoint of the load line, that is the standing feed.

In class C amplifiers the load line is always drawn through the point : anode current = 0, anode volts = mean H.T. volts effective on the anode. In class B amplifiers it is usually sufficiently accurate to draw the load line through the same point, although it should strictly be drawn through the point : anode current = standing feed, anode volts = mean H.T. volts. The other end of the load line as well as the slope are then determined by the considerations enumerated above.

4.2. Grid Bias and Cut-Off. If the ACT9 valve having the characteristics of Fig. 2/IX:4 is supplied with an H.T. of 10,000 volts and a bias of − 200 volts (i.e. the grid is made 200 volts more negative than the cathode) with 5,000 ohms in series, a current of 0·06 amps. will flow. If the negative bias is increased to about 230 volts the anode current is just reduced to zero. The valve is then said to be biased to *cut-off*.

5. Voltage Amplification.

Referring to Fig. 1, which shows the anode-voltage anode-current characteristics of a Marconi DA100 valve, suppose that a resistance of 12,000 ohms has been inserted in the anode circuit and that the H.T. voltage available is 2,100 volts.

For a current of 150 mA the voltage drop through the load resistance is 1,800, so that the load line must pass through the points $E_a = 2{,}100$, $I_a = 0$ and $E_a = 2{,}100 - 1{,}800 = 300$, $I_a = 150$ mA. (E_a = voltage between anode and cathode of valve, I_a = anode current.)

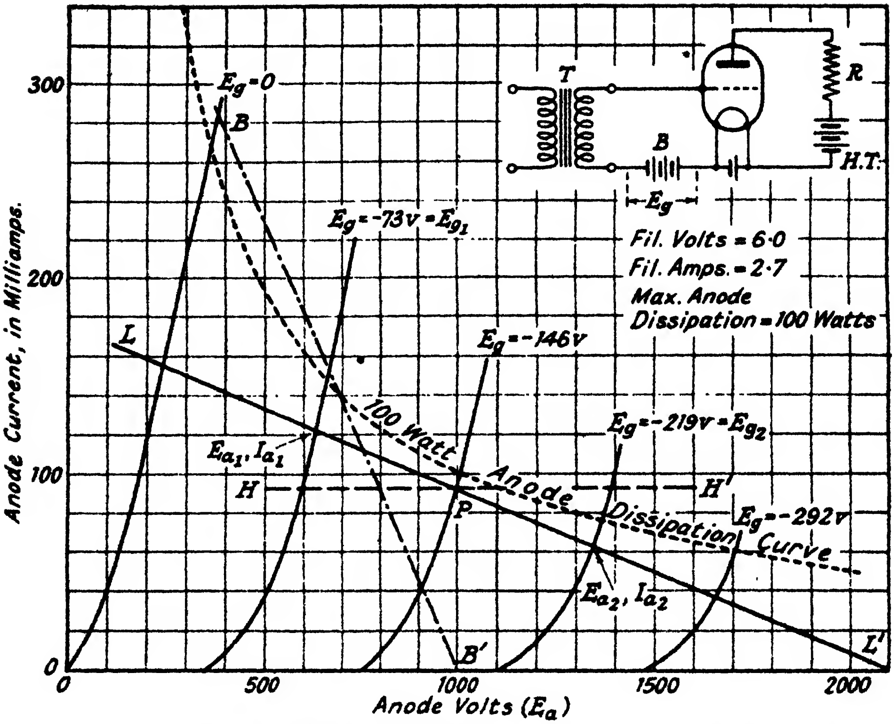


FIG. 1/IX:5.—Characteristics of Marconi DA100 (Triode) Valve with Load Line: illustrating Voltage Amplification.

(By courtesy of Marconi's Wireless Telegraph Company)

This load line is drawn in as LL'. Suppose now that by means of the circuit inset in Fig. 1 a steady bias of − 146 volts is applied to the grid of the valve, a current of 92 mA will evidently flow corresponding to the intersection of the anode-current anode-voltage curve, corresponding to a grid voltage of − 146 volts, with the load line. The anode volts are then 1,000.

The point P, defined in any of the following ways, is then called the ***working point*** of the valve. P may be defined by the anode volts (1,000) and the anode current (92 mA), *or* by the intersection of the − 146 anode-voltage anode-current characteristic with the load line or with the vertical through $E_a = 1{,}000$ volts or with the horizontal through $I_a = 92$ mA. The first way is the most usual and generally the most useful.

If, now, an alternating voltage 73 sin ωt is applied in series with the grid-bias battery B by means of the transformer T, the grid volts will swing from $E_g = -73$ to $E_g = -219$, and the anode

volts and anode current will swing along the load line from $E_{a_1}I_{a_1}$ to $E_{a_2}I_{a_2}$.

The anode-voltage swing and hence the swing of voltage across the load R is then from $E_{a_1} = 1{,}340$ volts to $E_{a_2} = 630$ volts, a total double amplitude swing of $1{,}340 - 630 = 710$ volts. The combination of valve and load has therefore provided a

$$\textit{voltage amplification} = \frac{710}{2 \times 73} = 4{\cdot}9$$

The current swing is then from $I_{a_1} = 62$ mA to $I_{a_2} = 122$ mA, and the peak current is 122 mA.

In general, if E_{g_1} and E_{g_2} are the limits of the grid excursion and E_{a_1} and E_{a_2} are the respective corresponding limits of the anode excursion (obtained, respectively, by projecting the intersection of the anode characteristics for E_{g_1} and E_{g_2} with the load line on to the axis of anode volts), then:

$$\text{The voltage amplification } \mu_e = \frac{E_{a_2} - E_{a_1}}{E_{g_1} - E_{g_2}} \qquad (1)$$

For instance, on Fig. 1, choosing the working point at P and the load line LL', and assuming a grid excursion of 73 volts,

$$E_{a_1} = 630 \text{ volts}, \qquad E_{a_2} = 1{,}340 \text{ volts},$$
$$E_{g_1} = -73 \text{ volts}, \qquad E_{g_2} = -219 \text{ volts},$$

so that the voltage amplification is

$$\frac{1{,}340 - 630}{-73 + 219} = \frac{710}{146} = 4{\cdot}9$$

which is the same result that was obtained before.

6. Voltage Amplification Factor of Valve = μ.

The voltage amplification when the load is of infinite impedance, that is, when the valve is working into open circuit, is called the voltage amplification factor of the valve, and is indicated by μ. It can be obtained at any point in the field of valve characteristics by drawing a load line parallel to the axis of anode volts across the field of anode characteristics (which is the load line corresponding to infinite anode load impedance) and determining the voltage amplification from equation (1)/IX:5 in the way described above for a finite load. For instance, drawing the line HH' in Fig. 1/IX:5 parallel to the axis of anode volts, for the DA100 valve in question at the working point used, and assuming a grid swing of ± 73 volts,

$$\mu = \frac{1{,}390 - 590}{2 \times 73} = 5{\cdot}5 = \text{open circuit voltage amplification of valve.}$$

Hence if one volt A.C. is applied between grid and cathode of a valve, the open circuit A.C. voltage across the anode circuit is μ volts.

7. Mutual Conductance of Valve = g_m.

The mutual conductance g_m, of a valve is the change of anode current per volt change of grid bias when the load impedance is zero, and the anode volts are kept constant.

$$g_m = \frac{I_{a_2} - I_{a_1}}{E_{g_2} - E_{g_1}} \quad . \quad . \quad . \quad . \quad (1)$$

where E_{g_1}, E_{g_2} and I_{a_1}, I_{a_2} are respectively corresponding values of anode current and grid voltage. g_m is usually specified in milliamps. per volt, so that I_{a_1} and I_{a_2} in the above formula should be in milliamps.

The mutual conductance can be obtained at any point in a field of valve characteristics, by drawing a vertical line (which is the load line corresponding to zero anode-load impedance) on the field of characteristics, and reading off the anode currents at the points of intersection of this line with the anode-current anode-voltage characteristics nearest to the point in question.

For instance, the mutual conductances of the PCA21 valve, whose field is shown on Fig. 1/IX:12 at points P_1, P_2, P_3 and P_4, are:

$$\text{At } P_1: \ \frac{1{\cdot}28 - 0{\cdot}4}{105 - 0} \times 10^3 = 8{\cdot}37 \text{ mA/volt}$$

$$\text{At } P_2: \ \frac{2{\cdot}0 - 1{\cdot}28}{175 - 105} \times 10^3 = 10{\cdot}3 \text{ mA/volt}$$

$$\text{At } P_3: \ \frac{3{\cdot}18 - 2{\cdot}0}{265 - 175} \times 10^3 = 13{\cdot}1 \text{ mA/volt}$$

$$\text{At } P_4: \ \frac{4{\cdot}25 - 3{\cdot}18}{335 - 265} \times 10^3 = 15{\cdot}3 \text{ mA/volt}$$

The PCA21 valve is a pentode.

7.1. Relation between g_m, μ and R_0. *By Thévenin's Theorem a valve can be considered as a generator of internal impedance R_0 and internal e.m.f. μe_g,* where R_0 is the internal anode impedance of the valve and e_g is the alternating e.m.f. applied to the grid.

When such a generator is short circuited, i.e. works into zero load impedance, the current which flows is $\frac{\mu e_g}{R_0}$, and if e_g is unity, the current is $\frac{\mu}{R_0}$.

In other words, if one volt A.C. is applied between grid and cathode of a valve, the short circuit A.C. current is $\frac{\mu}{R_0}$

Hence $$g_m \quad \frac{\mu}{R_0} \tag{2}$$

so that $$\mu = g_m R_0 \tag{3}$$

and $$R_0 = \frac{\mu}{g_m} \tag{4}$$

8. Power Amplification.

If the voltage swing across the anode load of one valve is applied between the grid and cathode of a second valve, further amplification of voltage results. Two valves so arranged are said to be in series, in tandem, or in cascade. The use of the term "series" here is poor nomenclature, but is very usual. Each valve with its associated anode and grid circuits is called a *stage.*

In the case of a number of valves in tandem, the power amplification of any one stage is given by the ratio

$$r = \frac{\text{Power delivered by valve to its anode circuit}}{\text{Power delivered by preceding valve to its anode circuit}}$$

The amplification or gain in decibels is then given by

$$\text{gain} = 10 \log_{10} r$$

Power delivered by a valve to the anode load.

Referring to Fig. 1 IX:5, and assuming as before a sinusoidal voltage swing 73 sin ωt applied to the grid, the power delivered to the load is given by $P = \frac{1}{2}$ peak voltage swing across load $\times$ peak current swing

$$\frac{1}{2}\left(E_{a_2} - E_{a_1}\right)\left(I_{a_1} - I_{a_2}\right) \tag{1}$$

$$\frac{1}{2}\left(1\ldots \quad 630\right)\left(\frac{122 - 62}{2}\right) \times 10^{-3} = 5{\cdot}37 \text{ watts}$$

9. Non-Linearity.

Anode-voltage anode-current curves should be and usually are drawn for values of grid voltage which increase by an equal number of grid volts from one curve to the next. If any load line is drawn intersecting these curves the projections of the points of intersection on to the axis of anode volts give the instantaneous values of anode volts corresponding to each instantaneous value of grid volts.

Towards the region of low values of anode voltage and high anode currents, anode characteristics corresponding to equal increments of grid voltage are not equally spaced from one another, so that the projection of the intersection of these curves with the load line are not equally spaced. This means that towards the top left-hand end of the load diagram equal changes of grid volts do not give rise to equal changes of anode volts and non-linear distortion is said to occur. A similar effect is observed at high values of anode volts and low values of anode current. This means that a sinusoidal grid voltage does not give rise to a sinusoidal anode voltage, but to a periodic wave of another shape. Such a wave can be analysed into a wave of the original frequency plus a number of sine waves of 2, 3, 4, etc., times the original frequency, and of appropriate amplitude and phase; see VIII:1 and VIII:3.3. In other words, the presence of non-linear distortion gives rise to a number of spurious frequencies, the presence of which can be extremely objectionable, when the input wave is originated by the sound form of a musical programme.

Examination of fields of characteristics of valves in commercial use shows that few, if any, are linear, and it is necessary to minimize non-linearity by the optimum location of load lines and appropriate adjustments of grid bias and grid swing. This is done partly by art and experience and partly by experiment: the valve is set up under what appears to be the optimum conditions, and adjustments are then made until the distortion products are a minimum.

10. Capacities in a Triode.

In a triode there are three important mutual capacities:

C_{ag}: between anode and grid
C_{gc} between grid and cathode
C_{ac} between anode and cathode

In valves with directly heated filaments the symbols C_{gf} and C_{af} will sometimes be used instead of C_{gc} and C_{ac} respectively.

When a triode operates as an amplifier, since in general the anode swings negative when the grid swings positive, a current flows through C_{ag}, the anode grid capacity, with the result that the input impedance of the valve no longer appears as a capacity of magnitude C_{gc}. Usually the anode swing is exactly 180° out of phase with the grid swing, in which case the grid input impedance appears as an effective capacity of magnitude.

$$C_e = C_{gc} + (1 + \mu_a)C_{ag}$$

where μ_e is the ratio of anode swing/grid swing, in other words, the effective voltage amplification of the valve in the particular circuit in which it is operating; see XXIII:9.2.

At high frequencies the value of C_e may be so large as to impose serious limitations on the performance of any coupling circuit with which the valve is driven, and since the fraction of C_e which is due to the feedback current is usually many times the value of C_{gc} (i.e. $(1+\mu_e)C_{ag} >> C_{gc}$), the presence of the anode-grid capacity is a serious disadvantage. Further, since it provides feedback of energy from anode circuit to grid circuit it is often a source of instability (tendency to oscillate), see X:19, XI:12 and XXIII:6. For this reason means have been devised to reduce the anode-grid capacity.

These led first to the screen-grid valve and then to the pentode.

11. The Screen-Grid Valve or Tetrode.

This has not come into general use because of the peculiar form of its characteristics, although there are one or two examples in use.

It consists of an otherwise normal triode in which an extra grid or screen has been inserted between the anode and the normal grid, which is now called the *control grid*, to distinguish it from the extra grid, which is called the *screen grid*. In operation the screen grid is maintained at a high positive potential with regard to the cathode by connecting it through a suitable resistance to the H.T. supply: by proper choice of this resistance the screen-grid current flowing through it drops the screen-grid potential to the required value below the H.T. voltage. To make it effective as a screen the screen grid is earthed through a condenser.

In a screen-grid valve the total *space current*, i.e. the number of electrons leaving the cathode, is determined by the potentials of control grid and screen grid, and for all practical purposes is entirely independent of the potential of the anode. (The curves of space-current screen-grid volts for a screen-grid valve have the same form as the anode-current anode-volts curve of a triode, and are substantially unaltered by any change of anode volts.)

For any given value of the potentials of control grid and screen grid, as the anode volts are increased from zero the anode current starts to increase, and since the total space current (anode current + screen-grid current) does not increase, the screen-grid current is decreased by exactly the amount that the anode current is increased.

11.1. Secondary Emission. If the anode voltage is increased

from zero, when the anode voltage reaches a certain value (less than the potential of the screen grid) the speed with which the electrons hit the anode reaches the point where *secondary* electrons are produced by impact. (They can be considered as being knocked off the anode by the electrons arriving from the cathode.) These secondary electrons are attracted to the electrode having the highest potential—i.e. to the screen grid—with the result that the anode current after first increasing begins to fall as the anode volts are increased further. Finally, just before the anode potential reaches that of the screen grid the anode current begins to increase again, rising rapidly and finally becoming nearly constant with further increase of anode volts, owing to cathode saturation.

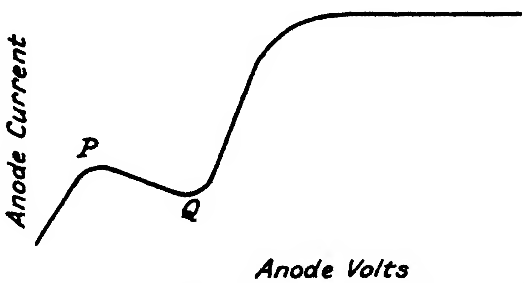


FIG. 1/IX:11.—Characteristic of Screen-Grid Valve.

The resulting anode-current anode-voltage curve is therefore of the form shown in Fig. 1. It is to be noted that the part of the characteristic between P and Q has a negative slope corresponding to a negative resistance. This negative resistance is an entirely real phenomenon, as can be shown by connecting an inductance and capacity in parallel across the anode-cathode circuit of a screen-grid valve biased to a point on the negative slope. The result is that the positive resistance of the associated circuit is neutralized and the circuit oscillates, taking energy from the anode circuit of the screen-grid valve. The initial pulse to start such an oscillation is provided either by the transients set up when the circuit is switched on, or by thermal agitation ; see XVIII:6.7 and IX:5.

The form of anode characteristic in a screen-grid valve results in a poor output, low efficiency and high distortion. The disadvantages of the screen-grid valve have been overcome in the pentode.

12. The Pentode.

The pentode is in effect a screen-grid valve in which a third

grid called the *suppressor grid* has been added between the screen grid and the anode. The suppressor grid is normally connected to the cathode and its function is to prevent the interchange of secondary electrons between screen grid and anode, which takes place in a screen-grid valve. This prevents the kink which occurs in the anode characteristic of a screen-grid valve.

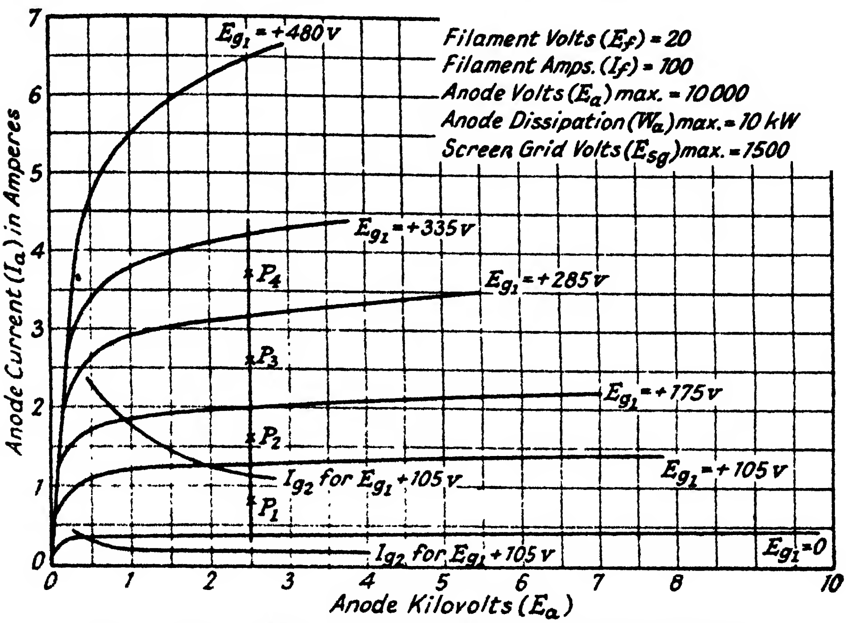


FIG. 1/IX:12.—Anode-Voltage Anode-Current Characteristic of Marconi PCA21 Pentode.

(By courtesy of Marconi's Wireless Telegraph Company.)

The anode-current anode-voltage characteristics of a Marconi PCA21 pentode are shown in Fig. 1/IX:12 and differ appreciably from those of a triode. In general, the working anode impedance of a pentode is very much higher than that of a triode and may be in the region of 200,000 ohms or even higher, while the anode impedances of triodes usually lie between 1,000 and 20,000 ohms, although triodes exist with anode impedances as low as 500 ohms and as high as 50,000 ohms. Since the anode impedance of a pentode, and of many triodes, falls to very low values with low H.T. and positive grid bias, it should be understood that the anode impedance of a valve (unless otherwise specified) is measured with working values of the applied voltages.

The screen grid of a pentode is usually biased from the H.T. supply either by means of a series resistance or a potentiometer,

and is earthed through a condenser. The suppressor grid is usually connected to the cathode.

The method of design of the output circuit for a pentode is identical with the method used for a triode, load lines being inserted in exactly the same way. It is generally found that the optimum load impedance is very much lower than the anode impedance; for instance, a pentode with an anode impedance of 200,000 ohms may require a load impedance of 7,000 or 10,000 ohms.

If the suppressor grid is disconnected from the cathode and a bias voltage introduced between the suppressor grid and the cathode, it is found that over a wide range of values of this bias voltage, the voltage amplification of the pentode from control grid to anode is directly proportional to the potential of the suppressor grid.

This means that pentodes can be modulated by varying the suppressor grid voltage; this system of modulation is known as *suppressor grid modulation*. In practice it is largely confined to small pentodes having outputs of the order of 10 to 50 watts; but so far it has not come into general use; see XIII:8 and XIX:10.

13. Mixers.

The fact that, in a valve with two grids in tandem, a voltage applied to one grid can be made to modulate the output of the anode circuit resulting from drive on the other grid, leads directly to the construction of hexodes, heptodes and octodes, which are used for modulation purposes in wireless receiving sets of the superheterodyne type; see XIX:10. Such valves are commonly called *Mixers*.

13.1. Hexode. Fig. 1(*a*)/IX:13 shows a type of mixer using four grids, two of which are connected together to constitute a

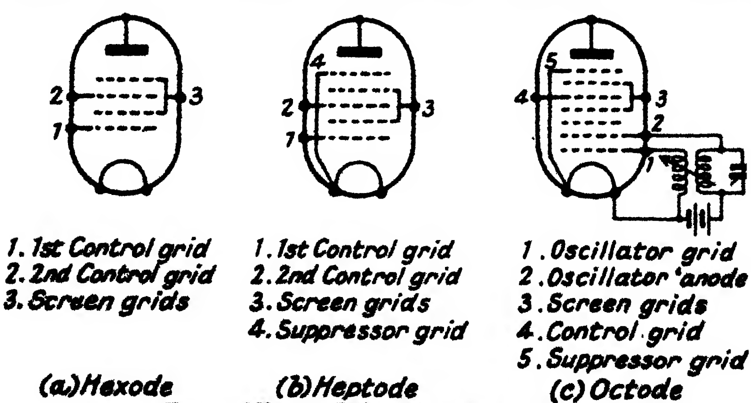


FIG. 1/IX:13.—Different Types of Mixer.

screen grid. In a radio receiver the received signal may be connected to the first control grid and the local oscillator to the second control grid, or vice versa.

It is evident that this mixer is the analogue of the screen-grid valve, since no suppressor grid exists between screen grid and anode.

13.2. Heptode. Fig. 1(*b*)/IX:13 shows a type of mixer which is the same as the hexode with a suppressor grid added.

13.3. Octode. This is a valve with six grids in which the two grids nearest the cathode constitute respectively the anode and grid of an oscillator. In Fig. 1(*c*)/IX:13 the circuit of a representative oscillator is shown connected to these two grids. By the arrangement shown the oscillator is said to be *electron coupled* to the anode circuit of the valve. Grids 3 and 5 are screen grids, 4 is a control grid to which the received signal is applied, while 6 is the suppressor grid.

14. Variable μ Valves.

These are usually pentodes which have an anode-current grid-voltage characteristic of the form shown in Fig. 1/IX:14.

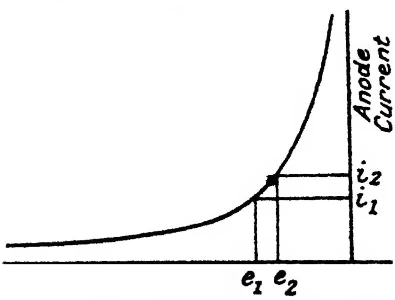


FIG. 1/IX:14.—Anode-Current Grid-Voltage Characteristic of Variable μ Valve.

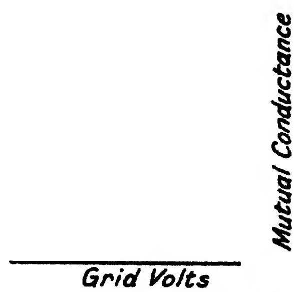


FIG. 2/IX:14.—Mutual Conductance Characteristic of Variable μ Valve.

The effective amplification of this valve *for small grid swings* is proportional to g_m, the mutual conductance, i.e. the tangent of the angle of slope of this characteristic, and as the slope of the curve varies widely with grid bias, by adjusting the grid bias the amplification of the valve may be varied.

This may be more obvious from the following argument. The value of external anode load used with a pentode is always very much less than the internal anode impedance of the pentode, as a result of which the anode current is almost unaltered by the presence of the anode load. The curve of Fig. 1/IX:14 therefore represents the practical behaviour of the valve with any practical value of anode load R_L.

If a change of grid voltage from e_1 to e_2 causes the anode current to vary from i_1 to i_2, the voltage amplification from grid to anode is

$$\mu_e = \frac{\text{anode voltage change}}{\text{grid voltage change}} = \frac{(i_1 - i_2)R_L}{e_1 - e_2} = g_m R_L$$

It is now obvious that $g_m = \dfrac{i_1 - i_2}{e_1 - e_2}$ is the tangent of the mean angle of slope of the curve in Fig. 1/IX:14 in the operating region considered.

It is evident from the above that any valve can be used to provide the feature of amplification varying with grid bias, but the special feature of a variable μ valve is that the curve relating the mutual conductance (and hence μ_e) and grid bias, approximates to a straight line, as in Fig. 2/IX:14.

15. Composite Valves.

Since most valve circuits have a common earth, where screening between the circuits is unimportant, it is possible to arrange any number of valves inside one envelope with a common cathode. While this practice has been carried to its logical limit by a German firm called Loewe, it is not usual to have more than two valves in one envelope, and these are usually valves which are required to work in close association.

There exist, therefore, Mixer-Triodes, Diode-Triodes, Double Diode-Triodes, Double Diode-Pentodes, Triode-Pentodes, etc. Double triode valves are also made, designed to constitute the two halves of a push-pull pair ; see X:8.

16. Short-Wave Valves.

Valves to operate at short waves, such as the CAT.21, are designed to have as small inter-electrode capacities as possible, and for this reason either the grid lead or the anode lead, or both, instead of being brought out through the base of the valve where they run near to the other electrode leads, as in normal low-frequency valves, are brought out through the top of the envelope.

In high-power valves extra provision is made for cooling the seals of the valve by means of air blast. In addition, in certain valves the grid seal is carried right round the envelope to provide a larger cooling surface. In some valves two leads are brought out for each electrode: in this way the inductance of the leads is halved.

17. Anode Dissipation.

Referring to Fig. 1/IX:5 and assuming as before: a load resistance of 12,000 Ω, H.T. volts 2,100, grid bias − 146 volts: it is evident that in the absence of grid swing (i.e. A.C. input voltage) the anode voltage is 990, say 1,000, and the anode current is 96 mA. The power input to the anode circuit of the valve proper, i.e. excluding the power dissipated in the load *R*, is 96 watts. This power is all expended in heating the anode of the valve. The power input does not change when grid swing is applied to the valve since the mean values of anode voltage and anode current do not change. When grid swing is applied, some of the 96 watts supplied to the anode circuit are supplied to the load in the form of A.C. power, the balance is expended in the valve and has the effect of heating the anode. In some valves in which the anode runs at a visible temperature, the anode can be seen to grow cooler when grid drive is applied and some of the anode input power is expended in the output circuit.

In small valves the anode maintains itself at a suitable temperature by radiation of heat, and the radiating properties of the anode, in conjunction with the temperature at which it can be run (governed by the material of which it is made, its size, and location in the envelope) determine the amount of power that can be dissipated in the anode circuit. In the case of the DA100 this is 100 watts, and the dotted line on Fig. 1/IX:5 is a plot of the permissible current at each anode voltage, for 100 watts input. In class A amplifiers, when assessing the permissible anode volts and anode current under steady conditions, it is usual to neglect the power supplied to the load, and to assume that these are limited by the rated dissipation of the valve, e.g. 100 watts in the case of the DA100. In class B and C amplifiers the power delivered to the load may sometimes be subtracted from the total input to the anode circuit to determine the effective anode dissipation. This may only be done in the case of amplifiers with a steady carrier drive on the grid (in the absence of modulation); see X:26.2.

17.1. Air-Cooled Anodes. In some medium-sized valves the permissible anode dissipation is increased by arranging that the anode of the valve constitutes part of the containing envelope of the valve and is fitted with cooling fins. Experiments are being made with a view to extending this practice to valves of higher power.

17.2. Water-Cooled Anodes. In high-power valves the anode, which again constitutes part of the valve envelope, is fitted

with a jacket through which water circulates. As the anode is at high potential above ground it is customary to feed the jacket through a long water path of high electrical resistance. This was at one time provided by means of rubber tubing arranged in a coil. The modern trend is, however, towards the use of porcelain or, more recently, polythene blocks made in one piece with a helical tubular water passage through them.

17.3. Cooling of Seals. The water-jacket is usually joined to the glass envelope with a copper glass seal which has to be kept cool during the operation of the valve. For this purpose a blast of air is directed on the seal. The seals at the points of entry of the grid and filament leads are also air cooled. In some cases filament seals are water cooled, but air cooling is becoming general.

18. Operation and Maintenance of Thermionic Valves.

Whenever power is applied to a radio transmitter the supplies to the high-power valves—water, air, L.T. grid bias and H.T.—must be applied in a definite order, otherwise the valves will either be destroyed or will have their life reduced.

The following order of procedure applies to all valves, whether or not they are equipped with all the features listed, except that on low-power valves the full filament voltage can be applied instantaneously. In the case of valves which are not equipped with all the features listed, the order of the list stands if the missing features are ignored.

18.1. Procedure for Applying Supplies to a Valve. See XV:I.

1. Turn on anode cooling water.
2. Turn on filament seal cooling air or water (air is tending to replace water in new designs).
3. Apply general air blast to valve including grid seal air cooling.
4. (Applies to large transmitting valves only.) Apply volts to filament starting with a low voltage so that filament current does not exceed 70% of normal, increasing volts until about 70% of normal current is reached. Since filament resistance is low at low-current values, this voltage is usually about 7% of normal voltage. After ten seconds the voltage is increased rather more rapidly, but current is not allowed to rise more than 10% above normal. The whole operation should take about 30 seconds. Due to the heating of filament leads, etc., the filament voltage should be readjusted after 5 minutes.

5. Apply grid bias.
6. Apply H.T.
7. Apply screen volts.

In the case of large transmitting valves the H.T. should initially be applied at not more than 70% of full H.T. and after a few seconds may be brought up to full H.T. See IX:18.3 and XV:1.1 for method to be adopted in putting a new valve into service.

In most large transmitters interlocks, associated with indicating meters and relays, are provided to control the order of application of supplies, and to make sure that each supply has reached the valve before the next can be applied. The maintenance of the interlocks and associated equipment in first-rate order is an important part of transmitter maintenance. If one of these safety devices fails a thousand pounds' worth of damage may be done in a second. Such things have happened. Where interlocks are not provided, or where the protection afforded is not complete, the responsibility for the correct order of procedure rests with the operating engineer. In small valves equipped with automatic grid bias, and/or with screen volts derived from the H.T., operations 5 and 6, or 6 and 7, or 5, 6 and 7 may be performed simultaneously.

With the qualifications given it is *absolutely essential that the above procedure be rigidly observed.*

The reason for the gradual application of filament voltage to large transmitting valves is because, when cold, the resistance of tungsten is only a small fraction of the resistance at the operating temperature, and the sudden application of the normal filament voltage would result in the filament passing a current many times its normal operation value. Further, the filament is brittle and the magnetic forces, consequent on the passage of a current of several hundred amperes, set up between the limbs of the filament are sufficient to cause mechanical distortion of the filament. The following table gives the normal filament voltages and normal currents, the starting voltages and currents, and the maximum permissible current for a number of high-power water-cooled transmitting valves which require the gradual application of filament volts.

18.2 Marked Volts (Filament Volts). In the case of high-power valves the manufacturers mark each valve with the value of filament voltage at which the guaranteed total emission is obtained. The effect of running the valves above this voltage is to reduce the life and render void the manufacturers' guarantee of life. *Valves*

TABLE I

Valve Type	Approx. Normal Filament Voltage	Nominal Filament Current (Amperes)	Approx. Starting Voltage	Upper Limit of Current during First 10 Secs. (Amperes)	Upper Limit of Current during following 10 Secs. (Amperes)
CAM.5	28	325	2·0	240	350
CAR.2	18–20	50	1·4	37	55
CAT.6	18–20	75	1·4	56	80
CAT.9	18–20	100	1·4	75	114
CAT.12, CAT.12A	30	220	2·0	160	240
CAT.14, CAT.14SW, CAT.14C	32·5	460	1·8	350	500
PCA.21	20	100	1·4	75	110
SS.1971	20	64	1·4	48	70
3Q/200A	20	59	1·4	45	65
4007A	20–21	50	1·4	37	55
4013C	13·5–14·5	36	1·0	27	40
4030C	25	250	1·8	190	280
4058B	19–20	58–61	1·4	45	65
4081A	20	59	1·4	45	65
4220C	22	41	1·5	30	45

must not, therefore, be operated above their rated filament volts. The term " marked volts " refers to this manufacturers' figure. Reducing the filament volts below the marked volts increases the life of the valve but reduces the total emission. Since transmitters are designed on the basis of their normally rated total emission, the effect of reducing the total emission is to introduce non-linear distortion. For reasons of economy, valve filaments are often operated below their marked volts, at such voltages that the resulting life and distortion represent what is considered to be the best compromise between the requirements of economy and fidelity. The amount of the reduction below the marked volts is in most cases about 8%, and in general the filament voltage should never be more than 3 volts below the marked volts.

Tungsten filaments decrease in diameter, due to evaporation, with a resultant reduction of emission, and as a consequence the filament volts have to be progressively increased during the life of the valve in order to ensure that the distortion does not increase above the specified figure.

If it is found that, as a consequence of lowering filament volts

or of ageing of the valve, the A.F. or R.F. output of the valve with normal grid drive drops, *the grid drive must on no account be increased*, since there is a serious risk of overheating the grid by putting in more power than the grid can dissipate. The correct procedure is to raise the filament volts until normal output is obtained with normal input.

18.3. New Valves: Introduction into Service of New High-Power Transmitting Valves and Valves which have been Out of Service for a Considerable Time. In the case of new valves, or of a valve that has been out of service for a period of more than 7 days, it is necessary to condition the valve by a gradual application of the high-tension voltage, and the conditioning should be carried out as follows:

An anode voltage of 5,000 volts should first be applied, and left on for 10 minutes. After this period the voltage may gradually be increased up to the full value in easy stages, and the whole operation should be completed in not less than 30 minutes. In the case, however, of valves which have been out of operation for a period of less than 7 days, this procedure may be accelerated, the first application of high-tension voltage being at 10,000 volts, and after 30 seconds it may be increased in one step to the full value.

During the ordinary operation of a transmitter, if the H.T. is interrupted, either deliberately or accidentally, for any reason (for example, due to the arc back from a mercury-vapour rectifier), the anode voltage may be reapplied immediately at a value not exceeding 70% of the normal, and 2 seconds thereafter may be increased to the full value.

Circumstances may arise when it becomes necessary, due to emergency, to reduce as far as possible the running-up period detailed above, and a concession may be made in the case of a valve which has completed a life of 1,000 hours, so that the running-up period may be reduced from 30 to 5 minutes, although the valve has not been in commission within the 7-day period mentioned above. It should be noted that this concession should not be used except in the most urgent cases.

It is intended that the above procedure should apply to valves type CAT.14, CAT.14C, CAT.20, CAT.20C, CAM.5, CAT.14SW, CAT.12A, PCA.21, 4067A, 4030C and 4Q/230A.

18.4. Gas Test. All transmitting valves on arrival at a transmitter should undergo a gas test. This consists in operating the valve at normal anode voltage, with grid bias giving a prescribed value of anode current, and measuring the reverse grid current.

Figures for the prescribed value of anode current and the permissible grid current are given by the manufacturers.

A list of the types of transmitting valves in general use in the B.B.C. with the valves of anode and grid voltage used for the gas test, together with the permissible reverse grid current and approximate anode current, is given in IX:21.

18.41. Pirani Test. Where it is not possible to set up a valve in a circuit in which normal H.T. can be applied so that the above type of gas test can be made, the Pirani test should be used.

This consists in passing a prescribed valve of current through the filament of the valve, with no other supplies applied, and measuring the voltage across the filament; this should lie within prescribed values. Figures for the Pirani test are given by certain manufacturers on request.

The principle of the test depends on the cooling effect on the filament of residual gas present in the valve.

18.5. Service Test on Spare Valves. All spare valves, as soon as is convenient after their arrival, should be put into service for a few days to check that they are satisfactory. Evidently any new valve, which is to be put into service, should be installed a sufficient time before the transmitter is required in service to enable it to be removed if unsatisfactory.

18.6. Emission Tests. Emission tests on high-power transmitter valves are usually carried out by the manufacturer. It is, however, convenient for engineers of transmitting stations to make emission tests on air-cooled valves of moderate power, when low emission is suspected. These tests take the form of a *total emission test for valves with tungsten filaments* and a *suppressed emission test for valves with thoriated tungsten filaments*.

18.61. Total Emission Test. In this test the anode and grid are connected together and 400 or 1,000 volts applied between anode and filament. The filament voltage is then gradually increased until a specified emission (anode current) has been reached. The filament voltage applied to give this emission is the operating voltage required by the valve. If this operating voltage lies outside certain limits the valve is rejected for low emission.

18.62. Suppressed Emission Test. In this test the grid is connected to the negative side of the filament. 1,000 volts are applied between anode and filament and the filament voltage is raised until a specified emission (anode current) is obtained. Under these conditions the filament voltage should not be greater than a specified value; if it is greater the valve is rejected for low emis-

sion. It should be noticed that this test does not give the required operating filament voltage.

18.63. Emission Tests on Medium-Size Valves and Small Valves. In the B.B.C. a special piece of apparatus called a valve bridge is provided for testing the emission of small valves. This is fitted with a selection of standard bases, means for applying the necessary voltages and meters for measuring anode current. The same bridge also measures mutual conductance by applying a standard 50 c/s voltage between grid and cathode of the valve and measuring the 50 c/s voltage developed across a known resistance in the anode circuit. This is done by feeding a second 50 c/s voltage from a slide-wire potentiometer, through 100 ohms, in phase opposition into the same resistance. The slide wire is adjusted until a vibration galvanometer, connected (through a condenser) between anode and cathode, reads zero. The mutual conductance is then read off the slide-wire dial.

18.7. Valve Cooling Water. To prevent corrosion and deposits in the supply pipes, and most particularly in the jackets of valve anodes, distilled water is used for water cooling, and the maintenance of the purity of the cooling water is an important item. Periodical tests, of the purity of the water circulating in the cooling system and also of the reputed pure water supplied by distillers, are therefore made.

Every possible precaution must be taken to avoid contamination of the distilled water. The carboys used for supplying distilled water must, when empty, be kept apart from all other carboys; carboys containing acid should be clearly marked, and carboys which have contained acid *must, on no account*, be used for the storage of distilled water.

The conductivity of good distilled water should be less than 10 micromhos/cm.[3] The water in circulation in a valve-cooling system should have a conductivity less than 100 micromhos/cm.[3]; it will usually be found to lie between 30 and 60 micromhos/cm.[3]

19. Total Emission, Peak Emission, Peak Anode Current, and Cathode Emission.

If the grid and any other electrodes in a valve, except the cathode, are strapped to the anode, and a source of high tension applied between anode and cathode, as the anode volts are used in potential with regard to the cathode, the total current leaving the cathode and flowing in the external circuit increases from zero in the way indicated in Fig. 1.

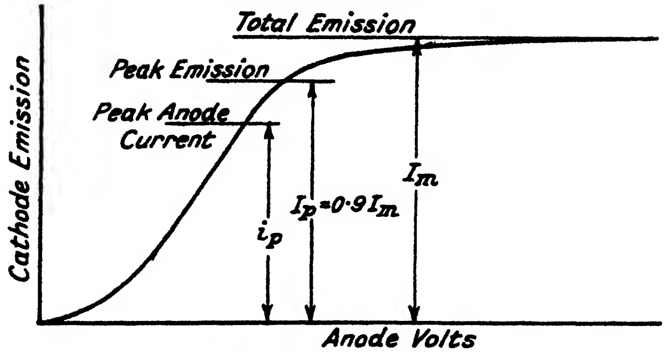


FIG. 1/IX:19.—Total Emission and Peak Emission of Valve.

It will be seen that after the cathode emission reaches a certain value, which has been designated I_m, further increase of anode voltage gives rise to no further increase in emission. The limiting value of emission is called the *total emission* and is sometimes supplied by manufacturers as part of the information relating to the performance of tungsten filament valves. A valve, when delivering its total emission, is said to be in *filament* (*cathode*) *saturation.*

It is not normally desirable to operate a valve in filament saturation, and for this reason manufacturers often specify a figure of *peak emission* I_p, which is 90% of the total emission and represents the maximum permissible value of the peak cathode current. The value of the peak emission is determined from consideration of distortion: evidently, if a valve is running into filament saturation it is not possible for the anode current to follow any wave form which may be applied to the grid.

It will be appreciated that since the cathode has to supply current to all other electrodes as well as the anode, when calculating the peak current taken by a valve, the current taken by all electrodes has to be taken into account. An example of the way in which this should be done is given in X:11.1. In this case the peak grid current is 0·9 amps., and the peak anode current is 12·2 amps.

The peak grid current may rise to a value as high as a quarter of the peak anode current, or even higher; see example 7 in X:99.

In the case of valves with oxide-coated or thoriated filaments, and also in the case of valves with separately heated cathodes (equipotential or homopotential valves), the total emission is so large that the limit of valve performance is set, not by the distortion due to filament saturation, but by the possibility of damage to the cathode due to an excessive value of average emission. For

these valves it is customary to specify a figure for the maximum permissible *cathode emission* which represents the sum of the readings of a set of D.C. meters placed one in series with each of the electrodes in the valve other than the cathode. It is, of course, true that a meter placed in series with the cathode return connection to the negative of the H.T. would read the cathode emission directly. The only objection to suggesting this as a method of measuring cathode emission is the fact that meters are not normally installed in such a position.

20. Grid Input Impedance of Valves.

At frequencies below a megacycle the grid (to cathode) input impedance of a valve is represented by a high resistance in parallel with a capacity, unless the valve is driven into the region of positive grid where grid current begins to flow. Normally, at these frequencies, the parallel resistance is so high that it can safely be neglected. When grid current flows it introduces a non-linear resistance in parallel which may drop to very low values (e.g. 1,000 ohms) at the instant that peak grid voltage is reached. The normal way of discounting the effect of grid current is to shunt the grid input circuit with such a low value of resistance that the non-linearity in the input voltage introduced by the non-linearity of the input impedance is adequately masked. For this purpose it is usual to use a shunt resistance of value equal to about a fifth of the minimum value reached by the grid-input circuit.

In some valves some of the electrons, drawn from the cathode by the anode, hit the grid and knock off from it more electrons than the number of electrons striking it. The resulting emission of electrons from the grid is called secondary emission and may introduce a component of input impedance which is represented by a negative resistance. This may give rise to oscillation ; see XI:5. The negative resistance is converted to a positive resistance by shunting the grid input by a resistance less than the secondary emission negative resistance. The same resistance that is used to mask non-linearity may be effective for this purpose.

At frequencies above a megacycle the parallel resistance component of the grid input impedance may fall to very low values (e.g. 600 ohms). One contributory cause of this is electron transit time in the valve. This resistance occurs independently of the presence of grid current, and is approximately inversely proportional to the square of the frequency. In a receiving valve resistances as low as 500 ohms are not likely to be reached until a

frequency of about 100 Mc/s, and some receiving valves have an input resistance at this frequency as high as 10,000 ohms.

21. Transmitting Valves.

A list of the more general types of transmitting valve in use in the B.B.C. is given below.

Type	E_a *Volts*	E_{sg} *Volts*	E_g *Volts*	I_a *(Approx.) Amperes*	I_g *(Approx.) Microamps*
Marconi Water-cooled Valves.					
CAM.2	10,000	—	– 280	0·5	10/60
CAM.3	12,000	—	– 1,450	1·25	10/75
CAM.4	12,000	—	– 1,050	1·0	80/1,000
CAM.5	12,000	—	– 1,350	4·0	200/1,500
CAT.6	10,000	—	– 40	1·0	30/150
CAT.9	12,000	—	– 120	1·0	20/125
CAT.12A	15,000	—	– 1,200	1·75	40/250
CAT.14, 14C	15,000	—	– 160	2·0	100/1,000
CAT.14SW	15,000	—	– 160	2·0	100/1,000
CAT.17C	12,000	—	– 80	3·0	100/1,000
CAT.20C	15,000	—	– 150	1·5	100/2,000
CAT.21	10,000	—	– 300	2·0	50/300
PCA.21	10,000	—	– 15	0·3	10/100
S.T. and C. Water-cooled Valves.					
3Q/200A	10,000	—	– 500	0·8	30/120
3Q/260E	6,000	—	– 96	1·0	30/100
3Q/331E	17,500	—	– 180	5·0	100/300
4Q/230A	15,000	2,000	– 20	0·6	30/120
4030C	12,000	—	– 20	3·0	100/700
4058A	12,000	—	– 200	1·0	40/200
4067A	17,500	—	– 180	5·0	100/300
4081A	10,000	—	– 50	1·0	30/150
4220C	12,000	—	– 40	0·9	30/150
SS.1971	10,000	—	– 120	1·5	40/200

The various quantities in this table apply to the gas test, see IX:18.4.

CHAPTER X

AMPLIFIERS

Main Conventions.

E_b = grid bias, working value of.
E_c = value of grid bias necessary to take valve to cut off.
E_{gn}, E_{gp} = maximum positive value of grid excursion.
$\hat{e}_g$ = peak grid volts = grid to cathode volts.
$\hat{e}_c$ = cathode peak volts (in an inverted amplifier) = cathode to grid volts.
$\hat{e}_e$ = effective peak grid volts = $\hat{e}_g - (E_c - E_b)$.
E = value of steady H.T. volts.
$\hat{e}_a$ = peak anode volts = anode to cathode volts.
$\hat{e}_{ag}$ = anode to ground peak volts (in an inverted amplifier).
i_p = peak anode current.
i_0 = standing feed.
$\bar{i}$ = mean anode current = driven feed.
i_f = peak fundamental frequency component of anode current.
$f = i_f/i_p$.
$u = i_f/\bar{i}$ = current utilization.
$g = \bar{i}/i_p$ = reciprocal of peak to mean ratio of anode current.
$h = \dfrac{\hat{e}_a}{E}$. = voltage utilization.
R_0 = internal anode impedance of a valve.
Z_c = circuit impedance presented towards the anode cathode circuit of a valve as measured by the ratio $\hat{e}_a/i_f$.
Z_L = slope of load line = $\hat{e}_a/i_p$.
P = power.
η = efficiency.
θ = angle of current flow.
C_{ac}, C_{af} = anode cathode, anode filament capacity (in a valve).
C_{ag} = anode grid capacity (in a valve).
C_{gc}, C_{gf} = grid cathode, grid filament capacity (in a valve).
F = rate of water flow in gallons per minute.

1. Class A Amplifiers.

Class A amplifiers are used when it is essential that the magnified replica of the input wave shall have instantaneous values which are at all times an exact (and the same) multiple of the instantaneous values of the input wave. In less exact terms the output wave is then a faithfully enlarged copy of the input wave. Where power considerations permit, class A amplifiers are sometimes used when

such rigid requirements are not essential; but their main application is to low-power audio-frequency circuits where evidently fidelity of reproduction is of first importance. They are also used as the high-frequency and intermediate-frequency amplifiers in receivers, and in a number of other cases when distortionless amplification is of importance.

A class A amplifier is constituted by a valve (and the associated circuits) so biased (i.e. with such a steady p.d. applied between grid and cathode) that the anode current, varying under the influence of the wave form applied to its grid circuit (i.e. between grid and cathode) never reaches zero.

Since waves to be amplified usually consist of both positive and negative swings of voltage and current about a mean value, this means that a class A amplifier must have a negative bias to a voltage above its cut-off which is at least as great as the maximum peak negative swing of driving voltage, and usually is rather larger, in order to ensure that the valve operates in the linear region of its field of characteristics.

Fig. 1 shows the characteristics for a Mazda AC/HL valve with a load line SS' corresponding to 360 volts H.T. and an anode resist-

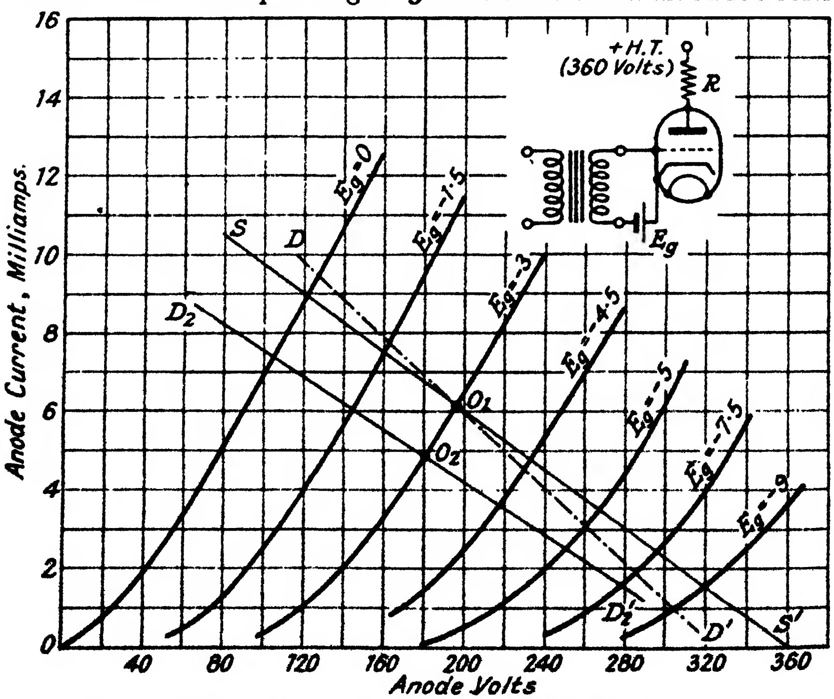


FIG. 1/X:1.—Anode-Current Anode-Voltage Curves for an AC/HL Valve.
(By courtesy of the Edison Swan Electric Company.)

ance load of 26,700 ohms. The corresponding circuit is shown inset in Fig. 1, where the anode resistance *Ra* is, of course, 26,700 ohms.

If the valve is biased with -3 volts the anode current which flows is given by the intersection of the anode characteristic for -3 volts with the load line, and is 6·1 mA.

The anode current of a valve is sometimes called the feed. The steady voltage effective on the anode is evidently only 198 volts, owing to the drop in voltage through the 26,700-ohm anode resistance.

If, now, a voltage of any wave form is superimposed on the bias so as to swing the instantaneous value of the grid voltage (i.e. voltage between grid and cathode) from $E_g = 0$ to $E_g = -6$, it will be evident that in the region traversed along the load line the spacing of anode characteristics is uniform or linear, i.e. distortionless reproduction will be obtained. This may be made clearer by reference to the top left-hand curve in Fig. 2, which is a plot for each point along the load line of anode current against grid volts.

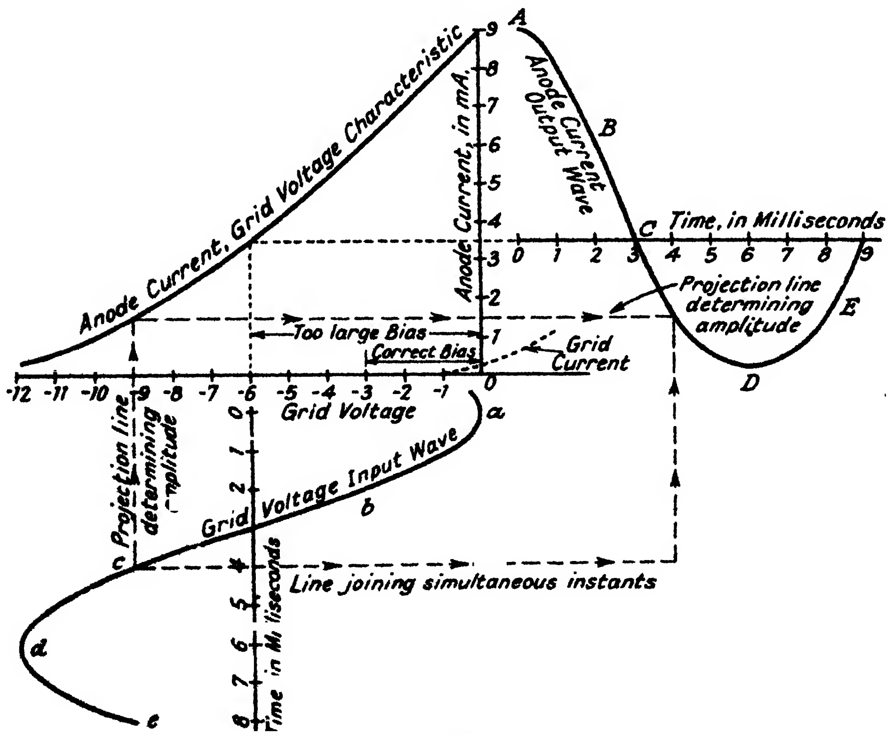


FIG. 2/X:1.—Anode-Current Grid-Voltage Curve of an AC/HL Valve with 26,700 Ω in its Anode Circuit and 360 volts H.T.
a, b, c, d, e = Sinusoidal Grid Voltage, and
A, B, C, D, E = Corresponding Anode Current Wave.

It will be seen that over the region from zero to − 6 grid volts the resultant curve is substantially a straight line. Since the voltage across the output resistance is given by the product of R and the anode current, the same curve can be made to represent the relation between input volts and output volts (i.e. grid volts and anode volts).

On Fig. 2 is also shown the effect of using too large a grid bias (i.e. larger than − 6 volts) and too high an amplitude of grid swing. The sinusoidal grid voltage wave a, b, c, d, e gives rise to the anode current wave A, B, C, D, E, which is evidently very different in form from a sine wave. The distortion in this case is evidently due to the bend at the bottom of the anode-current grid-voltage curve, and in colloquial terms the vålve is said to be *bottom bending*.

The projection lines indicate the method of derivation of curve A, B, C, D, E from the input curve a, b, c, d, e. The time scales are shown in milliseconds for clarity, but any arbitrary values of time scale could have been used provided they corresponded with one another.

If the grid is made positive with regard to the cathode, grid current begins to flow, in other words, the impedance looking into the grid circuit falls, so introducing non-linearity into the *input* circuit of the valve. For this reason in class A amplifiers the grid is not *usually* allowed to swing positive. This means that the negative grid bias must be at least as great as the peak grid drive.*

1.1. Phase Relation between Grid Volts and Anode Volts. Since in any correctly designed amplifier, with a few exceptions which need not be considered here, the anode load impedance, which is the impedance facing the anode of the valve, is resistive, i.e.without reactance; it follows that the alternating anode voltage is reversed in phase with regard to the alternating grid voltage. This applies to class A, class B and class C amplifiers, whether tuned or untuned.

This is because, as the grid becomes more positive the anode current increases, the voltage drop through the anode load increases, and the anode becomes more negative. Similarly, as the grid becomes more negative the anode becomes more positive.

The consequence is that the anode always reaches the most negative point of its excursion about its mean potential at the same instant that the grid reaches the most positive point of its excursion about its mean potential, and, at the instant when the anode current reaches its peak value. Any two of the quantities: maximum positive (limit of) excursion of grid: minimum positive (limit of) excursion of anode: peak anode current: when plotted on the anode-voltage anode-current field of the valve, therefore determine the upper end of the load line.

* See IX:20.

The minimum positive excursion of the anode is evidently equal to the mean value of the anode voltage minus the peak value of the anode-voltage swing. For this purpose the positive peaks of swing are normally assumed to be equal to the negative peaks, although in sound waves this is not always the case.

2. Resistance-Coupled Amplifier.

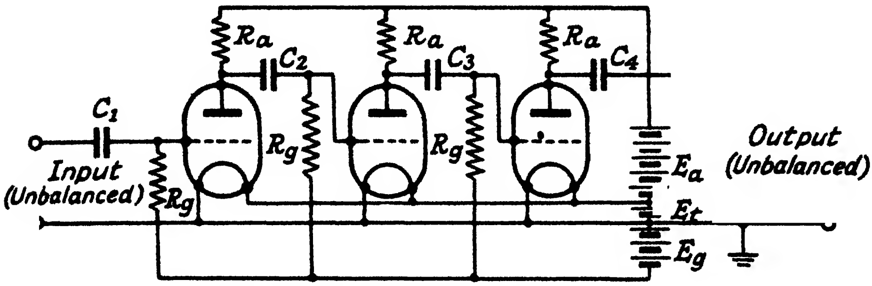


FIG. 1/X:2.—Resistance-Coupled Amplifier.

Fig. 1 shows how a number of valves may be connected in cascade so that each receives the amplified output of the preceding valve. Usually the values of resistances R_g, which in small amplifiers are called *grid leaks* and in large amplifiers are called *grid loads*, are very much larger than the values of the anode resistances R_a, although in large power amplifiers the reverse may be the case The amplification ratio per stage is given by

$$\frac{\mu R_p}{R_0+R_p}$$

where R_0 is the valve anode impedance and μ its amplification factor, while

$$R_p = \frac{R_a R_g}{R_a+R_g}$$

The gain per stage is

$$G = 20 \log_{10}\frac{\mu R_p}{R_0+R_p} \text{ db.}$$

The amplifier will transmit a band of frequencies substantially limited at the top end by the frequency at which the shunt reactance of stray capacities is equal to the circuit impedance (i.e. to R_p) and at the lower end by the frequency at which the reactance of C_1, C_2, etc., becomes equal to the impedance sum of the circuit (e.g. in the case of C_1, C_2, C_3, to $R_g+\frac{R_a R_0}{R_a+R_0}$). At these "cut off" frequencies the gain of the amplifier is 3 db. down on full gain ; see Fig. 8/XXI:5.

An amplifier with couplings between valves, as in Fig. 1, is said to be *resistance- capacity coupled* or *resistance coupled.*

3. Choke-Coupled Amplifier.

If in Fig. 1/X:2 the resistances R_a are replaced by chokes, the amplifier is said to be *choke-capacity coupled* or *choke coupled.* In such amplifiers the reactance of the choke should be made equal to at least five times the value of the *circuit impedance* at the lowest frequency to be transmitted, while the cut-off (defined as the frequency at which the gain is 3 db. below maximum), due to the condensers C_1 and C_2, etc., and the grid leaks, is designed to occur at a frequency less than a third of the lowest frequency to be transmitted. See, however, Fig. 9/XXI:5 and accompanying text for optimum values of C and L. The advantage of using choke capacity coupling is that there is no D.C. voltage drop due to the anode resistance, and the full value of the H.T. source is applied to the anode of the valve. The reaction of this on the load diagram is considered below. The disadvantage of choke coupling compared with resistance coupling is an increase in cost, size and weight.

The *circuit impedance* is the impedance of the valve anode impedance and any following resistance, e.g. grid leak, and/or anode resistance, if any, in parallel.

4. Load Lines for Resistance-Coupled Amplifier.

If the valves in Fig. 1/X:2 are AC/HL valves, the resistances R_a are 26,700 ohms, and E_a the voltage of the H.T. battery is 360 volts, then if the resistances R_g are presumed to be infinite the load line of Fig. 1/X:1 applies. In practice, such an assumption is usually justified. If it is not justified it is necessary to distinguish between the *static* and the *dynamic* load characteristic. The *static characteristic* results by plotting on the anode-voltage anode-current field the value of anode current against the value of voltage effective on the anode (i.e. between anode and cathode) as determined by the H.T. volts minus the D.C. voltage drop in R_a due to the anode current flowing through R_a. It is assumed that the changes in anode current are obtained by varying the grid bias and waiting until the current has reached a steady value before reading the current and volts effective on the anode. The *dynamic characteristic* results from plotting instantaneous corresponding values of grid bias and anode current when the rate of current variation is so great that the impedance of the condensers C_2, C_3, etc., is negligible. In this case the *slope* of the load line corresponds to the resistance

constituted by R_a and R_g in parallel, i.e. R_p. The dynamic load line must evidently pass through the point determined by the condition of the system with no A.C. grid drive. The load line is therefore drawn through the point on the field of characteristics corresponding to the grid bias and H.T. applied, and the anode current for an anode resistance R_a. For example, if $R_a = 26{,}700$ ohms and $R_g = 100{,}000$ ohms, the slope of the load line corresponds to a resistance of $\frac{100{,}000 \times 26{,}700}{126{,}700} = 21{,}100$ ohms. If the H.T. is 360 volts and the grid bias 3 volts, the load line is drawn through the point at which the $E_g = -3$ volts anode-current anode-voltage curve intersects the 26,700-ohms load line; this is the point O_1. The resulting curve is shown at DD' on Fig. 1/X:1. The locus of operation of the valve is then along DD', with O_1 as centre.

5. Load Line for Choke-Coupled Amplifier.

As indicated above, the circuit of a choke-coupled amplifier can be represented by the circuit of Fig. 1/X:2, with the resistances directly in the anode circuit replaced by chokes. In this case the dynamic characteristic only is of interest. This is here drawn on the assumption that the impedances of the chokes are infinite, and the impedances of the blocking condensers, e.g. C_2, C_3, etc., are zero. It will be evident also that, since the resistance of a choke is negligibly small, there is no D.C. resistance drop through an anode resistance, and in the absence of any potential applied to the grid other than the steady grid bias, the anode current or feed can be read directly from Fig. 1/X:1 (assuming AC/HL valves to be used) by entering the H.T. voltage and the grid bias. If, for instance, the H.T. voltage is 180 and the grid bias is -3, the feed is read off at O_2 the point of intersection of the vertical through 180 volts and the anode-current anode-voltage curve corresponding to $E_g = -3$ volts. The load line is then drawn through O_2. If, for instance, the value of R_g in Fig. 1/X:2 is 30,000 ohms, the load line is represented by D_2D_2'. The locus of operation of the valve is then along D_2D_2' with O_2 as centre. By measuring the change of current along D_2D_2' for a given change of anode volts, it can be seen that the slope of D_2D_2' corresponds to a resistance of 30,000 ohms.

The case where the impedance of the choke cannot be considered to be infinite, or where any other cause gives rise to an anode load having appreciable reactance, is dealt with in two papers: "Use of Plate Current—Plate Voltage Characteristics in Studying the Action of Valve Circuits" by E. Green and "Alterations to the Modulating

Panel at 2LO" by E. Green, J. L. Hewitt and T. G. Petersen. These appeared in the issues of *Experimental Wireless* and *The Wireless Engineer*, respectively, for July and August 1926 and August 1927.

6. Transformer-Coupled Amplifier.

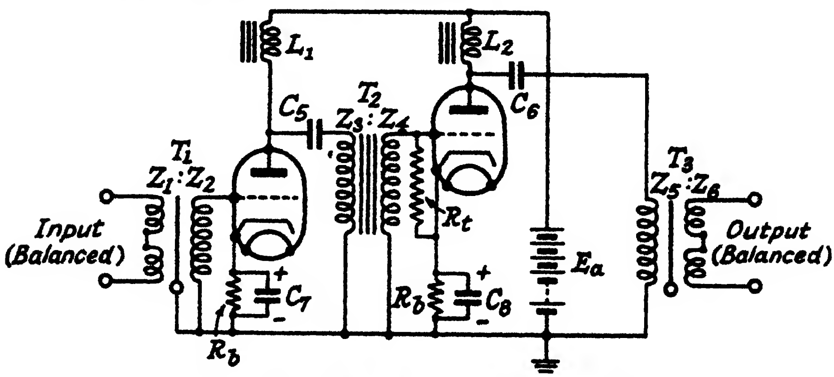


FIG. 1/X:6.—Transformer-Coupled Amplifier.

In Fig. 1 is shown a two-stage transformer-coupled amplifier, the coupling between the two valves being effected by T_2, which is called the *intervalve transformer*. In this case an input transformer T_1 and an output transformer T_3 are shown transforming the circuit from balanced to unbalanced at the input, and from unbalanced to balanced at the output. The input transformer also steps up the input voltage in the ratio of $\sqrt{\frac{Z_2}{Z_1}}$, where Z_1 is the line impedance and Z_2 is an impedance limited by the input capacity of the valve and the frequency range to be transmitted.

In the intervalve transformer, Z_3 is made equal to the optimum load impedance for the first valve, as determined from the load-line diagram, while Z_4 is determined as for Z_2 in the case of T_1. R_t is made equal to Z_4. In cases where the second valve takes heavy grid current presenting a non-linear impedance, the value of R_t must be dropped to a value low compared with the minimum resistance offered by the grid-cathode circuit (this occurs on positive voltage peaks), e.g. to a fifth of this minimum resistance. In this case also Z_4 is made equal to R_t. In general, as R_t and Z_4 are reduced the frequency range is increased, so that no serious disadvantage other than the loss of gain occurs as the result of reducing R_t to a low value.

In the output transformer Z_5 is determined as before from the load-line diagram, and Z_6 is made equal to the output load into which the amplifier has to work.

The gain of such an amplifier when working between matched impedances is given by

$$20 \log_{10} A \times B \times C \times D \times E \text{ decibels}$$

where $A = \sqrt{\dfrac{Z_2}{Z_1}}$ = voltage step up of input transformer T_1

$B = \dfrac{\mu Z_3}{R_0 + Z_3}$ = amplification ratio of first valve (μ and R_0 apply to first valve)

$C = \sqrt{\dfrac{Z_4}{Z_3}}$ = voltage step up or *step down* of intervalve transformer T_2

$D = \dfrac{\mu Z_5}{R_0 + Z_5}$ = amplification ratio of second valve (μ and R_0 apply to second valve)

$E = \sqrt{\dfrac{Z_1}{Z_5}}$ = voltage step down of hypothetical output transformer, having output impedance equal to the input impedance of the amplifier.

The reason for the form of E is very simple. *The gain of an amplifier is equal to* 10 *times the logarithm of the power amplification*, which is only equal to 20 times the logarithm of the voltage amplification in the case where the output impedance is equal to the input impedance. Provided the output load is always impedance matched to the amplifier output, changing the output impedance and load does not change the power delivered to the load, and so does not change the gain of the amplifier. It is therefore simplest to assume that the amplifier output impedance is equal to its input impedance, to calculate the voltage amplification on this basis and then to derive the gain as indicated. The quantity $A \times B \times C \times D \times E$ is the voltage amplification ratio on the assumption of equal input and output impedance.

It is very often useful to talk about voltage amplification, but this is best expressed as the voltage amplification ratio and can only be converted to decibels at the risk of inevitable misunderstandings in the most fortunate circumstances and gross errors in others. If, however, you have read and understood the above, you should be able to take this risk with impunity.

The load line of a transformer-coupled amplifier is drawn as for a choke-coupled amplifier, the slope of the load line corresponding

to the magnitude of the impedance facing the anode of the valve, e.g. Z_3 and Z_6 respectively in Fig. 1.

The object of the chokes L_1 and L_2 is to avoid the passage of D.C. through the windings of the transformers, since this has the effect of applying a magnetizing force to the core of the transformer, and so lowering the inductance of the transformer windings and dropping the efficiency of the transformers at low frequencies. Audio-frequency transformer cores are usually made of laminations (stampings) of some iron alloy, e.g. stalloy, permalloy, mumetal, rhometal, etc., while transformers for intermediate frequencies and high frequencies are usually made from compressed cores of a finely divided iron alloy held together with an insulating compound. Some high-frequency transformers are air cored. The *blocking* condensers, such as C_5 and C_6, serve the purpose of keeping H.T. voltage from the transformer. In other cases the chokes and stopping condensers are not used, but the transformer winding is connected directly between H.T. + and the valve anode.

7. Automatic Grid Bias.

It will be noted that separately heated cathode valves have been shown in Fig. 1/X:6 and that the heater battery and circuit has been omitted because this is an entirely independent circuit which need have no connection with the remainder of the amplifier. Further, no grid-bias batteries have been provided, but cathode resistances R_b, shunted respectively by condensers C_7 and C_8, have been provided. The anode current i_a of each valve, flowing through its cathode resistance, makes the potential of each cathode positive with regard to ground, by a p.d. equal to i_aR_b volts. Since each grid has a D.C. path to ground, this has the effect of making each grid i_aR_b volts negative with regard to the cathode of the valve. The resistances R_b are therefore effective in providing a grid-bias. The value of R_b is determined by deciding i_0 the required value of standing feed, determining from the valve characteristics the required bias E_g to give this feed, and then making $R_b = E_g/i_0$.

The condensers C_7 and C_8 are called *decoupling condensers*, and are electrolytic condensers with a capacity between 20 and 100 μF inserted to provide a low impedance shunt across R_b and so to prevent the cathodes from varying in potential with regard to ground when the anode current varies at audio frequency. It will be noted that, since when the grids of the valves go more positive the anode currents increase, in the absence of condensers C_7 and C_8 the cathodes of the valves go more positive, so reducing the change in p.d. between

grid and cathode. When the grids go negative, the anode current decreases and the cathodes go less positive, again reducing the change in p.d. between grid and cathode. Since the change in anode current and anode voltage is dependent on the change in p.d. between grid and cathode, the net effect of the resistances R_b, in the absence of the condensers C_7 and C_8, is to reduce the effective amplification of the stage, in other words, negative feedback is introduced, see XXIII. The value of the bias resistances R_b may be of the order of 1,000 ohms, so that very large capacities are necessary in order to provide at low audio frequencies an impedance low compared with the bias resistances. It has been assumed that the amplifier of Fig. 1/X:6 is an A.F. amplifier. In the case of radio frequencies very much smaller condensers are used, e.g. a condenser of 0·01 μF has a reactance of only 16 ohms at a megacycle.

Permissible Reactance of Decoupling Condensers.

If R_0 = the internal impedance of the valve
R_a = the impedance facing the valve anode
X = the reactance of the condenser
μ = the amplification factor of the valve

the voltage amplification of the stage from grid to anode is:

$$\frac{\mu R_a}{R_0+R_a-j(1+\mu)X}.\quad \text{See XXIII:9.11.}$$

If, therefore, the magnitude of $(1+\mu)X$ is a quarter of the magnitude of R_0+R_a, the gain reduction due to the cathode impedance is

$$20\log_{10}\left|\frac{R_0+R_a-j(1+\mu)X}{R_0+R_a}\right|$$

$$=20\log_{10}\left|\frac{1-j0{\cdot}25}{1}\right| = 0{\cdot}52\text{ db.}$$

Hence, to keep the gain reduction due to the cathode impedance below 0·5 db., the value of the reactance of the condenser at the lowest frequency to be transmitted must be less than a quarter of $\dfrac{R_0+R_a}{1+\mu}$.

8. Push-Pull Connection of Valves.

Fig. 1 shows two valves, V_1 and V_2, connected *differentially*, or in *push-pull*, both at their input and output. The valves are chosen so as to have as nearly as possible identical characteristics: windings $3\text{–}2_a$ and $3\text{–}2_b$ are equal: windings $5\text{–}4_a$ and $5\text{–}4_b$ are equal. The sense of the secondary windings of transformer T_1 is such that, when

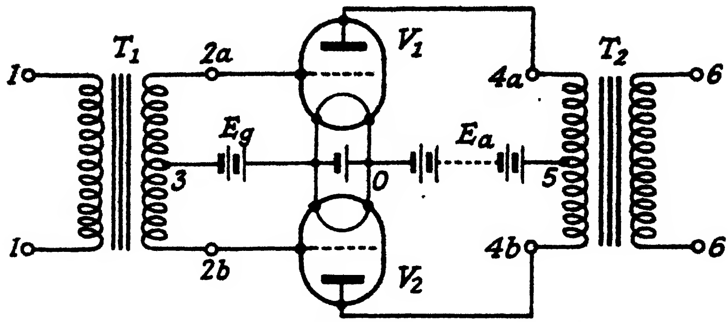


FIG. 1/X:8.—Push-Pull Connection of Valves.

the grid of V_1 is instantaneously driven more positive owing to the application of an A.C. voltage across terminals 1,1, the grid of V_2 is driven more negative, and vice versa. The sense of the primary windings of transformer T_2 is such that the magnetic fields through the transformer core, due to the steady anode currents (no drive feeds) of the two valves, are in opposition. The corollary of this is that an *increase* of the anode current of valve V_1 through winding $5\text{–}4_a$ produces an e.m.f. in the output winding 6,6, which is in phase with, and directly additive to, the e.m.f. produced by a *decrease* of the anode current of valve V_1 through winding $5\text{–}4_b$.

Owing to the arrangement of the input windings, a changing voltage across 1,1 applies a grid voltage to V_1, and causes an anode current to flow through V_1 and winding $5\text{–}4_a$, which increases simultaneously with the decrease of the anode current through V_2 and winding $5\text{–}4_b$. The application of a voltage across 1,1, therefore, produces changing currents through the primary windings of T_2, which give rise to the additive e.m.f.s in winding 6,6. Since the anode currents of V_1 and V_2 through windings $5\text{–}4_a$ and $5\text{–}4_b$ are in opposition, in order to represent their effects on the core, and therefore on the output circuit, in a single diagram it is necessary to show the anode current of one valve positive and that of the other valve negative. This may conveniently be done by drawing their dynamic anode-current grid-voltage characteristics with loads in their anode circuits equal to the load presented by transformer T_2 towards the anode of each valve. Since the grid of valve V_2 goes negative when that of V_1 goes positive, the axes of grid volts must be superimposed so that valves of positive grid swing for valve V_1 coincide with the same values of negative grid swing for valve V_2. The values of grid voltage corresponding to the grid bias E_g will then coincide. The resulting anode-current grid-voltage characteristics are drawn in

Fig. 2 for a grid bias $E_g = 8$ volts. These curves define, for any point on the axis of grid volts on Fig. 2, simultaneous values of current in the two primary windings. The net effective current in

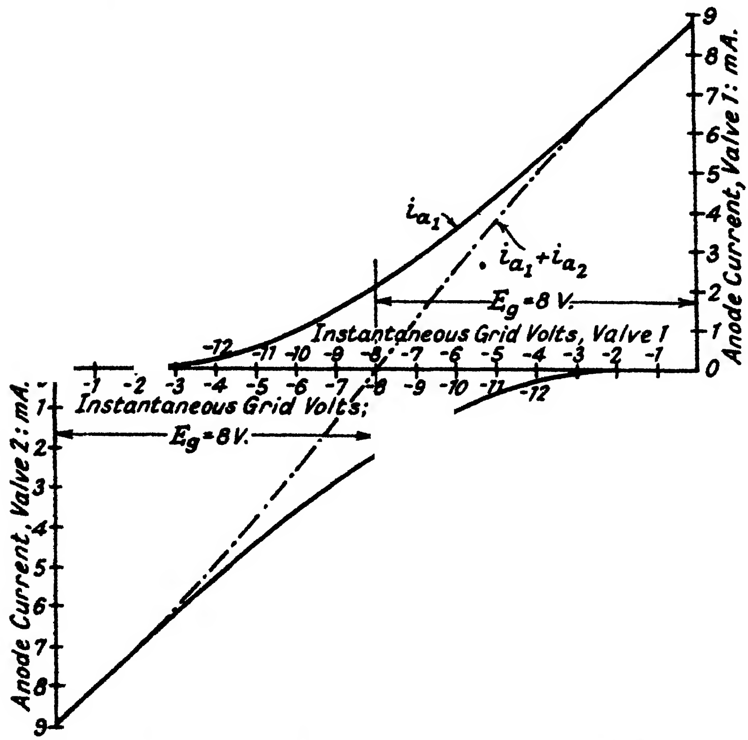


FIG. 2/X:8.—Anode-Current Grid-Voltage Curves of Two Valves in Push-Pull with Grid Bias $E_g = 8$ volts.

the primary winding is given by adding algebraically, for each point on the axis of grid volts, the currents defined by these curves.

The curve drawn through the resultant set of instantaneous effective currents is shown chain dotted. This curve represents the effective anode-current grid-voltage characteristic of the combination, and as can be seen from the practical example drawn in Fig. 2, it is very much more nearly a straight line than either of the original curves.

It is not exactly a straight line, however, and it can be shown that the effect of push-pull connection is to cause all even-order distortion products (e.g. $2f$, $4f$, $6f$, etc., $f_2 - f_1$, $f_1 + f_2$, etc., where f, f_1 and f_2 are frequencies supplied at the input) produced in each valve to subtract from those produced in the other valve. The even-order currents, which cancel in winding 6,6, add in the common

circuit from o to 5, i.e. provision has to be made for these to flow either through the H.T. source or through a special by-pass circuit. See XIV:12.

Odd-order products (e.g. f, $3f$, $5f$, etc., f_1, f_2, $2f_2 \pm f_1$, $2f_1 \pm f_2$, etc., where f, f_1 and f_2 are frequencies supplied at the input) do not cancel, but add in the output, and so appear undiminished in strength.

With a good balance of valves and transformer windings, even order harmonics may be reduced as much as 20 db. It is important to note that in push-pull amplifiers the even-order harmonics flow through the H.T. circuit, which must therefore present a low impedance towards the amplifier. In class B amplifiers it is particularly important that this impedance should be kept low. See XIV:12. Push-pull connection of valves may be applied to both class A and class B amplifiers.

9. Class B Amplifiers.

Referring to Fig. 2/X:8, the grid bias is 8 volts and the valve cut-off occurs with a value of grid voltage of -13. If, therefore, a grid swing of ± 5 volts is superimposed on the grid bias, each valve will just be driven down to cut-off, and by definition the amplifier is a class A amplifier, since plate current flows in each valve for all values of grid voltage which occur. Examination of the combined characteristic (the chain dotted line) shows, however, that even if the grid swing is increased to ± 8 volts, the amplifier will still have a reasonably linear characteristic, although during part of the traverse of the grid voltage each valve in turn ceases to pass anode current. The amplifier is then said to be a class B amplifier.

In practice, in order to economize in the *standing feed* (the anode current with no grid drive), a pair of valves such as those illustrated in Fig. 2/X:8 would be operated with a higher value of grid bias. In high-power amplifiers the peak grid swing would be made greater than the grid bias and the grids loaded with resistance to mask the non-linearity introduced by flow of grid current. The value of grid bias used is determined as a compromise between economy of standing feed and freedom from distortion, since it is found in practice, and examination of Fig. 2/X:8 will make it apparent, that the output curves of two valves marry to give an approximately linear overall characteristic only when the bias is such that the valve is not operating at cut-off, but with a small value of standing feed.

The advantages of class B operation over class A operation are:

(a) Reduced standing feed, and therefore reduced power input to the anode circuit;

(*b*) Increased output power.

The disadvantage of class B operation is that distortion is increased. This may, however, be reduced by the application of feedback; see XXIII.

It should be noted that an audio-frequency class B amplifier must always be a push-pull amplifier, since audio-frequency amplifiers are required to produce an output wave form which is a faithful copy of a complex input wave form.

10. Design of Push-Pull Output Transformers for Class A and Class B Amplifiers.

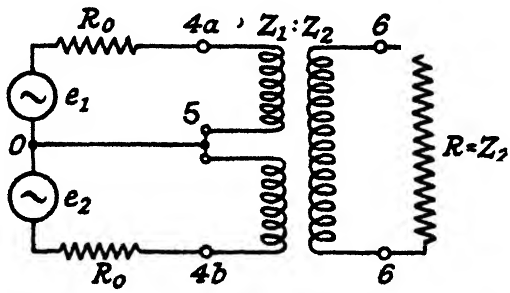


FIG. 1/X:10.—Push-Pull Output Transformer.

Class A. Consider the push-pull output transformer of Fig. 1, which is shown working from two equivalent circuits for class A operated valves, having anode impedances R_0 and internal e.m.f.s in their anode circuits equal to e_1 and e_2. These e.m.f.s e_1 and e_2 represent the odd term output products of the valves; they therefore add, and since the valves are assumed to be paired, are equal. The transformer is assumed to have an impedance ratio between terminals 4_a–4_b and terminals 6–6 of $Z_1 : Z_2$; so that the impedance offered to an e.m.f. applied between terminals 4_a–4_b, with an impedance Z_2 bridged across terminals 6–6, is Z_1. Under these conditions the impedance offered to an e.m.f. applied between terminals 4_a–5 with terminals 4_b–5 open circuited, is $Z_1/4$. When an e.m.f. is simultaneously applied between terminals 4_b–5 so as to cause a current to flow through Z_2 equal to, and in the same sense as the current due to the e.m.f. applied between 4_a–5, the impedance looking into 4_a–5 is doubled. Under these conditions, which are those which apply in the case of a class A amplifier, the impedance looking into 4_a–5 (and also looking into 4_b–5) is $Z_1/2$.

Hence in a class A amplifier the output transformer should be

designed for an impedance ratio, such that the impedance looking into the whole primary winding is twice the impedance which constitutes a suitable anode load for one valve acting alone. If there are n valves each side of the push-pull circuit the impedance looking into the whole primary winding should be $2/n$ times the load impedance required for one valve.

Class B. In this case, although there is an overlapping of the valve characteristics, the grid bias is usually such that, on high amplitudes of grid swing, each valve is out of action (i.e. takes no anode current) for such a period of time that it is permissible to approach the problem by considering each valve to operate alone during alternate periods.

In such case, in untuned class B amplifiers, it is necessary to consider only the impedance facing one valve when the other is biased back below cut-off, therefore passing no current, and so presenting infinite anode impedance to the circuit. The impedance facing one valve under these conditions is the impedance looking into 4_a–5 with 4_b–5 open circuited, and vice versa. This is $Z_1/4$ and must be made equal to Z_L.

Hence in a class B amplifier the output transformer should be designed for an impedance ratio, such that the impedance looking into the whole primary winding is four times the impedance which constitutes a suitable anode load for one valve acting alone. If there are n valves each side of the push-pull circuit, the impedance looking into the whole primary winding should be $4/n$ times the load impedance required for one valve.

It must be remembered that the value of Z_L for a class B amplifier will not necessarily be the same as that for a class A amplifier using the same valves and the same H.T. See Case II, X:11.1.

11. Power Output and Efficiency of Class B Amplifier (Untuned) with Sinusoidal Input.

Conventions.

A sinusoidal grid drive is assumed.

i_p = the peak current in one valve of a push-pull pair.

$i_p \sin \omega t$ = the instantaneous value of the current in one class B valve during the period of current flow.

$\hat{e}_a$ = the peak anode voltage on one valve in the class B amplifier.

$-\hat{e}_a \sin \omega t$ = the instantaneous peak anode voltage on one valve = the instantaneous peak voltage across the load impedance presented to one valve.

E = the H.T. volts effective on the anode of each valve.
Z_c = the load impedance presented to one valve.
$4Z_c$ = the impedance observed on a bridge connected from anode to anode of the push-pull pair, with the valves cold.
Z_L = the equivalent load line impedance.
$\bar{i}$ = the driven feed in one valve.
i_0 = the standing feed in the absence of drive.

It is assumed that a total of only two valves is used: on each side of the push-pull.

Power Output. Each valve operates alternatively and the power supplied by each valve *during its half-cycle of operation*, which is therefore equal to the continuous power output from both valves, is:

$$P = \tfrac{1}{2}\hat{e}_a i_p = \tfrac{1}{2}i_p^2 Z_c = \tfrac{1}{2}i_p^2 Z_L$$
$$= \tfrac{1}{2}\frac{\hat{e}_a^2}{Z_c} = \tfrac{1}{2}\frac{\hat{e}_a^2}{Z_L} \quad \ldots \quad (1)$$

Efficiency Assuming Ideal Valves Biased to Cut-Off (i.e. Neglecting Standing Feed). The anode input power to two valves is:

$$P_A = 2E\bar{i}$$

so that the efficiency

$$\eta = \frac{P}{P_A} = \tfrac{1}{4}\frac{i_p}{\bar{i}} \times \frac{\hat{e}_a}{E} \quad \ldots \quad (2)$$

For a series of current pulses of half-sine wave in form, the value of $i_p/\bar{i}$ is π; hence

$$\eta = \frac{\pi}{4}h \text{ where } h = \frac{\hat{e}_a}{E} \quad \ldots \quad (3)$$

In the case of a class B amplifier used as a modulator the value of h is proportional to the percentage modulation m, and if $\hat{h}$ is the value of h at 100% modulation *by a sinusoidal wave* the efficiency with a percentage modulation m is:

$$\eta_m = \frac{\pi m\hat{h}}{4} \quad \ldots \quad (4)$$

It will be appreciated that equation (4) is of limited application since sinusoidal modulation only occurs during tests. It does, however, provide a method of estimating the maximum power input which may be required for a modulator.

Performance of Class B Amplifier, taking into account Standing Feed. By a treatment along the lines of X:30 it may

be shown that the power output from two valves in push-pull, taking into account standing feed, is given more closely by:

$$P = i_f^2 Z_e = f_s^2 \dot{i}_p^2 Z_e = f_s \dot{i}_p^2 Z_L \quad . \quad . \quad . \quad (5)$$

and the efficiency is given by

$$\eta = u_s \frac{\hat{e}_a}{E} \quad . \quad . \quad . \quad . \quad (6)$$

where

$$f_s = 0{\cdot}5 + 0{\cdot}07 \frac{\dot{i}_a}{\dot{i}_p} \quad . \quad . \quad . \quad . \quad (7)$$

$$u_s = \frac{0{\cdot}5\dot{i}_p + 0{\cdot}07 i_0}{0{\cdot}318\dot{i}_p + 0{\cdot}266 i_0} \quad . \quad . \quad . \quad (8)$$

In this case Z_L is unequal to Z_e and is given by

$$Z_L = 2f_s Z_e \quad . \quad . \quad . \quad . \quad (9)$$

It is evident that Z_L is still substantially equal to Z_e and may be assumed to be equal to Z_e for all practical cases.

The power output and efficiency of a class B amplifier as described above, which is untuned, are the same as for the case of a tuned class B amplifier, given the same valves and conditions of working as far as bias, grid bias, H.T. and load lines are concerned. For clarity however it has been thought preferable to consider the untuned case separately. Tuned amplifiers are dealt with in section X:21 to 32.

11.1. Comparison of Power Output of Class A and Class B Amplifiers: Adjustment of Load Line for Maximum Output.

Case I. For this comparison the case of a push-pull amplifier using two S.T. & C. 4030C valves will be taken. In practice this is used in the class B modulating amplifiers of 100 kW medium-wave transmitters. As will appear, the use of such valves in class A is an uneconomical, and therefore unpractical, proposition, but serves as a good example of the advantage of class B. The example of the application of this valve to class A amplification, however, serves as an entirely valid example of the design of a class A amplifier for maximum output. The fact that the valve is operated in the region of positive grid voltage in no way invalidates the general application of the method of using load lines, but results only in the necessity for driving the grid-input circuit from a low-impedance source to offset the non-linearity in the input circuit consequent on the flow of grid current. The grid-current characteristics of the 4030C valve are shown in Fig. 1, and the anode-current anode-voltage characteristics in Fig. 2.

Class B Case. Considering the class B case first, suppose that

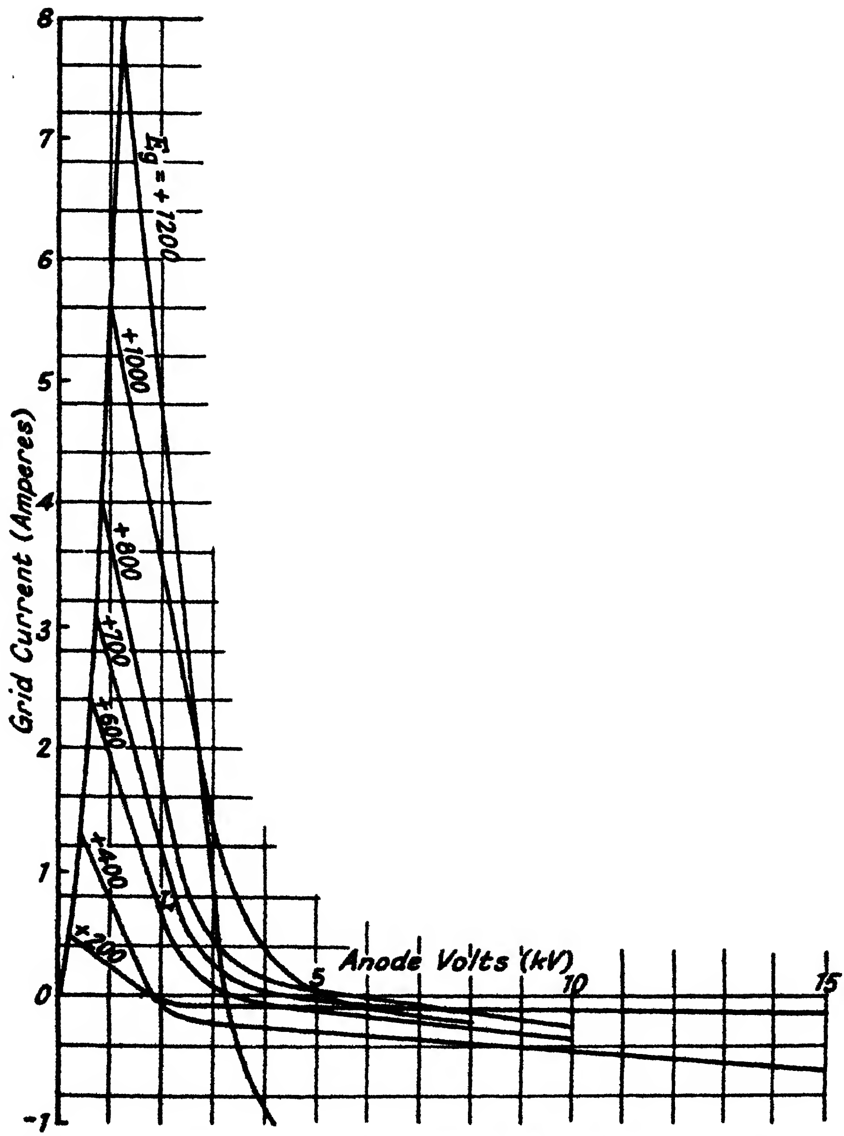


FIG. 1/X:11.—Grid-Current Anode-Voltage Characteristics of 4030C Valve for Different Values of Grid Voltage.

(By courtesy of Messrs. Standard Telephones & Cables.)

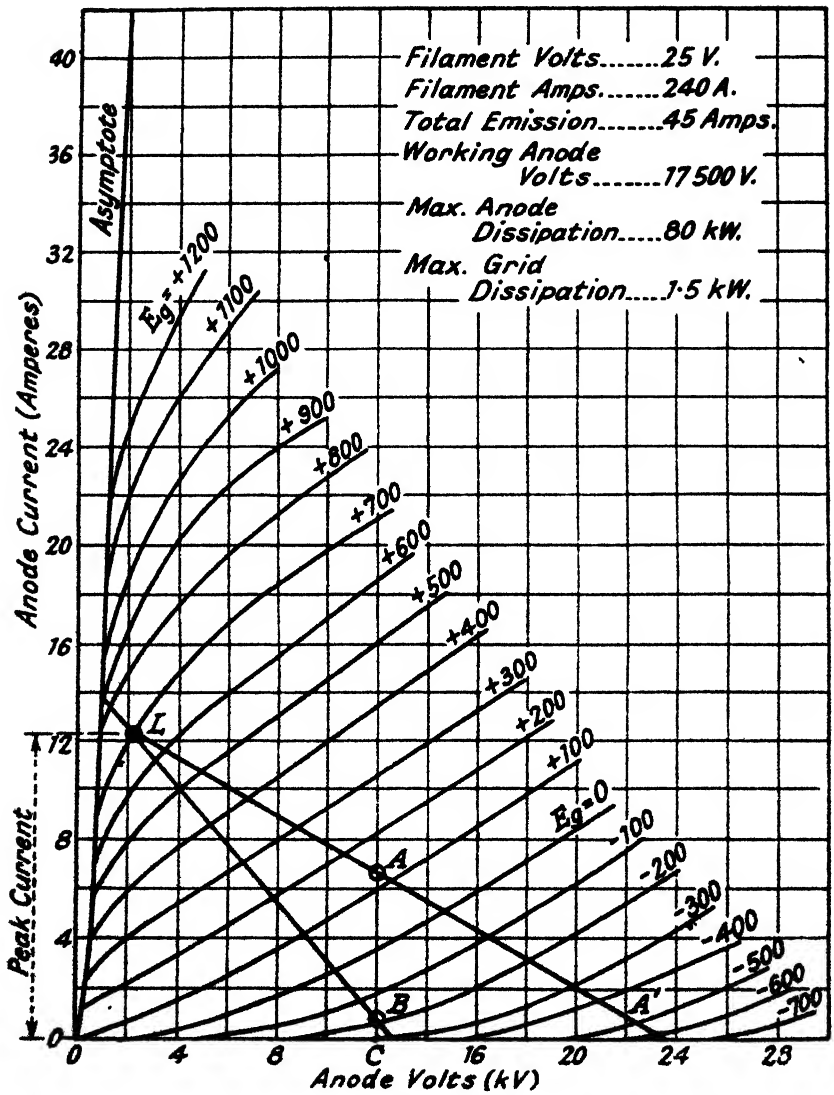


Fig. 2/X:11.—Use of 4030C Valve as a Class A and as a Class B Amplifier.
(By courtesy of Messrs. Standard Telephones & Cables.)

experiment determined that with the impedance of driving source available it is permissible for the impedance of the grid circuit to fall to 780 ohms on the positive peaks of grid current, e.g. if the grid goes to +700 volts, the grid current may rise to 0·9 amps., since 700/0·9 = 780. Referring to Fig. 1, such a condition is reached with an anode voltage of 2,200 volts, i.e. with 2,200 volts on the anode and +700 on the grid, the grid current is 0·9 amps. The impedance of the driving source must evidently be about

780/5 = 156 ohms ; see X:26.1.

Referring to Fig. 2, in order that +700 volts on the grid shall give rise to 2,200 volts on the anode, the load line must pass through the point: Anode volts = 2,200, Grid volts = +700, i.e. the point L. For class B operation, if the valve is biased to cut-off, the anode current will be zero, when the full H.T. is effective on the anode. If the H.T. is assumed to be 12,000 volts (because 12,000 volts is available) when the valve is biased to cut off, the other end of the load line passes through point C on the axis of anode volts. As explained above, in practice it is found, however, that in order that the curves of the two valves shall marry properly, a small standing feed is necessary. Experiment showed that with a standing feed of 0·8 amps. a satisfactory marriage resulted. The other end of the load line, therefore, passes through the point B: 12,000 volts, 0·8 amps. A standing feed of 0·8 amps. requires a grid bias of about − 170 volts, so that this is the practical value to use.

If, now, a sinusoidal grid swing of 700+170 = 870 volts peak value is applied to the valve (swinging up to the point L), the peak current through the load will be (the current at point L) $i_p = 12{\cdot}2$ amps., and the peak voltage across the load will be $\hat{e}_a = 12{,}000 - 2{,}200 = 9{,}800$. *Each valve during its half-cycle of operation* will therefore deliver a power

$$P_B = \tfrac{1}{2}\hat{e}_a i_p = \tfrac{1}{2}(9{,}800 \times 12{\cdot}2) = 59{\cdot}8 \text{ kW}$$

and since the valves operate alternately, this is the total continuous power supplied by both valves.

Class A Case. Considering the class A case, the maximum permissible anode dissipation is 80 kW for a 4030C valve, so that the standing feed must not exceed $\frac{80{,}000}{12{,}000} = 6{\cdot}67$ amps. (It is necessary to assume that the whole of the anode input power is dissipated in the anode in order to allow for intervals in the programme when none of the input power is converted to audio frequency.) Hence, if no factor of safety on anode dissipation is allowed, a load line

may be drawn through the point A: 12,000 volts, 6·67 amps. Assuming the same limitation on positive grid swing, as in the class B case, the load line for a class A operation should pass through the point L. The load line LAA' can then be drawn through the points L and A. The grid bias is about +130 volts. If, now, a sinusoidal grid swing of 700 − 130 = 570 volts peak value is applied to the valve (swinging up to point L), the peak current through the load is 12·2 amps., and the peak sinusoidal current will be 12·2 minus the standing feed = 12·2 − 6·7 = 5·5 amps. Similarly the peak sinusoidal voltage is 12,000 − 2,200 = 9,800.

The power output from one valve is therefore

$$\tfrac{1}{2}PA = \tfrac{1}{2}\hat{e}_a\hat{i} = \tfrac{1}{2}(5\cdot5 \times 9{,}800) = 26\cdot9 \text{ kW}$$

Since both valves are in operation simultaneously, the power output from two valves is

$$PA = 53\cdot8 \text{ kW}$$

On account of the permissible anode dissipation of a 4030C valve, in this case where the limitation on output is determined by a limitation on positive grid swing, there is evidently no advantage of class B over class A from the point of view of power output. From the point of view of the power expended in maintaining the standing feed there is, however, a large advantage, and this is a consideration of prime importance.

Power Expended in Maintaining Standing Feed. In the class A case this is 2 × 80 = 160 kW. In the class B case it is evidently given by the value of H.T. × the standing feed, i.e. 12,000 volts × 0·8 amps. = 9·6 kW per valve. This is 19·2 kW, less than an eighth of the power for the class A case.

With power at a halfpenny per unit, on a transmitter running 6,000 hours a year, the power cost of the class A amplifier would be $\frac{6{,}000 \times 160}{480}$ = £2,000 per annum. The advantage of the class B amplifier is therefore considerable.

Case II. Consider the case where the valves in the circuit of Fig. 1/X:8 are Marconi type DA100, and where 1,000 volts of H.T. are available.

Class A Case. Referring to Fig. 1/IX:5 which shows the valve characteristic of a DA100 valve, since the dotted curve marks the anode current limit imposed by the permissible anode dissipation, which is 100 watts, the maximum permissible standing feed is given by the intersection of this curve with the ordinate through E_a = 1,000 volts, and is evidently 100 mA. It will be assumed

that, to avoid grid current, the grid must not be driven positive. The load line LL' is drawn through the point $E_a = 1{,}000$ volts, $I_a = 96$ mA, which is in the neighbourhood of the maximum practical value of feed, allowing a small margin for safety.

The corresponding value of grid bias is -146 volts, so that the maximum permissible grid swing is 146 volts. It will be evident that if the slope of the load line through the point: 1,000 volts, 100 mA, is reduced, the output power for a grid drive of ± 146 volts about the point -146 volts (i.e. from -292 to zero grid volts) will be also reduced. On the other hand, owing to the curvature of the $E_g = -292$ volts characteristic, if the slope of the load line is increased, considerable non-linearity will occur. The load line therefore represents the optimum compromise, and the power output per valve is evidently $\frac{1}{2} \times 700 \times 54 \times 10^{-3} = 18{\cdot}9$ watts. The total power output is therefore $37{\cdot}8$ watts.

Class B Case. On Fig. 1/IX:5 draw the dotted load line BB' through the point $E_a = 1{,}000$, $I_a = 0$, *of slope equal to half the slope of the straight part of the* $E_g = 0$ *curve. This is equivalent to a load impedance of twice the anode impedance of the valve and corresponds to the impedance giving maximum output.* See X:14.

The mean power output over one half-cycle is evidently $\frac{1}{2}(1{,}000 - 380) \times 284 \times 10^{-3} = 88$ watts, and, since the other valve during its half-cycles of operation supplies the same mean power, this is the power output of the amplifier, which is evidently more than twice that in the class A case.

This increase in power has not been obtained without cost, since the load impedance has been lowered, and as can be seen by drawing anode-current grid-voltage curves for the two valves, the linearity of the circuit has been impaired. In so far as the non-linearity introduced is such as to introduce even-order harmonics, it is substantially eliminated by the use of the push-pull circuit.

It should now be noted that the class A and class B cases have been compared on what at first sight seems to be an unfair basis, since the class B case has been provided with an optimum load impedance, while the class A case has been provided with a load impedance which is more than twice the anode impedance. It was, however, pointed out that this impedance is the optimum impedance for the value of H.T. used. Can a greater output be obtained by using a lower impedance load in the class A case?

The most reasonable basis of comparison appears to be to assume the same load line BB' to be used for the class A case as for the class B case. For maximum output this means reducing the H.T. to

700 volts and the grid bias to about 75 volts, which gives 140 mA anode current and an anode dissipation just under 100 watts. To give the class A case every advantage, assume the anode current to swing from 280 mA down to zero: the corresponding anode voltage swing is from 380 to 1,000 volts, and the resulting power output is

$$\frac{1}{8}(1{,}000 - 380) \times 280 = 19{\cdot}30 \text{ watts per valve}$$

The total power output is therefore 38·6 watts for the class A case, compared with 88 watts for the class B case, using the same load line and a higher value of H.T.

The class B output is twice that of each of the class A cases. If the restriction on positive grid swing is removed the increase in class B output tends to be greater than that in class A output, *provided that it is permissible to operate the valve into a considerably lower load impedance.* This incidentally means that the criterion for optimum load impedance derived in X:14 can be violated because of the possibility of running the grid positive. To enable the grid to be run positive, the grid must of course be loaded with a suitable value of resistance to avoid a non-linear resistance being presented to the preceding valve, consequent on the variation of grid current throughout each cycle of the driving voltage. A suitable value of resistance is one equal to one-fifth of the minimum grid resistance defined by the ratio of peak grid voltage to peak grid current.

It should be noted that the limitation on class A output compared with class B output, for the same value of H.T. in each case, is due to the limit on standing feed imposed by the permissible anode dissipation. The ratio between the two power outputs (class A and class B) will therefore vary from valve to valve, and also with the operating conditions which are imposed.

12. Variation of Power Consumption of Anode Circuit of Amplifier as Grid Drive Varies.

Since, with the exception of certain telegraph and television circuits, all wave forms applied to the grid of an amplifier have passed through transformers or series condensers in some earlier part of the circuit, they all possess the characteristic that their mean value over any representative period of time, long compared to the time constants of the circuits constituted with such transformers and condensers, is zero. In other words, the area under the curve of the wave form on the positive side of the axis of time is equal to the area under the curve on the negative side.

The consequence is that, in a class A amplifier working over the linear part of its valve field (anode-current grid-voltage curves), the mean value of the feed should not depart from that of the standing feed when drive is applied to the grid. In practice, owing to non-linearity of the valve characteristics, the standing feed sometimes varies slightly as the drive varies.

In a class B amplifier, on the other hand, the positive parts of the wave cause an increase of plate current in one valve of the pair, while the negative parts of the wave cause an increase of plate current in the other half of the pair. The result is that the anode current rises with the application of drive to the input, and on speech or music programme transmissions fluctuates continually in accordance with the loudness of the programme.

13. Variation of Anode Dissipation as Grid Drive Varies.

Since in a class A amplifier the input power to the anode circuit is constant, and when drive is applied to the input circuit power is delivered from the anode circuit to the output circuit, the anode dissipation falls as the power output rises. In a class A amplifier with such a value of feed that its anode runs red hot, the anode can be seen to glow less brightly when the power output is steady at a high value.

In an ideal class B amplifier biased to cut-off, the anode dissipation is zero with no grid drive, and rises as the grid drive is increased. In practice, in a class B amplifier there is usually an appreciable value of standing feed, and the anode dissipation falls with *very* small values of grid drive as the drive is increased from zero, and then rises as the grid drive is increased further.

14. Theoretical Optimum Value of Anode Load Impedance.

A theoretical treatment exists to show that when the positive excursion of the grid is limited to a definite value, the maximum power output from a class A or class B amplifier can be obtained when the impedance facing the anode is twice the internal impedance of the valve. *Its application is, however, limited to valves in which the anode impedance does not appreciably vary for values of grid bias away from cut-off: it does not apply to valves with characteristics approaching those of pentodes.* This treatment is developed for the class A case in terms of ideal valve characteristics as follows:

Fig. 1 shows the anode-volts anode-current field of a choke-fed valve operated as a class A amplifier, with H.T. volts E_a, biased to a standing feed I_a. The load line PAQ is drawn through the point A defined by the co-ordinates $E_a I_a$, and the locus of the

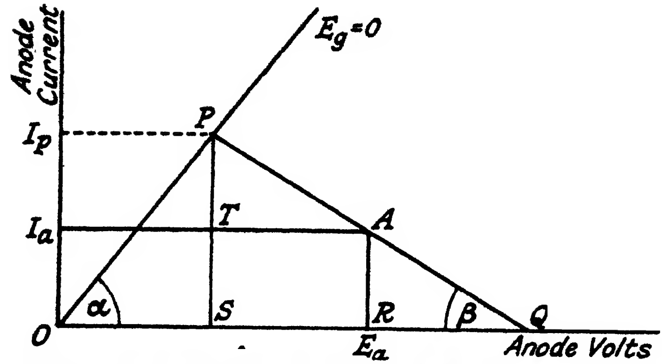


FIG. 1/X:14.—For Determining Optimum Load Impedance of a Valve.

operation of the valve is limited one end by the $E_g = 0$ anode-voltage anode-current curve (since to avoid grid current the grid must not run positive) and the other end by the axis of anode volts. This assumes that the valve is linear down to zero current, which is clearly an approximation.

The output power is then

$$P = \tfrac{1}{2}\frac{QS}{2}.\frac{PS}{2} = \tfrac{1}{2}SR.OS\tan\alpha$$

But $OS+SR = OR = E_a$ a constant. Hence $SR.OS$ and hence P is a maximum when $SR = OS$, since $\tan\alpha$ is a constant, being the reciprocal of R_0, the valve internal anode impedance.

If . $OS = SR$ then $OS = \tfrac{1}{2}SQ$

Therefore $\tan\beta = \tfrac{1}{2}\tan\alpha$

Hence $R = 2R_0$ where $R = \cot\beta$, the value of external anode load impedance corresponding to the slope of the load line.

The treatment for the class B case is similar.

This relationship must, however, be treated with considerable reserve, partly because it is only an approximation, but chiefly because it will sometimes be found in practice that substantially the same power output can be obtained for a given degree of distortion at maximum output, with an impedance facing the anode equal to three or four times the internal impedance of the valve. The anode load impedance should always be kept as high as possible, in order to keep the linearity as low and medium output powers as high as possible.

Finally, if grid current is permitted to flow (grid resistance loading being used), the maximum permissible positive excursion of the grid

is not determined by a particular anode-current anode-voltage curve, but by the spacing of such curves. And the load line which is drawn to provide the best compromise between linearity and power output will not necessarily correspond to an impedance equal to twice the valve anode impedance. This is particularly the case with certain types of valve characteristic, of which good examples are given by Fig. 2/X:11, and the characteristics of almost any pentode.

15. Tuned Amplifiers.

These may be classified as follows:

(*a*) *Drive Amplifiers.* These are designed to amplify a single frequency, and are used, for instance, to supply the drive to a modulated amplifier. When of low power these may sometimes be operated as class B or even class A amplifiers, but when of high power they are always operated as class C amplifiers. Since they are used to supply a steady carrier frequency, it is not necessary that the output should be proportional to the input over the range of levels from zero to full output.

(*b*) *Linear Amplifiers for Modulated Waves.* Low-power amplifiers of this kind may be operated as class A amplifiers. High-power amplifiers of this kind are operated as class B amplifiers; they can never be operated as class C amplifiers because in a class C amplifier the angle of current flow changes with change of carrier frequency grid drive, and consequently the output is not linearly proportional to the input. Under this heading come the amplifiers in a low-power modulated transmitter which amplify the modulated wave and supply it to the aerial. The final amplifier in such a chain is sometimes called the *power amplifier*.

(*c*) *Modulated Amplifiers.* These are class C amplifiers, driven on their grids with a carrier frequency in which the instantaneous amplitude of the H.T. or grid bias is varied in accordance with some other *modulating* wave form, so that the envelope of the carrier frequency output from the anode circuit is the same as that of the modulating wave form in question. It will be evident that a modulated amplifier has to conform to requirements of linearity analogous to but different from those of the linear amplifier for modulated waves. The linear amplifier for modulated waves has to preserve an envelope form, while the modulated amplifier has to create an envelope form corresponding to that of the modulating wave.

Class C Amplifiers. Class C amplifiers have practical application only as tuned amplifiers, and are described under this heading.

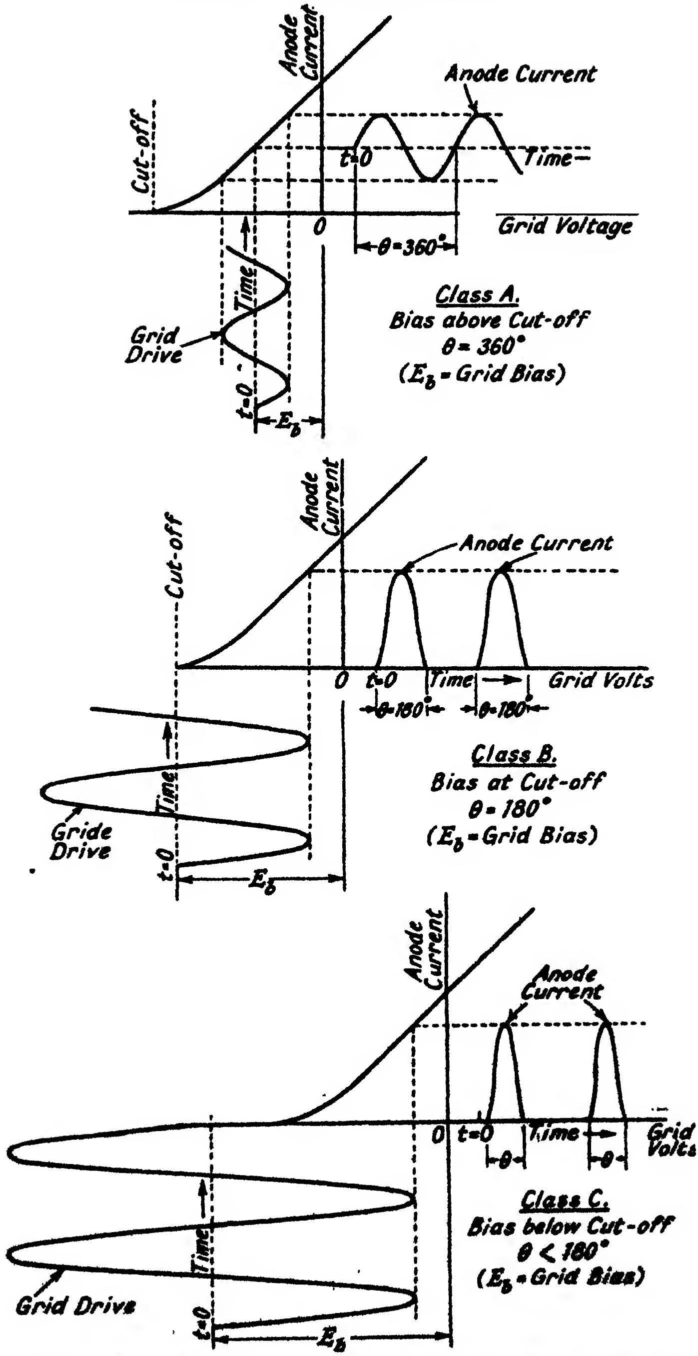


FIG. 1/X:16.—Conditions of Operation of Class A, B and C Tuned Amplifier. Different Values of Grid Voltage.

16. Definitions of A, B and C Tuned Amplifiers.

Definitions of class A, B and C amplifiers exist for tuned amplifiers on the assumption that they are amplifying a sinusoidal carrier wave (which may be of constant amplitude as in the case of a drive amplifier, or may be varying in amplitude in the case of an amplifier transmitting a modulated wave). These are consistent with the definitions for class A and B working already given, but are expressed in a form particularly applicable to the amplification of 'single-frequency' waves.

The definitions are given below, and are illustrated by the anode-current grid-voltage characteristic shown in Fig. 1.

A class A tuned amplifier is one biased above cut-off, so that current flows during the whole of every cycle of the driving wave. The angle of current flow θ is 360°, assuming a whole cycle to be represented by 360°, e.g. by one revolution of the generating vector.

A class B tuned amplifier is one biased to cut-off, so that current flows during alternate half-cycles only: the angle of current flow θ is 180°.

A class C tuned amplifier is one biased below cut-off, so that current flows during a part only of every alternate half-cycle: the angle of current flow is less than 180°.

It will be evident that a class B amplifier is the limiting case of a class C amplifier.

Class A tuned amplifiers are used in low-power circuits where preservation of wave form is of importance and power considerations are unimportant.

Class B tuned amplifiers are used in high-power circuits where preservation of envelope wave form is important, and power conservation is also important.

All amplifiers transmitting modulated waves must be either class A or class B.

Class C tuned amplifiers are used for carrier-drive amplifiers and for modulated amplifiers.

Although the anode-current wave form in a class C amplifier is extremely distorted, the voltage effective across the load in the anode circuit is normally restored to a sufficiently close approximation to its original sinusoidal form, by using a load impedance which is high at the driving frequency but low at all other frequencies. In the case of the final output circuits of a transmitter, which couple it to the feeder line leading to the aerial, and the circuits which couple the feeder to the aerial,* further suppression of harmonics of the driving

(carrier) frequency takes place by virtue of the inevitable nature of such circuits. If adequate suppression of harmonics does not take place in the essential parts of these circuits, additional filtering means are incorporated in these circuits.

17. Circuit Arrangement of Tuned Amplifiers.

As in the case of the amplifiers already considered, the design of tuned amplifiers involves:

(*a*) The determination of load line, grid bias and grid swing corresponding to the available value of H.T.

(*b*) The design of efficient coupling circuits between the input and the first valve, between valves, and between the output valves and the load, presenting the correct impedance towards the valve anode circuit.

In the case of tuned amplifiers, however, additional considerations are introduced by the design requirements of the selective circuits, and in the case of class B and C tuned amplifiers, by the peculiar relations between current and voltage which occur owing to the wave form of the anode current flowing through the input to the selective circuit connected to the anode circuit. The design of coupling circuits is explained in detail in VII:14.

Examples of particular arrangements occurring in tuned amplifiers are given in Fig. 1. These do not represent the practical case exactly, since in practice the batteries would normally be replaced by other sources of power supply, and the exact arrangement respectively to left and right of terminals 1,1 and 4,4 vary widely. Also the particular coupling circuits or impedance-matching circuits shown between 3,3 and 4,4 can evidently be replaced by any other suitable form of coupling circuit. The load to the right of 4,4 may be constituted by the grid circuit of another valve as shown at (*a*), by a concentric feeder as shown at (*b*), or by any other form of circuit to which it is required to supply high-frequency power. Finally, no means for neutralizing the anode grid capacity are shown. These are described in X:19.

Condensers C_1, C_2 and C_3 are *blocking condensers* of negligibly low reactance compared to the circuit impedance. These protect the remainder of the circuit from the D.C. potentials introduced by the grid bias and H.T. supplies.

Resistance R_1 is a grid leak or grid load, and, in association with the grid-bias supply, serves to maintain the grid at the correct potential with regard to the filament. In the case of linear amplifiers running into grid current it may serve to mask the non-linearity of

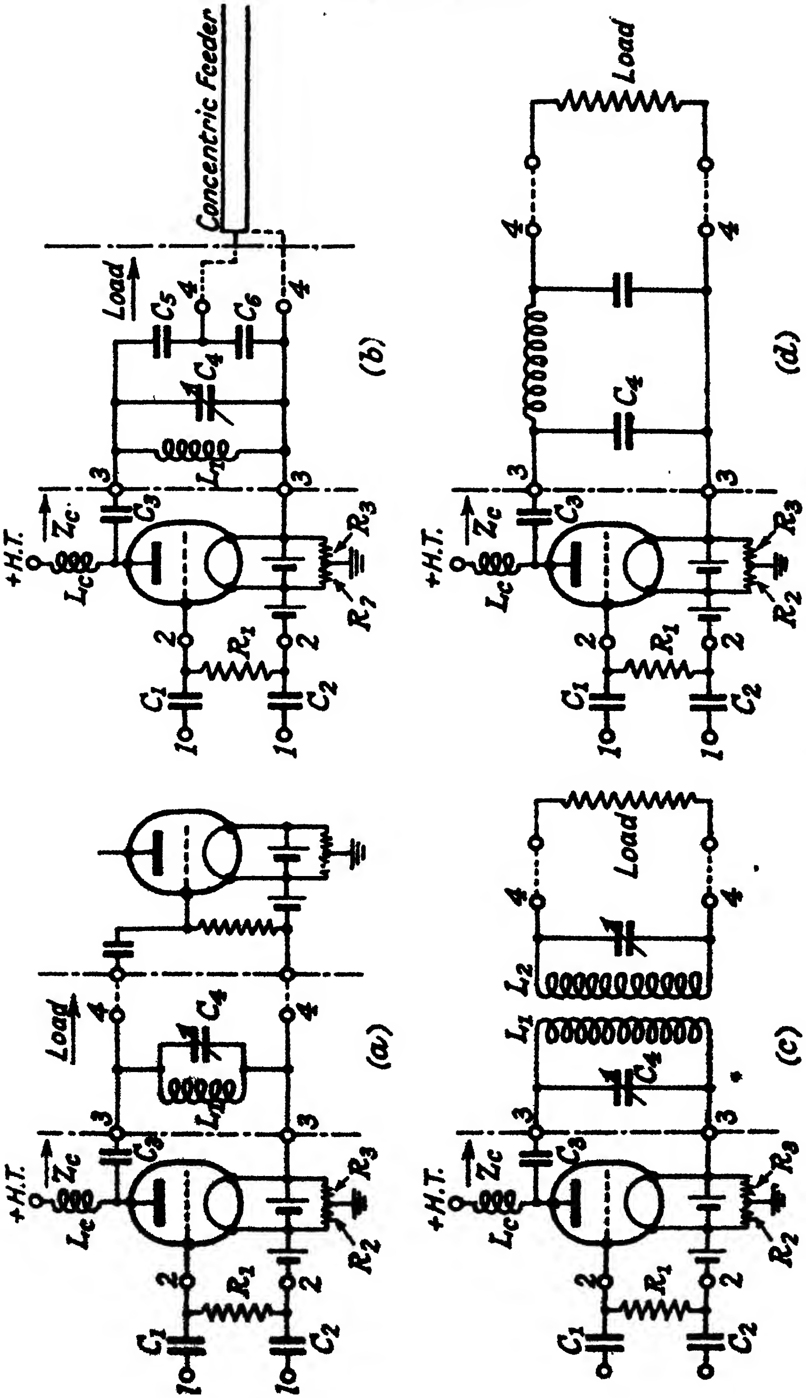


FIG. 1/X:17.—Examples of Tuned Amplifiers.

the grid circuit, in which case its value should be about a fifth of the minimum instantaneous resistance to which the grid falls; in such amplifiers the grid load R_1 should be shunted by an inductance.

Resistances R_2 and R_3 are called *centre-pointing* resistances and are bridged across the filament circuit, and earthed at their midpoint, so that the negative end of the H.T., which is connected to earth, supplies electrons equally to both ends of the filament of the valve, in order to distribute the emission of electrons as uniformly as possible over the surface of the filament. Their value is usually between 5 and 10 times the value of the filament resistance when hot.

Inductances L_c are the anode chokes and are made to have a reactance large compared to the tuning inductances L_1, or the reactance of C_4 in Figs. 1 (*a*), (*c*) and (*d*).

Any differences in circuit arrangement which may occur between class A, B and C tuned amplifiers are connected with the purposes for which they are used, rather than with their principle of operation. The circuits of Fig. 1 therefore serve to illustrate all these types of amplifier.

18. Make-up Current.

The current flowing through the stopping condensers C_3 in Fig. 1/X:17 is sometimes called the *make-up* current. The make-up current is equal to the varying current which flows through the anode cathode circuit of the valve. In the case of Class A amplifiers this current also flows through the local impedance presented at the valve anode. In the case of class B and C tuned amplifiers, the make-up current consists of the load current (of fundamental frequency) plus the harmonics of the fundamental frequency current which flow through the tuning condensers such as C_4, C_5 and C_6.

19. Neutralization of Anode-Grid Capacity of Valves.

The anode-grid capacity C_{ag} of a valve has a number of undesirable effects, of which the most important are:

(1) It tends to make the valve unstable when used as a radio-frequency amplifier, i.e. liable to oscillate owing to feedback from anode to grid.

At the frequencies involved in audio-frequency amplifiers the reactance of C_{ag} is comparatively high, so that the amount of feedback is usually (but not always) too small to be serious. When it is serious it is dealt with by some other method than by neutralization, e.g. by making any resonant circuit in the grid circuit resonate at a frequency different from any resonant circuit in the anode circuit.

(2) It adds a capacity effectively in parallel with the grid cathode capacity C_{gc}, of magnitude $(1+\mu_e)\ C_{ag}$, where μ_e is the effective voltage amplification of the valve from grid to anode when in its associated circuit.

The input capacity of the valve is therefore

$$C_{gc}+(1+\mu_e)C_{ag}$$

and in practice $(1+\mu_e)C_{ag}$ may be several times larger than C_{gc}.

(3) As a corollary of (2), in radio-frequency amplifiers, the capacity across the anode circuit of the previous stage varies as the anode circuit of the valve in question is tuned, since the effective amplification μ_e varies with the impedance in the anode circuit. *The consequence is that, if the previous stage is tuned with the stage in question out of tune, subsequent tuning of the stage throws the previous stage out of tune.*

(4) It may reduce the amplification of the stage, see X:19.2.

As has been explained in the section on valves, certain valves are made with screens between anode and grid in order to reduce the anode-grid capacity, but until recently these were only inserted in medium-size valves, and even at the present time, screen-grid valves and pentodes are not available for very high powers. In radio-frequency amplifiers it is therefore necessary to provide some means of neutralizing the effect of the anode-grid capacity.

Evidently the means employed should be such as to neutralize all of the above undesirable effects. These effects may be usefully and correctly regarded as being due to a current flowing from the anode of the valve through C_{ag} into the input circuit. This is a form of feedback. An obvious method of eliminating such feedback is to arrange a path by which a second current flows into the input circuit, of opposite phase to the current flowing through C_{ag}. Circuits which provide stability (i.e. freedom from tendency to oscillate) do not always eliminate feedback completely and therefore do not eliminate effects (2), (3) and (4) completely. Circuits which eliminate feedback eliminate all three of the undesirable effects.

19.1. Circuits which Provide Stability and Freedom from Feedback.

19.11. The Split-Anode Neut Circuit shown in Fig. 1 is the most common type of neutralizing circuit in use.

In this circuit C_s is the stopping condenser. The main anode tuning condenser or tank condenser is split into two fixed condensers, C_1 and C_2, condenser C_2 usually being about three or four times

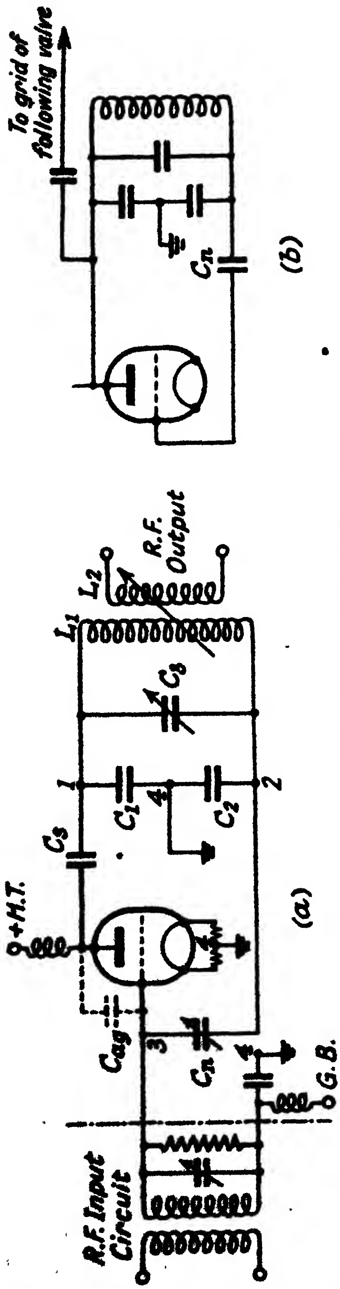


FIG. 1/X:19.—Split-Anode Neut Circuit.

as large as C_1 in order to permit the use of a reasonably large "neut" condenser which is shown at C_n. Tuning is effected by means of the variable condenser C_8 which, with the anode inductance L_1, is connected across C_1 and C_2. It is important to note that C_8 is not connected across C_1, if it were it would effectively vary the ratio between C_1 and C_2 when it was varied for tuning. The operation of this circuit can be regarded in two ways. From the point of view of stability it can be regarded as a bridge circuit. It is evident that the circuit can only oscillate if an R.F. current flowing through L_1, and so producing a voltage across points 1 and 2, gives rise to a p.d. between the grid and ground, that is, between points 3 and 4. (Note that point 4 appears in three places indicating a ground connection.)

If $\dfrac{C_{ag}}{C_n} = \dfrac{C_1}{C_2}$ the circuit represents a balanced bridge, with points 1 and 2 conjugate to 3 and 4, i.e. with infinite attenuation between these pairs of points. See XX:8. Currents in L_1 cannot give rise to feedback to the grid, so that normal oscillation at the frequency of the anode closed circuit (i.e. the circuit constituted by L_1, C_1, C_2 and C_8) cannot occur. From a stability point of view this circuit is excellent. From the point of view of reduction of feedback from anode to grid it is not quite so good. An alternative way of regarding the circuit is to consider the phase and the amplitude of the voltages between point 1 and ground and between point 2 and ground. If

L_1 has zero resistance these voltages are 180° out of phase. This can be seen very easily, since at tune the reactance of C_2 is greater than that of L_1, and the combination has an inductive reactance of magnitude X_L say, which is greater than the reactance of C_2 which will be called $-X_c$. If the voltage between point 1 and ground is V_1, the voltage between 2 and ground is $V_2 = \frac{-X_c}{X_L - X_c}V_1$, which is negative, since X_L is greater than X_c, that is, V_2 is 180° out of phase with V_1. Also under the conditions specified, since the same circulating current flows through C_1 and C_2, and the reactance is inversely proportional to capacity:

$$\frac{V_2}{V_1} = \frac{C_1}{C_2}$$

It follows that if $\frac{C_{ag}}{C_n} = \frac{C_1}{C_2}$, the potential of point 3, the junction point of C_{ag} and C_n and the grid will be at the same potential as point 4, the junction point of C_1 and C_2, that is, at ground potential.

In practice, however, L_1 has a resistance of magnitude corresponding to the load, which is usually equal to about a fifth of the reactance of L_1. In this case V_1 and V_2 are no longer exactly 180° out of phase. As a consequence, while the stability of the circuit is unaffected, the voltage fed back to the grid circuit from the anode circuit is no longer zero. The amount of feedback is usually not serious, however.

In certain valves the coupling between anode and grid is not represented by a pure capacity, but by a capacity in parallel with a (radio-frequency) conductance. In other words, the anode-grid capacity has loss in it. In this case, balance can be effected at one frequency: the carrier frequency: by inserting an appropriate resistance in series with C_n.

It is to be noted that when it is not necessary to introduce such a resistance the balance providing stability is independent of frequency. On the other hand, reduction of feedback from anode to grid varies with frequency, but this is comparatively unimportant. It is sometimes stated that the purpose of a resistance in series with C_n is to balance the stopper resistances. This is absurd because, firstly, the stopper resistances cannot be balanced by a resistance in such a position, and secondly, the stopper resistances are usually shunted by inductances of very low reactance at the carrier frequency.

19.12. Push-Pull Neut Circuit. Fig. 2 shows the arrange-

ment of neutralizing condensers in a push-pull amplifier. The principle of neutralization in this case can most simply be regarded

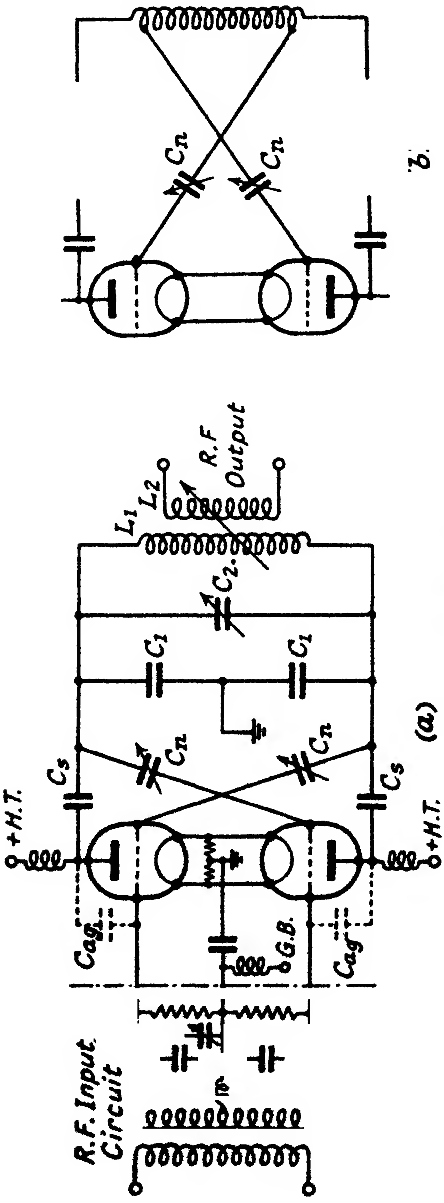


FIG. 2/X:19.—Push-Pull Neut Circui

as that of connecting each grid through a condenser C_n equal to its anode-grid capacity, to a point at potential with regard to ground

180° out of phase with its anode circuit. Such a point is very conveniently constituted by the anode of the valve constituting the other half of the push-pull. In this case, of course, the two condensers C_1 are equal. The circuit is otherwise self-explanatory. This circuit gives perfect stability and complete freedom from feedback, within the limits of accuracy of the balance, for all frequencies at which the reactance of the connecting wires is negligible. This proviso of course applies to all " neut " circuits, and the undesirable effects which occur at high frequencies where this requirement no longer holds are discussed under " Parasitic Oscillations " in XI:12.

19.13. Inductance-Capacity Potentiometer Neut Circuit. The above types of neut circuit are the only ones which are normally met, but evidently there are a variety of possible circuits which will fulfil the function of feeding back energy to the grid in reverse phase to that fed back through the anode grid capacity.

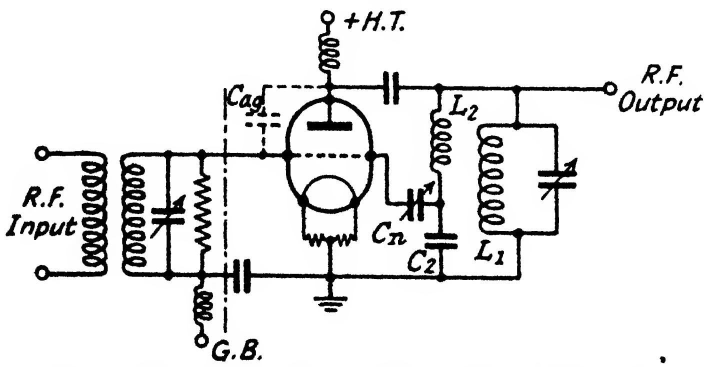


FIG. 3/X:19.—Neut Circuit avoiding Split of Anode Tuning Condenser.

Fig. 3 shows one such circuit which avoids splitting the anode tuning condenser. Phase reversal of the anode voltage is effected by the high impedance circuit L_2, C_2, in which the reactance of L_2 is about five times the reactance of C_2 at the carrier frequency. C_n is then the neut condenser. It will be noted that either C_n or C_2 can be varied to effect neuting, and as C_2 has one pair of plates at ground potential this may simplify construction. This circuit avoids the use of the large condenser C_2 in Fig. 1 which has to carry the full closed current circuit. Circuit L_2C_2 is not in resonance and carries only small currents. For all practical purposes this circuit adjusts to minimum feedback and maximum stability. The reduction of feedback is greater and the degree of stability *slightly* less than the circuit of Fig. 1.

Fig. 4 shows a circuit for the suppression of feedback through

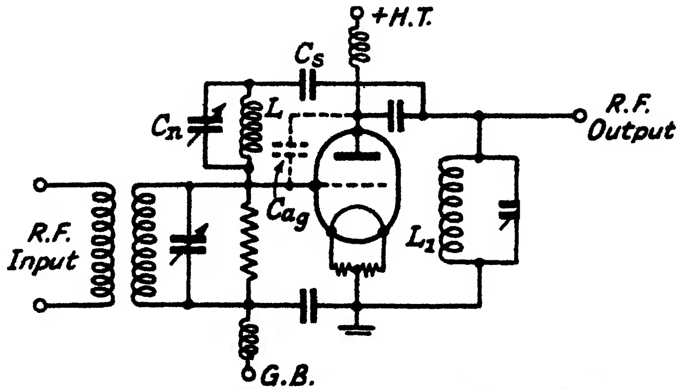


FIG. 4/X:19.—Tuned Rejector Suppression of Feedback.

C_{ag} which consists in tuning the anode-grid circuit to high impedance by means of a parallel inductance L_n and a parallel condenser C_n as shown. C_s is a blocking condenser of negligibly small reactance. C_n is then adjusted so that $C_n + C_{ag}$ anti-resonates with L_n. This circuit is most effective where the impedance from grid to ground is comparatively low, and can only be used at wavelengths where the stray capacity of L_n and C_n to ground is not serious. It is normally only used when extra requirements make the circuit of Fig. 1 impracticable.

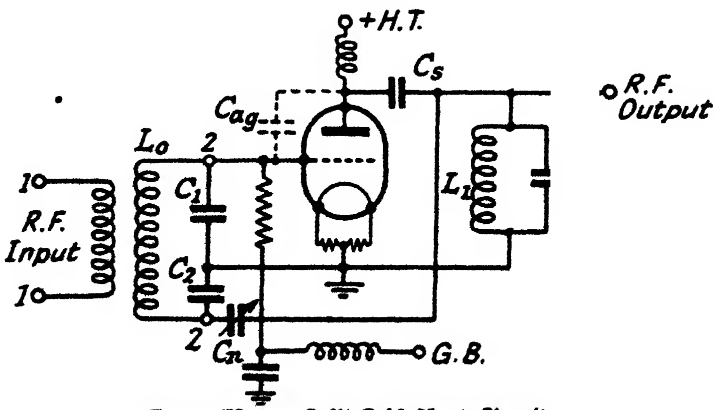


FIG. 5/X:19.—Split-Grid Neut Circuit.

19.2. The Split-Grid Neut Circuit. This circuit, which is shown in Fig. 5, has been left to the last because, although it gives the same degree of stability as the split-anode circuit, it does not eliminate feedback at all, and the feedback operates to drop the effective amplification of the stage. This may have an advantage

in reducing non-linearity of the carrier envelope in an amplifier for modulated waves, and for this purpose, the capacity C_{ag} may be supplemented and the value of C_n correspondingly increased to give increased stable negative feedback (see XXIII), but normally the loss of amplification is considered to be a disadvantage.

For balance the normal circuit is adjusted so that $\frac{C_{ag}}{C_1} = \frac{C_n}{C_2}$. In this case points 1 and 2 constitute the diagonally opposite points of a balanced bridge of condensers, of which the other diagonal is represented by the anode and ground. Voltages between the anode and ground can therefore produce no p.d. across the ends of inductance L_0, and hence oscillations cannot be maintained between anode and grid circuit. For a given anode voltage, V_a, however, the voltage fed back to the grid, i.e. to point 1, is the same as the voltage fed back to point 2 and is equal to

$$V_f = \frac{\frac{1}{C_1}}{\frac{1}{C_1}+\frac{1}{C_{ag}}} V_a = \frac{C_{ag}}{C_1+C_{ag}} V_a = \frac{C_n}{C_2+C_n} V_a \qquad (1)$$

If μ_e is the effective amplification of the valve as determined by its field of anode characteristics and the load line for which the output circuit is designed, the amplification round the feedback path is

$$\mu\beta = \frac{\mu_e C_{ag}}{C_1+C_{ag}} \quad . \quad . \quad . \quad . \quad (2)$$

Reference to the chapter on Feedback shows that under this condition the effective amplification is therefore reduced from μ_e to

$$\mu'_e = \frac{\mu_e}{1+\mu\beta} = \frac{\mu_e}{1+\frac{\mu_e C_{ag}}{C_1+C_{ag}}} \quad . \quad . \quad . \quad (3)$$

Example. $C_{ag} = 50\ \mu\mu\text{F}$, $C_1 = 300\ \mu\mu\text{F}$, $\mu_e = 8$

then $$\mu'_e = \frac{8}{1+\frac{8 \times 50}{350}} = 3{\cdot}74.$$

The above values of C_{ag}, C_1 and μ_e are such as might occur in practice, so that the reduction of amplification with the circuit may be considerable. For this reason the circuit should not be used unless R.F. feedback is required for the purpose of improving envelope linearity or gain stability. This circuit provides a very neat way of applying R.F. feedback while still retaining the feature of neuting

to provide freedom from oscillation. This circuit is not very popular, because in practice it has been found difficult to stabilize, although there is no obvious reason why this should be the case.

19.3. Special Neut Circuit in High-Power Tetrodes. (B.B.C. Patent No. 518,390.) Although by earthing the screen grid of a tetrode (screen-grid valve) through a condenser, the screen grid may be held substantially at ground potential, and unwanted reaction from anode to control grid so reduced to a value which is tolerable for most purposes, in high-power circuits the cost of a condenser of adequate size may be appreciable.

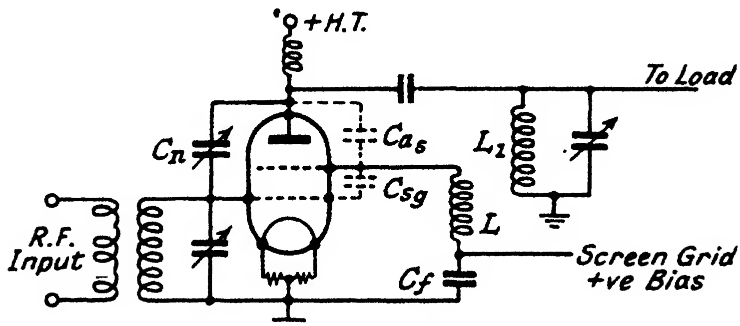


FIG. 6/X:19.—Tetrode Neut Circuit.

In certain cases a form of neutralizing circuit has been used which is illustrated in Fig. 6. The internal valve capacities are shown dotted: C_{as} is the anode to screen-grid capacity, C_{sg} is the screen grid to control-grid capacity. Anode to control-grid capacity may exist, in which case it is in parallel with the neut condenser C_n, *which is connected from anode to control grid*, and so introduces capacity in the place where it is least wanted in a normal circuit. C_f is a smoothing condenser of lower reactance than the inductance L, so that the screen grid is located at the junction of L and C_{as} on the inductance capacity potentiometer from anode to ground constituted by L and C_{as}. Since the reactance of C_{as} is higher than the net reactance of L and C_f, the current fed through C_{as} to the screen grid drives the screen grid 180° out of phase with the anode volts. The control grid has therefore potentials induced in it from two points in opposing phase: from the anode through C_n, and from the screen grid through C_{sg}. Hence by suitable adjustment of C_n the two effects can be neutralized. This circuit adjusts for minimum feedback which constitutes a condition of maximum stability. It is comparable in performance with Fig. 3.

20. Adjustment of Neut Condensers.

The neut condenser in Fig. 1/X:19 may be adjusted *either* so that high attenuation (theoretically infinite) is introduced between the terminals of the anode tank circuit inductance (e.g. L_1 in each of Figs. 1 to 4 of X:19) and the grid-cathode circuit, i.e. *to give maximum stability*, *or* so that maximum attenuation is introduced between anode to ground and grid to ground, i.e. to give *minimum feedback*.

The maximum stability condition is generally used.

In the circuits of Figs. 2, 3, 4 and 6/X:19 the maximum stability and minimum feedback conditions are identical.

The circuit of Fig. 5/X:19 is adjusted for maximum stability, and *there is no reduction of feedback.*

The attenuations which respectively determine the degree of stability and of freedom from feedback are determined entirely by the characteristics of the circuit considered as a passive structure, that is with no H.T. applied to the valve anode circuit. The attenuation is therefore independent of direction: if the attenuation from anode to grid is high, the attenuation from grid to anode is high. This consideration leads directly to what is possibly the most obvious method of neutralization.

20.1. Method 1. Direct Adjustment of Loss.

1. Remove H.T. from anode of valve by taking out fuse or disconnecting link, etc.
2. Insert R.F. milliammeter (e.g. 0–250 mA) in series with L_1 to read I_{L_1}, the R.F. circulating current in L_1. (Alternatively, a small lamp such as a pocket-torch lamp may be bridged across one or two turns or a fraction of a turn of L_1. For short waves a lamp is preferable.)
3. Apply *low* drive to grid of valve. Tune anode circuit to maximum value of I_{L_1}, taking care not to exceed current rating of meter—if meter reaches full-scale deflection, drive must be reduced before tuning is completed.
4. Adjust C_n until I_{L_1} falls to a minimum. If C_n does not tune sharply to a minimum, increase grid drive with C_n in the position for minimum I_{L_1}, and then readjust C_n, moving it slowly and watching I_{L_1} carefully to see that the meter is not burnt out.

As an alternative indication of the R.F. current through L_1, the grid volts applied to the following valve can be used by observing the grid current of the following valve with its bias reduced to zero.

The following valve is then used as a valve voltmeter: evidently any type of valve voltmeter can be used in this position.

The above method of neuting, using the R.F. current in L_1 as an indicator, adjusts the circuits of Figs. 1 and 5/X:19 to maximum stability. (It is assumed that the circuit of Fig. 5/X:19 is driven at 1,1, and not between grid and ground.) It adjusts Figs. 2, 3, 4. and 6/X:19 to minimum feedback and maximum stability. It is therefore true to say that this method adjusts all circuits to maximum stability, and those circuits in which the conditions of maximum stability and minimum feedback are the same are adjusted also to minimum feedback. In subsequent discussions this corollary will be understood but not specified.

If the circuit in Fig. 1(*a*) X:19 is replaced by that of Fig. 1(*b*) X:19 so that the drive of the following valve, instead of being taken from L_2, is taken from the anode direct, L_2 being omitted, C_n is adjusted for minimum feedback if it is adjusted for minimum grid drive on the following valve.

Other circuit variations and the resultant circuit condition: stability or minimum feedback: can evidently be worked out.

20.2. Method II. Grid Current Method.

1. Remove H.T. from the anode of the valve as before.
2. Bring up grid drive slowly until the maximum rated grid current of the valve is reached, or until normal drive is reached, whichever gives the lower value of drive.
3. Adjust the neut condenser until varying the anode tuning condenser causes no variation of grid current.

This method adjusts all circuits to maximum stability and is non-applicable to Fig. 5/X:19, because the condition of maximum stability is not a condition of zero feedback.

It can evidently be used only in cases where the grid current is of sufficient magnitude to give a clear indication on the meter.

The method depends on the fact that energy is fed from the grid circuit to the anode circuit through the high reactances supplied by the anode-grid capacity C_{ag} and the neut condenser C_n, and the energy transfer is a maximum when the impedance presented by the output circuit is high (and with some value of positive reactance). Hence, as the anode tuning condenser is varied, the energy transfer, and therefore the grid voltage and the grid current vary. When the neut condenser is varied so that no voltage is generated across L_1, there is no energy transfer from grid circuit to anode circuit, and the grid voltage and grid current are therefore independent of the setting of the anode tuning condenser.

20.3. Method III. Anode–Current Grid–Current Method. This is carried out with full H.T. and bias, and normal carrier drive, and should only be used on large transmitters if the approximate position of the neut condenser has been previously found by some other method. This is because, with the neut condenser far from balance, large oscillations may occur and give rise to arcing and breakdown. Starting with the neut condenser in the position as found it is adjusted until, as the anode tuning is varied, the anode current reaches a minimum at the same time that the grid current reaches its maximum.

This method always adjusts for minimum feedback from anode to grid. It cannot be used for the split-grid neut circuit of Fig. 5/X:19, which is incapable of adjustment for zero feedback from anode to grid; see X:19.2.

The principle of this method may be explained as follows:

Considering the circuit of Fig. 2/X:19, when condensers C_n are too large, the feedback current flowing through the whole grid input circuit (which, being in tune, is resistive) leads nearly 90° on the voltage applied to the grid by the drive (because the reactance of $C_n - C_{ag}$ is much larger than the impedance of the grid input circuit) and so contributes a voltage in the grid circuit leading 90° on the voltage supplied by the drive. As the anode tuning condenser is increased, the anode voltage lags more and more on the grid drive, so that the feedback voltage is brought more and more into phase with the grid-drive voltage. The grid-drive voltage therefore increases, reaching a maximum when the rate of increase of gross grid voltage, due to change of phase of the anode voltage, is equal to the rate of fall of gross grid voltage, due to the fall of feedback current caused by the drop in anode impedance, and therefore of anode peak volts, as the anode tuning condenser is increased from the position of tune. The maximum grid current therefore occurs at some value of anode tuning condenser greater than the tune value. Similarly, if condensers C_n are too small, the maximum grid current occurs at some value of anode tuning condenser which is too small.

Since it is impossible to watch both grid-current and anode-current meters simultaneously, the best practical procedure is to note separately the anode-tuning condenser dial readings D_a and D_g at which anode current and grid current are respectively a minimum and a maximum. If D_g is greater than D_a (assuming condenser capacity to increase with increase of dial reading), then C_n must be reduced, and vice versa.

This method can also be used with the circuits of Figs. 1, 3, 4, 5 and 6/X:19 with substantially the same effect, and although theoretically the points D_g and D_a can never be made to coincide (because the feedback can never be reduced to zero) in these circuits, in practice very close approximation to coincidence can be obtained.

20.4. Method IV. Anode Current Method. This is carried out with full H.T. and normal drive, and is only applicable to cases where an adequate capacity variation is available in the tuning condenser.

The tuning condenser is varied each side of the tuning point until the anode current ceases to rise any further. If the current with high capacity is less than the current with low capacity the neut condenser is too small, and vice versa. When the circuit is correctly neuted, the current variation is symmetrical about the tuning point. This effect is due, in the presence of incorrect neuting, to reversal of the phase of the feedback as the anode circuit passes through tune.

In single-sided amplifiers this method of neuting adjusts for minimum feedback. In push-pull amplifiers it adjusts for both minimum feedback and maximum stability.

Other methods of neuting are sometimes used. It is always permissible, for instance, to drive the anode circuit and adjust for zero drive at the grid, but such methods only have application to particular experimental circuits, and the above methods are the most usual in operating transmitters.

20.5. Method of Neuting to Use. The method of neuting to use is the simplest which gives satisfactory operation, and for all amplifiers except modulated amplifiers, or the final amplifiers of Doherty transmitters, any of the above methods will be found to give reasonable results, where they can be applied.

Method IV is considered by some to give the most accurate adjustment to minimum feedback, and for this reason it is particularly suitable to the neuting of modulated amplifiers or the final amplifiers of Doherty transmitters; in both these cases the presence of feedback is undesirable. In modulated amplifiers, unless the grid drive is modulated 100%, the presence of feedback in random phase introduces phase modulation. Method IV is however not always applicable because sufficient capacity variation is not always available in the tuning condensers.

Method III may be used where sufficient capacity variation is not available to use Method IV, and where it is required to adjust a single-sided circuit to minimum feedback.

Methods III and IV have the advantage that they can be used with the transmitter on power and with normal carrier drive, and require no circuit modification. They are therefore quick to use, and although they adjust to minimum feedback, are generally found to provide an adequate degree of stability.

Methods III and IV must not be used the first time a transmitter is brought up on power after a circuit adjustment which is likely to have affected the neut appreciably. If this is done there is a danger of the circuit going into violent oscillation before the neut is made.

For this reason, after a circuit change which is likely to have affected the neut, either Method I or Method II must be employed to obtain a preliminary setting of the neut condenser. After this the transmitter can be brought up on power, and if thought necessary the final neut can be obtained by means of either of the Methods III or IV. This restriction applies although Methods I and II adjust to maximum stability and an adjustment to minimum feedback is required. It will be appreciated, however, that this distinction (between minimum feedback and maximum stability) is of importance only in the case of the circuits of Figs. 1 and 5/X:19, the second of which will normally never be met in practice. Even in the case of the circuit of Fig. 1/X:19 adjustment by Methods I and II brings the neut condenser to a position of maximum stability and so prevents the occurrence of violent oscillations before the final neut is applied on power. In the case of small transmitters of 1 kW output and less, the preliminary neut may be omitted and neuting carried out directly on power.

In certain transmitters, where oscillation is found to occur in a stage after the stage has been neuted by Methods I or II, the stage is neuted finally by adjusting the neut condenser until oscillation disappears. This method cannot be recommended as a general practice, since it can evidently only be applied when experience has established that the oscillations which occur do no damage. Further, if a stage oscillates after it has been neuted by Methods I or II it is an almost certain indication that feedback exists in the circuit through some other path than that through the anode-grid capacity; and this should be traced and eliminated.

Method I is generally considered preferable to Method II because it is simpler in conception and is more fundamental; also it is always applicable. Method II sometimes fails because there is not sufficient grid current present to give a sensitive indication on the meters which are available.

The stability tests described in XI:12 are primarily intended to

show up oscillations at frequencies other than the fundamental frequency. Since these are carried out with the circuit in neut, any oscillation occurring at the fundamental frequency is an indication of the presence of feedback extra to the valve capacity as indicated above. While as an emergency measure such oscillations may be eliminated by adjustment of the neut condensers, the circuit should be examined at the first opportunity to find and eliminate any feedback through capacities between circuit elements and through mutual earth impedance.

20.6. Neuting of Short-Wave Amplifiers. Before neuting short-wave amplifiers it is essential that both the grid-lead and cathode-(filament)-lead neutralizing condensers are correctly adjusted; if this is not done the circuit will go out of neut when these condensers are adjusted. These are normally adjusted to values specified by the manufacturer, but where values are not given they should be adjusted to cancel the reactance of the associated conductor by placing an R.F. current meter in the lead and adjusting for maximum current.

21. Design of Class A Tuned Amplifiers.

The design of these is very simple and proceeds exactly as in the case of the class A amplifiers already considered. Referring to Fig. 1/X:17, Z_e, the optimum impedance required to face the anode of the valve at terminals 3,3, is determined by drawing load lines on the field of anode-voltage anode-current characteristics for the valve, until the best compromise condition is found.

The coupling circuit between 3,3 and 4,4 is then designed to pass the necessary band of frequencies and to work between Z_e and the impedance provided by the output load L connected to terminals 4,4. This design is dealt with in VII:14.

An exception occurs in the case of the circuit of Fig. 1(*a*)/X:17 where there is evidently no choice of impedance: the impedance facing the valve is given by the parallel impedance of the load and the impedance of the parallel tuned circuit at resonance.

In practice also, the circuit at (*d*) would have a parallel-tuned circuit bridged across 3,3 for fine adjustment, to eliminate any residual reactance presented by the valve owing to the difficulty of exact adjustment of the network between 3,3 and 4,4.

22. Class B and C Tuned Amplifiers.

The class B amplifier can conveniently be treated as the limiting case of a class C amplifier with 180° angle of current flow.

It is therefore only necessary to consider the case of a class C amplifier.

The determination of optimum load line is carried out exactly as in the case of class A amplifiers, except that in the case of a drive amplifier the limitations imposed by requirements of linearity do not exist.

If, however, the coupling circuit is designed to present a true impedance Z_L towards the valve, the valve will not work along the load line corresponding to Z_L because of the current wave form in the anode circuit. A calculation must therefore be made to determine what impedance Z_c must be presented at terminals 3,3 (Fig. 1/X:17) by the input of the coupling circuit in order that the valve shall work along any required load line of slope Z_L.

This calculation is made on the assumption that the impedance presented to the anode of the valve is finite and of the required value (to be determined) in the band of transmitted frequencies, and is zero at *harmonic* frequencies of magnitude two, three, four, etc., times the midband frequency: the carrier frequency.

All coupling circuits must therefore conform approximately to the requirements imposed by this assumption, and examination of the coupling circuits used in practice shows that the impedance falls away at frequencies above and below the frequency to which the circuit is tuned. In practice, with the types of coupling circuit in common use, assuming a kVA to kilowatts ratio of 5, the impedance at 3,3, the input to the coupling circuit, at the second harmonic of the carrier frequency is a nearly pure reactance of magnitude equal to just under a tenth of the impedance presented at the carrier frequency. At the third harmonic the reactance is a fifteenth of the impedance, and so on.

For purposes of this calculation, the wave applied to the grid of the valve, when a modulated wave, is treated as a sinusoidal wave of frequency equal to the carrier frequency, and of varying amplitude, rather than as the corresponding band of frequencies constituted by carrier plus sidebands. The justification for this method is that in practice it is found to give the right answer. In the case of a drive amplifier the grid drive consists of a sinusoidal carrier frequency wave and no discussion is necessary.

A further assumption is made which is justified by the fact that the back e.m.f. in the anode circuit is substantially sinusoidal, and is almost exactly 180° out of phase with the grid drive. (These conditions are departed from as the Q and kVA/kW ratio of the output circuit are lowered.) This is that the *form* of the current

wave in the anode circuit of a valve biased to cut-off or below cut-off, is independent of the variation with frequency of the impedance in the anode circuit of the valve, and is the same as if that impedance were a pure resistance. On this assumption, the current which flows in the anode circuit consists of a series of fractions of sine waves determined completely by specifying i_p the peak current, and θ the angle of current flow. See Fig. 1(*b*) and 1(*c*)/X:16.

An error which has been perpetrated by manufacturers of short-wave transmitters is to neglect to provide a low impedance path from anode to cathode at the harmonic frequencies. Such omissions occur in push-pull stages, and the net result is a loss in efficiency and the somewhat inconvenient fact that the whole of the output circuit on the primary side of the coupling (assuming an inductive output coupling circuit to be used) assumes a large potential to ground at the harmonic frequencies. The seriousness of the situation is somewhat reduced because at short waves stray capacities provide a partially adequate path back to the cathodes.

The reason for the omission is usually not faulty design, but because when the circuit was set up it was found for some reason or other to be unstable and was stabilized by removing, for instance, the earth connection at the centre of the (balanced) anode tuning capacity. The proper procedure of course would be to locate the feedback path which evidently is due to an impedance in the ground (probably cathode to ground) lead, and remove it by providing a low impedance to ground at the frequency of oscillation.

22.1. Definition of Angle of Current Flow. Fig. 1/X:16. shows three idealized anode-current grid-voltage characteristics of a valve supplied with a grid bias E_b and driven on its grid with a sine wave of voltage. The amplitude of the (sinusoidal) grid drive, in conjunction with the magnitude of the grid bias, determines whether the valve is operated in class A, B or C, and also determines the form of the anode-current wave, which is indicated on the right of each of the three diagrams. The angle of current flow is equal to 360° × the fraction of each cycle of the driving sine wave voltage during which current flows. The angle of current flow is indicated by θ.

22.2. Relation between Angle of Current Flow and Anode Current Ratios : Peak Fundamental to Peak, Peak Fundamental to Mean, and Mean to Peak Ratio. For any value of θ such a series of fractions of a sine wave (i.e. as shown in Fig. 1/X:16) constitutes a periodic wave which is capable of being analysed into

a series of frequencies of appropriate amplitude and phase. The most important of these is $\hat{i}_f$, the peak amplitude of the fundamental frequency component of the anode current: the lowest of these frequencies. This is of the same frequency as the original frequency driving the grid. It is this frequency which, flowing through the (anode load) impedance presented by the input to the coupling circuit (e.g. through terminals 3,3 in Fig. 1/X:17) generates the required voltage at the carrier frequency across the input to the coupling circuit and so determines the amplitude of the "amplified" (or

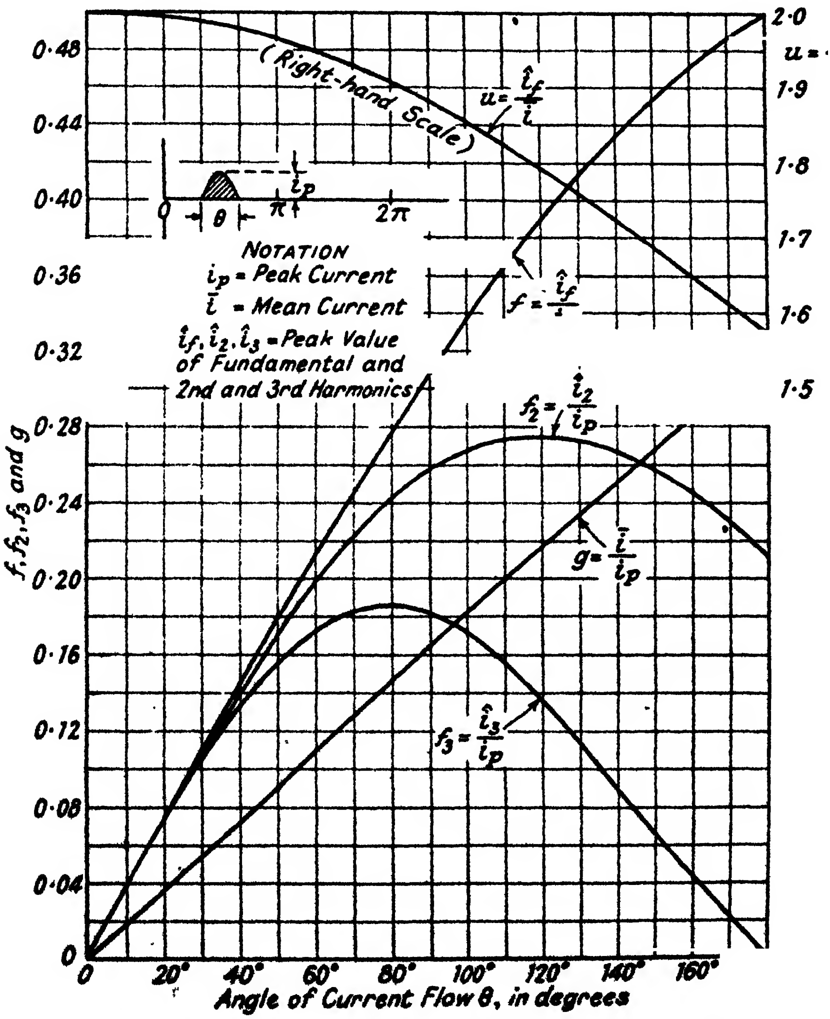


FIG. 1/X:22.—Relations between $\hat{i}_f$, $\bar{i}$ and $\hat{i}$ in terms of

reconstructed) carrier-frequency voltage constituting the output from the valve: i.e. the anode peak volts. The other currents i_2 i_3, i_4, etc., of frequency equal to two, three, four, etc., times the carrier frequency, also flow through the input coupling circuit, but generate little voltage across it, because at these frequencies the coupling circuit presents negligible impedance.

Fig. 2/X:22 shows the anode-current wave form of a class C amplifier driven with a sinusoidal-grid voltage and operating at an angle of current flow of 120°.

In this figure

i_p = the peak current.

i_f = the peak amplitude of the fundamental frequency component of the anode current

$\bar{i}$ = the mean anode current = the driven feed.

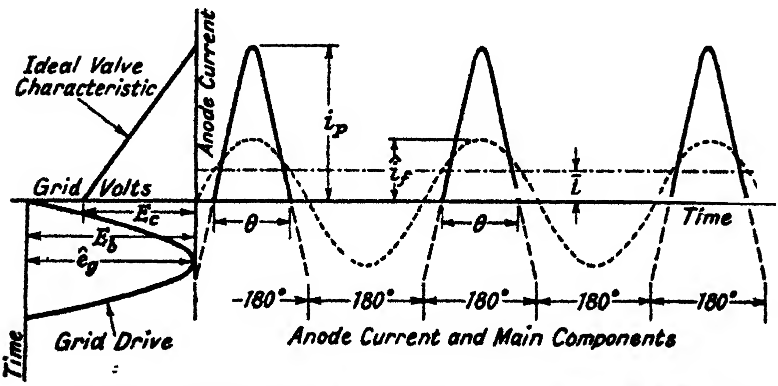


FIG. 2/X:22.—Illustrating relative values of i_p, i_f and $\bar{i}$ for $\theta = 120°$.

There are three important ratios which are represented by f, u, and g:

$$f = \frac{i_f}{i_p} = \text{the peak-fundamental-to-peak ratio}$$

$$u = \frac{i_f}{\bar{i}} = \text{the peak-fundamental-to-mean ratio} \qquad (1)$$

$$= \text{the current utilization}$$

$$g = \frac{\bar{i}}{i_p} = \text{the mean-to-peak ratio.}$$

The method of calculating the values of these ratios f, u and g from the angle of current flow is given in CVII:3.

Each of the ratios f, u and g is plotted on Fig. 1 for different values of the angle of current flow θ. On this figure are also plotted

the values of $f_2 = \frac{i_2}{i_p}$ and $f_3 = \frac{i_3}{i_p}$. f_4, f_5, etc., are shown in Figs. 6, 7 and 8/VIII:1.

It will be noted that g, the mean-to-peak ratio, is the reciprocal of the peak-to-mean ratio, and it may be wondered why the peak-to-mean ratio has not been shown instead of g. The reason is convenience in representation, since it is quite simple to plot a quantity which goes down to zero, but rather difficult to show one which goes up to infinity.

The ratio u, the peak-fundamental-to-mean ratio, is sometimes called the *current utilization.*

22.3. Relation between Load Line in Class C Amplifier and Impedance Facing Anode. If the impedance presented at terminals 3,3 of Fig. 1/X:17 by the coupling circuit at the carrier (driving) frequency is Z_c, the peak voltage generated across it by the passage of the fundamental frequency component of the anode current is

$$\hat{e}_a = \hat{\imath}_f Z_c \qquad . \quad . \quad . \quad . \quad (2)$$

$\hat{e}_a$ is then the amount by which the anode voltage falls at the moment of peak current (since at the carrier frequency Z_c is a resistance free from reactance).

Since the impedance facing the anode varies with frequency it is not obvious, but it is a fact of experience, that when the amplifier is properly adjusted (so that Z_c is a pure resistance) the peak anode current (i_p as distinguished from $\hat{\imath}_f$) occurs at the instant that the anode volts are a minimum, i.e. when the voltage across 3,3 in Fig. 1/X:17 is a maximum, that is equal to $\hat{e}_a$.

Hence, i_p and $\hat{e}_a$ determine a point on the load line such that

$$\frac{\hat{e}_a}{i_p} = Z_L = \text{the slope of the load line} \quad . \quad . \quad (3)$$

From (1)
$$i_p = \frac{\hat{\imath}_f}{f} \qquad . \quad . \quad . \quad . \quad (4)$$

Hence, from (2) and (4)

$$Z_L = \frac{\hat{e}_a}{i_p} = \frac{\hat{\imath}_f Z_c}{\hat{\imath}_f/f} = fZ_c \qquad . \quad . \quad . \quad (5)$$

Therefore
$$Z_c = \frac{Z_L}{f} \qquad . \quad . \quad . \quad . \quad (6)$$

Hence to design a class C amplifier it is only necessary to determine Z_L from a load line drawn on the field of valve characteristics, to determine Z_c by dividing Z_L by the value of f (read from Fig. 1/X:22) corresponding to the angle of current flow chosen (see X:25), and to

design the coupling network to have an impedance ratio from Z_e to L, where L is the impedance into which the coupling network is required to work.

In the case of a drive amplifier the grid drive is then adjusted in accordance with the instructions given below under X:26, so that the required peak current is not exceeded. In a linear amplifier for modulated waves the grid drive is adjusted so that the grid never becomes more positive than the anode, that is to say, so that the quantity—peak grid volts minus the grid bias—is not greater than the difference between the H.T. voltage effective on the anode and the anode peak volts. This is discussed immediately below. In the case of an anode modulated amplifier, see XIII:4, the grid drive is adjusted so that the amplifier is driven into anode limitation with the maximum value of the H.T. which occurs, see X:27.

At the peak of the positive grid swing, when the anode current is at its peak value the drop through the anode load is a maximum and the anode potential is a minimum. If the grid drive is large enough it is possible for the grid to become more positive than the anode during a small fraction of the cycle. If this happens in a linear amplifier, two undesirable things happen. First, the grid current rises to an excessive value, and if the regulation of the driving stage is not adequate the grid voltage may be dropped in a non-linear way. Secondly, while the grid is more positive than the anode it robs the anode of electrons, so that the anode current may actually fall while the grid voltage is rising. Both these effects lead to non-linearity and must be avoided in linear amplifiers. (The fall of the anode current with increase of grid volts in this case is not to be confused with the same effect due to anode limitation, see X:27 and in particular Fig. 1(*b*)/X:27, although both causes may act together.)

In modulated amplifiers and in class C amplifiers where linearity of the envelope does not have to be preserved, the grid may be driven more positive than the anode, provided excessive grid dissipation does not take place.

Before considering the angle of current flow, it is necessary to derive expressions for the power output and efficiency of a class C tuned amplifier.

23. Power Output of a Class C Tuned Amplifier.

The power output of a class C tuned amplifier is given by

$$P = \tfrac{1}{2}i^2 Z_e = \tfrac{1}{2}f^2 i_p^2 Z_e = \tfrac{1}{2}f^2 i_p^2 \frac{Z_L}{f} = \tfrac{1}{2}f i_p^2 Z_L . \qquad (1)$$

obtained by substituting for $\hat{\imath}_f$ and Z_c from equations (4) and (6)/X:22 respectively.

From equation (3)/X:22 $i_p Z_L = \hat{e}_a$

Therefore $$P = \tfrac{1}{2} f \hat{e}_a i_p = \tfrac{1}{2} \hat{e}_a \hat{\imath}_f \quad . \quad . \quad . \quad . \quad (2)$$

which is obvious, but serves to show how the relations in (1) may be determined alternatively.

23.1. Power Output of a Class B Amplifier. In a class B amplifier $\theta = 180°$, and, from Fig. 1/X:22, $f = 0{\cdot}5$, so that

$$P = 0{\cdot}25 \hat{e}_a i_p = 0{\cdot}125 i_p^2 Z_c = 0{\cdot}25 i_p^2 Z_L \quad . \quad . \quad (3)$$

24. Anode Efficiency of a Class C Tuned Amplifier.

This is the ratio obtained by dividing the output power P given by equation (2)/X:23 by the input power to the anode circuit given by the product of the H.T. voltage E and the feed or mean current $\bar{\imath}$.

Hence the efficiency

$$\eta = \tfrac{1}{2} f \frac{\hat{e}_a i_p}{E \bar{\imath}} = \tfrac{1}{2} \frac{\hat{e}_a}{E} \times \frac{\hat{\imath}_f}{\bar{\imath}} = \tfrac{1}{2} hu \quad . \quad . \quad . \quad (1)$$

where h · *the voltage utilization* . . . (2)

and $= \dfrac{\hat{\imath}_f}{\bar{\imath}}$ the current utilization.

Once the angle of current flow has been determined, the value of u can be read from Fig. 1/X:22, while h is determined from the values of $\hat{e}_u$ and E.

24.1. Anode Efficiency of a Class B Amplifier. In a class B amplifier $\theta = 180°$, and, from Fig. 1/X:22

$$u = \frac{\pi}{2} = 1{\cdot}57$$

so that $$\eta = \tfrac{1}{2} \times 1{\cdot}57 \times h = 0{\cdot}785h \quad . \quad . \quad . \quad (3)$$

This formula is applicable to both tuned radio frequency amplifiers and to push-pull (untuned) audio-frequency amplifiers, when driven by sinusoidal waves.

24.2. Anode Efficiency of a Class B Tuned Amplifier Transmitting a Modulated Wave. 1. *Efficiency on No Modulation.* From equation (3) the efficiency is $0{\cdot}785h$ where h is the voltage utilization. In the unmodulated carrier condition the anode peak volts are half the value to which they will rise at 100% modulation. If the maximum permissible voltage utilization is $\hat{h}$, and the amplifier is correctly adjusted to read this utilization at 100% peak modulation, the voltage utilization in the carrier condition is $\tfrac{1}{2}\hat{h}$, and the efficiency $$\eta_c = 0{\cdot}785 \times \tfrac{1}{2}\hat{h} = 0{\cdot}3925\hat{h} \quad . \quad . \quad . \quad (4)$$

This is the efficiency which is observed during the adjustment of anode conditions on the amplifier.

2. *Efficiency on Modulation.* During modulation the carrier power increases by a factor $\left(1+\frac{m^2}{2}\right)$ where m is the percentage modulation, while the mean feed remains unchanged, assuming conditions of linearity to exist. The power input to the anode circuit is unchanged, and the efficiency

$$\eta_m = 0{\cdot}3925\bar{h}\left(1+\frac{m^2}{2}\right) \quad . \quad . \quad . \quad (5)$$

On normal modulation this efficiency cannot be observed from moment to moment. It can only be determined over a period of time by measuring the mean modulation, using some form of integrating modulation meter. In practice the reverse procedure is sometimes adopted. The mean power supplied to the transmitter in a given time is measured on a recording wattmeter and the mean modulation is found from equation (5).

The mean power consumed depends of course on the type of programme transmitted and how the modulation is controlled, but is usually found to correspond to a mean modulation depth between 15% and 20%. Where limiters of the non-distorting type (e.g. amplifiers fitted with automatic volume control) are equipped and driven into limitation, the effective mean modulation may be more than double this value.

25. The Angle of Current Flow.

The angle of current flow is described and defined in X:22.1.

25.1. Determination of θ, the Angle of Current Flow. The angle of current flow is determined by the instant in the positive E.F. cycle of the grid drive (corresponding to the negative half-cycle of the anode swing) at which anode current begins to flow and the instant at which anode current stops flowing. These instants are determined on the correct assumption that the anode volts and grid volts are exactly 180° out of phase.

Conventions.

f = the driving (carrier) frequency.

$e_g \sin \omega t$ = the grid voltage ($\omega = 2\pi f$)

$-e_a \sin \omega t$ = the anode voltage.

(The driving voltage is assumed to pass through zero and to be increasing in a positive direction at times $t_0 = 0$ or n/f, where n is any positive integer.)

t_1 = time at which anode current begins to flow.

t_2 = time at which grid reaches maximum positive value and anode reaches minimum positive value (i.e. anode peak volts reach their maximum).

$\psi = \omega(t_1 - t_0)$ = angle of positive half-cycle of grid voltage (and negative half-cycle of anode voltage) which elapses before current begins to flow.

$\theta = 2\omega(t_2 - t_1) = 180° - 2\psi$ = angle of current flow.

E = H.T. volts.

E_b = value of working grid bias: usually negative.

E_c = value of grid bias necessary to take valve to cut-off with H.T. volts E. This is also usually negative bias.

E_{gf} = magnitude of instantaneous value of grid bias at time anode current starts to flow.

E_{af} = instantaneous value of anode volts at time anode current starts to flow.

Note that $\psi \quad 90° - \frac{\theta}{2}$.

Throughout this discussion the quantities E_b and E_c will be treated as positive quantities, although, of course, they are normally negative. This is a standard convention, and when substitution of the actual negative quantities is made in the formulae evolved, the correct answer appears. For instance, the quantity $E_c - E_b$ appears frequently; if, in a particular case $E_b = -300$ and $E_c = -100$, then $E_c - E_b = +200$.

Fig. 1 shows a field of valve characteristics with two related curves showing corresponding instantaneous values of anode volts and grid volts. The instantaneous value of anode volts is plotted against a time scale O_1T_1, and the instantaneous value of grid volts is plotted against a time scale O_2T_2. These time scales are identical and are only drawn in different positions for convenience in representation, and given different designations for convenience in reference. The anode-voltage amplitude scales of the valve field and of the anode-voltage variation curve are identical. The grid-voltage amplitude scale of the grid-voltage variation curve is determined by the intercepts of the (anode-voltage anode-current) valve characteristics with the normal (shown dotted) through the midpoint of the time scale O_2T_2: the grid voltage marked on each characteristic determines the grid-voltage amplitude at the point of intersection. Confusion may be avoided if it is realized that two-dimensional paper is here being made to portray a three-dimensional relationship, and if each of the sine-wave diagrams are imagined to be rotated about the normal (in the plane of the paper) through the centre of the time base.

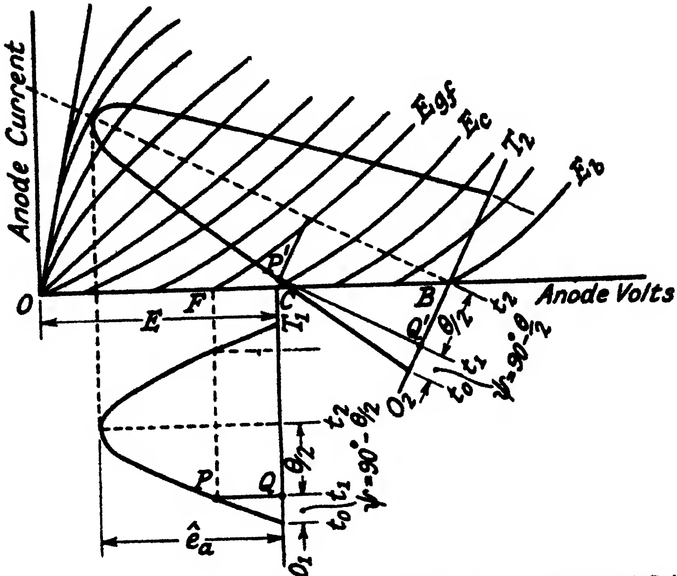


FIG. 1/X:25.—Determination of Angle of Current Flow in Terms of Grid Bias, Cut-off Bias, Peak Grid Volts, Peak Anode Volts, and μ of Valve as defined.

P and P' represent respectively the corresponding instantaneous values of grid drive and anode volts at the instant that anode current starts. The points corresponding to the instant that anode current stops are not shown, but are evidently symmetrically related to P and P': the current stops flowing at an instant located by a time interval before the end of each positive half-cycle of grid voltage which is equal to the time interval after the beginning of each positive half-cycle which locates the instant at which current starts to flow.

By inspection:

$$PQ = \hat{e}_a \sin\psi = \hat{e}_a \sin\left(90^\circ - \frac{\theta}{2}\right) = \hat{e}_a \cos\frac{\theta}{2} \quad . \quad . \quad (1)$$

$$P'Q' = \hat{e}_g \sin\psi = \hat{e}_g \sin\left(90^\circ - \frac{\theta}{2}\right) = \hat{e}_g \cos\frac{\theta}{2} \quad . \quad . \quad (2)$$

But $\quad PQ = OC - OF = E - E_{af} = \mu(E_{gf} - E_c)$

$$= \mu(P'Q' - \overline{E_c - E_b}) = \mu\left(\hat{e}_g \cos\frac{\theta}{2} - \overline{E_c - E_b}\right). \quad (3)$$

Hence

$$PQ = \hat{e}_a \cos\frac{\theta}{2} = \mu\left(\hat{e}_g \cos\frac{\theta}{2} - \overline{E_c - E_b}\right)$$

$$\therefore \hat{e}_g \cos\frac{\theta}{2} - \frac{1}{\mu}\hat{e}_a \cos\frac{\theta}{2} = E_c - E_b \quad . \quad . \quad (4)$$

Therefore

$$\theta = 2\cos^{-1}\frac{E_c - E_b}{\hat{e}_g - \frac{1}{\mu}\hat{e}_a} \tag{5}$$

Note that

$$E - E_{af} = \mu\left(\hat{e}_g \cos\frac{\theta}{2} - \overline{E_c - E_b}\right)$$

$$= \mu\left(\hat{e}_g\frac{E_c - E_b}{\hat{e}_g - \frac{1}{\mu}\hat{e}_a} - (E_c - E_b)\right)$$

$$= (E_c - E_b)\frac{\mu}{\frac{\mu\hat{e}_g}{\hat{e}_a} - 1} \tag{6}$$

= the instantaneous value of the anode voltage at the instant that anode current starts to flow (and stops flowing).

For approximate determinations of the angle of current flow the general value of μ (e.g. as given by the manufacturer, or as determined at the middle of the load line from the valve characteristics, see IX:6) for the valve may be used. For accurate determinations the value of $E - E_{af}$ should be found from equation (6), using the general value of μ. The true value of μ should then be determined from the field of valve characteristics in the region FC in Fig. 1. This region is evidently determined by making $OF = E - E_{af}$, while C is the point where the anode-current anode-voltage characteristic corresponding to the cut-off bias meets the axis of anode volts.

25.2. Effect of Changing the Angle of Current Flow, θ. As has been stated already, except when used as modulated amplifiers, class C amplifiers have no application to cases where linearity of the envelope is of importance, and in such cases, if power conservation is necessary, no choice of type is possible: a class B amplifier must be used and the angle of current flow must be 180°. A rare exception to this occurs in the case of low-power modulated transmitters in which the percentage modulation is less than 100% owing to some limitation in the modulator. In this case, by operating the following stages as class C amplifiers, the effective depth of modulation can be increased: such a procedure must of course be applied with caution, since it inherently leads to envelope non-linearity, because the angle of current flow varies with percentage modulation.

In the case of drive amplifiers of sufficient power handling capacity to make the efficiency of importance, an increase in efficiency can be obtained by biasing the valve back so that the angle of

current flow is less than 180°. The increase in efficiency to be obtained by so doing can never exceed 20–25%, and if there is any reduction in power output it will be less than this when the filament consumption is taken into account. The limit of efficiency follows from the fact that the efficiency is proportional to the current utilization u (see equation (1)/X:24), which has a value of 1·57 at $\theta = 180°$ and only rises to 2·0 at $\theta = 0$. Intermediate values of θ give intermediate values of u. The efficiency of a class C amplifier is always determined by equation (1)/X:24, but the effect of reducing the angle of current flow in any particular design of amplifier, in so far as it effects peak anode volts, peak anode current, and power output, depends on the way in which the change is made.

The effects of reducing the angle of current flow in three different ways will now be considered; the effects of increasing the angle of current flow in corresponding ways then follow by analogy. The same three cases are dealt with analytically in X:25.22 below.

Case I. Anode impedance unchanged and angle of current flow reduced by increasing the grid bias and then increasing the grid drive so that the maximum positive excursion of the grid remains unchanged.

In this case the effect is to reduce the anode peak volts, increase the anode peak current and reduce the power output. This may be seen either by reference to the analytical treatment in X:25.22 below or by drawing load lines on a valve field. The load line for the lower angle of current flow cuts the line of constant (maximum) positive excursion of the grid at a point which corresponds to a higher peak anode current and a lower negative going excursion of the anode and therefore corresponds to a lower value of anode peak volts. The reason for this is of course that the load line for the lower angle of current flow is steeper than the load line for the initial angle of current flow.

Case II. Anode impedance unchanged and angle of current flow reduced by increasing the grid bias and then increasing the grid drive so that the anode peak volts remain unchanged.

Since the anode peak volts are unchanged the power output is evidently unchanged. It is evident that the grid drive must be greater than in Case I so that the anode peak current is greater than in Case I. *It will be evident that this step could only be taken provided that by so doing the permissible grid dissipation and peak anode current are not exceeded.*

Case III. Angle of current flow reduced by increasing the grid bias and then increasing the grid drive so that the maximum positive excursion

of the grid remains unchanged. Anode circuit of the valve adjusted so that the load line remains unchanged.

If the load line remains unchanged and if the maximum positive excursion of the grid remains unchanged, the anode peak current and the anode peak volts are unchanged. Since, in order to maintain the same load line with a reduced angle of current flow, the impedance facing the anode of the valve must be increased, it is evident that the power output is reduced because the anode peak volts are unaltered. Incidentally the practical way of realizing such a change is evidently to make the changes in grid bias and grid drive and then to adjust the anode circuit until the anode peak volts are restored.

As a preliminary to understanding the behaviour of class C amplifiers from this point of view, it is useful to compare the performance of a class B amplifier with that of a class C amplifier ; this is done below.

25.21. Comparison of Performance of Class B and Class C Tuned Amplifiers. *Definition of Effective Grid Peak Volts in a*

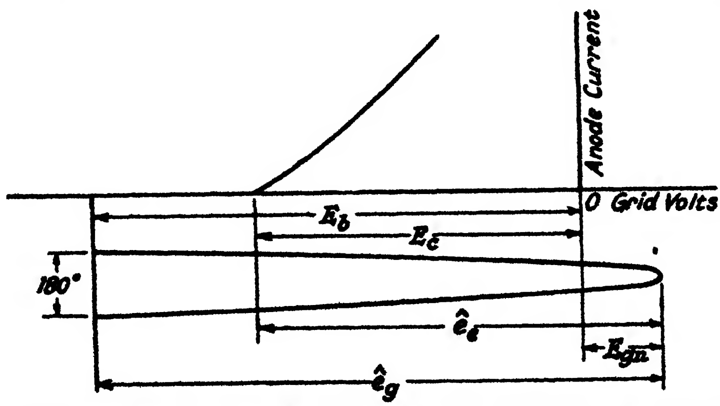


FIG. 2/X:25.—Definition of Effective Peak Grid Volts in Class C Amplifier.

Class C Amplifier. Fig. 2 shows part of the anode-current grid-voltage characteristic of a valve operated as a class C amplifier. In this figure :

E_c = the grid-bias voltage necessary to take the valve to cut-off.

E_b = the working value of grid bias, assumed equal to, or greater (more negative) than E_c.

$\hat{e}_g$ = the peak value of the R.F. voltage applied to the grid, the grid peak volts.

$\hat{e}_e$ = the effective grid peak volts, as defined below.

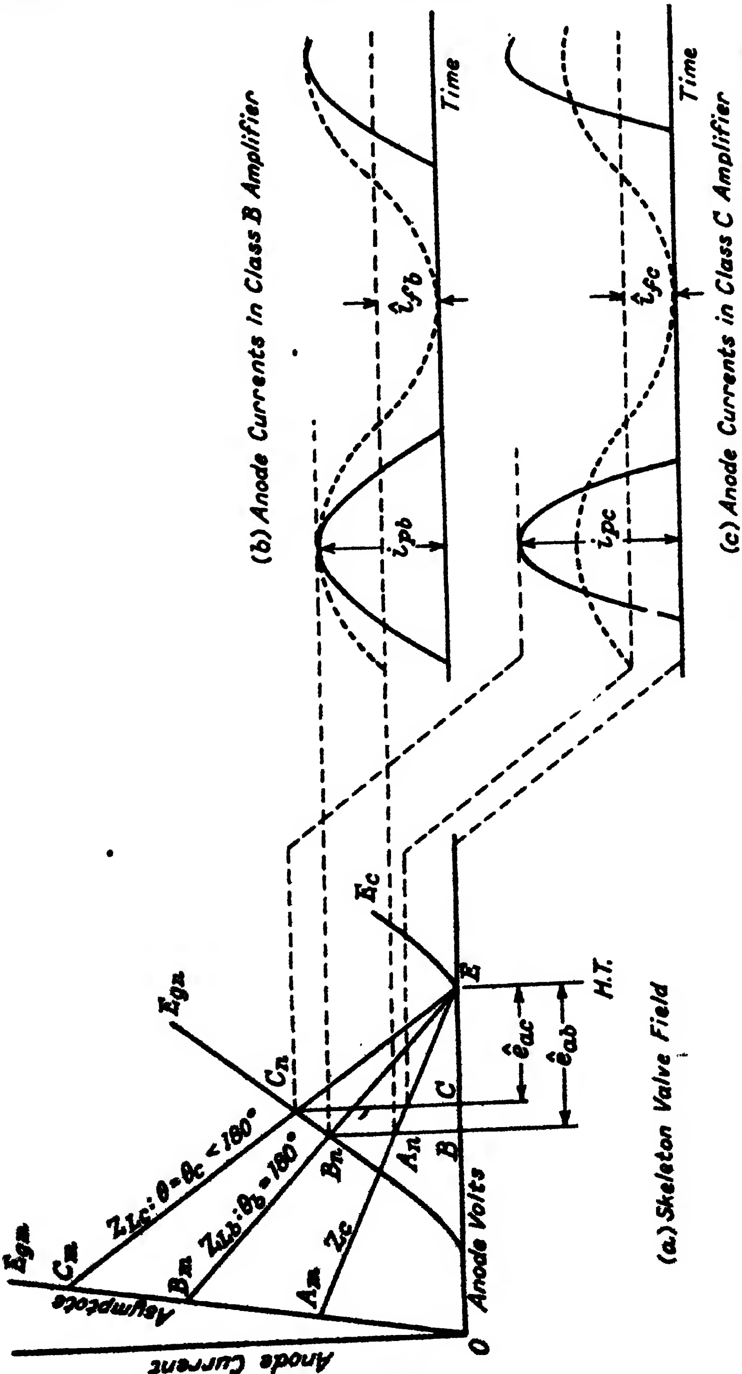


FIG. 3/X:25.—(a) Skeleton Valve Field. (b) Anode Currents in Class B Amplifier. (c) Anode Currents in Class C Amplifier.

E_{gn} = the maximum positive excursion of the grid, i.e. the maximum positive value of the grid-cathode voltage.

The effective grid peak volts = the amount of excursion of the grid in a positive direction above the cut-off bias

$$= \hat{e}_e = \hat{e}_g - (E_c - E_b)$$
$$= E_{gn} - E_c$$

Fig. 3 (*a*) shows the anode-voltage anode-current field of a valve with one anode-voltage anode-current characteristic. This characteristic corresponds to the value of grid voltage E_{gn}. The line marked asymptote should be neglected in this discussion.

The line EZ_c which looks like a load line, is not a load line, but is given an angle of slope the cotangent of which is equal to the impedance Z_c facing the anode circuit at the carrier frequency. The line EZ_{Lb} is the true load line corresponding to the case when the valve is biased as a class B amplifier and has an anode load Z_c. In this case, when the valve is driven so that its grid peaks up to the value E_{gn}, in other words when $\hat{e}_g - E_c = E_{gn}$, the peak current is determined by the point of intersection of the line E_{gn} and the line EZ_{Lb}, that is B_n. The peak current is then $i_{pb} = BB_n$ and the peak fundamental current is $i_{fb} = BA_n$, the peak anode volts being $\hat{e}_{ab}$.

Since (as can be seen by reference to Fig. 3 (*b*) or by looking up the value of f corresponding to $\theta = 180°$ on Fig. 1/X:22) $i_{fb} = \frac{1}{2}i_{pb}$, the slope of EZ_{Lb} is twice the slope of EZ_c.

It is important to note that the only point on the load line EZ_{Lb} which has been determined by this analysis is the point B_n which is reached at the same time as the peak of the grid volts and anode current. *Normal or nominal load lines in class B and C amplifiers therefore determine only the relation between peak anode volts, peak grid volts and peak anode current.* See X:27. This, however, is sufficient for nearly all practical cases which arise. The above argument is based on the good approximation that the impedance in the anode circuit is Z_c at the fundamental carrier frequency and zero at all other frequencies which are harmonics of the carrier frequency. The peak current i_{pb}, therefore, rises to such a value that the voltage drop $\hat{e}_{ab}$ through the load, due to the fundamental frequency component of current, reduces the instantaneous value of the anode voltage to the point B, the value required to give rise to the current i_{pb} with grid volts E_{gn} and anode volts equal to $E - \hat{e}_{ab}$. $\hat{e}_{ab}$ is then the value of the anode peak volts.

If without changing Z_c, the bias is increased to reduce the angle of current flow to some value θ_c less than 180°, the drive being

increased so that the peak volts again peak up to E_{gn} the ratio of peak current to peak fundamental $\frac{i_{pc}}{i_{fc}}$ is increased as compared with the ratio $\frac{i_{pb}}{i_{fb}}$ in the class B case, to some value greater than 2. In exact terms, $\frac{i_{pc}}{i_{fc}} = \frac{1}{f_c}$ where f_c is the value of f corresponding to θ_c as read from Fig. 1/X:22. The load line of the valve is therefore changed to some position such as EZ_{Lc} such that the ratio of the slope of EZ_{Lc} to that of $EZ_e = \frac{i_{pc}}{i_{fc}} = \frac{1}{f_c}$. The value of peak current $i_{pc} = CC_n$ adjusts itself as before so that the peak fundamental current flowing through the load drops the anode volts to the value which will give a current equal to i_{pc} with a bias E_{gn}, and anode volts equal to $E - \hat{e}_{ac}$, where $\hat{e}_{ac}$ = the anode peak volts.

The relation between $\hat{e}_{ac}$ and $\hat{e}_{ab}$ may be derived from the geometry of Fig. 3 (*a*) as follows:

Let Z_{Lb} and Z_{Lc} be apparent impedances corresponding to the slopes of EZ_{Lb} and EZ_{Lc} respectively, then

$$i_{pb} = BB_n = \frac{\hat{e}_{ab}}{Z_{Lb}} = \frac{\hat{e}_{ab}}{f_b Z_e} \tag{7}$$

$$i_{pc} = CC_n = \frac{\hat{e}_{ac}}{Z_{Lc}} = \frac{\hat{e}_{ac}}{f_c Z_e} \tag{8}$$

and

$$\frac{\hat{e}_{ab} - \hat{e}_{a}}{i_{pc} - i_{pb}} = R_n \tag{9}$$

where R_n is the value of anode impedance defined by the line E_{gn} between points B_n and C_n.

Substituting the above values of i_{pb} and i_{pc} from (7) and (8) into (9):

$$\hat{e}_{ac} = \frac{1 + \frac{R_n}{f_b Z_e}}{1 + \frac{R_n}{f_c Z_e}} \times \hat{e}_{ab} \quad . \quad . \quad . \tag{10}$$

Also from (7), (9) and (10):

$$\frac{i_{pc}}{i_{pb}} = \frac{\hat{e}_{ac}}{\hat{e}_{ab}} \times \frac{f_b Z_e}{f_c Z_e} = \frac{f_b Z_e + R_n}{f_c Z_e + R_n} \tag{11}$$

Relation between Effective Grid Peak Volts and Anode Peak Volts. Referring to Fig. 4, the slope R_e will be called the mean anode impedance at the point B or C.

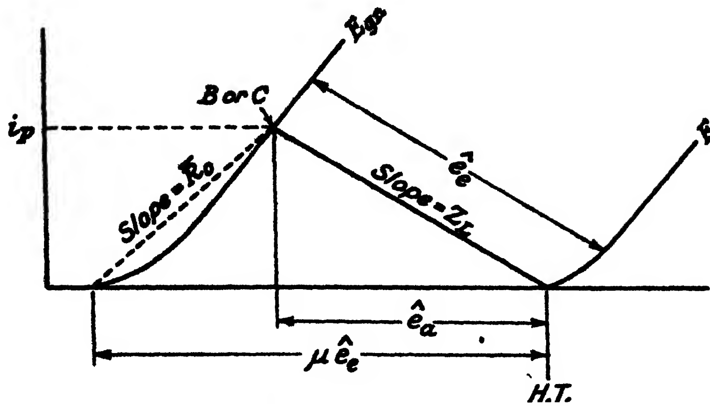


FIG. 4/X:25.—Relation between Effective Peak Grid Volts and Anode Peak Volts.

Then, since
$$i_p = \frac{\hat{e}_a}{Z_L} = \frac{\mu\hat{e}_e - \hat{e}_a}{R_0}$$

$$\hat{e}_a = \frac{Z_L\mu\hat{e}_e}{R_0+Z_L} \tag{12}$$

25.22. Detailed Examination of the Effect of Changing the Angle of Current Flow in a Class C Amplifier.

Conventions.

θ_1 = initial angle of current flow.

f_1 = value of f (see Fig. 1/X:22) corresponding to θ_1.

θ_2 = final angle of current flow.

f_2 = value of f corresponding to θ_2.

$\hat{e}_{a_1}$ and $\hat{e}_{a_2}$ = initial and final values of anode peak volts respectively.

i_{p1} and i_{p_2} initial and final values of anode peak current respectively.

$\hat{e}_g$ = grid peak volts.

R_n = slope of anode-voltage anode-current characteristic corresponding to grid voltage $E_{gn} = \hat{e}_g - E_b$.

Z_e = impedance presented towards anode of valve by output circuit.

E_b = the grid bias.

$\hat{e}_{e_1}$ and $\hat{e}_{e_2}$ = the initial and final values of effective grid peak volts respectively.

Case I. Z_e constant and angle of current flow changed by varying E_b and $\hat{e}_g$ so that the maximum positive excursion of the grid is kept constant.

In this case the effect of reducing the angle of current flow from θ_1 to θ_2 $(\theta_2 < \theta_1)$ is to reduce the peak anode volts and increase the peak anode current in the ratios defined by:

$$\hat{e}_{a_2} = \frac{1+\dfrac{R_n}{f_1 Z_c}}{1+\dfrac{R_n}{f_2 Z_c}}\hat{e}_{a_1} \text{ and } \hat{i}_{p_2} = \frac{f_1 Z_c + R_n}{f_2 Z_c + R_n}\hat{i}_{p_1} \qquad (13)$$

This follows directly from equations (10) and (11) and the considerations from which they were derived.

The power output at the carrier frequency is reduced in the ratio:

$$\frac{\hat{e}^2_{a_2}}{\hat{e}^2_{a_1}}.$$

Case II. Z_c constant and angle of current flow changed by varying E_b and $\hat{e}_g$ so that $\hat{e}_{a_1}$ is unchanged: i.e. $\hat{e}_{a_2} = \hat{e}_{a_1}$.

In this case the reduction of angle of current flow from θ_1 to $\theta_2(\theta_2 < \theta_1)$ must be carried out so that the effective grid peak volts are increased in the ratio determined by:

$$\hat{e}_{e_2} = \frac{1+\dfrac{R_n}{f_2 Z_c}}{1+\dfrac{R_n}{f_1 Z_c}}\hat{e}_{e_1} \qquad (14)$$

The peak anode current is then increased in the ratio defined by:

$$\hat{i}_{p_2} = \frac{f_1 Z_c + R_n}{f_2 Z_c + R_n} \times \frac{\hat{e}_{e_2}}{\hat{e}_{e_1}}\hat{i}_{p_1} = \frac{f_1}{f_2}\hat{i}_{p_1} \qquad (15)$$

The power output at the carrier frequency is evidently unchanged.

Case III. Z_c adjusted (as the angle of current flow is varied) so that the load line Z_L and the peak anode current $\hat{i}_p$ is kept constant (equal to $\hat{i}_{p_1}$).

It is assumed again that θ_1 is reduced to θ_2 $(\theta_2 < \theta_1)$.

The condition specified is that the value of Z_c shall always be changed in accordance with the equation $Z_c = \dfrac{Z_L}{f}$ (where f is read from Fig. 1/X:22 corresponding to each angle of current flow), so that the valve always operates along the load line Z_L. Z_L is therefore constant.

It is to be noted that once Z_L is chosen, the value of $\hat{e}_e$ for a given value of peak current is fixed independent of the angle of current flow. The value of $\hat{e}_e$ is therefore unchanged.

From equation 1/X:23 the power output is changed from

$\frac{1}{2}i_{p_1}^2 f_1 Z_L$ to $\frac{1}{2}i_{p_2}^2 f_2 Z_L$ (where $i_{p_2} = i_{p_1}$), that is, the power output is reduced in the ratio f_2/f_1. The value of Z_c is increased in the ratio f_1/f_2 and hence the peak anode volts are unchanged.

For clarity of enunciation and to enable the explanation to be easily related to Fig. 3/X:25 the above discussion has been conducted in terms of reducing the angle of current flow from θ_1 to θ_2 (i.e. $\theta_2 < \theta_1$). The formulae of course apply if $\theta_2 > \theta_1$, but the accompanying explanation must be correspondingly modified, reduction being replaced by increase, and vice versa.

25.3. Choice of Angle of Current Flow. The small increase in efficiency to be obtained by reducing the angle of current flow may be illustrated by taking an example where θ is reduced to the point where the power output is halved, as compared with the class B case. The most useful case to consider for this purpose is Case III, in which the load impedance is varied as the angle of current flow is changed, so that the valve still operates under the optimum conditions which it is reasonable to assume as an initial condition. Under these conditions, since the power output is proportional to the value of f, and since f is 0·5 for the class B case, to reduce the power output to half, the value of f must be reduced to 0·25 which corresponds to an angle of current flow of 71°, see Fig. 1/X:22. The angle of current flow must therefore be reduced to 71°.

With an angle of current flow of 71° the value of u is 1·925 compared with 1·57 in the case of a class B amplifier. Assuming a voltage utilization of 0·9 the resulting anode efficiencies are:

Class B: $\eta = \frac{1}{2}hu = \frac{1}{2} \times 1{\cdot}57 \times 0{\cdot}9 = 70{\cdot}6\%$

Class C: $\theta = 71°$: $\eta = \frac{1}{2}hu = \frac{1}{2} \times 1{\cdot}925 \times 0{\cdot}9 = 86\%$

This improvement in efficiency is appreciable, but from the point of view of total economy must be accepted with reservation, since the overall efficiency of the valve is not given by the anode efficiency, but by the ratio:

$$\frac{\text{Output power}}{\text{Power input to anode circuit plus Power dissipated in filament circuit}}$$

The overall efficiency therefore may be reduced by reducing the angle of current flow, since the power output is reduced and the filament power is not reduced. The steps in valve-power handling capacity are however of finite size, and no smaller valve may exist, which, working at an angle of current flow near to 180°, is capable of supplying the required power output. If this can be obtained from a particular available valve when working at a lower angle of current flow, a saving in the power input to the anode circuit results.

Having chosen the smallest valve which is capable of giving the required power output with any angle of current flow less than 180°, it is a truism to say that the angle of current flow which should be used is the lowest which provides the required power output.

The case considered above, where the angle of current flow is reduced so much as to halve the power output, is an extreme one, designed to show that even when the angle of current flow is reduced by an amount greater than ever occurs in practice the increase in efficiency is not large. A more practical case will now be considered in which the angle of current flow in a CAT14 valve is reduced from 180° to 140°, the power output at 180° angle of current flow being 100 kW.

$$\text{At } 180°,\ u = 1{\cdot}57 \text{ and } f = 0{\cdot}5$$
$$\text{At } 140°,\ u = 1{\cdot}72 \text{ and } f = 0{\cdot}435$$

On the basis of Case III, which is the practical case of most interest, the power output at 140° is $100 \times 0{\cdot}435/0{\cdot}5 = 87$ kW.

Assuming a voltage utilization of 0·9 the efficiencies are:

$$\text{At } 180°,\ \eta = 0{\cdot}5 \times 0{\cdot}9 \times 1{\cdot}57 = 70{\cdot}6\%$$
$$\text{At } 140°,\ \eta = 0{\cdot}5 \times 0{\cdot}9 \times 1{\cdot}72 = 77{\cdot}3\%$$

The power inputs to the anode circuit are therefore:

$$\text{At } 180°,\ P_{in} = 100/0{\cdot}706 = 142 \text{ kW}$$
$$\text{At } 140°,\ P_{in} = 87/0{\cdot}773 = 112{\cdot}5 \text{ kW}$$

Since the filament consumes 15 kW (460 amps. at 32·5 volts) the total power inputs are 157 and 127·5 kW and the overall efficiencies of the valve under the two conditions are:

$$\text{At } 180°,\ \eta' = 100/157 = 63{\cdot}7\%$$
$$\text{At } 140°,\ \eta' = 87/127{\cdot}5 = 68{\cdot}3\%$$

In this case there is still an increase in efficiency when the filament consumption is taken into account; but, what is more important, there is a power saving of 142 − 112·5 kW = 29·5 kW. Hence, if 87 kW power output is adequate, it is better to use the smallest angle of current flow which will give this output, rather than, for instance, reducing the power output by increasing the anode impedance, keeping the angle of current flow at 180°.

25.4. Determination of Grid Bias and Grid Drive to Produce a Required Angle of Current Flow. The normal procedure is to draw the load line and so to determine E_{gp}, the maximum positive excursion of the grid, and $\hat{e}_a$, the anode peak volts. The cut-off bias E_c with the value of H.T. used is then read off from the valve field and the value of μ with zero anode current

and an anode voltage swing from anode volts = H.T. to anode volts = 2/3 H.T. is determined.

Then
$$\hat{e}_g = E_{gp} - E_b \quad . \quad . \qquad (16)$$
and from (5)/X:25,
$$\theta = 2 \cos^{-1} \frac{E_c - E_b}{\hat{e}_g - \frac{1}{\mu}\hat{e}_a}$$
$$\therefore \; E_c - E_b = \hat{e}_g \cos\frac{\theta}{2} - \frac{\hat{e}_a}{\mu}\cos\frac{\theta}{2}$$
$$\therefore \; E_b = E_c \quad \left(E_{gp} - \frac{\hat{e}_c}{\mu}\right)\cos\frac{\theta}{2} + E_b \cos\frac{\theta}{2}$$
$$\therefore \; E_b = \frac{\left(E_{gp} - \frac{\hat{e}_a}{\mu}\right) \quad \theta}{1 - \cos\frac{\theta}{2}} \qquad (17)$$

All the quantities on the right of equation (17) are known and hence E_b can be found. The value of $\hat{e}_g$ is then given by equation (16).

26. Practical Design of a Class C Amplifier.

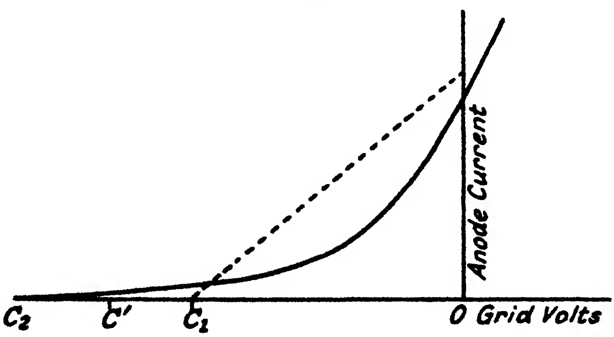


FIG. 1/X:26.—Anode-Current Grid-Voltage Characteristic.

The above treatment of class C amplifiers is to some extent an approximation, and is based on the assumption that the anode-current grid-voltage characteristic of a valve is of the form indicated, as represented, for instance, by the dotted line on Fig. 1 instead of by a curved line, such as the full line in the same figure. In the case of the dotted line there is no doubt as to where cut-off occurs and also the anode current is linearly proportional to grid volts. In the case of the full-line curve, while cut-off undoubtedly occurs

at the point C_2, the current rise is so small up to the point C_1 that if, for instance, the valve is biased back to C', it is evidently effectively operative as a class C amplifier, since no appreciable output will be obtained until the drive considerably exceeds the voltage corresponding to the distance between C' and C_1. In fact, for the purpose of determining the angle of current flow, some such point as C_1 should be considered to be the cut-off point of the curve characteristic. In practice this is assumed to be the case, but ***the final bias point which is used must always be determined by experiment.***

In the case of class C amplifiers other than modulated amplifiers, the use of a low angle of current flow may be regarded as a measure of economy to bridge the gap between available valve sizes. When the cost of valve replacements and filament power are taken into account it is usually cheaper to use a small valve at a high angle of current flow than a large valve at a low angle of current flow delivering the same power. In a straight class C amplifier the bias and grid drive are chosen to provide an angle of current flow which will supply the required power with the anode impedance corresponding to the chosen load line.

The first step therefore in the design of any class C amplifier is to draw the load line on the valve field, and to determine the limits of the excursion of anode volts and anode current. In a linear amplifier for modulated waves the load line must be drawn as far as possible in the linear region of the field, and the limits of excursion are determined by the linear region of the field traversed by the load line. A linear amplifier must also be operated with 180° angle of current flow.

In a class C amplifier linearity of the valve field is unimportant and the limits of excursion are determined by the grid drive available and the anode dissipation and the peak value of anode current permissible; see IX:19. These limitations also apply in the case of a class B amplifier, but whereas linear amplifiers may not be driven into anode limitation, it is usual to drive class C amplifiers into anode limitation to obtain a maximum efficiency (by making the voltage utilization, that is, the ratio of peak anode volts divided by H.T. volts, as high as possible) as well as a maximum power output. In all cases the load line is drawn with such a slope as to give the optimum compromise between the requirements of power output and efficiency. In the region of efficiencies which are generally considered as acceptable these requirements are mutually antagonistic.

In class B amplifiers it is usually a good enough approximation to draw the load line through the point on the valve field: anode

volts = H.T. volts, anode current = zero: and in class C amplifiers the load line should always be drawn through this point. In the case of a class B amplifier a slightly closer approximation to the load line may be made by making a guess at the standing feed; in the absence of other indications this may be taken at about a fifteenth of the peak anode current. In this case the load line may be drawn through the point: anode volts = H.T. volts, anode current = standing feed.

In the case of a class B amplifier, once the load line is drawn and the grid excursion is determined, the operating conditions of the valve are fixed. The peak current and anode peak volts are determined from the load line, and the circuit impedance facing the valve anode is equal to twice the impedance corresponding to the load line.

In the case of a class C amplifier, after drawing the load line, values of grid bias and grid drive are chosen (on the assumption of a compromise cut-off point for the valve) by applying the rigid theory above (i.e. equations (1)/X:22 to (14)/X:25), so that an angle of current flow results which will

(1) make the valve work along the load line drawn;
(2) with the chosen value of excursions of grid and anode peak volts and the chosen value of anode peak current, give a power output about 10% in excess of that required.

The valve must then be faced with an anode impedance determined by the slope of the load line and the value of f derived from the angle of current flow according to equation (5)/X:22.

The circuit is then set up and bias and drive of the calculated amounts are applied. Owing to the curvature of the valve characteristics, see Fig. 1, the form of the anode current will not be sinusoidal. The form of the anode current in the case of a class B amplifier, for instance, will be that shown by the dotted line in Fig. 2 instead of that shown by the full line.

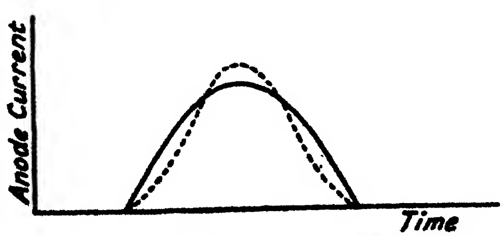


FIG. 2/X:26.—Anode-Current Wave Form.

As a consequence, the value of f, the ratio of peak fundamental to peak current, will be smaller than predicted by Fig. 1/X:22.

The result of this is that the peak current will be greater, and the peak anode volts and power output will be less than the predicted values. If, however, an adequate factor of safety has been provided by designing for a peak cathode current equal to 80% * of the total emission (and generally the load line will determine a value of peak anode current appreciably smaller than this), the increase in peak current can be neglected. If any increase in power output is required it can be obtained by increasing the angle of current flow, keeping the effective grid peak volts constant, and adjusting the value of impedance facing the anode of the valve so that the load line is unchanged. This corresponds to a change in the reverse sense to that considered in Case III and so provides an increase of power output without changing the peak anode current, except in so far as the curvature of the valve characteristics modifies the valve performance. Provided, however, that the original assumption of a cut-off point was reasonable, this effect is only of secondary importance. Finally, if after increasing the angle of current flow to 180°, the power output is still too small, the impedance facing the anode circuit may be reduced. This can only be done provided the increase in peak current does not cause it to exceed the total emission of the valve.

The method of initial adjustment of a class C amplifier when used as a modulated amplifier is given in XIII:4.2.

26.1. Grid Loading of Linear Class B Amplifiers. When a class B tuned amplifier is used for the amplification of a modulated wave, in order that the envelope form shall be preserved, it is necessary that the amplifier shall be linear in amplitude. It is therefore customary to mask the non-linearity of the grid circuit due to grid current by shunting the grid circuit with a resistance equal to about a fifth of the minimum instantaneous resistance to which the grid falls. This minimum resistance is determined by finding from the load-line diagram the corresponding maximum positive value of the grid and minimum positive value of the anode. These values are then entered on the grid current characteristics of the valve (e.g. see Fig. 1/X:11) to give the maximum instantaneous grid current. The ratio of the instantaneous peak values of grid volts and grid current gives the minimum resistance to which the grid circuit falls.

The grid circuits of untuned class B amplifiers must be loaded by the same amount when linearity is required.

* Or 90% of manufacturers' figure for permissible peak cathode emission.

26.2. Choice of Valve for Use in Class C Amplifiers. The valve used in a class C amplifier must

(*a*) Be capable of supplying the required power output with the H.T. volts available.

(*b*) Be capable of withstanding the corresponding grid and anode voltages at the operating frequency.

(*c*) Have sufficiently small interelectrode capacities.

(*d*) (In the case of linear class B amplifiers) Have such characteristics that a region exists through which a load line can be drawn affording a linear input-output amplitude characteristic.

The power output which the valve is capable of giving is determined finally from the peak anode current, the peak anode volts and the angle of current flow. See equation (2)/X:23. A preliminary determination of the power output should however be made, in terms of the permissible anode dissipation of the valve, assuming a probable angle of current flow. The way in which this is done is given below.

Conventions.

P = power output at radio or audio frequency.

P_D permissible anode dissipation of valve.

P_A anode input power $= P+P_D$.

η = anode efficiency $= P/P_A$.

$\hat{i}_p$ = permissible peak anode current = 0·8 × total emission of valve = 0·9 × peak cathode emission.

i_p = working value of peak anode current.

$f = \dfrac{\text{peak fundamental frequency current}}{\text{peak current}} = \frac{1}{2}$ for an angle of current flow of 180°.

Z_L = impedance corresponding to slope of load line.

E = H.T. volts available.

$\hat{e}_a$ = anode peak volts, assumed to be $0{\cdot}9E$.

h = voltage utilization.

Class C Amplifier. From equation (1)/X:24

$$\eta = \tfrac{1}{2}\frac{\hat{e}_a}{E}\cdot\frac{\hat{i}_f}{\hat{i}} = \tfrac{1}{2}hu$$

where $u = \dfrac{\hat{i}_f}{\hat{i}}$ and is determined from the angle of current flow by reference to Fig. 1/X:22, and $h = \dfrac{\hat{e}_a}{E}$ assumed to be 0·9.

Therefore $\eta = 0{\cdot}45u$. But $\eta = \dfrac{P}{P_A} = \dfrac{P}{P+P_D} = 0{\cdot}45u$.

Therefore $P = 0{\cdot}45u(P+P_D) = \dfrac{0{\cdot}45uP_D}{1 - 0{\cdot}45u}$ . . . . (1)

Class B Amplifier. When $\theta = 180°$, $u = 1{\cdot}57$ and $0{\cdot}45u = 0{\cdot}707$.

Therefore $$P = \frac{0{\cdot}707P_D}{0{\cdot}293} = 2{\cdot}41P_D \quad . \quad . \quad . \quad (2)$$

This equation applies whether the amplifier is tuned or untuned, provided it is driven so that $\hat{e}_a/E = 0{\cdot}9$.

Linear Class B Amplifier for Modulated Waves. The unmodulated carrier anode voltage is only half that at 100% peak modulation so that $\eta = 0{\cdot}785 \times 0{\cdot}45 = 0{\cdot}353$, say.

$$P = 0{\cdot}353(P+P_D) = \frac{0{\cdot}353P_D}{1 - 0{\cdot}353}$$

$$= 0{\cdot}546P_D \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Class B Push-Pull Audio-Frequency Amplifier. The power P must be taken as that corresponding to 100% sinusoidal modulation, when the amplifier constitutes part of a transmitter, e.g. a modulator, and in other cases that corresponding to a sinusoidal anode voltage equal to the highest value which will occur.

The efficiency $\eta = 0{\cdot}785h$, where $h = \dfrac{\hat{e}_a}{E}$

Therefore $$\frac{P}{P+P_D} = 0{\cdot}785h$$

Therefore $$P = \frac{0{\cdot}785hP_D}{1 - 0{\cdot}785h} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In this case the value of h varies widely in different cases and must be inserted in equation (4) as determined (by considerations of linearity) for each particular case.

In the case of a linear class B amplifier for modulated waves, and also for a modulated amplifier, the value of P to be entered in equation (3) is the unmodulated carrier power supplied by the valve.

It is then necessary to verify that the peak current permits an adequate power output to be obtained. In the case of a straight class C amplifier there is no ambiguity about what is meant by the power output required. A linear class B amplifier must be capable of delivering four times carrier power without non-linearity and with normal H.T. A modulated amplifier must be designed to give four times carrier power with twice normal H.T. The load line should therefore be drawn through a point on the axis of anode volts corresponding to twice normal H.T. These verifications are

constituted by normal design procedure which involves the drawing of load lines so that the peak cathode current does not exceed 0·8 of the total emission. This is done, by inspection, so that a reasonable compromise is effected between the requirement that the product i_pZ_L should be large, and the requirement that the efficiency as determined by $\frac{\hat{e}_a}{E}$ should be high. In general, in a class C amplifier and in a modulated amplifier, $\frac{\hat{e}_a}{E}$ should not be less than 0·9, and not less than 0·45 in a linear class B amplifier, provided linearity can be obtained in this condition.

When, by drawing load lines, the values of i_p and Z_L have been chosen, the power output of a class C amplifier is given by

$$P = \tfrac{1}{2}fi_p^2Z_L \qquad . \quad . \quad . \quad . \quad (5)$$

In the case of a *linear class B amplifier* for modulated waves, $f = \frac{1}{8}$, so that the carrier power is given by

$$P = \tfrac{1}{4}P_p = \frac{i_p^2Z_L}{16} \qquad . \quad . \quad . \quad . \quad (6)$$

In the case of a *modulated amplifier* a fair approximation to the carrier power may be obtained by putting $f = 0{\cdot}5$ (corresponding to an angle of current flow of 180°), see XIII:4, in which case the carrier power is also given by

$$P = \frac{i_p^2Z_L}{16} \qquad . \quad . \quad . \quad . \quad (7)$$

In the case of a class C amplifier, the value of f may be determined from the equation

$$f = \frac{2P}{i_p^2Z_L} \qquad . \quad . \quad . \quad . \quad (8)$$

where P is now the required power output. If f comes out larger than $\frac{1}{4}$, the valve is incapable of giving the required power output. If f is lower than $\frac{1}{4}$, attempts should be made to find a valve with a lower filament consumption to provide the power. Strictly, the annual costs of each valve should be determined, taking into account the cost of power consumed in anode and filament circuits, and the cost of valve replacements. Other things being equal, the valve with the lower annual charges should then be used.

Class B Audio-Frequency Amplifier (Untuned). In this case the power *per pair of valves is*

$$2P = \tfrac{1}{2}i_p^2Z_L \qquad . \quad . \quad . \quad . \quad (9)$$

where i_p is the peak current in one valve.

It is evidently rather pedantic to talk about the power output from one valve when each valve substantially amplifies one-half of the wave only.

It will be evident that in selecting any valve for use in a class B or class C amplifier preference should be given to valves having an asymptote (of the anode-voltage anode-current characteristics, see Fig. 1/X:27) with a steep slope, since such a valve enables a high power output to be obtained with good efficiency.

In default of valves with rectilinear anode-current grid-voltage characteristics, valves for use in push-pull amplifiers should be chosen to have anode-current grid-voltage characteristics which follow a square law: the anode current should be proportional to the square of the grid voltage. This affords cancellation of second harmonics.

26.3. Determination of Working Conditions of a Class C Amplifier which is not driven into Anode Limitation. This refers to the practical case where it is required to examine a class C amplifier which is set up and working and where normal indications only are available. These are assumed to be provided by the measurements of anode peak volts on a peak voltmeter, the driven feed $\bar{i}$ and standing feed on the anode feed meter, and the H.T. volts and grid bias, and by the field of anode-current anode-voltage characteristics for the valves used in the stage under examination. Class C here embraces class B.

The important quantities to be determined are given below in the order in which they are found.

(*a*) *The anode load impedance* Z_c, which, when divided into the anode peak volts, gives i_f, the peak value of the fundamental frequency component of anode current. The anode load impedance is determined as given in X:26.31, below.

(*b*) *The value of* $u = \dfrac{i_f}{\bar{i}}$.

(*c*) *The angle of current flow* θ, obtained by entering the value of u in Fig. 1/X:22.

(*d*) *The value of* $f = \dfrac{i_f}{i_p}$, obtained by entering the angle of current flow θ in Fig. 1/X:22.

(*e*) *The anode peak current* i_p, which with the anode peak volts determines the load line.

The anode peak current is given by $i_p = \dfrac{i_f}{f}$.

(*f*) *The load line.* This is drawn through the points on the valve field: anode volts = H.T. volts, anode current = 0, and anode volts = H.T. minus anode peak volts, anode current = anode peak current.

(*g*) E_{gp}, the maximum positive excursion of the grid. This is found from the valve field as the value of grid volts corresponding to the anode-current anode-voltage characteristic drawn through the point already plotted: anode volts = H.T. minus anode peak volts, anode current = anode peak current.

(*h*) E_c = the cut-off grid bias. This may be read from the valve field or may be determined experimentally by finding the bias required to reduce the anode current to about a fifteenth of the peak anode current with the value of H.T. used.

(*j*) the peak grid drive. This is given by $\hat{e}_g = E_{gp} - E_b$, where E_b is the steady value of grid bias, e.g. if $E_{gp} = +1{,}200$ and $E_b = -300$, then $\hat{e}_g = 1{,}500$ volts.

A general check on the accuracy of these determinations may then be applied by calculating the angle of current flow from equation (5)/X:25 and seeing how nearly it agrees with the value determined above.

26.31. Determination of Anode Load Impedance. Assuming that no R.F. bridge is available, the valve itself may be used to measure the anode load impedance. For this purpose the valve should be biased to cut-off with the value of H.T. used, i.e. so that the anode current is reduced to about a fifteenth of the estimated value of peak anode current. The valve is therefore biased as a class B amplifier. The valve should then be driven with a value of grid drive equal to about half that at which anode limitation occurs. This value is not critical but is chosen to use as much of the linear part of the load line as possible without running into excessive anode dissipation. (In the case of a linear class B amplifier for modulated wave, half the peak anode current may represent the maximum possible peak current with steady drive.) The peak current should be estimated as being equal to π times the driven feed.

The anode peak volts and the driven feed are then observed, when the impedance facing the anode is given by the anode peak volts divided by 1·57 times the driven feed.

The circuit is then restored to its operating condition and the quantities determining the performance of the amplifier are found in the order given in paragraphs (*a*), (*b*) and (*c*), etc., above.

It will, of course, be evident that if the amplifier is a linear class B amplifier for amplifying modulated waves, the quantities characterizing its performance will normally be determined by the above procedure for the unmodulated carrier condition. The anode peak volts and peak current at 100% peak positive modulation will then be twice the values found as above.

Two sources of inaccuracy occur in the above measurements. The first is due to the fact that the valve characteristics supplied for a typical valve will not represent exactly the performance of any particular valve. The second is due to the unavoidable approximation involved in determining the cut-off bias, and also due to the non-linearity of the valve at low values of anode current. It will be appreciated, however, that inaccuracy due to these causes is fundamental, and unavoidable under normal practical conditions of working. Such inaccuracy has been inherent in all methods of lining up amplifiers used in the past, and it will be found that it does not constitute a bar to the successful practical adjustment and operation of class B and C amplifiers.

It is evident that the principles above may be applied at any time to provide the necessary information to enable a class B or class C amplifier to be set up to any required operating condition.

27. Anode Limitation.

Fig. 1 shows a field of anode-current anode-voltage characteristics for a 4030C valve. It will be noted that on the left of the field the characteristics are all asymptotic to a line *OA* marked " asymptote ". Suppose the valve is supplied with an H.T. voltage E and a grid bias E_b, is driven with a grid swing peaking up to E_{gp1} and is faced with such an impedance in its anode circuit that EP is the effective load line determining the relation between the peak volts (grid and anode) and peak current. Evidently the operating point determining the peak anode current and minimum anode volts is P.

Under these conditions the peak current is AP and the peak anode volts are equal to EA. The point P is a real point on the load line since it represents the instantaneous value of the anode current and the anode volts at the instant that the grid voltage reaches its peak value.

It is *approximately* true that, if the grid is driven more positive, by increasing the value of the grid drive so that the grid voltage peaks up to some value such as E_{gp2}, the peak anode volts do not increase but remain constant at a value equal to EA. Under this condition the valve is said to be driven into anode limitation. It

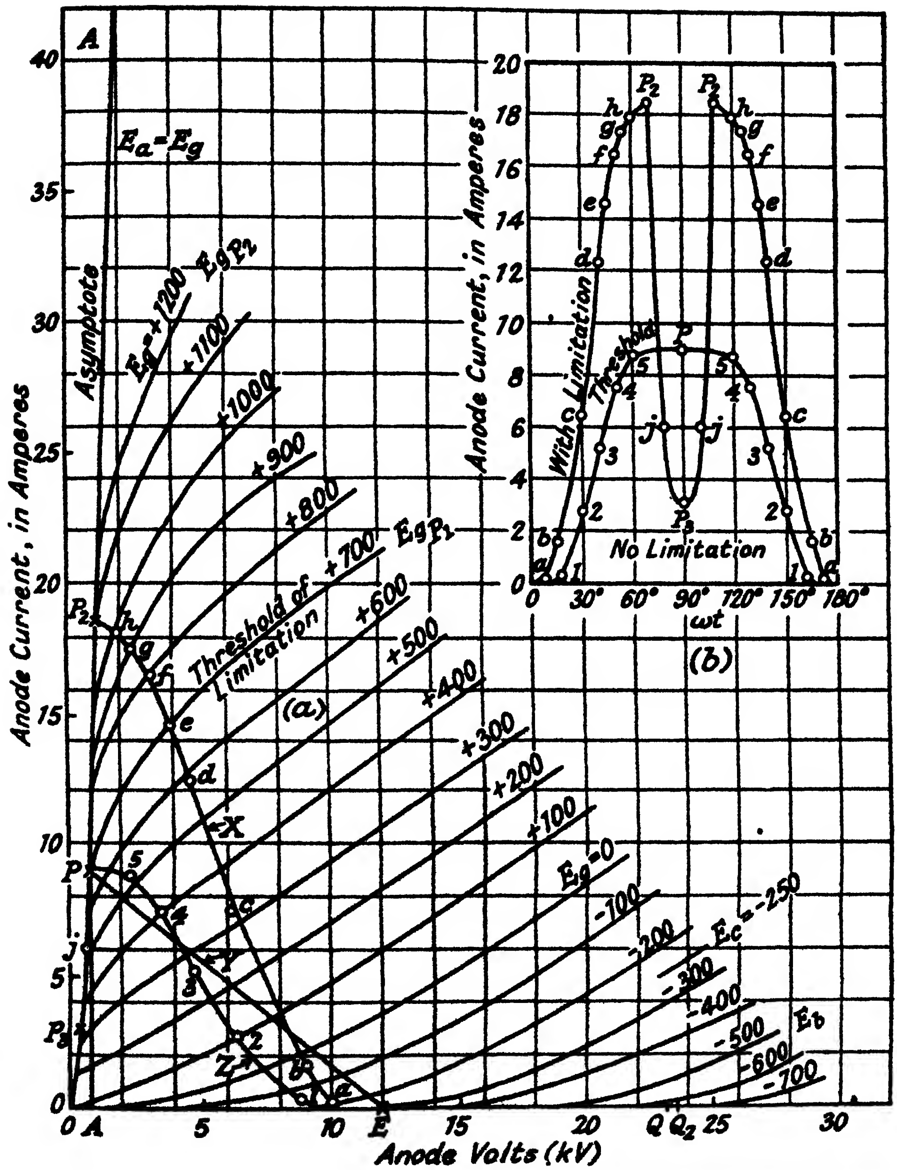


FIG. 1/X:27.—(*a*) 4030C Valve Field with True Load Lines.
(*b*) Corresponding Anode-Current Wave Forms.

X = Approximate True Load Line with Anode Limitation.
Y = Normal Load Line at Threshold of Limitation.
Z = True Load Line at Threshold.

(By courtesy of Messrs. Standard Telephones & Cables.)

is, however, not even approximately true to say that the peak current remains constant at AP.

For nearly all practical purposes it is useful to assume that, when a valve is driven into anode limitation, the peak volts remain constant at the value at which anode limitation begins, e.g. in Fig. 1 at the value EA. The error consequent on this assumption is normally negligible.

The peak value of the anode current may be found by plotting the *true load line*. This is a plot, at every instant of the drive cycle, of the relation between the instantaneous value of the anode volts and the anode current, or of the anode volts and grid volts. The true load line can only be found by a series of successive approximations, of which the first gives the peak anode current with a sufficient degree of approximation for all practical purposes. The description of how this series of approximations might be made shows how the anode volts are limited to a value substantially equal to the value represented by EA in Fig. 1. *In practice such a series of approximations would not be made as it is too laborious and does not give precision results, for reasons given below.*

The method of approximation can best be demonstrated by means of an example.

Valve at Threshold of Limitation. Suppose that the valve represented by Fig. 1 is operated under the following conditions.

The impedance facing the anode $= Z_e = 2{,}765\ \Omega$

H.T. volts $= E = 12{,}000$

Grid drive $= \hat{e}_g = 1{,}200$

Grid bias $= E_b = -500$

Hence, from Fig. 1 $E_c = -250$, also $\mu = 36$

Max. positive grid excursion $= E_{gp} = \hat{e}_g + E_b = +700$

The angle of current flow

$$= \theta = 2\cos^{-1}\frac{E_c - E_b}{\hat{e}_g - \dfrac{\hat{e}_a}{\mu}} = 2\cos^{-1}\frac{250}{1{,}200 - \dfrac{\hat{e}_a}{36}}$$

Then to find the point P representing the upper end of the load line, assume any value of $\hat{e}_a$, calculate the value of θ, find f from Fig. 1/X:22. Then plot $E - \hat{e}_a$ on valve field and erect vertical to meet the characteristic marked $E_{gp} = +700$ and so find a trial value of i_p the peak current. The value of $\hat{i}_f = \hat{e}_a/Z_e$. Now determine another value of f by dividing $\hat{i}_f$ by i_p. Repeat with a new assumed value of $\hat{e}_a$ until the two values of f coincide. The inter-

section of the specified vertical with the characteristic marked $E_{gp} = +700$ then defines the point P.

It will be found that, if this process is carried out with the conditions above, the point P is at the point shown in Fig. 1. Also $\theta = 147°$, $f = 0{\cdot}45$, $i_f = 4{\cdot}05$, $i_p = 9{\cdot}0$, $\hat{e}_a = 11{,}200$.

The above procedure is important because it represents the inverse of normal procedure. The normal procedure is to draw a load line and then to find the load impedance and other conditions which will ensure that this load line is used. The present procedure is to choose the load impedance and other conditions and then find what load line is being used.

So far the example has only shown how to draw the nominal load line for the case where anode limitation does not occur, although the operating conditions have been chosen (by the writer) so that the threshold of anode limitation has been reached. As a preliminary to drawing the true load line for the case of anode limitation, the load line for the present case will be drawn. This is done very simply as follows:

The instantaneous grid voltage is

$$e_g = E_b + \hat{e}_g \sin \omega t = -500 + 1{,}200 \sin \omega t$$

The instantaneous anode voltage is

$$E - e_a = E - \hat{e}_a \sin \omega t = E - EA \sin \omega t = 12{,}000 - 11{,}200 \sin \omega t$$

Now give ωt any convenient values covering the range from zero to 360° and calculate the corresponding values of grid voltage and anode voltage. These values when plotted on Fig. 1 give the true load line. The calculation is tabulated on the next page and the results are plotted on Fig. 1.

It is evident that the load line during the first half of each positive half-cycle is the same as that during the second half. A similar symmetry exists during each negative half-cycle. Such symmetry always occurs when the load is non-reactive. Hence, for non-reactive loads it is only necessary to plot the curve from $-90°$ ($= 270°$) to $+90°$.

The values of e_g and e_a from columns 4 and 6 respectively are plotted against one another in Fig. 1 and appear as the curve $QE12345P$ which constitutes the true load line. The values of instantaneous anode current at each of these points, as read from Fig. 1, are tabulated in column 8 and plotted against ωt in Fig. 1 (*b*), which then shows the form of the anode-current wave.

To the extent that the departure of this wave from a sinusoid changes the value of f from the value given in Fig. 1/X:22, a funda-

1	2	3	4	5	6	7	8
ωt	sin ωt	1,200 × sin ωt	e_g	11,200 × sin ωt	e_a	Point Plotted	i_a
0	0	0	− 500	0	12,000	*E*	0
16° 30′	0·284	340	− 160	3,180	8,820	1	0·2
30°	0·5	600	+100	5,600	6,400	2	2·8
40°	0·643	772	+272	7,200	4,800	3	5·2
50°	0·766	918	+418	8,570	3,430	4	7·5
60°	0·866	1,040	+540	9,700	2,300	5	8·8
90°	1·0	1,200	+700	11,200	800	*P*	9·0
120°	0·866	1,040	+540	9,700	2,300		8·8
150°	0·5	600	+100	5,600	6,400		2·8
160° 30′	0·284	340	− 160	3,180	8,820		0·2
180°	0	0	− 500	0	12,000		0
210°	− 0·5	− 600	− 1,100	− 5,600	17,600		0
240°	− 0·866	− 1,040	− 1,540	− 9,700	21,700		0
270°	− 1·0	− 1,200	− 1,700	− 11,200	25,200	*Q*	0
300°	− 0·866	− 1,040	− 1,540	− 9,700	21,700		0
330°	− 0·5	− 600	− 1,100	− 5,600	17,600		0
360°	0	0	0	0	0		0

mental error is revealed in the standard method of adjusting class C amplifiers described in this chapter. However, since distortion at low-current amplitudes increases the value of f, while the distortion at high-current amplitudes below the threshold of anode limitation decreases the value of f, the errors subtract and in practice can be ignored.

Anode Limitation Case. This is the case where the valve, instead of being driven only to the threshold of anode limitation as above, is driven well into limitation.

Suppose that, with other conditions unchanged, the peak grid volts are now increased to 1,700, so that the maximum positive excursion of the grid is now

$$E_{gp_s} = +1{,}200 \text{ volts.}$$

First Approximation. Assume a value for the peak anode volts very slightly larger than the value previously assumed. To keep the number of approximations as low as possible this value should be such as to make the minimum positive value of the anode about half of the minimum positive value of the anode at the threshold of anode limitation: that is with a grid drive reaching the threshold of anode limitation. In an extreme case the anode may, however, swing negative. At the threshold of anode limitation (the case just considered) the anode peaks down to +800 volts. It will be assumed

that the anode peaks down to +400 volts, so that the anode peak volts are:

$$\hat{e}_a = 12{,}000 - 400 = 11{,}600 \text{ volts.}$$

The instantaneous grid voltage is then:

$$e_g = -500 + 1{,}700 \sin \omega t$$

The instantaneous anode voltage is:

$$\hat{e}_a = 12{,}000 - 11{,}600 \sin \omega t$$

The values of e_g and e_a are tabulated below for different values of ωt.

1	2	3	4	5	6	7	8
ωt	sin ω	1,700 × sin ωt	e_g	11,600 × sin ωt	e_a	Point Plotted	i_a
0	0	0	− 500	0	12,000	*E*	0
10	0·174	296	− 204	2,020	9,980	*a*	0·2
15	0·259	440	− 60	3,000	9,000	*b*	1·6
30	0·5	850	+350	5,800	6,200	*c*	6·4
40	0·643	1,092	+592	7,470	4,530	*d*	12·3
45	0·707	1,200	+700	8,200	3,800	*e*	14·6
50	0·766	1,300	+800	8,900	3,100	*f*	16·5
55	0·819	1,390	+890	9,480	2,520	*g*	17·5
60	0·866	1,470	+970	10,050	1,950	*h*	18·1
70	0·940	1,600	+1,100	10,900	1,100	P_2	18·5
80	0·985	1,675	+1,175	11,400	600	*j*	6·0
90	1·0	1,700	+1,200	11,600	400	P_3	3.0

The values of e_g and e_a from columns 4 and 6 respectively are plotted against one another in Fig. 1 and appear as the curve $Q_2EabcdefghP_2jP_3$, which constitutes the first approximation to the load line.

The values of the instantaneous anode current at each plotted point, as read from Fig. 1, are tabulated in column 8 and plotted against ωt in Fig. 1 (*b*), which then gives the first approximation to the anode-current wave.

Second Approximation. The current wave in Fig. 1 (*b*) in conjunction with the following half-period during which no anode current flows, is then analysed by one of the graphical methods given in CVII:2 and the peak value of the fundamental frequency component of current ($\hat{i}_f$) is so found. A new value of anode peak volts is then found by multiplying $\hat{i}_f$ by Z_e. A new assumed value of anode peak volts must then be taken intermediate between the previously assumed value and the value just calculated. The second approxi-

mation then follows the same course as the first, as do any further approximations which may be necessary.

Very little consideration is required to see that the anode peak volts can never exceed the threshold of limitation anode peak volts by any appreciable amount. For instance, under the conditions assumed, the peak volts can never exceed EA by more than a small percentage. Any increase in the peak volts above the threshold value demands a current wave with a larger dip in the middle which tends to reduce i_f. Hence, when the grid drive is increased above the threshold of limitation value, i_f is increased by the increase in the value of peak current and reduced by the increase in the width of the dip. The net result is the limiting effect observed.

In practice the limitation is even sharper than is indicated by the above analysis because, for the present purpose, the performance of the valve is not truly represented by the asymptote, see X:27.1 below.

In practice, when the grid is more positive than the anode the anode current falls faster than is indicated by the asymptote, so that the dip is deeper than is obtained by the above analysis.

It should therefore be stressed again that, although this analysis has been conducted on a quantitative basis, it is of little value for highly precise determinations of peak anode volts, although it can be used to give a close approximation to the peak anode current.

Conclusions. A class C amplifier which is to be driven into anode limitation may be designed by standard methods to deliver a required value of anode peak volts to a given circuit impedance at the threshold of limitation.

As the grid drive is increased the anode peak volts and the output power will remain substantially constant at the value obtained at the threshold of limitation.

As the grid drive is increased, the anode peak current increases and may increase to a value considerably greater than the value at the threshold of limitation.

A safe estimate of the peak anode current may be obtained by applying the first approximation exactly as above, *using the value of anode peak volts at the threshold of limitation.* This will give a value of peak anode current slightly larger than that obtained in practice. For simple rule, see XIII:4.

These findings have an immediate application to the case of class C modulated amplifiers where it is customary to assess the peak current at 100% peak positive modulation by doubling the peak current in the carrier condition as estimated by standard

methods, which in effect determine only the peak current at the threshold of limitation. If the modulated amplifier is driven so hard that at 100% p.p.m. it is still driven into anode limitation, the peak anode current at 100% p.p.m. may be larger than the value so assessed.

27.1. Asymptote. The line which has been referred to as the "asymptote" is the curve plotted between anode current and anode voltage with the grid at the same potential as the anode. An enlarged view of the valve field with all the anode-current anode-voltage characteristics completed down to zero anode volts and zero anode current would, in most (if not all) cases show the characteristics approaching very close to the asymptote, but never quite reaching it until the origin.

The practice of regarding this anode-volts anode-current characteristic as the asymptote of the valve characteristics therefore constitutes an engineering approximation which is justified by its usefulness.

The further assumption that this "asymptote" is a straight line, which is used in describing the operation of modulated amplifiers in XIII:4, is a further engineering approximation. The fact that this is only an approximation may account for a small part of the improvement in linearity obtained in modulated amplifiers by the use of grid-current bias. See XIII:2.

28. Variation of Anode Current as Anode Circuit is Tuned.

By a dispensation of providence, when the anode tuning condenser of a class C amplifier is varied, the impedance presented towards the valve anode reaches maximum, and a reactanceless condition, at the same value of tuning condenser. When the tuning condenser is at the position giving maximum impedance the valve is therefore faced with a pure resistance load (which is evidently very desirable) and the circuit is said to be in tune. See VI:2.2.

The peak value of current flowing through the anode circuit always reaches a value, as read from the valve characteristic, corresponding to the peak grid volts and the instantaneous value of anode volts, at the instant when the grid voltage reaches its peak value. The instantaneous value of anode volts at this time is equal to the H.T. volts minus the peak voltage swing across the load. The peak voltage swing across the load is equal to the peak fundamental current multiplied by the load impedance. In other words, as the anode load increases with constant grid drive, the voltage drop across it increases, so that the peak current drops, and hence the mean

anode feed drops. The anode feed is therefore a minimum (and the peak anode voltage a maximum) when the anode impedance is a maximum at the carrier frequency, that is, when the circuit is in tune.

It is customary to use the condition of minimum anode current as a criterion of a circuit being in tune : to tune a circuit, the anode tuning condenser is adjusted until the anode feed is a minimum.

29. Variation of Grid Current as Anode Circuit is Tuned.

Assuming a perfectly neutralized amplifier with no feedback from any part of the anode circuit to the grid circuit, as the anode-circuit tuning is varied the grid current varies, reaching a maximum when the anode circuit is in tune.

The reason for this is that the *mean* effective anode voltage, *averaged only over the periods when the grid is positive*, is a minimum when the anode circuit is in tune, so that the anode robs the grid of less electrons, and the grid current is therefore a maximum.

Evidently, as the anode circuit impedance is varied by tuning, the anode peak volts vary and the mean anode voltage is unchanged. Since, however, during the periods when the grid swings negative the anode swings positive, and no grid current flows when the grid swings negative, the increase of anode voltage during these periods does not affect the grid current. The decreases of anode voltage during the periods of positive grid swing do, however, contribute to increase of grid current when the impedance facing the anode is increased; and so the anode negative swings are increased.

(This effect, which applies to the tuning of a stage and the grid current of the *same* stage, is not to be confused with the fact that the grid current on any stage rises to a maximum as the previous stage is brought into tune.)

30. Working Conditions of Class B Tuned Amplifier taking into account No-drive Standing Feed.

• All the above formulae have neglected the standing feed, and have assumed valves to be biased to cut-off, in addition to being linear down to cut-off. In practice, a valve is never linear down to cut-off, so, academically speaking, these assumptions are unwarranted. The practical solution is to obtain substantial linearity by biasing the valve so that a standing feed exists in the absence of drive.

A more exact solution is given below, taking into account the standing feed. In the example of a power amplifier for modulated

waves considered later, it is shown, however, that the errors introduced by the two assumptions above are not large, and the conventional or practical method of direct application of the equations already derived has the merit of simplicity, and generally gives results which are quite accurate enough. In certain cases, however, it is useful to be able to obtain good correlation between measurements made in different parts of the circuit in different ways, and for this purpose the treatment below is applicable.

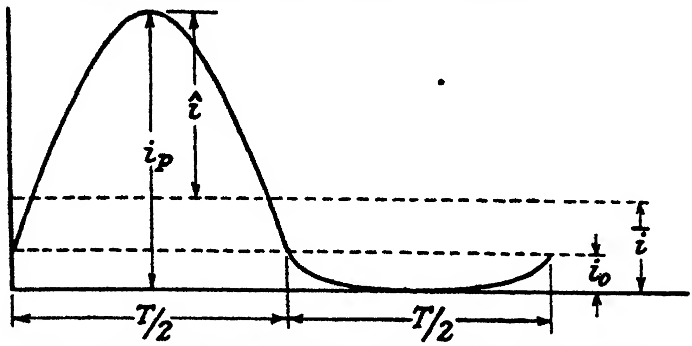


FIG. 1/X:30.—Anode Current during One Cycle of Carrier.

Fig. 1 shows the anode current of a valve during one cycle of the drive frequency applied to its grid. In this figure

i_p = the peak anode current.
i_0 = the standing feed in the absence of drive.
$\bar{i}$ = the driven feed = the mean anode current with drive applied to the grid.
$\hat{i} = i_p - i_0$, so that the instantaneous value of anode current in the positive half-cycle is $i = i_0 + \hat{i} \sin \omega t$.
i_f = the peak value of the fundamental current, i.e. of the carrier frequency component of current flowing in the anode circuit.
E = the H.T. volts.
T = the duration in seconds of one cycle of the carrier.

The mean value of current in the positive half-cycle is

$$\bar{i}_1 = i_0 + \frac{2}{\pi}\hat{i}$$

The mean value of current in the negative half-cycle is

$$\bar{i}_2 = i_0 + \bar{i}_n$$

where $\bar{i}_n$ = the mean value of negative current due to the applied drive.

If the wave form were sinusoidal in the second half-cycle, then

i_n would be equal to $-\frac{2}{\pi}i_0$, and if the wave form were of square form, then i_n would equal $-i_0$. In practice, the wave form lies somewhere between these two forms, and a good approximation is to make

$$i_n = -\frac{5}{6}i_0 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Therefore $$\bar{i}_2 = i_0 - \frac{5}{6}i_0 = \frac{1}{6}i_0 \quad . \quad . \quad . \quad . \quad (2)$$

Hence, the mean current over a cycle with drive is

$$\bar{i} = \frac{1}{T}\left[\frac{T}{2}\left(i_0 + \frac{2}{\pi}\hat{i}\right) + \frac{T}{2}\cdot\frac{i_0}{6}\right]$$
$$= \frac{7}{12}i_0 + \frac{\hat{i}}{\pi}$$

Therefore $$\hat{i} = \pi\left(\bar{i} - \frac{7}{12}i_0\right) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Also, by definition,

$$i_p = \hat{i} + i_0 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The peak fundamental current is then given by adding the separate components of fundamental frequency contributed by currents flowing respectively during positive and negative half-cycles.

That due to the positive half-cycle is evidently: $\frac{1}{2}\hat{i}$.

That due to the negative half-cycle is approximately the mean of the answers resulting from considering the current form to be sinusoidal and a square wave; i.e.

$$\frac{1}{2}\left(\frac{1}{2} + \frac{2}{\pi}\right)i_0 = 0{\cdot}57i_0$$

Hence: $$i_f = \frac{1}{2}\hat{i} + 0{\cdot}57i_0$$
$$= \frac{1}{2}(i_p - i_0) + 0{\cdot}57i_0$$
$$= \frac{1}{2}i_p + 0{\cdot}07i_0 \quad . \quad . \quad . \quad . \quad . \quad (5)$$

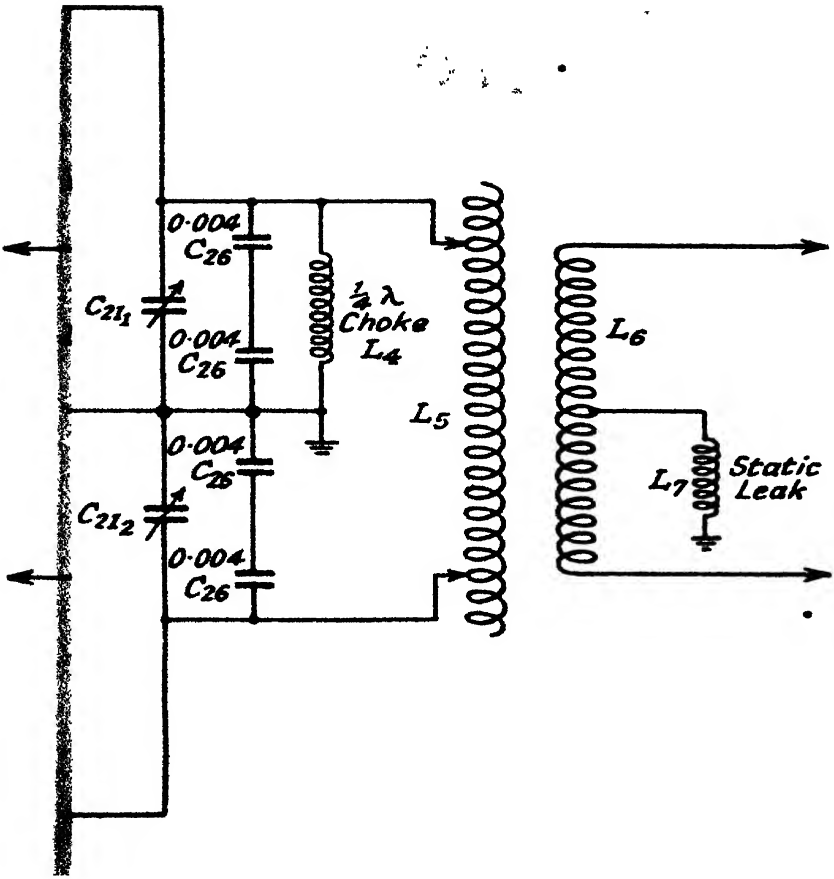
Hence: $$f = \frac{i_f}{i_p} = 0{\cdot}5 + 0{\cdot}07\frac{i_0}{i_p} \quad . \quad . \quad . \quad (6)$$

$$u = \frac{i_f}{\bar{i}} = \frac{\frac{1}{2}i_p + 0{\cdot}07i_0}{\frac{7}{12}i_0 + \frac{\hat{i}}{\pi}} = \frac{0{\cdot}5i_p + 0{\cdot}07i_0}{0{\cdot}584i_0 - 0{\cdot}318i_0 + 0{\cdot}318i_p}$$

(because $\hat{i} = i_p - i_0$)

$$= \frac{0{\cdot}5i_p + 0{\cdot}07i_0}{0{\cdot}318i_p + 0{\cdot}266i_0} \quad . \quad . \quad . \quad . \quad (7)$$

$$g = \frac{\bar{i}}{i_p} = \frac{0{\cdot}318i_p + 0{\cdot}266i_0}{i_p} = 0{\cdot}318 + 0{\cdot}266\frac{i_0}{i_p} \quad . \quad (8)$$



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The quantities f, u and g may then be used in the standard equations for power output, efficiency, etc.

31. Examination of a Practical Tuned Amplifier.

The example chosen is the final stage of a medium wave transmitter working on 373 metres. This amplifies the modulated output of the penultimate stage.

The circuit of this is shown in Fig. 1, set up for operation on 373 metres to deliver a nominal unmodulated carrier power of 130 kW.

This circuit calls for little comment. L_1 and C_1 constitute the anode tuning of the previous stage, L_2 is the grid choke, and L_3 is the grid decoupling inductance. Incidentally, it will be seen that one type of choke is specified as 60 m. and anotner as 14 m. This only means that the chokes were originally wound so as to constitute quarter-wave networks at 60 metres and 14 metres wavelength respectively (see XVI:4). The figures against condensers give their values in microfarads unless $\mu\mu$F are specifically stated. The condensers in series with R_1, the grid load, were inserted since the resistance used was slightly inductive at 373 metres. M_g and M_a are respectively grid current and anode current meters. S are stoppers : it will be noticed that a stopper (see XI:12.3, para 4 (*b*)) is inserted in series with each neut condenser C_n : this is merely an oddity due to the mechanical arrangement of the circuit : stoppers in this position were found necessary by experiment.

L_6 and L_7 are important elements which have so far not been discussed : these are static leaks to ground designed respectively to prevent leakage across the anode stopper condensers from charging up L_5 and atmospheric electricity from charging up L_6 and the output circuit to a high potential above ground. L_6 is made a quarter-wave choke at the operating frequency so that it does not introduce any appreciable unbalance to ground. It is not so important for L_7 to have a very high impedance since it is connected to the midpoint of L_6. Since, however, L_6 may not be exactly balanced about its nominal midpoint, L_7 should have a fairly high reactance to prevent unbalance in the output circuit.

Resistances R_2 are anode current limiting resistances inserted to limit the anode current in the event of valve failure introducing a short from anode to ground.

The operating conditions of this stage, as determined from meter readings, were :

E_b = grid bias = − 256 volts.

i_0 = standing feed = 1·75 amps.
$\bar{i}$ = mean anode current per valve = 5·95 amps. = driven feed.
H.T. volts = 16,800, but the voltage effective on the anodes is less than this by the voltage drop of 5·95 amps. flowing through the 50-ohm anode current limiting resistances R_2.
E = H.T. volts effective on anode = 16,800 − (50 × 5·95) = 16,500.
$\hat{e}_g$ = grid peak volts in carrier condition = 500.
$\hat{e}_a$ = anode peak volts in carrier condition = 7,000.
e_a = instantaneous value of anode volts.

The main circuit consists of four CAT14 valves in parallel push-pull with 16,500 volts H.T. effective on their anodes and a grid bias of 256 volts. Referring to Fig. 2, which shows the anode-current anode-voltage characteristics of a typical CAT14 valve, it is seen that, with the above values of H.T. and grid bias, a standing feed of about 2 amps. is to be expected in the absence of drive, so that the stage is operated substantially in class B. In the practical case under consideration the standing feed was 1·75 amps. per valve; a small variation from any representative valve curves is naturally to be expected. In the carrier condition (i.e. with no modulation) the current rises to 5·95 amps., with grid peak volts 500, and anode peak volts 7,000.

Determination of carrier conditions by conventional method, developed in X:24.

In the conventional method the value of i_0 is neglected, and the anode current is assumed to consist of a series of half-sine waves occurring every other half-cycle. In this case

$\hat{i}_p = \pi\bar{i} = \pi \times 5·95 = 18·7$ amps.

$\hat{i}_f = f\hat{i}_p = \frac{1}{2}\hat{i}_p = 9·35$ amps. (see Fig. 1/X:22 for value of f in a class B amplifier, i.e. when $\theta = 180°$)

Power output per valve $= \frac{1}{2}\hat{e}_a\hat{i}_f = \frac{1}{2} \times 7,000 \times 9·35 = 32·7$ kW.
Power output from 4 valves $= 32·7 \times 4 = 130·8$ kW.

Impedance facing one valve $= Z_e = \dfrac{\hat{e}_a}{\hat{i}_f} = \dfrac{7,000}{9·35} = 750$ ohms.

Impedance facing one side of push-pull $= \frac{1}{2} \times 750 = 375$ ohms.
Impedance presented by complete circuit facing the anodes, as measured by an R.F. bridge connected from side to side,

$= 2 \times 375 = 750$ ohms.

Efficiency $= \frac{1}{2}\frac{\hat{e}_a}{E}\cdot\frac{\hat{i}_1}{\bar{i}} = \frac{\pi}{4}\cdot\frac{\hat{e}_a}{E}$ (see equation (1)/X:24) . (1)

$$= \frac{\pi}{4}\cdot\frac{7{,}000}{16{,}500} = 33{\cdot}3\%$$

The load line along which each valve works passes through the point $e_a = 16{,}500 - 7{,}000 = 9{,}500$; $i_p = 18{\cdot}7$, and has a slope corresponding to an impedance Z_L which is related to the impedance Z_c facing each valve, by the relation

$$Z_L = fZ_c = \tfrac{1}{2}Z_c = 375 \text{ ohms.}$$ See equation (6)/X:22

The above conditions all occur when unmodulated carrier is being transmitted.

At 100% peak positive modulation the peak current and peak volts are doubled: $\hat{e}_a = 14{,}000$ and $i_p = 2 \times 18{\cdot}7 = 37{\cdot}4$ amps. The full corresponding load line is shown dotted on Fig. 2.

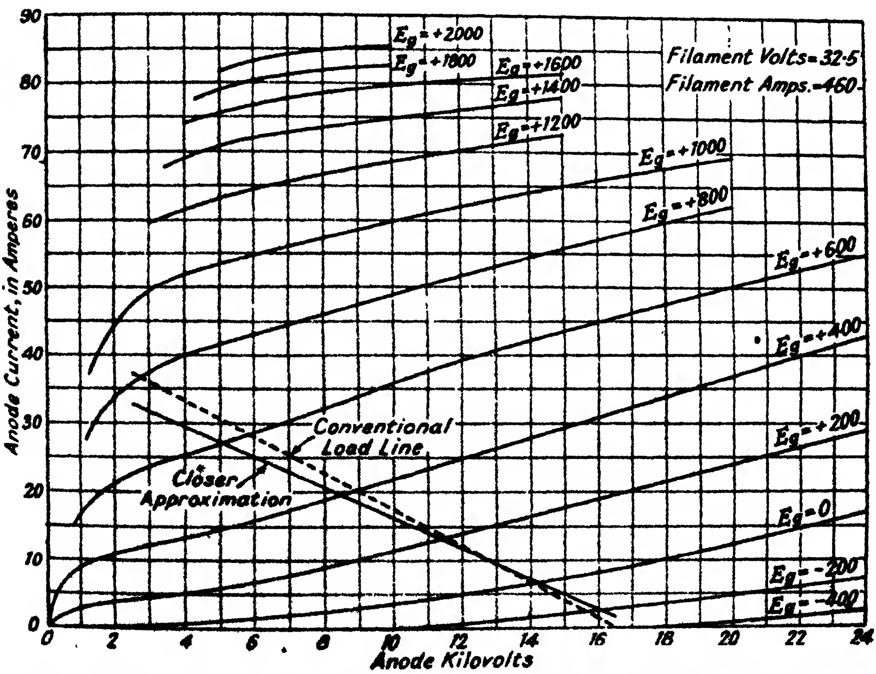


FIG. 2/X:31.—Characteristics of CAT14 Valve, with Conventional and Closer Approximation Load Lines at 100% p.p.m.

(By courtesy of Marconi's Wireless Telegraph Company.)

At 100% p.p.m. the power output is increased in the ratio $1 + m^2/_2 = 1{\cdot}5$ (m being the percentage modulation) while the value of $\bar{i}$ is unaltered, so that the efficiency is increased in the same ratio.

Determination of carrier condition by closer method of approximation, developed in X:30.

As before, $i_0 = 1{\cdot}75$ amps. and $\bar{i} = 5{\cdot}95$ amps.

Hence from (3)/X:30 $\hat{i} = \pi(5{\cdot}95 \quad \frac{7}{12}1{\cdot}75) = 15{\cdot}5$ amps.

from (4)/X:30 $i_p = 15{\cdot}5 + 1{\cdot}75 = 17{\cdot}25$ amps.

and from (5)/X:30 $\hat{i}_f = \frac{1}{2} \times 17{\cdot}25 + 0{\cdot}07 \times 1{\cdot}75$
$= 8{\cdot}75$ amps.

Power output per valve $= \frac{1}{2}\hat{e}_a\hat{i}_f = \frac{1}{2} \times 7{,}000 \times 8{\cdot}75$
$= 30{\cdot}6$ kW.

Power output from 4 valves $= 30{\cdot}6 \times 4 = 122{\cdot}4$ kW.

Impedance facing one valve $= Z_c = \dfrac{\hat{e}_a}{\hat{i}_f} = \dfrac{7{,}000}{8{\cdot}75} = 800$ ohms.

Impedance facing one side of push-pull $= \frac{1}{2} \times 800 = 400$ ohms.

Impedance presented by complete circuit facing the anodes, as measured by an R.F. bridge connected from side to side,

$$= 2 \times 400 = 800 \text{ ohms.}$$

$$\text{Efficiency} = \frac{\text{Power output per valve}}{\text{Power input to valve anode}}$$

$$\frac{30{,}600}{16{,}500 \times 5{\cdot}95} = 31{\cdot}2\%$$

The load line along which each valve works passes through the point $e_a = 9{,}500$, $i_p = 17{\cdot}25$, and has a slope corresponding to an impedance $Z_L = \dfrac{\hat{e}_a}{\hat{i}} = \dfrac{7{,}000}{15{\cdot}5} = 450$ ohms. The value of Z_L differs from $\frac{1}{2}Z_c$ owing to the contribution to the peak fundamental current occurring during the negative half-cycle, which is almost exactly one ampere in amplitude. If this were subtracted the value of Z_c would be

$$\frac{7{,}000}{8{\cdot}75 - 1} = 900 \text{ ohms}$$

In this case the conventional method gives a result 6% high on power and efficiency, 6% low on impedance, 16% high on slope of load line, and nearly 9% high on peak current. The load lines corresponding to the conventional approximation and the more accurate approximation are drawn on Fig. 2.

32. Design of Output Coupling Circuit for a Push-Pull Class B or C Tuned Amplifier.

From Z_L the load-line slope, by means of equation (6)/X:22,

determine the impedance Z_c required to face each valve. If n is the number of valves in parallel constituting the bank of valves on one side of the push-pull, the impedance facing one side of the push-pull

$$Z_b = \frac{1}{n} Z_c \quad . \quad . \quad . \quad . \quad (1)$$

The output coupling circuit is then designed to present an impedance $\frac{2}{n} Z_c$ *as observed looking into the coupling circuit, from the anodes of the valve bank on one side of the push-pull to the anodes of the valve bank on the other side of the push-pull.*

This follows because a class B or C tuned push-pull amplifier is analogous to a class A amplifier, since each side of the push-pull is effectively supplying drive to the output circuit throughout the whole of each R.F. cycle.

33. Practical Measurement of Valve Anode Efficiency.

For the present discussion the definitions established in X:37 will be used.

It will be clear that the anode efficiencies which have so far been discussed are the valve anode efficiencies as defined in X:37. These are assumed to be measured in the carrier condition.

The valve anode efficiency can evidently be measured by measuring the anode peak volts on a peak voltmeter, and the H.T. volts effective on the valve anodes. The anode efficiency is then given by equation (1)/X:24.

$$\eta = \frac{1}{2}\frac{\hat{e}_a}{E}u$$

The value of $u = \frac{\hat{\imath}_f}{\bar{\imath}}$ must therefore be known. If the angle of current flow is known, the value of u can be read from Fig. 1/X:22. The angle of current flow, if unknown, can be calculated from equation (5)/X:25; this involves measuring the grid bias, grid peak volts and anode peak volts and determining the cut-off bias of the valves either from valve curves, or better, by increasing the bias on the stage and reading anode feed with no drive on the stage. The value of μ must be determined as described in X:25.1.

Alternatively the value of u may be determined without reference to the angle of current flow by direct determination of $\hat{\imath}_f$ and $\bar{\imath}$. This is the more accurate method but involves the use of an R.F. bridge to measure the impedance facing the valve anodes. If this

impedance is Z_c, then $i_f = \frac{\hat{e}_a}{Z_c}$. $\bar{i}$ is evidently the value of the driven anode feed with normal carrier drive. Where possible, the second method of finding u should be used.

In principle, *the valve-anode circuit efficiency* might be measured by the obvious method of measuring the power input to the valve-anode circuit, which is evidently $P_1 = \frac{1}{2}\hat{e}_a^2/Z_c$, and the power output from this circuit which can be measured in a manner analogous to the input power. For instance, if the valve-anode circuit drives a feeder which presents an impedance Z_f towards the valve-anode circuit, the power output equals the power supplied to the feeder, and is given by

$$P_2 = \frac{1}{2}\frac{\hat{e}_f^2}{Z_c} = i_f^2 Z_c$$

where $\hat{e}_f =$ the peak volts measured across the feeder input, and $i_f =$ the R.M.S. current entering the feeder.

Measurement of peak volts is a much more accurate process than measurement of current at radio frequencies, and therefore the current measurement is of little value: the peak volts should be used as an indication of the power.

The valve-anode circuit efficiency would then be given by $\eta = \frac{P_2}{P_1}$.

Usually, however, it will be found that the valve-anode circuit efficiency is so high that such a method is not sufficiently accurate to give a reasonable indication of the efficiency. A method which will probably give a better indication is to calculate the power dissipated in the inductances of the circuit by measuring the coil resistances and the currents flowing through the coils. Although the measurement of the R.F. currents in the coils is admittedly inaccurate, the total error by this method is usually less than by the previous method. If P_L is the total power loss in the coils, and P_1 the input power to the valve-anode circuit, calculated as above, the anode circuit efficiency is given by

$$\eta = \frac{P_1 - P_L}{P_1}$$

33.1. Measurement of Valve Anode Efficiency by Temperature Rise of Anode Cooling Water. The principle on which this depends is that the sum of the R.F. output power of the valves and the power supplied to the cooling water must be equal to the anode input power to the valves.

The power in kilowatts dissipated in the water flowing through the valve may be determined from the fact that 0·318 kW raises 1 gallon of water 1° Centigrade in one minute.

Hence, if T_1 is the temperature of the inlet water to the valve anode jackets in degrees Centigrade, T_2 is the temperature of the water as it emerges from the jackets, and the water flow is F gallons per minute, the power supplied to the water is

$$0{\cdot}318F(T_2 - T_1) \text{ kW}$$

In practice, however, it is not satisfactory to use this figure to determine the anode dissipation, because on the one hand a certain amount of power is conducted, convected and radiated into the surrounding air, while on the other hand, the valve filament may contribute to the heating of the water.

Dead Loss Test. The proper procedure is therefore to carry out a dead loss test on the stage to be measured. For this purpose the water flow is kept constant at its normal value and, with no grid drive on the valves, the anode current is adjusted to a series of increasing values by adjusting the grid bias. The anode current is held constant at each value until the outlet water has reached a stable temperature, when the values of T_1 and T_2, as defined above, and the water flow meter, are read. Since, under the above conditions, the anode dissipation is equal to the product of the anode current and the H.T. volts, it is possible to plot a curve between anode dissipation and the value of $F(T_2 - T_1)$. This curve serves as a calibration of the stage; subsequently by observing the values of T_1, T_2 and F, the value of anode dissipation can be determined from this curve.

The effect of the above is to furnish a method of measuring the anode dissipation under all conditions, e.g. under working conditions, for instance in the carrier (no modulation) condition, which is usually the condition of interest.

The valve anode efficiency of the stage when driven is then given by:

$$\eta = \frac{\textit{D.C. Power Input to Valve Anode minus Anode Dissipation}}{\textit{D.C. Power Input to Valve Anode}}$$

Use of Thermometers at High Potential Points. To obtain a reliable indication of the temperature of the outlet water from a valve anode jacket, it may be necessary to mount the thermometer on the valve anode jacket assembly which is at high potential. If a mercury thermometer is used, the mercury thread constitutes a point of electrical high density which gives rise to a brush discharge

that burns out the thermometer. This can be prevented quite satisfactorily by shrouding the thermometer in a close-fitting copper screen connected to the metal of the high-potential point in question. The screen should completely enclose the thermometer except for a slot left along the scale to enable readings to be made.

Alternatively an alcohol thermometer should be used.

34. Anode Peak Volts and Anode Peak Current in a Modulated Amplifier or other Amplifier Driven into Anode Limitation.

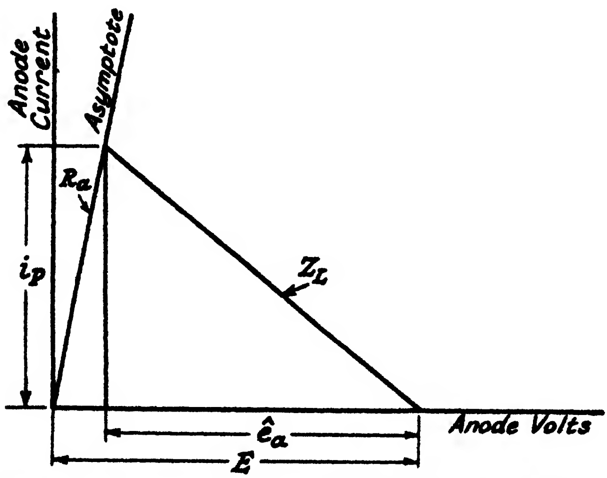


FIG. 1/X:34.—Load Line of an Amplifier Driven into Anode Limitation.

The R.F. anode peak voltage of a hard-driven amplifier is normally determined by the slope of the asymptote of the valve characteristics, the working load line and the value of the H.T. volts. (See X:27 for exact method.) The expressions for the voltage output may be derived as follows.

Conventions.

$\hat{e}_a$ = anode peak volts.
R_a = the slope of the asymptote of the valve,
Z_L = the slope of the load line along which the valve is working.
i_p = the peak current at the threshold of anode limitation.
E = the H.T. volts.

Referring to Fig. (1)

$$i_p = \frac{\hat{e}_a}{Z_L} = \frac{E - \hat{e}_a}{R_a} \qquad (1)$$

Therefore $$\hat{e}_a R_a = Z_L E - Z_L \hat{e}_a$$

Therefore $$\hat{e}_a = \frac{Z_L}{R_a+Z_L} E \qquad . \quad . \quad . \qquad (2)$$

A hard-driven amplifier therefore behaves like a generator of internal impedance R_a and internal e.m.f. E.

It is important to note that since R_a has a value which corresponds to a very low resistance, the effective internal impedance of a hard-driven amplifier is very low.

A modulated amplifier is a hard-driven amplifier in which the H.T. volts are varied.

If $E = E_0 + mE_0 \sin \omega t$ where m is the depth of modulation

$$= \frac{Z_L E_0}{R_a+Z}[1 + m \sin \omega t] \qquad (3)$$

The peak current is evidently not equal to i_p as defined above, but for most practical purposes may be determined by the rule given in italics in XIII:4. For more precise determinations, if they are ever required, the methods of X:27 may be used.

35. Inverted Amplifier.

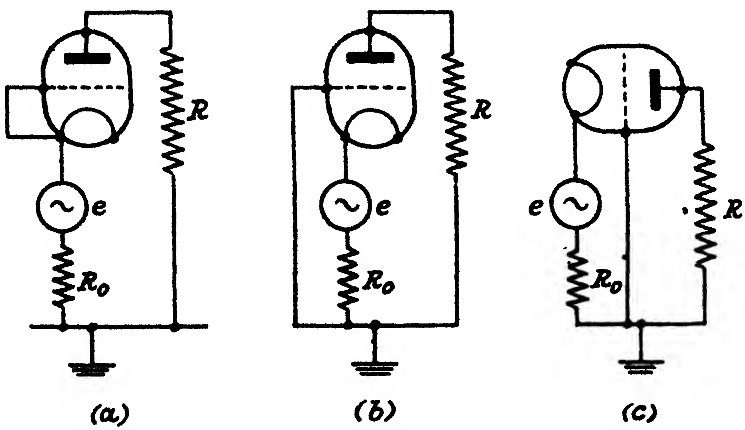


FIG. 1/X:35.—Principle of Inverted Amplifier.
(By courtesy of C. E. Strong and Messrs. Standard Telephones & Cables.)

Fig. 1 (*a*) shows a generator of internal e.m.f. e, and internal impedance R_0, connected in series with the cathode circuit of a valve with its grid strapped to its cathode (which, in the type of valve normally involved, is usually a directly heated filament) and with impedance R constituting the load in its anode circuit. It is evident that the anode cathode circuit of the valve is in series with the generator, and that the power delivered to the load R is equal to

$\frac{R}{R+R_0'+R_0}$ times the power output of the generator, where R_0' is the internal impedance of the valve.

If the grid instead of being strapped to the cathode is connected to ground as in Fig. 1 (*b*) through suitable bias arrangements, not shown, it is evident that the e.m.f. e introduces a potential between grid and cathode equal to e minus the voltage drop in R_0 due to the current which flows into the cathode-anode circuit of the valve and the load R. The effect of this is to vary the value of the internal anode resistance of the valve. This variation is often conventionally represented by an internal e.m.f. in the cathode-anode circuit of the valve. It is evident that this e.m.f. is in series with the e.m.f. due to the generator just as the internal impedance R_0' of the valve is in series with the internal impedance R_0 of the generator. The valve, therefore, behaves as an amplifier, in which the driver (driving source) and the amplifier anode circuit are in series. The same current passes through both the valve-anode circuit and driver, and the sense of this current is such that e_c the p.d. effective between cathode and ground is less than the e.m.f. e by the amount of the drop in R_0, the internal resistance of the driver. Negative feedback therefore occurs, and unless the value of R_0 is very much lower than $R_0'+R$ the value of e_c may be very much less than that of e.

The arrangement is called an *inverted amplifier* and the circuit of Fig. 1 (*c*) which is identical with that of 1 (*b*) shows the most usual conventional representation of this circuit.

It will be evident from the above argument that a fraction of the power delivered by the driver circuit represented by the generator R_0 is supplied to the output load.

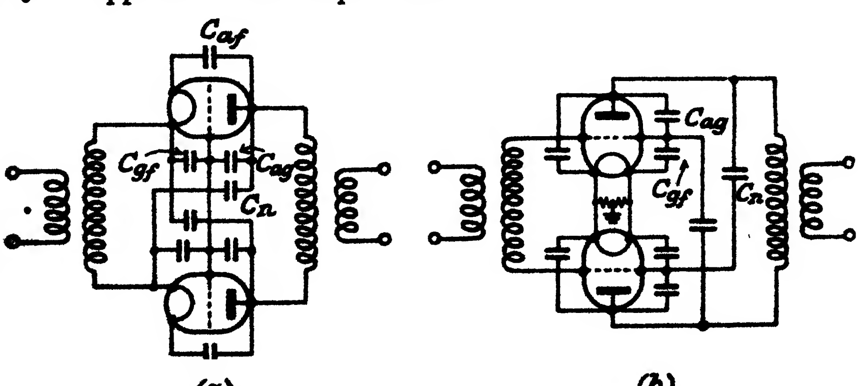


FIG. 2/X:35.—Comparison of Capacity Introduced across the Output Circuit by (*a*) Inverted Amplifier and (*b*) Normal H.F. Amplifier.
(By courtesy of C. E. Strong and Messrs. Standard Telephones & Cables.)

The particular application of this circuit is for amplification at short waves and ultra-short waves, where the valve capacities become important because of the tuning inductance, and therefore, if too large, set a limit to the value of output impedance (facing the valve-anode circuit) which can be realized. Fig. 2 (*a*) shows the arrangement of valve capacities and neutralizing condensers C_n in a push-pull inverted amplifier, and Fig. 2 (*b*) shows the same for a normal high-frequency amplifier. Typical values of valve capacities which might occur in a practical amplifier are $C_{ag} = 61\ \mu\mu F$, $C_{af} = 15\ \mu\mu F$, and $C_{gf} = 45\ \mu\mu F$. These are the capacities of the 4030C valve which is used in the inverted modulated amplifiers in two short-wave senders at the B.B.C. station at Daventry. It will be seen that the total capacity effectively in shunt across the anode tuning inductance in the case of the inverted amplifier is less than half that in the case of the normal high-frequency amplifier, so that the minimum output circulating current is halved, and the maximum permissible output circuit inductance and therefore possible maximum output impedance is twice that for the normal amplifier.

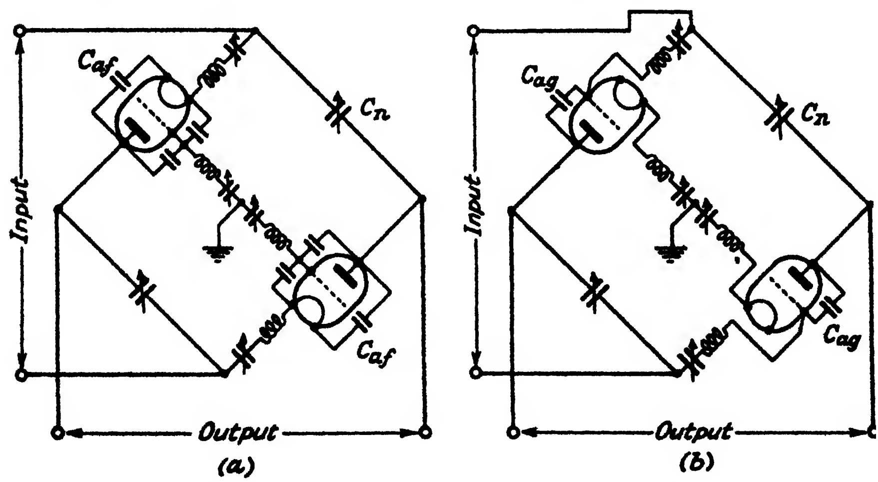


FIG. 3/X:35.—(*a*) Neutralization of C_{af} in Inverted Amplifier.
(*b*) Neutralization of C_{ag} in Normal H.F. Amplifier.

(By courtesy of C. E. Strong and Messrs. Standard Telephones & Cables.)

A feature which is claimed for the inverted amplifier is that the grid acts as a screen between the input and output circuits and that reaction due to the inter-electrode capacities is avoided. This is only partially true, since the reaction due to the grid anode capacity is replaced by reaction due to the anode-cathode capacity. The latter, however, is very much smaller: in the case of the 4030C valve

C_{af} is a quarter of C_{ag}, so that it is certainly true to say that reaction is reduced. Neutralization of the anode-cathode capacities is, however, necessary. Fig. 3 (*a*) shows the arrangement of neuting capacities C_n for a push-pull inverted amplifier represented in bridge form, the equivalent circuit for the normal high-frequency amplifier being shown at (*b*). An alternative method of capacity neutralization which is used in single-sided (single-ended) inverted amplifiers is to tune the capacity to high impedance with a parallel inductance. This method has the advantage that it does not increase the output capacity, but it has the disadvantage that the inductance has to be adjusted for each working frequency.

In the inverted-amplifier case, the inductance of the cathode-to-ground and grid-to-ground leads are tuned out by series variable condensers, just as in the normal amplifier the inductance of the grid- and cathode-to-ground leads are tuned out by series variable condensers.

35.1. Necessity for Modulating the Drive Stage. As can be seen from Fig. 4, since the load current flows through L_c, the

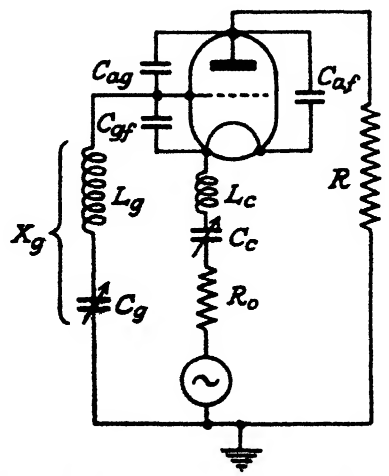


FIG. 4/X:35.—Inverted Amplifier showing Grid and Cathode Inductance Neutralizing Condensers.

(By courtesy of C. E. Strong and Messrs. Standard Telephones & Cables.)

cathode inductance, if this were not tuned out it would give rise to feedback having a component in quadrature with the driving voltage. Since the magnitude of the quadrature component varies with modulation, ***unless the driving valve is also modulated an exactly equal amount***, the effective cathode-to-ground driving voltage is

therefore phase modulated as the result of unneutralized reactance in the cathode lead.

Again, since the reactance of the grid-to-ground lead is always much less than the reactance of either C_{ag} or C_{gf}, it can be seen from Fig. 4 that the grid is effectively situated at the midpoint of a potentiometer of which one arm is constituted by C_{ag} and the other arm by X_g, the unbalance component of C_g and L_g. If X_g is negative, the potentiometer is a pure capacity potentiometer and the grid is driven with a feedback voltage in phase with the anode voltage, so that pure negative reaction occurs. If, on the other hand, X_g is of positive reactance less than the reactance of C_{ag}, then the grid is driven 180° out of phase with the anode by the feedback current, and positive feedback occurs. This argument has assumed the reactance of C_{gf} to be so high as to be negligible and has demonstrated that no quadrature component of feedback occurs under this assumption. The effect of C_{gf}, however, is (assuming the cathode reactance to be neutralized by C_c) to feed back quadrature components into the cathode circuit, since the cathode is tapped on to a potentiometer between grid and ground, of which the grid-cathode arm is the condenser C_{gf} and the cathode-ground arm is the resistance R_0. Similarly the feedback through C_{af} from the anode also introduces quadrature components into the cathode circuit.

When an inverted amplifier is used as a modulated amplifier, since the driver amplifier supplies power into the main load in series with the main amplifier, in order to obtain 100% modulation, quite apart from considerations of phase modulation, it is evidently necessary to modulate the drive amplifier.

35.2. Summary. The advantages of the inverted amplifier are, first, its lower output capacity and, secondly, the use of smaller neutralizing capacities which simplifies mechanical design and removes the resonant frequency of balancing condenser circuits further from the frequency of operation. Its disadvantages are its low gain and consequent high-power input, and the necessity for modulating the driver stage as well as the inverted amplifier, when used as a modulated amplifier.

The above outline is adapted from an article by C. E. Strong in *Electronics* for July 1940.

35.3. Circuit of Practical Inverted Amplifier and Driver. Fig. 5 shows the circuit of the driver and final stage of one of the senders at the B.B.C. station at Daventry. The driver stage is a straight amplifier while the final stage is an inverted amplifier.

Drive to the grids of the driver stage is applied to the input

coupling L_1L_2 tuned by C_2., Inductances L_3 are chokes applying bias to the grids of valves V_1; the condensers C_3 are grid-bias smoothing condensers; resistances R_1 are filament centre-pointing resistances. H.T. is applied to the driver stage, through choke L_4, to the centre of the anode-tuning inductance L_5 which is earthed through smoothing condenser C_4. The anode-tuning condenser is C_5; C_n are neut condensers.

Drive to the final stage is taken off from inductance L_5 via the coupled inductance L_6 through a short feeder line to inductance L_7. This is magnetically coupled to inductances L_8, which are connected to the centre-pointing resistances R_1 of the filaments of the inverted amplifier, and are tuned by means of condenser C_8. Inductances L_{13} are chokes of reactance high compared with C_8 which provide the return path to ground for the anode current. Condenser C_7, which in this case has a centre-point earth, is known as the grid-to-grid condenser, and is adjusted to neutralize the reactance of the grid leads. Bias is applied through chokes L_9; condensers C_9 are the grid-bias smoothing condensers, the anode-output circuit is of the same form as that of the driver stage, condensers C_n constituting the anode-to-cathode neut condensers.

It will be noted that there is no low-impedance path at second-harmonic frequency, from the anodes to the filaments; although the centre point of the anode-tuning inductance is earthed this only supplies a low-impedance path to earth from the anodes to ground if the coupling between the two halves of L_{10} is high; there is no centre-point connection from the filaments to ground since the inductance of chokes L_{13} is high at second-harmonic frequency. The net result is a reduction in valve-anode efficiency. This is a definite fault in design which should not be tolerated in modern transmitters. It is true that stray capacities from anode to ground, and from filaments to ground, do provide a path for second-harmonic frequency currents, but this path is not of low enough impedance for the full efficiency of the output stage to be realized.

35.4. Design of Inverted Amplifier and Driver Stage. The final stage is designed from the point of view of its anode voltage, impedance facing its anode circuit, grid bias and grid drive, just as in the case of a normal high-frequency amplifier. This applies whether the inverted amplifier is a modulated amplifier, a class C amplifier, or a class B amplifier for amplifying modulated waves.

In the case of a single-sided amplifier with one valve, the driver stage is then designed to supply a power equal to the power supplied to the cathode-grid circuit (see X:37) plus the power supplied to the

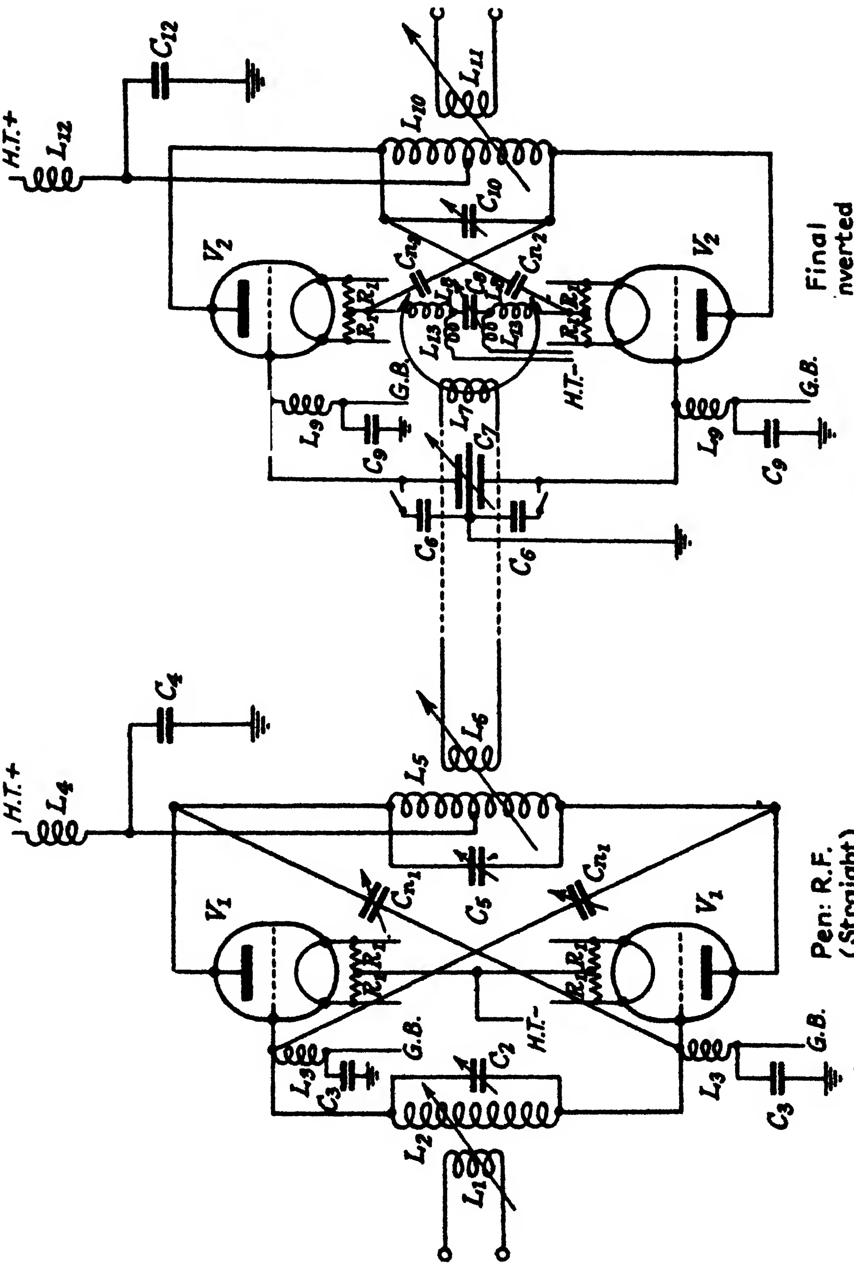


FIG. 5/X:35.—Inverted Amplifier and Driver Stage.

cathode-anode circuit. A good approximation to the power supplied to the cathode-anode circuit is given by:

$$P_c = \tfrac{1}{2}\hat{e}_c\hat{\imath}_f \qquad . \quad . \quad . \quad . \quad (1)$$

where $\hat{e}_c$ = the cathode-ground (grid) peak volts

$\hat{\imath}_f$ = the peak value of the component of anode current (of the inverted amplifier) of fundamental frequency.

It is important to note that for all practical purposes the whole of P_c is supplied to the output load in the anode of the inverted amplifier.

(Academically, two generators in series can only be said to deliver all their power to a common load if the ratio of internal e.m.f. to internal resistance is the same for both. Such an analogy is not directly comparable with the present case, and it is not very fruitful to pursue the argument far.)

The value of $\hat{\imath}$ is determined by the peak anode volts on the inverted amplifier divided by the impedance facing the anode, and so is known from the design of the inverted-amplifier stage, as of course is also the value of $\hat{e}_c$.

The impedance presented by the cathode-to-ground input circuit of the inverted amplifier is given approximately by:

$$Z_{cg} = \frac{\hat{e}_c}{\hat{\imath}_f + \hat{\imath}_{fg}} = \frac{\hat{e}_c}{\hat{\imath}_f + 2\bar{\imath}_g} \qquad . \quad . \quad . \quad (2)$$

where $\hat{\imath}_{fg}$ = the peak value of the fundamental frequency component of grid current

$\bar{\imath}_g$ = the mean value of grid current.

The value of $\bar{\imath}_g$ can most simply be determined by setting up the amplifier as a straight amplifier working with the same anode and grid conditions as the inverted amplifier.

In the case of a single-sided amplifier with n valves in the inverted amplifier, the equations still apply if $\hat{\imath}_f$ is made equal to the peak value of the current of fundamental frequency flowing through the anode load, i.e. n times the peak fundamental frequency current flowing through one valve. $\bar{\imath}_g$ is then equal to the total D.C. grid current.

In the case of a push-pull amplifier equations (1) and (2) must be modified to conform to the definition of $\hat{e}_c$ which is still most conveniently specified as the cathode-to-ground drive because that is more easily measured than the cathode-to-cathode grid drive. The power input to the push-pull anode circuit is then given by:

$$P_c = \hat{e}_c\hat{\imath}_f \qquad . \quad . \quad . \quad . \quad (3)$$

where $\hat{e}_c$ = the cathode-to-ground drive on one side of the push-pull
$\hat{i}_f$ = the peak value of the fundamental frequency current through the anode load.

The total power input to the two cathode-grid circuits is of course given by the product of $\hat{e}_c$ and the total D.C. cathode-grid current.

For the purpose of determining the impedance of the driving circuit, the cathode-to-cathode input impedance is twice the cathode-to-ground impedance as determined for a single-sided amplifier. See equation (2) and qualifying remarks for case where a number of valves are used. The value of $\bar{i}_g$ to be used in equation (2) is evidently the mean grid current to one side of the push-pull only.

When the design on the above basis is complete, the impedance facing the anode should be increased in the ratio $\left(1+\frac{\hat{e}_c}{\hat{e}_a}\right)$ where $\hat{e}_a$ is the value of anode peak volts determined in the initial design of the inverted amplifier.

35.5. Relations in a Class A Inverted Amplifier. As a preliminary to dealing with the class C case it is useful to examine the very much simpler case of a class A amplifier.

Assume the circuits of Fig. 1 (*b*) or 1 (*c*)/X:35 and the following conventions:

e = the internal e.m.f. of the driving generator.
R_s = the internal impedance of the driving generator.
e_{ag} = the load volts = the anode volts with regard to ground.
e_c = the volts between cathode and ground.
R = the load impedance.
R_a' = the internal impedance of the valve.
i = the current flowing in series through R_s, R_a and R.
$Z_{cg} = e_c/i$ = the effective cathode-to-ground input impedance of the inverted amplifier.
μ = the amplification constant of the valve.
$\mu_c = e_{ag}/e_c$.
$\mu_c' = e_{ag}/e$ = the gross overall voltage amplification.
e_a = the volts between anode and cathode (equivalent to anode output volts in a normal amplifier).

Then $$i = \frac{e+\mu(e-iR_s)}{R_a'+R+R_s}$$

Therefore $$i = \frac{(1+\mu)e}{R_a'+R+(1+\mu)R_s} \qquad (4)$$

$$e_c = e - iR_s = e - \frac{(1+\mu)R_s e}{R_a'+R+(1+\mu)R_s}$$

$$= \frac{R_0'+R}{R_0'+R+(1+\mu)R_0} e \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\mu_c = \frac{e_{ag}}{e_c} = \frac{Ri}{e_c} = \frac{(1+\mu)R}{R_0'+R} \quad . \quad . \quad . \quad . \quad (6)$$

$$\mu_e' = \frac{e_{ag}}{e} = \frac{Ri}{e} = \frac{(1+\mu)R}{R_0'+R+(1+\mu)R_0} \quad . \quad . \quad (7)$$

$$Z_{cg} = \frac{e_c}{i} = \frac{R_0'+R}{1+\mu} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

It is therefore evident from equations (5) *and* (6) *that, as in all cases of feedback, the effect of the negative feedback in reducing* μ_e' *the overall voltage amplification, as defined by equation* (7), *operates to reduce* e_c *the cathode-grid drive, without reducing* μ_c *the effective amplification from cathode-grid circuit to load, which is defined by equation* (6). *The effective amplification from cathode grid to load (i.e.* μ_c*) is in fact greater than in a normal amplifier, in the ratio* $\frac{1+\mu}{\mu}$ *because the cathode-grid e.m.f. is in series aiding with the internal e.m.f. generated in the valve.*

It should be noted further that

$$e_{ag} = e_a + e_c \quad . \quad . \quad . \quad . \quad . \quad (9)$$

35.6. Relations in a Class C Inverted Amplifier. These cannot be analysed in the same way as a straight class A amplifier, but all the necessary conclusions can be derived very simply. These apply also to the unmodulated condition of a modulated amplifier.

Assume that the operating conditions of the amplifier have been determined as for a normal non-inverted amplifier, and that these have determined the following:

Z_e = the impedance facing the anode circuit.
$\hat{e}_c$ = the cathode-grid drive from cathode to ground (= the grid-to-ground peak volts in the normal amplifier).
$\hat{e}_a$ = the anode (to cathode) peak volts.
$\hat{i}_f$ = the peak value of the fundamental frequency component of anode current.
P_c = R.F. input power to cathode.
P_n = R.F. power output from normal amplifier operated under these conditions = $\hat{e}_a^2/2Z_e$.

$\hat{e}_{ag}$, the volts effective across the impedance facing the anode circuit = $\hat{e}_a + \hat{e}_c$, so that in order to maintain the value of $\hat{i}_f$ at the

value found in the initial design, the value of Z_c must be increased in the ratio defined by:

$$Z_w = \frac{\hat{e}_a+\hat{e}_c}{\hat{e}} = \left(1+\frac{\hat{e}_c}{\hat{e}_a}\right)Z_c \qquad (10)$$

where Z_w is the working value of impedance facing the anode circuit.

The cathode input impedance, Z_{cg}, and driving power are then determined as in X:35.4, remembering that the driving power is P_c, the power required to drive the cathode circuit, as determined from equation (1) or (3).

35.61. Power Output from an Inverted Amplifier. This is evidently given by any of the expressions:

$$P = \frac{\hat{e}^2_{ag}}{2Z_w} = \frac{\hat{e}^2_a}{2Z_c\left(1+\frac{\hat{e}_c}{\hat{e}_a}\right)} = P_n\left(1+\frac{\hat{e}_c}{\hat{e}_a}\right) \quad . \quad . \quad (11)$$

$$\therefore\ P = \tfrac{1}{2}\hat{e}_{ag}\hat{\imath}_f = \tfrac{1}{2}\hat{\imath}_f(\hat{e}_a+\hat{e}_c) = P_n+P_c \quad . \quad . \quad (12)$$

where $P_n = \frac{1}{2}\hat{\imath}_f\hat{e}_a$ = the normal power output from the valve anode circuit

and P_c = the power input to the grid circuit.

Equations (11) and (12) may be regarded as applying to a single-sided amplifier, when all voltages are measured to ground. The reader may retain the same conventions and multiply by 2 in order to make the equations apply to push-pull working; alternatively, the conventions for $\hat{e}_{ag}$, $\hat{e}_a$ and $\hat{e}_c$ may be changed.

In the case of an inverted amplifier used as a modulated amplifier, if the drive stage is not modulated to the same depth as the main (inverted) amplifier the relations of equations (11) and (12) apply only in the carrier condition. This, however, is the one condition in which they are useful.

It will now be clear that, since it is not permissible to modulate the drive stage of a modulated amplifier to the same depth as the main amplifier, because a modulated amplifier must be driven into anode limitation at all times, it is never possible to secure 100% modulation with an inverted amplifier used as a modulated amplifier. See X:35.63.

35.62. Power Amplification of an Inverted Amplifier. This is evidently the ratio of the power output to the required driving power and is best expressed as a power ratio rather than in decibels. It is given by

$$A = \frac{P_n+P_c}{P_c} \quad . \quad . \quad . \quad . \quad (13)$$

As an example, the realizable value of this ratio with a CAT14 valve is about 5. This means that nearly a fifth of the power in the output circuit is supplied by the driver stage. Since the same current flows through the cathode and anode circuit, it follows that nearly a fifth of the voltage in the output load is contributed by the driver stage. The importance of modulating the driver stage, when the output stage is a modulated amplifier, is now very clear.

35.63. Percentage Modulation Realizable with an Inverted Amplifier. If the driver stage is modulated to a depth m when the output stage is modulated 100%, with an amplification ratio of 5, the output is modulated to a depth

$$m_0 = \frac{4+m}{5} \qquad (14)$$

e.g. if the driver stage is modulated 50% the value of m is 0·5 and $m_0 = 0{\cdot}9$; the percentage modulation of the output is only 90%. A further disadvantage of the inverted amplifier therefore appears.

35.7. Operation and Adjustment of Inverted Amplifiers. Owing to the interaction of an inverted amplifier with its driver stage, special precautions have to be taken in adjustments on these stages which in certain cases have to be treated as a whole. The order of lining up an inverted amplifier is as follows:

1. *Adjust Grid-Grid Condenser.* In a push-pull amplifier, this is constituted by the two grid-to-ground condensers, but, since it is found in practice that the ground connection may be omitted, it is referred to as the grid-grid condenser. This is adjusted until the correct cathode drive (as judged by grid current) is obtained on the inverted amplifier with the normal anode current in the drive stage. If the grid-grid condenser is too small, a high drive-stage anode current will occur with too small a cathode drive on the inverted amplifier. If the grid-grid condenser is too large, too large a value of cathode drive will occur (owing to oscillation) with too low a value of anode current on the drive stage.

2. *Adjust Cathode Tuning Condenser.* This is the condenser neutralizing the reactance of the cathode lead. It is adjusted until the tuning of the driver stage (as judged by minimum anode current) is unaffected by a change of anode volts on the two stages. The driver stage is tuned with half H.T. and the H.T. is then increased to 10,000 volts. When the cathode tuning condenser is at its correct value no change in tuning of the driver stage should result.

After the cathode tuning condenser is adjusted, the tuning of the driver stage is restored to the setting found in 6 below.

3. It may be necessary to repeat 1, followed by 2.

4. *Adjust Anode-Cathode Neut Condensers on Main Amplifier.* These may be adjusted by one of the methods given in X:20.

5. *Neut Drive Stage.* This is done by the grid-current method: the H.T.

is removed and the neut condensers adjusted until varying the anode circuit tuning causes no variation of grid current.

6. Tune the anode circuits of drive stage and final stage independently. These are best tuned cold. Input drive is applied with no H.T. and the inductance components adjusted until the circulating current is a maximum, as indicated by an absorption wave meter. Drive to the anode circuit is obtained in each case by slightly de-neuting the stage concerned. The tuning settings are then noted.

While the preferred method of tuning is as indicated in 6, it is possible to tune the driver stage from normal indications (minimum anode current). The final stage cannot be tuned to minimum anode current, but may be tuned to maximum grid current.

For further information on grounded grid amplifiers, see articles by P. A. T. Bevan in the *Wireless Engineer* for June and July 1949.

36. Short-Wave Amplifiers.

The behaviour of short-wave amplifiers is identical in principle with that of medium- and long-wave amplifiers, but owing to the higher frequency, the reactance of all conductors and the admittance of all incidental capacities, e.g. valve capacities and strays, is of sufficient magnitude to affect the performance considerably. For this reason all connections should be kept as short as is physically possible, the layout and assembly of components being expressly designed for this end. Stray capacities must also be kept to a minimum.

Even when this is done, the reactance of grid input leads and cathode-ground leads is too high at frequencies of the order of 10 Mc/s and higher. For this reason, condensers are inserted in series with the grid and cathode leads and are adjusted at each wavelength so as to cancel the lead reactance. If the manufacturer has not specified the value of the lead reactance, the most satisfactory method of adjusting the condenser is to make a model of the lead in question and adjust the condenser to series resonate this lead by measurement on a bridge, due allowance being made for the connecting bridge leads. See also XVI:13.2 and 13.3.

37. Valve Anode Efficiency, Valve Anode Circuit Efficiency and Output Efficiency of an Amplifying Stage.

$$\text{Valve anode efficiency} = \frac{\text{R.F. power output of valves}}{\text{D.C. power input to valve anodes}}$$

The R.F. power supplied by a valve all goes into the input of the circuit connected to its anode. The output from this circuit is,

however, less than its power input, owing to (very small) losses in the resistance of coils and loss in stray capacities across the circuit.

Valve anode circuit efficiency $= \dfrac{\textbf{R.F. power output of stage}}{\textbf{R.F. power output of valves}}$

Output efficiency of an amplifying stage = Valve anode efficiency × Valve anode circuit efficiency

$$= \frac{\textbf{R.F. power output of stage}}{\textbf{D.C. power input to valve anodes}}$$

In any particular short-wave transmitter it may be found that the output efficiency falls off towards high frequencies, because the inevitable stray capacity sets a limit to the magnitude of the tuning inductance it is permissible to use. This in turn limits the value of anode impedance which can be realized with a given amount of referred resistance (i.e. transferred in series with the anode inductance by coupling with the secondary circuit).

If the referred resistance is kept very low, the normal value of valve anode efficiency may be obtained. In practice, this merely means that the output coupling must be reduced until the correct anode conditions are observed. When this is done it will be found that the kVA/kW ratio will sometimes be considerably higher than 5; for instance, on frequencies between 10 and 20 Mc/s it may lie between 15 and 20.

The valve anode circuit efficiency depends to a negligibly small extent on the R.F. resistance of the anode tuning inductance, to a small extent on losses introduced into the anode tuning coil by electro-magnetic coupling with stray masses of conducting material in the vicinity, and probably, to a larger extent, on the dielectric loss of the circuit, which is introduced at every point where an electrostatic line of force enters a conducting point which is connected to earth through a resistance path. For this reason, to keep losses to a minimum it is found necessary to screen all semi-conductors, such as the ground and walls of the containing cubicle, with a continuous screen well bonded together and to ground. The following figures serve to show how unimportant is the radio-frequency resistance proper of the anode tuning coil. They are taken from one of the senders at the B.B.C. station at Daventry.

Operating frequency Mc/s = 9·58.

Anode Tuning Coil.

Inductance = 2·56 μH.

Reactance = $L\omega$ = 154 ohms.

Length of conductor = 209 inches.
Diameter of conductor = 3 inches.
R.F. resistance (from length and diameter) = 0·018 ohms.
Anode peak volts = 10,000.
Power output = 80 kW.
Impedance facing anodes = 2,500 ohms.
Resistance in series with anode tuning coil transferred from secondary circuit = 9·5 ohms.
kVA/kW ratio = 16·2.

It is evident that if there were no dielectric and electromagnetic losses, the anode circuit efficiency would be equal to unity minus the R.F. resistance divided by the transferred resistance, so that it would be about 99·9%.

Even when dielectric and electromagnetic losses are taken into account, the anode circuit efficiency is seldom worse than 95% and is usually in the neighbourhood of 98% or 99%.

It will be also evident that the heat dissipated in the anode tuning coil should be negligible. In practice it is sometimes found that the anode tuning coil gets warm. It is safe to say that with normal design this is never due to the proper R.F. resistance of the conductor, but is due to poor contacts in joints.

38. Grid Dissipation in a Class C Amplifier.

In establishing the working conditions of a valve it is essential that the manufacturer's figures for rated grid dissipation should not be exceeded.

The most reliable method of determining the grid dissipation is by measurements of the grid peak volts $\hat{e}_g$ and mean grid current $\bar{i}_g$ under working conditions, that is, with the working values of grid bias, H.T. and anode loads. $\bar{i}_g$ is the value of grid current as measured on the normal D.C. grid-current meter. With a sinusoidal grid drive the power supplied to the grid circuit is then given by

$$P_g = \hat{e}_g \bar{i}_g \quad . \quad . \quad . \quad . \quad (1)$$

This is an approximate figure, but is a safe one, because it errs in the direction of giving slightly too high a value of grid dissipation. It is derived from the fact that most of the grid current flows during a very small fraction of a cycle. The form of the grid-current wave therefore *approximates* to that of a series of pulses of amplitude A and duration p, separated by intervals T. From Fig. 3/VIII:1 the equivalent Fourier series for such a wave is:

$$S = A\left[\frac{p}{T} + \sum_{n=1}^{n=\infty} \frac{2}{\pi n} \sin \frac{n\pi p}{T} \cos n\omega t\right]$$

The value of $\bar{i}_g$ is therefore proportional to $A\frac{p}{T}$ while the peak amplitude of the fundamental frequency component of grid current $\hat{i}_{fg}$ is proportional to

$$\left[\frac{2}{\pi n} \sin \frac{n\pi p}{T}\right]_{n=1} = \frac{2A}{\pi} \sin \frac{\pi p}{T}$$

When $\frac{p}{T}$ is very small, $\sin \frac{\pi p}{T} = \frac{\pi p}{T}$ and $\hat{i}_{fg}$ is proportional to

$$\frac{2A}{\pi} \times \frac{\pi p}{T} = \frac{2pA}{T}$$

Hence
$$\frac{\hat{i}_{fg}}{\bar{i}_g} = \frac{2\frac{pA}{T}}{A\frac{p}{T}} = 2$$

Therefore
$$\hat{i}_{fg} = 2\bar{i}_g \quad . \quad . \quad . \quad . \quad (2)$$

(It is to be noted that if p is not small compared to T, $\hat{i}_{fg}$ is proportional to $\frac{2A}{\pi} \sin \frac{\pi p}{T}$, which is less than $\frac{2A}{\eta} \cdot \frac{\pi p}{T}$. The assumption above therefore leads always to a value of grid dissipation larger than the actual dissipation.)

The grid dissipation is therefore given by

$$P_g = \tfrac{1}{2}\hat{e}_g \hat{i}_{fg} = \hat{e}_g \bar{i}_g \quad . \quad . \quad . \quad . \quad (3)$$

as stated in (1).

It may be noted that although the grid current contains currents of frequency other than $\hat{i}_{fg}$, these are wattless currents, since no grid voltage exists at these frequencies.

39. D.C. Amplifiers.

When it is required to build an amplifier to deal with frequencies down to zero frequency, the connections from the anode of each stage to the grid of the following stage must be conductive : it is not possible to use series condensers to insulate the grid of the following stage from the high voltage on the anode of the preceding stage. Some other artifice must therefore be used to ensure that the potentials of grid, cathode and anode of each stage have the required relative values.

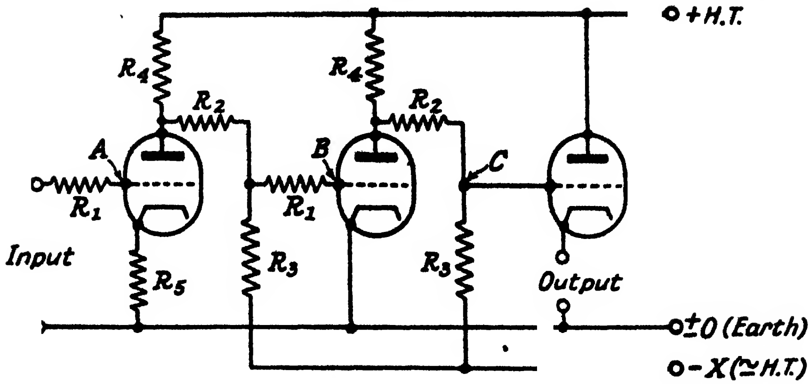


FIG. 1/X:39.—Simple D.C. Amplifier.

Fig. 1 shows a D.C. amplifier in which the grid of each following stage is maintained at its correct value by means of a resistance potentiometer connected between the anode of the previous stage and a source of high negative potential. This potentiometer is constituted by resistances R_1 and R_2, and it is evident that this potentiometer introduces a loss in the amplifying circuit which is reduced as the voltage value of the negative source is increased.

This form of D.C. amplifier is the one which will probably be found to have most general application since it does not involve abnormal values of H.T. and its output is not at a high potential with regard to its input.

A modification of this circuit is sometimes employed in which the resistances R_1 are replaced by neon tubes, which have the advantage that their A.C. resistance is lower than their D.C. resistance so that the loss in amplification due to the potentiometer can be reduced. The use of this expedient is not recommended.

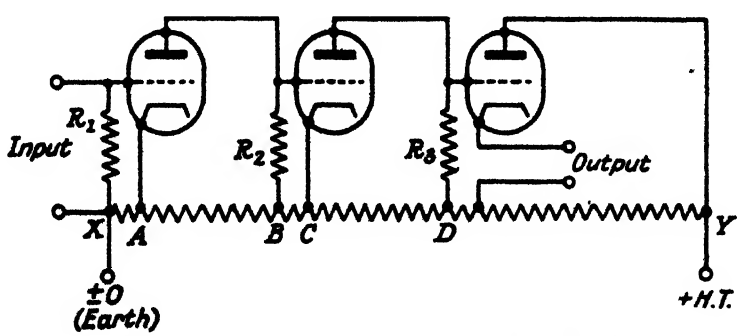


FIG. 2/X:39.—D.C. Amplifier with Series H.T.

Fig. 2 shows an extremely simple type of D.C. amplifier which has the advantage over that of Fig. 1 in that only one source of high voltage is required. It has, however, the serious disadvantages that the output is at a high D.C. potential relative to the input and that the resistance *XABCDY* absorbs a considerable amount of power : the values of resistances *AB* and *CD* must be respectively much less than resistances R_2 and R_3.

Fig. 3 shows a type of push-pull amplifier. The circuit is symmetrical about the centre (earth) line. Common cathode resistances

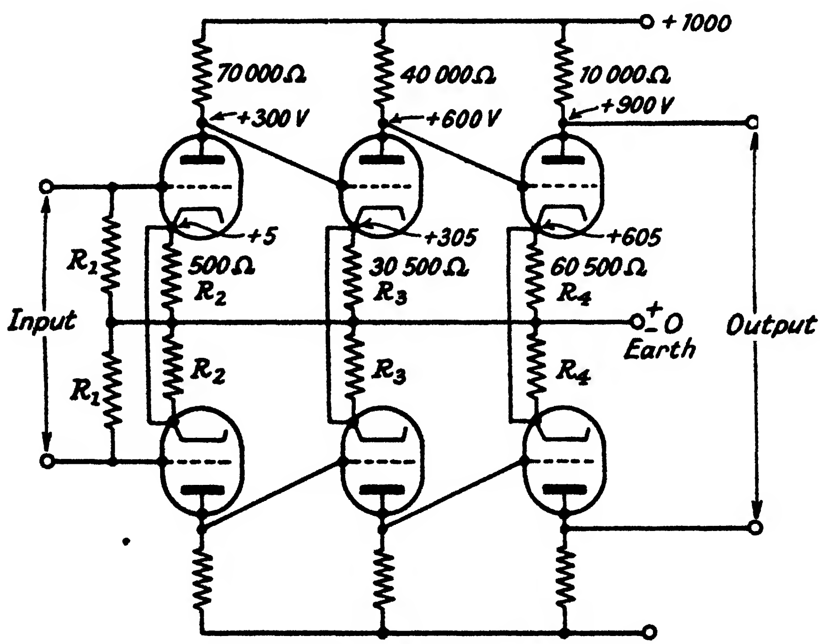


FIG. 3/X:39.—Push-Pull D.C. Amplifier.

have been replaced by two resistances for the sake of clarity in calculating voltages. The D.C. potentials with regard to ground have been indicated on the assumption of an H.T. of 1,000 volts and the use of valves which, with − 5 volts grid bias, and 295 volts between anode and cathode, draw an anode current of 10 milliamps. It is assumed that all valves are to be operated under this condition.

It will be seen that the effect of the cathode resistances is to adjust the steady potentials only and that they are not effective in introducing degeneration in the push-pull path.

The output of this circuit is at high potential with regard to its input circuit, and a high value of H.T. is required.

99. Examples.

1. A resistance-coupled class A amplifier valve has an anode resistance of 30,000 ohms and is coupled through a large condenser to a grid load (of the following valve) of 50,000 ohms. (*a*) If the condenser reactance is negligible, what is the load impedance facing the anode of the valve? (*b*) If the condenser reactance is not negligible, what must be the minimum value of the condenser in order that the series reactance of the anode load at 30 c/s shall be not greater than a fifth of its resistance? What is the anode load impedance under this condition? (*c*) What is the ratio of the anode peak volts to the grid peak volts on the following valve at 30 c/s and at a very high frequency, with the condenser having the value determined under (*b*)? (*d*) What is the fall in response in decibels at 30 c/s?

[A. (*a*) 18,750 Ω.

(*b*) If X_c is the magnitude of the condenser reactance, the anode load impedance

$$= \frac{30{,}000(50{,}000 - jX_c)}{80{,}000 - jX_c} = \frac{30{,}000[4 \times 10^9 + X_c^2 - j30{,}000X_c]}{80{,}000^2 + X_c^2}$$

Equating the resistance component to five times the reactance component $X_c^2 - 150{,}000X_c + 4 \times 10^9 = 0$

$\therefore X_c = \frac{1}{2}(150{,}000 - \sqrt{2{\cdot}25 \times 10^{10} - 1{\cdot}6 \times 10^{10}}) = 35{,}250$ ohms,

the positive value of the root being excluded because it corresponds to a high reactance which gives poor coupling to the grid load. From the reactance chart the value of condenser is about $0{\cdot}14$ μF.

The anode load impedance is then $20{,}600 - j4{,}150$.

(*c*) Ratio: grid volts/anode volts at 30 c/s

$$= \frac{50{,}000}{50{,}000 - j35{,}250} = 0{\cdot}82 \underline{/35^\circ}$$

Ratio at a very high frequency = unity.

(*d*) Fall in response $= 20 \log_{10} 0{\cdot}82 = 1{\cdot}73$ db.]

2. An amplifier consists of: An input transformer of impedance ratio 600 : 100,000 Ω working into 100,000 Ω the grid load of a valve with $\mu = 50$ and anode impedance $R_0 = 50{,}000$ Ω. This valve is resistance coupled to the grid of the next valve, the anode resistance being 100,000 Ω and the grid leak 200,000 Ω. The second valve is a pentode with a mutual conductance of 5 mA/volt and is choke-capacity coupled to an output transformer with an impedance ratio of 10,000 : 7·5 Ω terminated in a resistive load of 7·5 Ω. What is

the gain of the amplifier in decibels, at frequencies at which the reactance of condensers and choke can be neglected ?

$$\left[\text{A. Gain} = 20 \log_{10} \left[\sqrt{\frac{100{,}000}{600}} \times 50 \times \frac{66{,}700}{50{,}000 + 66{,}700} \times 5 \times 10^{-3} \quad 10{,}000 \times \sqrt{\frac{600}{10{,}000}}\right] = 71{\cdot}1 \text{ db.}\right]$$

There was a catch in this question : the gain of an amplifier always refers to power amplification and is most simply calculated by making the output impedance equal to the input impedance. See X:6.

3. A push-pull class A amplifier is to contain two valves and to work into a resistive impedance of 1,000 ohms. The linear range of the chosen load line extends from $e_a = 300$ volts, $i_a = 150$ mA to $e_a = 1{,}700$ volts, $i_a = 30$ mA. If the amplifier is correctly designed from these data, what is (*a*) the value of H.T., (*b*) the standing feed per valve, (*c*) the anode dissipation per valve, (*d*) the maximum sinusoidal power output of the amplifier and anode efficiency with this output, (*e*) the impedance facing each valve, (*f*) the impedance ratio of the output transformer, whole primary to whole secondary ?

[A. (*a*) H.T. $= \frac{1}{2}(1{,}700+300) = 1{,}000$ volts.

(*b*) Standing feed $= \frac{1}{2}(150+30) = 90$ mA.

(*c*) Anode dissipation $= 1{,}000 \times 90 \times 10^{-3} = 90$ watts.

(*d*) $P_m \quad \frac{1}{8}(1{,}700-300) \times (0{\cdot}150-0{\cdot}030) \times 2 = 42$ watts.

$$\eta_{max.} = \frac{42}{2 \times 90} \quad 23{\cdot}3\%.$$

(*e*) Impedance facing each valve $= \dfrac{1{,}700-300}{0{\cdot}15-0{\cdot}03} = 11{,}650$ ohms.

(*f*) Impedance of whole primary $= 2 \times 11{,}650 = 23{,}300$ ohms.

Impedance ratio of output transformer $= 23{,}300 : 1{,}000$ ohms. The above load line should be drawn on the DA30 valve field in Fig. 1/IX:5.]

4. A push-pull class B audio-frequency amplifier is to contain two valves and to work into a resistive impedance of 1,000 ohms. The H.T. is 1,000, and the maximum linear range of the chosen load line extends to the point $e_a = 400$ volts, $i_a = 280$ mA. If the amplifier is correctly designed from these data, standing feed being neglected, what is : (*a*) the maximum sinusoidal power output from the amplifier, (*b*) the impedance facing each valve, (*c*) the impedance ratio of the output transformer, whole primary to whole secondary ? (*d*) The efficiency when delivering maximum output ?

[A. (*a*) $P_{max.} = \frac{1}{2}(1{,}000 - 400) \times 280 \times 10^{-3} = 84$ watts.

(*b*) Impedance facing each valve $= \dfrac{1{,}000-400}{0{\cdot}28} = 2{,}140$ ohms.

(*c*) Impedance ratio of the output transformer
$= 4 \times 2{,}140 : 1{,}000 = 8{,}560 : 1{,}000$ ohms.

(*d*) $\eta_{max.} = \dfrac{\pi}{4} \times \dfrac{600}{1{,}000} = 47\%.$]

5. A push-pull class B *tuned* amplifier is designed for the same conditions, valves and load line, etc., as in example 4. Answer the same questions as in example 4, the output transformer being replaced by the output coupling or matching circuit.

[A. (*a*) $P_{max.} = 2 \times \frac{1}{2} f \hat{e}_a i_p$ and $f = \frac{1}{2}$ for $\theta = 180°$ corresponding to class B. Hence $P_{max.} = \frac{1}{2}(1{,}000-400) \times 280 \times 10^{-3} = 84$ watts.

(*b*) Impedance facing each valve is $Z_c \quad \dfrac{Z_L}{f} = 2 \times \dfrac{600}{0{\cdot}28}$
$= 4{,}280$ ohms.

(*c*) Impedance ratio of output coupling $= 2 \times 4{,}280 : 1{,}000$
$= 8{,}560 : 1{,}000$ ohms.

(*d*) $\eta_n \quad = \frac{1}{2} u \dfrac{\hat{e}_a}{E}$ and $u = \dfrac{\pi}{2}$ for $\theta = 180°$

hence $\quad \eta_{max.} = \dfrac{\pi}{4} \times \dfrac{600}{1{,}000} = 47\%.$]

6. If the class B tuned amplifier in example 5 is used to amplify a modulated wave, what is (*a*) the peak current and anode peak volts in the carrier condition ? (*b*) the power output P_0 and the efficiency η_0 in the carrier condition ? (*c*) the driven feed in the carrier condition ? Have you any comments on the performance ?

[A. (*a*) $i_p = 140$ mA, $\hat{e}_a = 300$ volts.
(*b*) $P_0 = 21$ watts, $\eta_0 = 23{\cdot}5\%$.
(*c*) Driven feed $= i_p/\pi = 44{\cdot}5$ mA.

Comment. The efficiency in the carrier condition is low and would not be tolerated on a high-power valve. It would be increased either by driving the valve harder, if grid-current linearity and total emission permit. If not, the efficiency would be increased at the expense of power output by working the valves into a higher load impedance.]

·7. Design a push-pull class C amplifier using two 4030C valves with characteristics as shown in Fig. 2/X:11 to operate on 12,000 volts H.T., the peak anode current to be 24 amps., the peak anode

volts to be 10,000, and the power output to be 93·5 kW. What quantities must be determined and what are their values?

[A. (*a*) θ, u and g, obtained from f by entering f in Fig. 1/X:22 f obtained as follows: Power output $= 93{,}500$ watts $= f \times 10{,}000 \times 24$. Hence $f = 0{\cdot}39$ and $\theta = 120°$, so that $u = 1{\cdot}79$ and $g = 0{\cdot}217$.

(*b*) Driven feed per valve $= gi_p = 0{\cdot}217 \times 24 = 5{\cdot}21$ amps.

(*c*) Anode dissipation per valve $= 5{\cdot}21 \times 12{,}000$ watts minus $\frac{1}{2} \times 93{\cdot}5$ kW $= 15{\cdot}75$ kW.

(*d*) Impedance facing each valve $= \dfrac{Z_L}{f} = \dfrac{10{,}000}{0{\cdot}39 \times 24}$

$= 1{,}070$ ohms.

(*e*) Push-pull impedance facing anodes

$= 2 \times 1{,}070 = 2{,}140$ ohms.

(*f*) Anode efficiency $= \dfrac{93{\cdot}5 \times 10^3}{12{,}000 \times 2 \times 5{\cdot}21} = 74{\cdot}8\%$.

(*g*) Maximum positive excursion of grid $= E_{gp}$.

From Fig. 2/X:11, plotting the point: anode volts $= E - \hat{e}_a$ $= 12{,}000 - 10{,}000 = 2{,}000$, anode current $= i_p = 24$ amps.: $E_{gp} = +1{,}200$ volts.

(*h*) Value of μ at zero anode current for anode voltage swing from H.T.: to two thirds of H.T.

From Fig. 2/X:11, $\mu = \dfrac{14{,}000 - 10{,}000}{100} = 40$. This value of u is not obtained from exactly the prescribed region; but is taken from points where all voltages are clearly defined.

(*i*) Cut-off bias at 12,000 volts $= E_c$. From Fig. 2/X:11, $E_c = 280$ volts.

(*j*) Value of grid bias E_b. From equation (17)/X:25

$$E_b = \frac{280 + \left(1{,}200 - \dfrac{10{,}000}{40}\right)\cos 60°}{1 - \cos 60°} = \frac{280 + 950 \times 0{\cdot}5}{0{\cdot}5}$$

$= 1{,}510$ volts.

$\hat{e}_g = E_b + E_{gp} = 2{,}710$ volts. The grid-to-grid drive is therefore 5,420 volts.]

8. What is the peak grid current in example 7?

[A. Peak grid current occurs when the grid is maximum positive $= +1{,}200$ volts, and the anode is least positive $= +2{,}000$ volts. From Fig. 1/X:11 the peak grid current is just over 4·8 amps., which is a third of the peak anode current. The rule that the peak filament emission (in this case $= 24 + 4{\cdot}8$ amps.) should not exceed

0·9 of the total emission applies. As the total emission of the 4030C valve is 45 amps., there is more than enough margin.]

9. How would you determine the grid dissipation in example 7 ?

[A. The instantaneous positive excursion of the grid is $\hat{e}_g \sin \omega t - E_b = 2{,}710 \sin \omega t - 1{,}510$. The instantaneous value of anode volts = H.T. $- \hat{e}_a \sin \omega t = 12{,}000 - 10{,}000 \sin \omega t$. By putting ωt successively equal to 0°, 20°, 40°, etc., a series of corresponding instantaneous values of grid and anode volts are obtained. By entering these in Fig. 1/X:11, corresponding instantaneous values of grid current are obtained and plotted against the corresponding value of ωt. The mean height of this curve averaged over 360° gives $\bar{i}_g$, the mean value of the grid current. The grid dissipation is then given by $\hat{e}_g \bar{i}_g$, and of course should not exceed the rated grid dissipation of the valve which for the 4030C valve is 1·5 kW.]

10. What advantages result from using push-pull (*a*) in an audio-frequency amplifier amplifying a band of frequencies, (*b*) in a class C tuned R.F. amplifier, (*c*) in a class B tuned R.F. amplifier for modulated waves, (*d*) in an anode modulated class C tuned R.F. amplifier ?

[A. (*a*) Reduction of second-order harmonics and consequent reduction of distortion, the possibility of working valves in class B, cancellation of magnetic effect of anode feed in primary winding of output transformer and consequent avoidance of expensive chokes and coupling condensers.

(*b*) Reduction of second-order harmonics of the driving frequency, and, if used in a radio transmitter, the reduction of the amount of harmonic suppression circuits.

(*c*) as (*b*) and also reduction of second-order harmonics in the envelope of the modulated wave.

(*d*) as (*b*) only.]

11. (*a*) Can the design principles of tuned R.F. amplifiers be applied to audio-frequency amplifiers ?

(*b*) Can the principles of design of audio-frequency amplifiers be applied at radio frequency ?

[A. (*a*) Certainly, if an audio-frequency amplifier is required to amplify a single frequency only, or a very narrow band of frequencies.

(*b*) These principles *are* applied when it is required to amplify a band of radio frequencies, but special types of coupling have to be used between valves to secure as wide a band-width as possible having regard to the valve capacities. See VII:14.31.]

12. A push-pull R.F. amplifier for modulated waves is to operate on 10,000 volts H.T., is to use one valve only on each side of the

push-pull, and is to supply 100 kW of unmodulated carrier with a valve anode efficiency of 35%. If the peak grid current is assumed to be 30% of the peak anode current, what peak emission and anode dissipation is required from the valves, and, assuming the amplifier to be correctly designed, what will be the impedance looking into the push-pull circuit facing the valve anodes, as measured on an R.F. bridge?

[A. The valve anode efficiency in the carrier condition $= \frac{1}{2}u\frac{\hat{e}_a}{E}$ $= 0{\cdot}35$ where $E = 10{,}000$ and $u = 1{\cdot}57$, since the amplifier is for modulated waves and is therefore a class B amplifier. Hence $\hat{e}_a = \dfrac{0{\cdot}35 \times 20{,}000}{1{\cdot}57} = 4{\cdot}470$. The amplifier power output in the carrier condition $= 2 \times \frac{1}{2}f\hat{e}_a\hat{i}_p = 0{\cdot}5 \times 4{,}470\hat{i}_p = 100{,}000$ watts. Hence $\hat{i}_p = 44{\cdot}9$ amps. in the carrier condition, and at 100% p.p.m. $\hat{i}_p = 89{\cdot}8$, so that the required peak emission per valve is $89{\cdot}8 \times 1{\cdot}3$ $= 117$ amps. The required anode dissipation per valve is $\dfrac{50}{0{\cdot}35} - 50$ $= 94{\cdot}3$ kW. The impedance facing one valve $= \dfrac{\hat{e}_a}{f\hat{i}_p} = \dfrac{4{,}470}{0{\cdot}5 \times 44{\cdot}9}$ $= 198$ ohms. The impedance looking out from anode to anode $= 2 \times 198 = 396$ ohms.]

13. A load line drawn on a valve field has a slope of 1,000 ohms. If this valve is to work in a push-pull amplifier, what should be the impedance of the push-pull circuit facing the valve anodes in order that the valve should work along this load line (*a*) in the case of a class A amplifier; (*b*) in the case of a class B amplifier—untuned; (*c*) in the case of a tuned class B amplifier; (*d*) in the case of a class C amplifier with an angle of current flow of 120°?

[A. (*a*) 2,000 ohms. (*b*) 4,000 ohms. (*c*) 4,000 ohms.
(*d*) 5,130 ohms $= 1{,}000/0{\cdot}39 \times 2$, since the value of f is $0{\cdot}39$.]

14. If the class C amplifier of example 7 is to be driven as an inverted amplifier, what power must the driver stage supply if the working conditions of the valves in the main amplifier are to be kept the same as those in example 7. What will be the push-pull impedance of the circuit facing the anodes as measured on an R.F. bridge?

[A. The power input to the push-pull cathode circuit $= \hat{e}_g\hat{i}_f = \hat{e}_g f\hat{i}_p = 2{,}710 \times 0{\cdot}39 \times 24 = 25{\cdot}3$ kW.

The power input to the grid should be calculated by the method given in example 9. The power to be supplied by the driver stage is then equal to the power input to the grid plus 25·3 kW. The push-pull impedance facing the anode circuit

$$= 2{,}140(1 + 2{,}719/10{,}000) = 2{,}720 \text{ ohms.}$$

The power output $= 93{\cdot}5(1 + 2{,}710/10{,}000) = 119$ kW.]

CHAPTER XI

OSCILLATORS

1. Early Views on Oscillators.

THE behaviour of an oscillator can be viewed in more than one way and it is not possible to say that these ways are all applicable to all types of oscillator. In the following, several ways of regarding the working of oscillators will be explained, with some indication of which methods are particularly applicable to any given case.

One of the earliest man-made contrivances which operated on a principle analogous to certain types of oscillator was the pendulum; a later development was the balance-wheel of a watch.

The balance-wheel of a watch consists of a small lightly pivoted wheel connected to a spiral spring in such a way that as the wheel rotates in either direction from its equilibrium position it exerts a force on the spring deforming it from its rest configuration and causing it to coil or uncoil. In either case, when the spring is deformed potential energy is stored in the spring. If the wheel is displaced from its equilibrium position and released, the spring exerts a force on the wheel, causing it to rotate so that the potential energy in the spring is converted to kinetic energy in the wheel. This transference of energy continues until the spring assumes its equilibrium form and position, after which the wheel continues to rotate under the impetus of its momentum, deforming the spring in the opposite direction from the original deformation and transferring its own kinetic energy to potential energy in the spring. This transference of energy continues until all the kinetic energy in the wheel is expended and the wheel comes to rest. The (half) cycle of events is then repeated in the opposite direction until the wheel comes to rest in a position just short of the position of initial displacement. If there were no friction the wheel would come exactly back to its position of initial displacement and the cycle of events would be repeated indefinitely. In practice, friction exists and the wheel is maintained in continuous oscillation by providing an escapement mechanism which serves the twofold function of supplying the energy lost in friction (which is of interest to the present discussion) and of communicating a step-by-step movement to part of the watch mechanism synchronous with the oscillation of the balance-wheel (this is not of interest to the present discussion).

It is important to note that the escapement maintains the wheel

in oscillation by subjecting it during suitable intervals to impulses which impart an acceleration to the wheel in the same direction as its velocity. These impulses during their period of operation are in the same sense as that of a sinusoidal force in phase with the wheel movement, which is also substantially sinusoidal in form, except for the modification of movement caused by the impulses.

If a charged condenser C is connected to an inductance L of low resistance, a closely analogous cycle of events occurs. The condenser discharges through the inductance, building up a current in the inductance until the condenser is completely discharged. During this process, energy in the dielectric of the condenser is transferred to the magnetic field of the inductance. When the condenser is completely discharged, current continues to flow in the same direction through the inductance with the result that the energy in the magnetic field of the inductance is transferred to the electric field of the condenser. This process continues until the energy in the magnetic field is completely transferred to the condenser, and the condenser is charged in the opposite sense from its original charge. After this, the (half) cycle of events is repeated in the opposite direction until the condenser is charged in the initial sense and to nearly the same degree. If there were no resistance in the circuit, and no dielectric loss, the final charge on the condenser would be equal to the initial charge, and the cycle of events would be repeated indefinitely. In practice there is attenuation, and the oscillation dies away according to the law:

$$e = \hat{e}_0 e^{-\frac{R}{2L}t} \sin \omega t \qquad (1)$$

$$i = \hat{i}_0 e^{-\frac{R}{2L}t} \sin \omega t \quad . \quad . \quad . \quad . \qquad (2)$$

where e and i are respectively the instantaneous values of voltage across inductance and capacity, and current in any part of the circuit.

$\hat{e}_0$ and $\hat{i}_0$ are the peak values of voltage and current at time $t = 0$

e = the base of Naperian logarithms = 2·71828

R = the resistance of the circuit in ohms

L = the value of inductance in Henrys

t = the value of time in seconds

$\omega = 2\pi f$

f = the natural frequency of the circuit defined by

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \quad . \quad . \quad . \qquad (3)$$

It is to be noted that the natural frequency of the circuit is different from the resonant frequency of the circuit. The natural frequency of the circuit is normally of very little importance and only acquires practical importance, if any, in the case of the shock excitation of circuits such as may occur in the tuned circuits of receivers due to atmospherics, or Johnson noise.

It may be remarked in passing, that equations (1) and (2) define a sinusoidal oscillation dying away logarithmically with time. The quantity $\frac{R}{2L}$ which determines the rate of decay of current and voltage in the circuit may be called the decay index of the circuit. This quantity also only acquires practical importance in special cases. (See also VI:5.)

By putting $R = 0$ in equation (3)

$$f = \frac{1}{2\pi\sqrt{LC}} = \text{the resonant frequency of the circuit,}$$

and the decay index, $$\frac{R}{2L} = 0$$

In other words, reference to equations (1) and (2) shows that if R is made equal to zero, the circuit oscillates steadily at its resonant frequency without any decay of current and voltage: the oscillation is maintained at steady amplitude.

The simplest way to neutralize the resistance R is to inject into the resonant circuit composed by L and C an e.m.f. $\hat{e}_f \sin \omega t$, which equals and opposes the back e.m.f.s set up by the circulating current in the resonant circuit in flowing through R. Under these conditions, no net back e.m.f. is set up round the circuit due to L and C, since the back e.m.f.s due to L and C are always equal and opposite. It is of course true that the percentage of R contained in L may be greater than that in C, so that the voltages *across* L and C are not necessarily equal; this does not invalidate the previous statement.

The neutralization of R therefore requires the injection into the closed circuit constituted by L and C of a sinusoidal e.m.f. in phase with the circulating current through L and C. The normal method of trying to do this is to connect L and C, the combination of which will henceforward be called the *oscillatory circuit*, to the input of an amplifier, and couple the output of the amplifier to the oscillatory circuit in any convenient manner so that the output of the amplifier injects the required e.m.f. into the oscillatory circuit. Such an arrangement is indicated in Fig. 1.

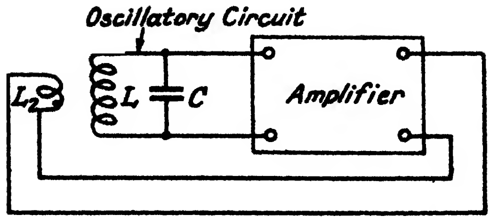


FIG. 1/XI:1.—Basic Principle of Elementary Oscillator.

The output of the amplifier is connected to L_2, which is coupled to L, the inductance of the oscillatory circuit. When an oscillation is started in the oscillatory circuit by any means, it applies a sinusoidal e.m.f. to the input of the amplifier, with the result that the output of the amplifier passes a current through L_2 and injects a sinusoidal e.m.f. into L. If this e.m.f. is in phase with the current in the oscillatory circuit, one of three things may happen:

(*a*) If the e.m.f. induced in L is less than the back e.m.f. due to the resistance R of the oscillatory circuit, the oscillation will decrease in amplitude, and if the condition persists, will die away altogether.

(*b*) If the induced e.m.f. equals the back e.m.f. due to R, the oscillation will be maintained.

(*c*) If the induced e.m.f. is greater than the back e.m.f. due to R, the oscillation will build up.

It is immediately evident that, if R and the amplifier gain are constant and independent of current or voltage amplitude, the problem of exactly adjusting the feedback to provide the correct e.m.f. is impossible of solution: the oscillation will either die away or build up until something limits it.

In practice, in all oscillators the amount of feedback is adjusted so that for small amplitudes of oscillation the oscillation tends to build up. As the oscillation increases in amplitude, the losses introduced in the amplifier circuit (owing to a fall in the effective amplification factor of valves with increase of grid swing, and due especially to grid current limitation) drop the gain of the amplifier, and the oscillation continues to build up until the amplifier gain and consequent feedback have fallen to such values that the resistance of the oscillatory circuit is exactly neutralized: the oscillator then settles down to deliver an output of steady amplitude.

It should be noted that when the e.m.f. induced into the oscillatory circuit is equal to the back e.m.f. due to the resistance, the voltage induced into L is equal to the voltage which is causing the current to

flow, and the voltage amplification round the loop constituted by oscillatory circuit, amplifier and feedback circuit, is unity.

It will be noticed that, since the e.m.f. induced in L is in phase with the current in L, and the oscillatory circuit presents an impedance which is a pure resistance at the point where the induced e.m.f. is injected, *the phase shift round the loop circuit constituted by the oscillatory circuit, amplifier and feedback is zero.*

Hence, if the phase shift round the circuit is zero at the resonant frequency of the oscillatory circuit, the system will oscillate at the resonant frequency of the oscillatory circuit.

What happens if the phase shift round the circuit is not zero at the resonant frequency of the oscillatory circuit ? *The answer is, that the circuit will still oscillate, but at the frequency at which the phase shift round the circuit is zero. This of course will only happen if the voltage amplification round the loop is unity : if the loop amplification is less than unity the circuit will not oscillate.*

Since the e.m.f. fed back by the feedback circuit neutralizes the resistance R, the feedback circuit is sometimes said to introduce into the oscillatory circuit negative resistance equal to R.

Reference to *Radio Engineering* by F. E. Terman, shows that earlier designers of oscillators attached great importance to the necessity for making an oscillator oscillate at the resonant frequency of the oscillatory circuit in order to secure constancy of frequency. The arguments are rather difficult to follow and do not appear to be relevant to the present discussion. There are, however, types of oscillator in which, other things being equal, this is true : e.g. in any oscillator in which the change of loop phase shift with frequency, is a maximum at the resonant frequency of the oscillatory circuit. In general, the more rapidly loop phase shift varies with frequency the greater the frequency stability of the oscillator. This follows because, if the phase shift varies due to any circuit variation (H.T. volts grid bias, etc.), the frequency has only to shift a small amount to restore the phase shift round the loop to zero.

1.1. Standard Oscillator Circuits. Fig. 2 shows a number of standard oscillator circuits in use which can be used where a frequency stability of less than 1 in 1,000 is adequate. By taking the special precautions described in the next section this may be improved to 1 in 20,000 without temperature control.

All supply circuits, including chokes and stopping condensers, are omitted. The oscillators in most common use are the Hartley, the Tuned Anode and the Tuned Anode with Resistance Feedback.

The tuned-grid tuned-anode oscillator is one which sometimes

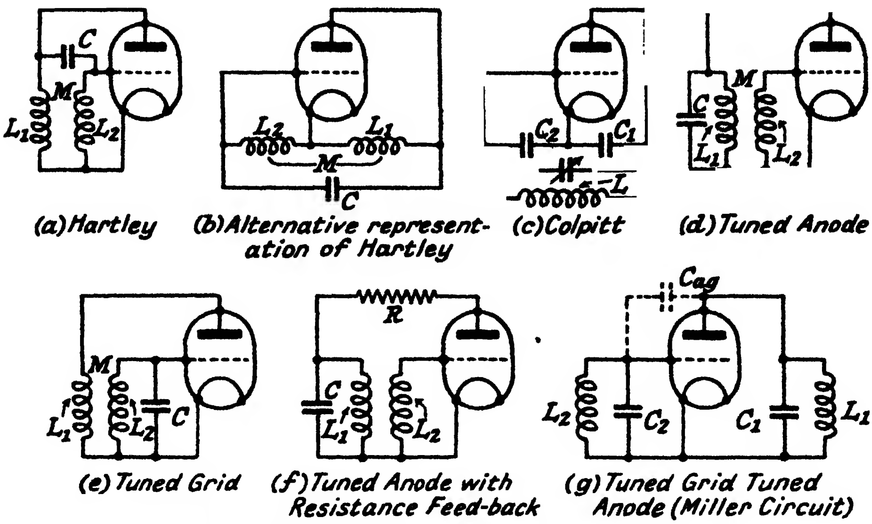


FIG. 2/XI:1.—Fundamental Oscillator Circuits.

occurs in high-frequency amplifiers when it is not wanted, owing to back coupling introduced through the anode grid capacity C_{ag}.

Where a higher stability is required than is afforded by the simple circuits of Fig. 2, it is necessary to apply the technique described in XI:1.2 to 4 below.

1.2. Frequency Stabilization of Oscillators.

Provision of Separator. The most serious cause of frequency variation in an oscillator is variation of the impedance of the output load which introduces a variable shunt across the loop path, and so shifts the frequency. To prevent this it is essential in all types of oscillator to provide a separate valve between the oscillator and the load. This is a class A amplifier with its grid circuit tapped across a low impedance point of the *grid circuit* of the oscillator. The *grid circuit* is chosen, because the wave form here is more free from harmonics than in the anode circuit. A low impedance point is chosen because any impedance variations reflected through the separator valve (which, small as they are, may be of appreciable importance) are in this way made less effective in causing frequency shift.

Fig. 3 shows a tuned anode oscillator with its H.T. supply circuit and a separator stage. V_1 is the oscillator valve and V_2 is the separator valve, which has been shown as a triode, but in practice a pentode would be used on account of its lower anode grid capacity. L_1C_1 is the frequency determining network and C_2, C_3 and C_4 are stopping condensers of low reactance compared with the associated

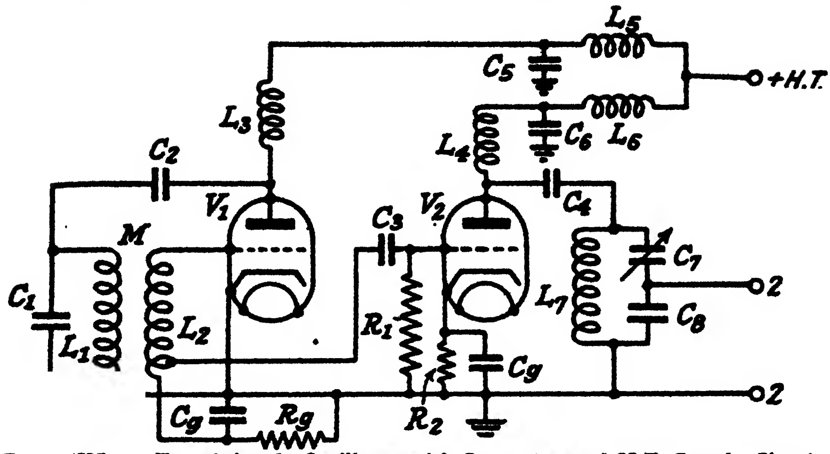


FIG. 3/XI:1.—Tuned Anode Oscillator with Separator and H.T. Supply Circuits.

impedances. R_1 is a grid leak, R_g is an automatic (grid current) bias resistance, C_g is a decoupling condenser, L_3 and L_4 are chokes. L_5 and L_6, C_5 and C_6 provide individual filtering for the H.T. to each stage : this is important. L_7, C_7 and C_8 constitute a capacity tap output circuit providing a step down of impedance from the valve to the output terminals 2,2.

It will be noted that the loop phase shift is affected by the reactance of the supply circuit constituted by C_2 and L_3, and to a less extent by L_5 and C_5, and by R_1 and C_3, and the input impedance of the separator valve.

C_g and R_g constitute a leaky condenser providing grid-current bias. As the oscillation builds up the bias increases, and so reduces the loop amplification and limits the amplitude of oscillation. Such an arrangement may be used with any of the circuits of Fig. 2.

This is a very important point : in all oscillators some deliberate method of limiting the amplitude of oscillation should be introduced. This method incidentally is very far from being the best, since any change in valve parameters may change the grid current, and so change the impedances bridged across L_2 and the phase round the circuit, and hence the frequency. An improved method of limitation is described in XI:3.

The classical method of operating the circuits of Fig. 2 is with zero grid bias, so that the flow of grid current introduces a resistance shunt across the grid coil, which falls in value (owing to rise of grid current faster than in direct proportion to the voltage) with increase of grid swing, and so introduces a varying phase shift. The immediate effect of this is that as the resistance so introduced varies,

the phase shift round the circuit changes, and the frequency of oscillation shifts continually as the oscillation builds up. When the oscillation has settled down to a steady amplitude no further frequency shift occurs, unless the grid current changes. Amplitude limitation in this way by means of grid current should not be used.

Stabilization of H.T. The most serious change of grid current occurs owing to change of the H.T. volts on the anode. Hence, to maintain stability of frequency, the H.T. volts on the anode must be kept constant. A secondary effect of change of H.T. volts is a change in the amplification factor of the valve with a consequent change in effective grid input capacity, see XXIII:9.2, which again introduces a change in phase shift round the circuit.

Stabilization of Filament Volts. In the case of directly heated filaments, the grid current is affected by change of filament volts, so that stabilization of filament volts is desirable. In the case of valves with separately heated cathodes, the changes of filament volts, which occur from normal supply sources: batteries, mains and machines, are unimportant.

Voltage Step Down from Anode to Grid. The effect of grid current variation on frequency can be further reduced by introducing a voltage step down from anode to grid of such magnitude that the voltage amplification round the circuit for incipient oscillation is very little in excess of unity. This also has the effect of reducing harmonics in the oscillator output.

Low Impedance facing Grid. A further measure is to make the grid coil of as low a reactance as is consistent with the maintenance of oscillations. This reduces the effect of variations in grid input impedance of the valve, and imposes the same circuit requirements as the last measure.

Resistive Impedance facing Grid. A measure which has not been generally accepted, of which the success is limited by grid input capacity, is to insert a resistance attenuator between the grid coil and the grid, so that as far as possible the variable limiting resistance, introduced by the grid current, is introduced across a non-reactive circuit.

Resistance Feedback. The circuit of Fig. 2(*f*)/XI:1 was at one time credited with a degree of frequency stability superior to the other circuits of Fig. 2/XI:1. The reasons for this are not clear, but are probably connected with the fact the feedback resistance R, which was made adjustable, provided a convenient method of introducing a voltage step down from anode to grid. In practice, using mica condensers and air-core coils, observations on an oscillator of

this type showed it to have a frequency variation of 0·05% due to battery variations and valve replacements, and a frequency shift of 0·01% per degree Centigrade change in room temperature. When fitted with dust-core coils the corresponding figures were 0·2% and 0·075% respectively. The performance of dust-core coils of course varies from one type to another, so that the only useful information that can be obtained from these figures is that the frequency stability with dust-core coils is less than that with air-core coils.

The above was an audio-frequency oscillator constructed about 1930. In 1942 an oscillator of the Colpitts type was constructed for use at the B.B.C. station at Daventry, using a porcelain former for the coil and condensers of the sputtered ceramic type, to operate in the neighbourhood of 6 megacycles. This had a frequency stability of $\pm 1/50{,}000$ without temperature control, although none of the other methods of frequency stabilization here enumerated had been applied. It should be mentioned, however, that the mains supply which supplied power to the oscillator was unusually steady.

Use of Anode Limitation instead of Grid Current Limitation. As an alternative to grid current limitation, if a valve with a sufficiently high mutual conductance is used, by providing it with bias as in a normal amplifier and operating the anode into a sufficiently high impedance with adequate voltage and impedance step down to the grid, the valve can be made to run into anode limitation before grid current flows. With such a form of limitation the vagaries due to grid current are absent. H.T. stabilization is however still necessary.

Stabilization of Elements in Oscillatory Circuit. Another source of variation is change of inductance and capacity elements with temperature. This is reduced by special design of elements and by temperature control. Air condensers are built of invar (a steel alloy with a low-temperature coefficient of expansion) with a specially rigid type of construction, coils are wound on porcelain formers, or in the case of high-frequency coils, are built of heavy conductors and made self-supporting.

Temperature Control. It is normally not worth while using temperature control with the simple types of oscillator shown in Fig. 2/XI:1, but where this is done, the main elements controlling the phase shift through the circuit are contained in a heat-insulated oven which is maintained at constant temperature by means of a thermostat. (See XI:4.4.)

2. Present Views on Oscillators.

It is assumed that the reader is familiar with the subject matter of the preceding section, which is important and relevant to the discussion below, quite apart from the fact that the era of usefulness of the types of oscillator described is by no means ended.

An oscillator consists of three essential parts : an amplifier with feedback, a phase shift device, and a limiting device at any convenient points in the loop path. *Oscillation occurs at the frequency at which the phase shift round the loop path is zero.*

The phase-shift device must have as high a variation of phase shift with frequency as possible ; in other words, the phase-shift-frequency slope must be high. Further, the phase shift must be constant with time and independent of temperature and battery voltages applied to the amplifier. The reason for the high slope of the phase shift device is a simple one. Under normal conditions of steady oscillation the phase shift round the loop circuit at the oscillatory frequency is zero. If for any reason, due to change of battery voltages, change of temperature, or any other cause, the phase shift in any other part of the circuit changes, the frequency shifts until the change in phase shift round the circuit is again zero. If the rate of change of phase shift in the circuit, with frequency change, is large, the frequency only has to shift a small amount to restore the condition of zero loop phase shift. Hence, the higher the phase-shift-frequency slope the more stable the oscillator.

The amplitude limiting device must be so arranged as to limit without changing the phase shift in the circuit. A method of doing this which has come into use consists in introducing a variable μ valve in the loop circuit, and in controlling the bias of this valve by the amplified and rectified output of the oscillator. Evidently the simplest arrangement is to use a variable μ valve as the oscillator valve, and to amplify and rectify the output of the oscillator, feeding back the rectified D.C. voltage to the grid circuit of the oscillator valve in such sense that as the output increases the grid becomes more negative and the gain of the valve drops.

2.1. Series Resonance Oscillator. Before giving an example of an oscillator embodying the above principles, it is necessary to consider some improvements in oscillator design due to G. G. Gouriet, which are aimed at reducing the effect of variation of valve parameters on frequency. Briefly, these are twofold : the elimination of Miller capacity by the use of cathode feedback ; and the use of a circuit in which the input to and the output from the valve

are shunted with low reactances which tend to mask any variation in valve impedances.

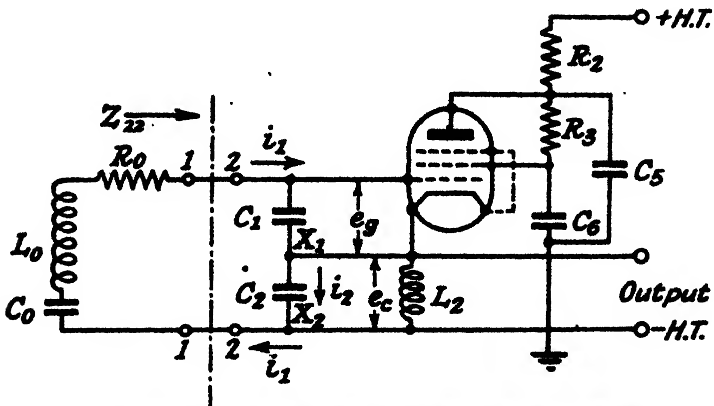


FIG. 1/XI:2.—Cathode Feedback Amplifier with Series Resonant Circuit.

The circuit principle is illustrated in Fig. 1 in which a *series* oscillatory circuit constituted by L_0 and C_0 is used, R_0 representing the resistance in L_0. The condensers marked C_1 and C_2 are condensers of as low reactance as is consistent with the condition for oscillation derived below. L_2 is an inductance of reactance large compared with the reactance of C_2 and having a low D.C. resistance. R_2 and C_5 provide H.T. filtering, R_3 and C_6 provide screen bias and decoupling. The output is taken from L_2. It will be noted that the major differences between this circuit and the circuit of Fig. 2/XI:1 are: the use of a series oscillatory circuit, which enables the circuit connected to the valve to be maintained at low impedance, and the fact that C_5 holds the valve anode at ground potential.

The circuit can be considered to be maintained in oscillation because the valve and its associated circuits creates an impedance Z_{22} looking into 2,2 (with the circuit on the left removed), which is constituted by a reactance in series with a *negative* resistance. The magnitude of the impedance Z_{22} can be derived as follows:

Let $-X_1$ = the reactance of condenser C_1.
$-X_2$ = the reactance of condenser C_2.
e_{22} = the voltage across 2,2.
i_2 = the current through C_2 flowing in the cathode circuit of the valve with sense as indicated. The reactance of L_2 is so high as to be negligible.
e_g = the voltage between grid and cathode.

e_c = the voltage of the cathode above ground = the voltage across C_2.

g_m = the mutual conductance of the valve.

If a sinusoidal current i_1 flows through terminals 2,2

$$e_g = -jX_1L_{22}$$
$$i_2 = e_g g_m = -jX_1 i_1 g_m$$

The reactance of L_2 is made very large compared to X_2, so that :

$$e_c = -jX_2(i_1 - jX_1 i_1 g_m)$$
$$e_{22} = e_g + e_c = (-jX_1 - jX_2 - X_1X_2g_m)i_1$$

therefore the impedance looking into 2,2 is

$$Z_{22} = \frac{e_{22}}{i_1} = -jX_1 - jX_2 - X_1X_2g_m \qquad (4)$$

That is, the impedance facing the oscillatory circuit is equal to the series impedance of X_1 and X_2 in series with a negative resistance of magnitude $X_1X_2g_m$. When this negative resistance is equal to R_0 the circuit is maintained in stable oscillation. If greater than R_0, the oscillation builds up ; if less than R_0, the oscillation decays. The system will oscillate at a frequency f, such that :

$$\left(jL_0\omega + \frac{1}{jC_0\omega}\right) - jX_1 - jX_2 = 0 \qquad . \qquad (5)$$

$$j\left[L_0\omega - \frac{1}{\omega}\left(\frac{1}{C_0} + \frac{1}{C_1} + \frac{1}{C_2}\right)\right] = 0$$

$$\omega^2 = \frac{1}{L_0}\left(\frac{1}{C_0} + \frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$= \frac{1}{L_0C_0}\left(1 + \frac{C_0}{C_1} + \frac{C_0}{C_2}\right)$$

$$f = \frac{1}{2\pi\sqrt{L_0C_0}}\sqrt{1 + \frac{C_0}{C_1} + \frac{C_0}{C_2}} \qquad . \quad . \quad . \qquad (6)$$

$$= f_0\sqrt{1 + \frac{C_0}{C_1} + \frac{C_0}{C_2}} \qquad . \qquad (7)$$

where f_0 is the resonant frequency of L_0 and C_0.

In the practical oscillator shown in Fig. 1/XI:3 below, the mean value of C_0 is 260 $\mu\mu$F, the value of C_1 is very nearly 3,000 $\mu\mu$F, while the value of C_2 may be 6,000, 10,000 or 16,000 $\mu\mu$F.

Taking the lowest value of C_2,

$$\frac{C_0}{C_1} = 0{\cdot}087, \text{ and } \frac{C_0}{C_2} = 0{\cdot}043$$

Therefore $$f = f_0\sqrt{1.13} = 1{\cdot}06 f_0$$

The oscillation frequency is of course determined as before by the frequency at which the loop phase shift is zero. In view of the form of the analysis, it is convenient in this case to use also an alternative criterion which is expressed by equation (5): the frequency will always adjust itself so that the reactance facing the oscillatory circuit at 2,2 neutralizes the reactance of the oscillatory circuit. It is assumed that the reactance frequency slope of Z_{22} is negligibly small. Such being the case, the stability increases as S, the slope of the *reactance* frequency curve of the oscillatory circuit, increases. By plotting the value of $L\omega_0 - \frac{1}{C\omega_0}$ it may be checked that the value of the slope is given by

$$S = 2\pi\left(L_0 + \frac{1}{C_0\omega^2}\right) = \text{ohms change of reactance per unit change of frequency, i.e. per c/s} \quad . \quad (8)$$

Hence, the greater the value of L_0 and the smaller the value of C_0 the greater the stability of the circuit.

At the resonant frequency,

$$\omega = \omega_0 \text{ and } L_0 = \frac{1}{C_0\omega^2} \text{ and } S_0 = 4\pi L_0 \quad . \quad . \quad (9)$$

and at f the oscillating frequency

$$S = 2\pi\left(L_0 + \frac{1}{C_0\omega^2}\right) = 2\pi\left(L_0 + \frac{f_0^2}{f^2} \times \frac{1}{C_0\omega^2}\right)$$
$$= 2\pi L_0\left(1 + \frac{f_0^2}{f^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

It will be noticed that the lower the oscillating frequency, the greater the value of S and the better the stability. The optimum stability does not occur by making the circuit oscillate at the resonant frequency, but at the lowest possible frequency; because the phase-shift-frequency slope increases as the frequency is reduced. The stability at the resonant frequency may however be used as a reference of stability. The slope S forms a convenient criterion of the performance of the oscillatory circuit, since all reactance changes are introduced by the circuit to the right of terminals 2,2 in

Fig. 1/XI:2 and a change of dX ohms in the reactance presented at terminals 2,2 to the oscillatory circuit causes a change in frequency equal to dX/S cycles per second.

A relative figure of merit for the stability of the oscillatory circuit is then given by the ratio S/S_0. This is unity if the oscillator oscillates at the resonant frequency of the oscillatory circuit, less than unity if at a higher frequency, and greater than unity if at a lower frequency.

The absolute figure of merit for the oscillatory circuit is of course S. The value of S/S_0 only indicates how much better or worse the oscillator is than it would be if made to oscillate at the resonant frequency of the oscillatory circuit.

From (9) and (10)

$$\frac{S}{S_0} = \frac{1}{2}\left(1 + \frac{f_0^2}{f^2}\right) \quad . \quad . \quad . \quad . \quad (11)$$

In the case of the oscillator shown in Fig. 1/XI:3 as determined above

$$\frac{f_0^2}{f^2} - \frac{}{1 \cdot 13} = 0 \cdot 885$$

so that

$$\frac{S}{S_0} = \frac{1 \cdot 885}{2} = 0 \cdot 9425$$

This means that if the frequency stability is 100 parts in a million at the resonant frequency, it is 94·25 parts in a million at the oscillating frequency.

It may be remarked that if inductances are substituted for the condensers C_1 and C_2 of equal reactance to X_1 and X_2 respectively, the value of $S/S_0 = \frac{1}{2}(1 + 1 \cdot 06^2) = 1 \cdot 065$. The improvement, in comparison with $S/S_0 = 0 \cdot 94$, is however quite unimportant, and since commercial condensers of good stability are available for use as C_1 and C_2, while the construction of stable inductances is expensive, the use of condensers is preferred.

3. Design and Performance of a Practical Oscillator.

Fig. 1 illustrates the practical design of a variable frequency oscillator, constructed in accordance with the above principles to cover a continuous frequency range approximately from 0·7 to 1·4 Mc/s. This particular oscillator is used in conjunction with a frequency multiplier ($\times$ 4) to provide continuous coverage over the frequency range from 2·8 to 5·6 Mc/s. In order to supply the initial drive for short-wave transmitters, which require a range from

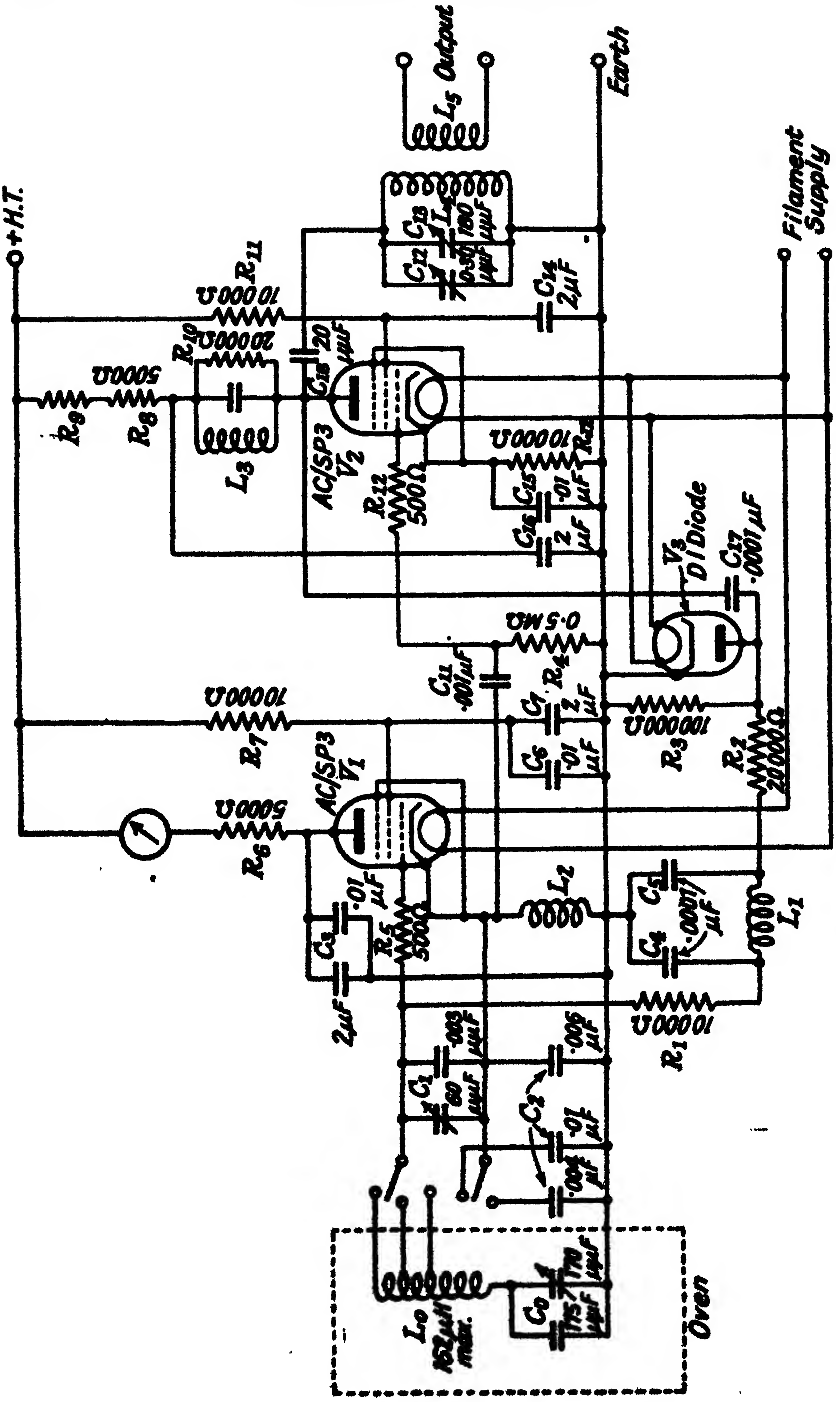


FIG. 1/XI:3.—Variable Frequency Oscillator of High Frequency Stability.
(By courtesy of the B.B.C.)

5·6 to 22·4 Mc/s, a second multiplier is connected to the output of the first multiplier, which is capable of multiplying by 2 or 4 and so supplying the frequency ranges 5·6 to 11·2 Mc/s and 11·2 to 22·4 Mc/s. The oscillatory circuit constituted by L_0 and C_0 is contained in a temperature controlled oven with a temperature stability of approximately $\pm 0{\cdot}2°$ C. for a change of ambient temperature of $\pm 3°$ C.

Loss. The coil constituting L_0 has a Q of approximately 170 at 1 Mc/s when mounted in the oven, and to keep down losses in the oscillatory circuit, connections between L_0, C_0 and the main oscillator circuit are made in low-loss conductors.

Frequency Stability. The frequency stability of this oscillator is as follows:

Variation with time. Short time stability (*1 hour*): $\pm 3/10^6$ after oven has been in operation for at least 48 hours.

Long time stability: $\pm 10/10^6$ after 7 days' operation.

Frequency—Temperature Coefficient of Frequency Determining Network: Approximately $30/10^6$ for $1°$ C. increase in temperature.

Frequency Variation with Variation of Ambient Temperature: $-10/10^6$ for $5°$ C. change in ambient temperature.

H.T.-Frequency Coefficient: $-5/10^6$ for $+40\%$ H.T. voltage change.

L.T.-Frequency Coefficient: $-5/10^6$ for $+10\%$ L.T. voltage change.

Change of Frequency with Change of Valves: oscillator valve: $\pm 15/10^6$ — average of 8 valves. Separator valve: $\pm 13/10^6$ — average of 4 valves.

Change of Frequency with Change of Valve Capacities: *Grid cathode capacity*: $-23/10^6/+1$ $\mu\mu$F. *Cathode-earth capacity*: $-0{\cdot}7/10^6/+1$ $\mu\mu$F.

The Atlantic City tolerance for the carrier frequency stability of short-wave broadcast transmitters is $30/10^6$. The latest form of oscillator in use in the B.B.C., which uses separate coil and condenser units for each frequency range, meets this requirement easily; its frequency stability over a week is $\pm 10/10^6$.

The circuit of Fig. 1 is self-explanatory: V_1 is the oscillator valve and V_2 is an amplifying stage acting as a separator. A point of special interest is the D.C. feedback circuit constituted by the connection from the separator anode through C_{17}, the diode V_3, and the filter composed of R_3, L_1, C_4 and C_5. The output of the filter

supplies a D.C. bias to the grid resistance R_1, which is proportional to the output of the oscillator, and so supplies a limiting action owing to the variation of the amplification factor of V_1 with grid bias.

In the complete oscillator, as used in practice, instead of the separator stage feeding into the output circuit constituted by L_4, L_5, C_{12} and C_{13}, it is capacity coupled to the grid of a third AC/SP3 valve which drives a fourth AC/SP3 valve connected to an output circuit of similar form to L_4, L_5, etc. Both these valves have a lower value of screen volts than the separator stage and are biased nearer to cut-off, so that the percentage of second harmonic in their output is high. The anode circuit of the third valve is tuned to twice the oscillator frequency, while the anode circuit of the fourth valve: the output circuit, is tuned to four times the oscillator frequency. Each of these stages therefore constitutes a frequency doubler, so that the final output is four times the oscillator frequency.

In the latest design of oscillator of this type, instead of tapping the inductance L_6, four frequency ranges are provided by means of four separate inductance-capacity units switched into circuit in turn: each unit then corresponds to the combination of L_6 plus C_6.

Accuracy of Setting. Where high frequency stability is required a high degree of precision is required in the accuracy of setting of the variable condenser C_1 which controls the frequency. For this purpose a high-grade drive for the condenser is essential. A suitable form of drive which has been used is the Muirhead Type D80A.

4. Crystal Oscillators.

Quartz crystals exhibit a phenomena known as piezo electricity, which can best be explained in terms of certain natural axes of the crystal. These axes may be referred to respectively as the optical, mechanical and electrical axes. The crystals are in the form of long prisms of hexagonal cross-section. The unsymmetrical configuration of facets closing each end of the crystal is irrelevant, except in that in complete crystals there is a point at each end.

The optical axis is on a line parallel to the length of the prism drawn through the points at each end of the crystal, and is so called because if a beam of plane polarized light traverses the crystal parallel to this axis, the plane of polarization is rotated.

There are three electrical and three mechanical axes perpendicular to the optical axis. These are illustrated in Fig. 1 (*a*) or (*b*):

XX, $X'X'$, $X''X''$ are the electrical axes. YY, $Y'Y'$, $Y''Y''$ are the mechanical axes.

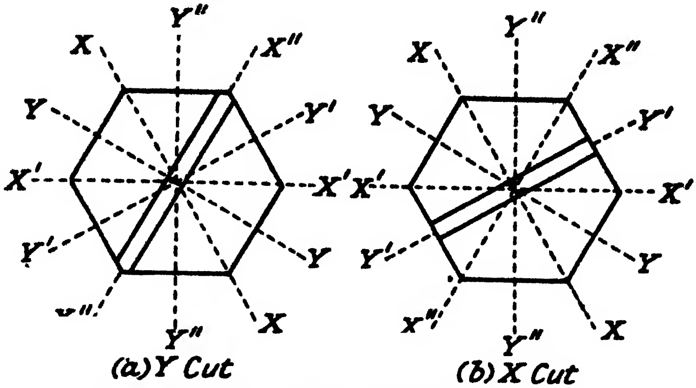


FIG. 1/XI:4.—Cross-sections of Quartz Crystal perpendicular to Optical Axes showing X and Y Cut Crystals. Electrical Axes: XX, $X'X'$, and $X''X''$. Mechanical Axes: YY, $Y'Y'$, and $Y''Y''$.

(By courtesy of the B.B.C.)

Alternating potential differences applied along the electrical axes cause alternating compression and extension to occur along the mechanical axes.

Owing to the high elasticity (stiffness) and low damping of quartz, sections cut from quartz crystals exhibit very sharp mechanical resonance characteristics under the influence of electric forces, applied between electrodes in contact with, or very close to, the faces of these sections.

Fig. 1 (*b*) shows an X-cut section which is cut with its longer faces normal to an X (electrical) axis. The mode of vibration of

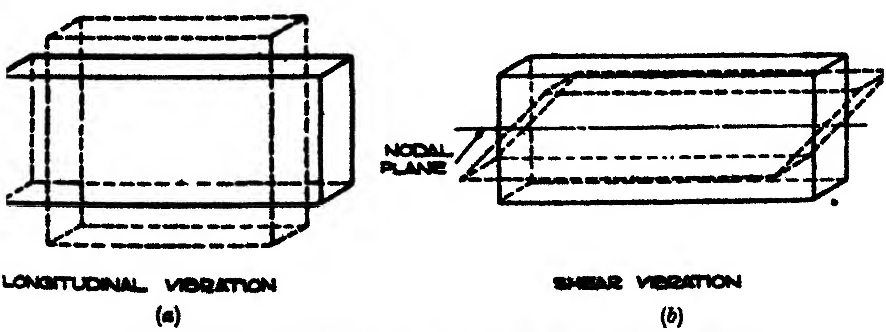


FIG. 2/XI:4.—Illustrating Longitudinal and Shear Vibration.

(By courtesy of the B.B.C.)

such a section under the influence of an A.C. p.d. applied between electrodes on or near its flat faces (i.e. along an electrical axis) is

illustrated in Fig. 2 (*a*). This vibration illustrates the fundamental behaviour of the piezo electric effect.

Fig. 2 (*a*) shows a *Y*-cut section which is cut with its longer faces normal to a *Y* (mechanical) axis. Electrodes applied to the flat faces now produce an A.C. p.d. along a ***mechanical axis.*** The result is a shear vibration as indicated in Fig. 2 (*b*).

The cut which is used in practice is called the A_T cut and is described later. A_T-cut crystals are often clamped in position by means of supports located at the ends of the axis of shear, i.e. on the nodal line.

By Lenz's law the movement of the crystal gives rise to back e.m.f.s opposing the p.d. causing vibration so that the mechanical impedance of the crystal gives rise to a corresponding electrical impedance between the electrodes. The form of the impedance may be simulated by *R*, *L* and *C* in the electrical circuit of Fig. 3 which represents the equivalent electric circuit of a quartz crystal in its holder. In this circuit C_a is the capacity between electrodes and the faces of the crystal section, C_q is the inherent capacity

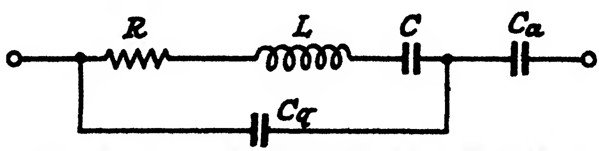


FIG. 3/XI:4.—Equivalent Electrical Circuit of Quartz Crystal Plate in its Holder.

between the faces of the crystal, while *R*, *L* and *C* represent the (electrical) motional impedance of the crystal.

The magnitude of C_a depends on the magnitude of the air-gap between the electrodes and the crystal and on the method by which the crystal is mounted, while the magnitudes of *L*, *C*, *R* and C_q depend on the way the crystal is cut, the size of the crystal and the type of vibration involved.

The effective resistance depends on the method of mounting: a representative value of *Q* for a clamped A_T cut crystal in vacuum is 250,000 at a megacycle, while the *Q* of a high-quality dust-core coil at the same frequency is about 150. The most important feature of the crystal equivalent circuit is, however, the *L* to *C* ratio. Whereas the practically realizable value of *L* in an oscillator operating at a megacycle is certainly below 300 microhenrys, the value of *L* realizable at a megacycle in the equivalent circuit of the A_T crystals in use is about 4 Henrys: more than 10,000 times as great. Since the slope of the reactance frequency curve at the

series resonance frequency of L and C is given by $S_0 = 4\pi L$ (see equation (9)/XI:2) the slope of the reactance curve of a crystal operated at the series resonance frequency may be as much as ten thousand times that of a normal L, C, circuit.

This means that if a crystal is substituted directly for a series resonant circuit in the oscillator circuit of Fig. 1/XI:3, the change in frequency consequent on a change in any of the elements of the circuit other than the crystal, is of the order of 1/10,000th of the change in frequency when a normal L, C, circuit is used. The

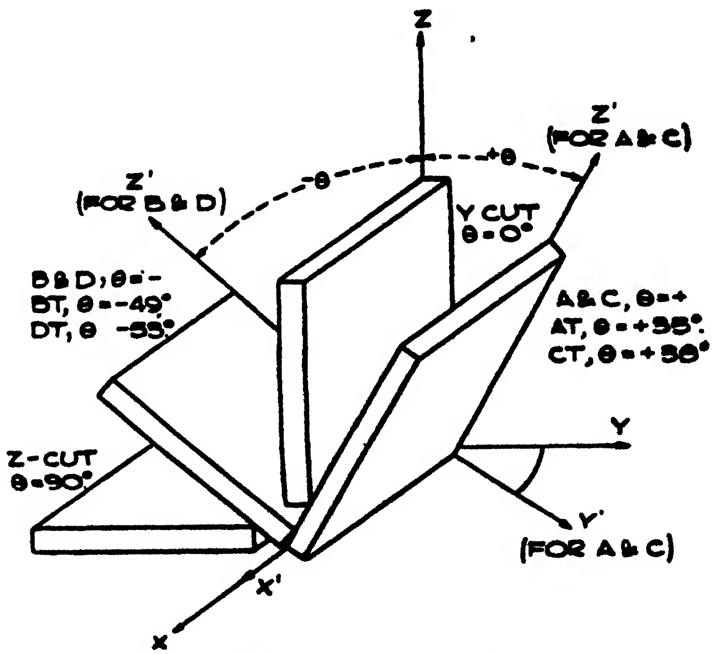


FIG. 4/XI:4.—Planes of AT, BT, CT, and DT Cuts.
(By courtesy of the B.B.C.)

frequency stability of the circuit is therefore determined substantially by the frequency stability of the crystal.

The series resistance of the clamped A_T-cut crystals at a megacycle is in the neighbourhood of 100 ohms.

4.1. The A_T Cut Plate. The resonant frequency of a crystal varies with temperature and in a search for a section which would yield a reduced temperature coefficient, cuts have been made in various planes. The series of cuts which have yielded the most useful practical results are illustrated in Fig. 4. The variation of temperature coefficient with the angle θ of Fig. 4 is shown in Fig. 5.

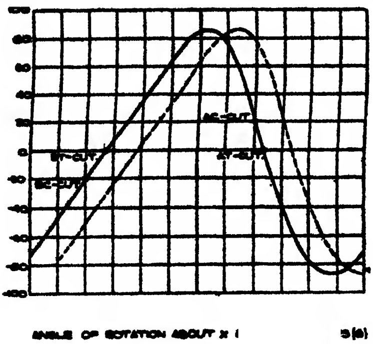


FIG. 5/XI:4.—Variation of Temperature Coefficient with Angle θ.
(By courtesy of the B.B.C.)

It is seen that when $\theta = +35°$ or $-49°$ the temperature coefficient is zero.

Two systems of notation are in use for describing the type of cut used. In England, the National Physical Laboratory and the Radio Section of the Post Office refer to angular Y cuts, in the planes indicated in Fig. 4, as Y_θ cuts, and to plates cut to specific angles for zero temperature coefficient as $Y_{\theta t}$ cuts. In the notation employed by the Bell Laboratories in the U.S.A., modified Y-cut plates have the following description :

Cuts between $\theta = 0°$ and $180°$:

High frequency mode : A cuts. Low frequency mode : C cuts.

Cuts between $\theta = 180°$ and $360°$:

High frequency mode: B cuts. Low frequency mode: D cuts.

Y_θ-cut crystals have two modes of oscillation : a shear mode (the high frequency mode) in which the near face of the plates in Fig. 4 moves parallel to the Z and Z' axis ; and a longitudinal mode along the direction of the X axis in Fig. 4. Hence the terms "high-frequency mode" and "low-frequency mode", above. Plates cut at the specific angles giving zero temperature frequency coefficient are designated by adding the suffix T, hence the term "A_T cuts". In A_T cuts $\theta = +35°$ and in B_T cuts $\theta = -49°$. Normally, coupling occurs between the two modes of oscillation of a plate, but at certain angles, $\theta = 38°$, for instance, this coupling is absent. The cuts where the coupling is zero are distinguished by the suffix C in place of T, e.g. A_C and B_C cuts. The values of θ for the A_T and A_C cuts are more nearly equal than for the B_T and B_C cuts. In practice, therefore, the A_T cut is used. Even so, the inter-mode coupling occurring in all A_T cut plates is still sufficient to make the use of large dimension ratios necessary, and in practice this limits the application of the A_T cut to plates having frequencies greater than 600 kc/s. (The dimension ratio is the ratio of mean length of side to thickness.)

The upper limit of frequency for an A_T cut is about 1 Mc/s when clamped, and 3 to 4 Mc/s when unclamped. In order to secure

greater mechanical stability it is customary to clamp crystals by supports at the ends of the longitudinal median of shear oscillation. (Crystals of other types of cut can be obtained at frequencies ranging from 20 kc up to about 15 Mc, but are not suitable for the production of extra high-frequency stability.)

The size of an A_T-cut plate for medium waves is about 1 in. square and 0·06 in. thick.

4.2. Frequency Stability of Crystal Oscillator using A_T-Cut Plates. As already explained, for all practical purposes this is determined by the stability of the crystal. Although theoretically it should be possible to produce an A_T-cut crystal with a temperature coefficient of zero, in practice the frequency temperature coefficient of an oscillator using an A_T-cut crystal is very close to $\pm 0{\cdot}02$ in 10^6 per degree Centigrade.

The best-grade Crystal Oven in the B.B.C. in 1943 was the CP2, which has a temperature variation of $\pm 0{\cdot}005°$ C. for $\pm 5°$ C. variation in room temperature. This corresponds to a maximum frequency variation of ± 1 in 10^9, when using an A_T-cut crystal as above.

The CP17E oven had a temperature variation of $\pm 0{\cdot}01°$ C for $\pm 5°$ C. variation in ambient temperature. This corresponds to a maximum frequency variation of ± 2 in 10^9, when using a high-grade crystal as above.

In addition, there was a low-grade oven with a temperature variation of $\pm 0{\cdot}05°$ C. for $\pm 5°$ C. variation in ambient temperature.

4.3. Crystal Oscillator Circuits.

4.31: Series Resonance Type. A recent (1943) form of crystal oscillator is illustrated by the circuit of Fig. 1/XI:3 with L_s and C_s replaced by a crystal assembly: a crystal mounted between electrodes in a suitable housing: the electrodes constitute the circuit connections of the crystal. The values of C_1 and C_2 are changed to provide the appropriate value of negative resistance facing the crystal, which is considerably higher in the case of a crystal than in the case of an L, C, circuit, e.g. about 100 ohms instead of about 5 ohms.

Considering the circuit of Fig. 3/XI:4, the reactance presented at the electrodes of a crystal, neglecting the series capacity C_a, is of the form shown in Fig. 6. It possesses a series resonant frequency f_1 just below an anti-resonant frequency f_2. Due to the high value of C/C_q in Fig. 3/XI:4 these two frequencies are very close together.

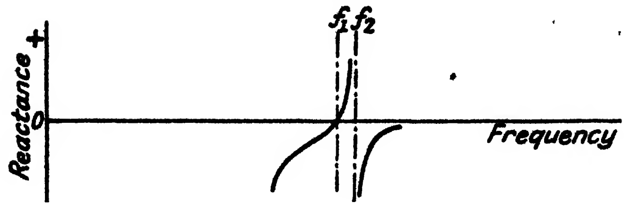


FIG. 6/XI:4.—Form of Reactance Frequency Curve for a Crystal in the Neighbourhood of One Mode of Oscillation.

In the circuit of Fig. 1/XI:3 the crystal operates at a point of low reactance at a frequency just above its resonant frequency, i.e. between f_1 and f_2 in Fig. 6.

An alternative circuit for a crystal oscillator is shown in Fig. 8, in which the crystal operates at a point of high reactance very near to its anti-resonant frequency, so that the oscillator corresponds to a tuned-anode tuned-grid oscillator, the back coupling being provided by stray capacity from anode to grid, and by the residual anode-grid capacity in the valve, which persists in spite of the screen between anode and grid.

4.32. The Meacham Crystal Bridge Oscillator Circuit. This is shown in Fig. 7. It consists of a bridge circuit containing resistances in two arms, a lamp in the third arm and a crystal in the fourth. The resistances R_1 and R_2 are as closely as possible

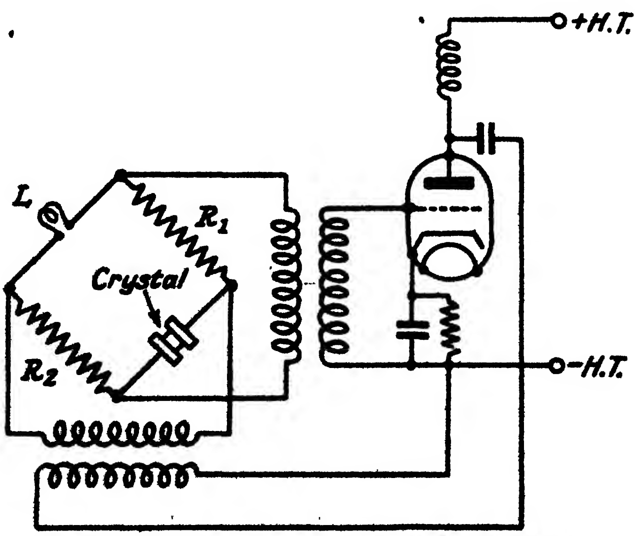


FIG. 7/XI:4.—Meecham Crystal Bridge Oscillator.

equal to the series resistance of the crystal, and the resistance of the lamp L is chosen so that when cold the bridge is just off balance, the resistance of the lamp being less than R_1 and R_2. The input to and output from the bridge are provided by means of transformers, connected respectively in the anode and grid circuits of an amplifying valve.

When the circuit oscillates, the heating of the lamp increases its resistance and brings the bridge nearer to balance, so introducing limitation of the amplitude of oscillation.

The crystal is operated at a frequency just below series resonance, with the result that since the phase shift through the bridge reverses at series resonance, feedback at harmonic frequencies is negative, resulting in a pure wave form.

It is claimed that the effect of the bridge is to produce a phase-shift-frequency characteristic considerably steeper than that produced by direct insertion of the crystal in the circuit, with the result that increased stability of operation is obtained as far as variation of valve parameters are concerned.

The performance of a crystal operating in a Meacham bridge oscillator circuit is superior to that when operating in the series resonant oscillator circuit.

4.33. Miller Circuit Crystal Oscillator. This is an early type of crystal oscillator which is still in use in the B.B.C. Its circuit is shown in Fig. 8 (*a*).

In Fig. 8 (*b*) is shown the input circuit with the crystal and mounting replaced by its equivalent circuit. It will be evident from this that the crystal and mounting effectively constitutes, in com-

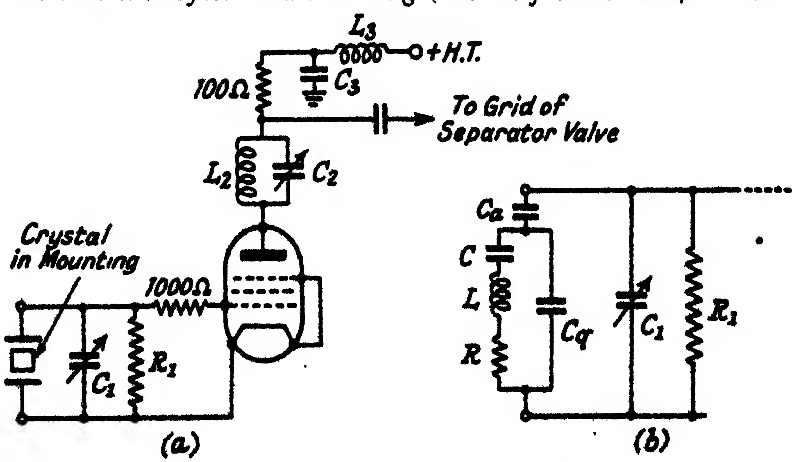


FIG. 8/XI:4.—Miller Circuit Crystal Oscillator.

bination with condenser C_1, a parallel tuned circuit. The oscillator is therefore of the tuned-grid tuned-anode type, feedback being provided through the anode grid capacity of the valve. Since the resistance R in the equivalent circuit is very low compared to the reactance of L, the circuit has a very high Q and the variation of phase shift round the loop circuit, with change of frequency, is very high, resulting in a high-frequency stability.

Condenser C_1, which has a maximum value of about 50 $\mu\mu$F, is normally variable. Fine adjustment of frequency can then be obtained by means of this condenser.

L_3 and C_3 are part of the H.T. decoupling. The 100-ohm resistance in the anode circuit provides coupling to the grid of the separator or output valve of the oscillator. The reason for taking the output coupling from this point, instead of from the anode of the oscillating valve, is to prevent any variation in the input impedance of the separator valve from introducing phase shift into the loop path, and so causing variation of frequency. With the arrangement shown, the input impedance of the separator valve is shunted by 100 ohms, which is comparatively a very low impedance, so that variations of the input impedance of the separator valve are heavily masked.

This circuit has the disadvantage that both the anode and grid of the oscillating valve are faced with high impedances, so that any variation of the grid or anode impedance of the valve gives rise to change of frequency. For this reason, in more recent oscillators, this circuit has been replaced by the series resonance type of circuit, and Meacham bridge type.

For further information on crystal oscillators, see *Quartz Crystals for Electrical Circuits* by R. A. Heising (pub. van Nostrand).

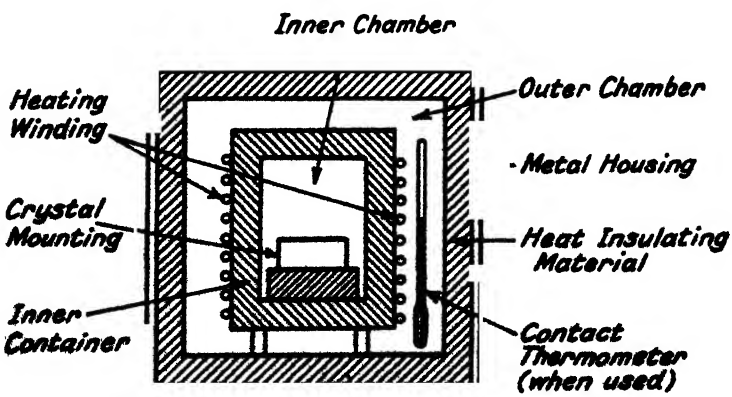


FIG. 9/XI:4.—Arrangement of Crystal Oven.

4.4. Crystal Ovens. The general arrangement of a typical crystal oven is shown in Fig. 9. The oven is contained in an outer metal housing lagged internally with any suitable form of heat-insulating material. Inside this is a metal container which constitutes the crystal oven. The mounting containing the crystal is supported inside the inner container. A heating winding is conveniently wound round the inner container, and some temperature indicating device is located in the outer chamber between the inner container and the lagged metal housing. This temperature-indicating device may be either a contact thermometer, or a bridge as shown in Fig. 10.

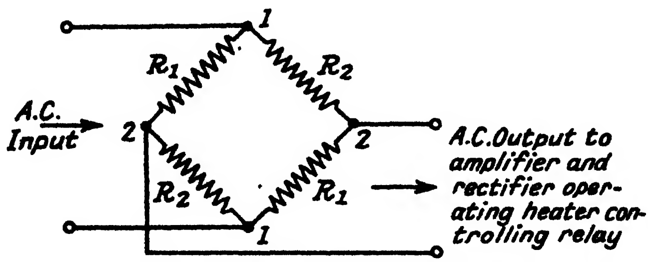


FIG. 10/XI:4.—Temperature Bridge.

The contact thermometer consists of a mercury thermometer with two contacts, one making permanent contact with the mercury, and the other making contact only at or above a predetermined temperature which is fixed by the position of the contact in the thermometer tube. It is evident that such a thermometer can be used to switch on the heater when the temperature falls below the predetermined value, and to switch off the heater when the temperature rises above the predetermined value.

The resistance bridge shown in Fig. 10 can be made to do the same thing. The bridge consists of two equal resistances R_1 made of one metal, e.g. copper, and two other equal resistances R_2 made of some other metal, e.g. Eureka, Constantin, or nickel-silver. The values of the resistances R_1 are made nearly, but not quite equal to the resistances R_2, so that the bridge is just off balance. Since the temperature coefficients of the two sets of resistances are different, it is evident that any change of temperature will vary the degree of balance of the bridge. If, therefore, an A.C. voltage is applied across terminals 1,1, the voltage across terminals 2,2, will vary with temperature. An A.C. voltage is normally applied across terminals 1,1 by a transformer connected to the A.C. mains, and the output from the bridge is connected to an amplifier and a rectifier, operating

a relay which switches the heating winding on and off. The resistances R_1 and R_2 may conveniently be wound round the inner container between turns of the heating winding.

The effect of such an arrangement, when in operation, is evidently to keep the mean temperature of the outer chamber constant, but the instantaneous temperature of the outer chamber fluctuates with an amplitude which depends on the amount of energy dissipated by the heating winding, the magnitude and location of the heat capacities around the heating winding, the ambient temperature, and any backlash in the temperature-indicating device including the effects of conduction, radiation and convection in the path from the heater to the temperature-indicating device. Since the whole purpose of the oven is to produce constant temperature in the inner chamber, it is desirable, first, that the amplitude of the temperature variation in the outer chamber should be kept small, and secondly, that the temperature variation transmitted through the walls of the inner container should be attenuated as much as possible.

The first requirement can be met by operating the oven at as low a temperature as possible above the ambient (room) temperature and by using as much lagging as possible to line the metal housing. Both these contribute to making the required power dissipation of the heating winding small, with the result that each successive rise of temperature contributed by the heating winding is made small.

The second requirement can be met by choosing material for the walls of the inner container for which the ratio Heat Capacity divided by Thermal Conductivity is a maximum; and also by making the heat cycle short. The period of the heat cycle can be made short by arranging that the path from the heating winding to the temperature-indicating device is one of high thermal conductivity, and by making the heat capacity of the temperature-indicating device as low as possible. Care must, however, be taken to ensure that the temperature assumed by the heating device is a true measure of the temperature assumed by the outer surface of the inner chamber. Since air has a low thermal conductivity, an ideal method of meeting both the requirements above would be to use the resistance bridge as the temperature-indicating device, and to mount its windings and the heating winding on a thin metal screen situated in the outer chamber and completely surrounding the inner chamber. The method of construction described has, however, proved adequate for present requirements of temperature stability.

For detailed information on temperature control of crystal ovens, see *Temperature Control* by A. J. Ansley (pub. Chapman & Hall).

5. Dynatron Oscillators.

It has already been noticed that, if a tuned circuit is shunted across a negative resistance, oscillation results. In Fig. 1/XI:2 a negative resistance was produced across terminals 2,2 by means of feedback. There are other methods of producing negative resistance, one of these is by means of secondary emission : the origination of secondary electrons at a surface in a thermionic valve due to the impact of primary electrons, i.e. electrons arriving from the cathode. When each primary electron gives rise to more than one secondary electron, the direction of the current entering the surface in question is reversed. In a thermionic valve, secondary emission may take place either at the grid, in which case the grid current is reversed or decreases with increased positive grid volts, or at the anode, in which case if the secondary electrons can find an electrode of sufficiently high potential to which to escape, the anode current during at least a part of the range of H.T. volts decreases as the H.T. volts are increased (see IX:11.1).

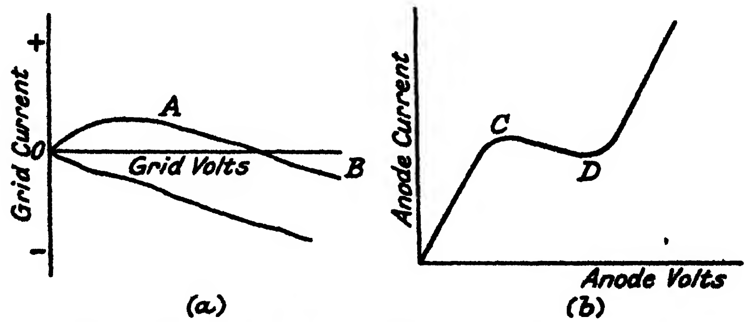


FIG. 1/XI:5.—Negative Resistance Curves due to Secondary Emission.

Fig. 1 (*a*) illustrates two types of grid-current grid-voltage curve which sometimes occur. The part of the upper curve from *A* to *B* represents a negative resistance, while the lower curve presents a negative resistance all along its length. Fig. 1 (*b*) represents the form of the anode-voltage anode-current characteristic of a screened grid valve. This presents a negative resistance over the part of the curve from *C* to *D*. If a negative resistance constituted by either of the above means is shunted across a parallel tuned circuit as shown in Fig. 2, oscillation will occur, provided the impedance presented by the tuned circuit across 2,2 *is not less than* the magnitude *R* of the negative resistance. It will be noted that this con-

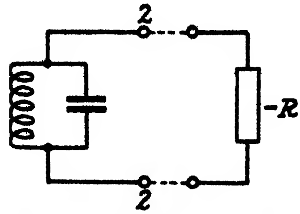


FIG. 2/XI:5.—Negative Resistance Shunted across a Tuned Circuit.

dition for oscillation is the inverse of that laid down for the circuit of Fig. 1/XI:2.

The importance of the dynatron oscillator does not consist in its usefulness as an instrument, but in the possibility of it occurring unwanted in amplifier circuits and so giving rise to parasitic oscillations. It can always be eliminated by shunting the negative resistance with a positive resistance of smaller magnitude, or by putting in series with the negative resistance a resistance of larger magnitude.

6. The Two Types of Negative Resistance.

Two types of negative resistance occur in oscillator circuits respectively due to parallel feedback and series feedback. These

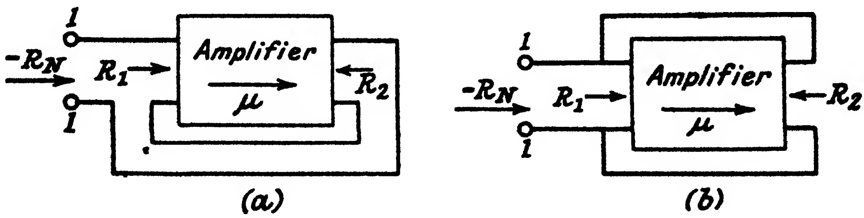


FIG. 1/XI:6.—Types of Negative Resistance.

are illustrated in Fig. 1/XI:6. In the series type at (*a*) the circuit will oscillate when R_N is greater than the positive resistance connected across it, i.e. across 1,1. In the parallel type at (*b*) the circuit will oscillate when R_N is less than the positive resistance connected across 1,1. See XXIII:15.

7. Resonant Line Oscillators.

Deliberate use of these is becoming of increasing importance at ultra short-wave lengths, but on medium and short waves their importance lies in the possibility of parasitic oscillations being caused owing to the circuit configuration unwittingly conforming to the circuit requirements for such oscillators.

In this type of oscillator the resonant circuit that controls the frequency is supplied by a resonant transmission line. Fig. 1/XI:7 shows three simple circuits which will oscillate. In Fig. 1(*a*)/XI:7 L_a and L_g are quarter-wave chokes at a frequency not far removed

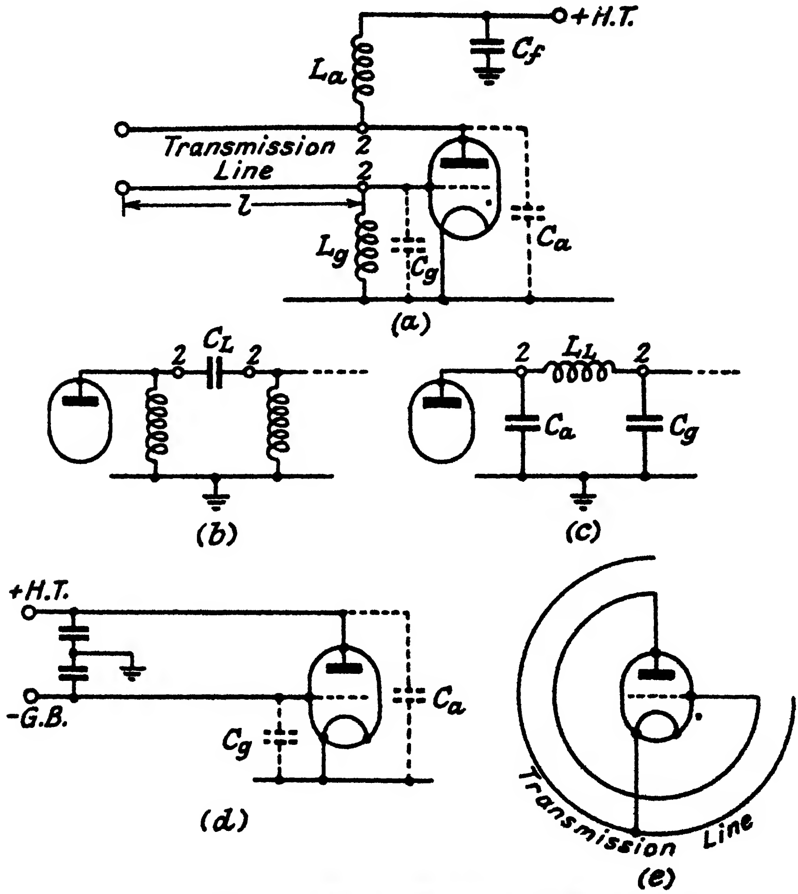


FIG. 1/XI:7.—Types of Resonant Line Oscillators.

from the oscillating frequency. C_a and C_g are the stray capacities to ground respectively of anode and grid, C_f is a filter condenser of negligibly small reactance. If l is the length of line and λ the wavelength corresponding to the frequency of oscillation: the impedance presented at 2,2 is $-j \cot \frac{2\pi l}{\lambda}$ if the line is open circuited at 1,1 and $j \tan 2\pi l/\lambda$ if the line is short circuited at 1,1, see equations (35) and (38)/XVI:1. In either case, the impedance presented by the transmission line at 2,2 may be a capacitative or an inductive

reactance. If the line impedance at z,z is capacitative at a frequency below the frequency at which L_a and L_g resonate with their self-capacities plus C_a and C_g respectively, the circuit connected between anode and grid is of the form shown at (*b*), where C_s is the effective capacity looking into the line.

Hence it may be seen that a frequency exists at which 180° phase shift takes place from anode to cathode so that the phase shift round the loop is zero and the circuit will oscillate. Similarly, the circuit at (*c*) applies when the line impedance is inductive at a frequency above the frequency resonance of L_a and L_g with their associated capacities: in this case the phase shift from anode to grid is also 180° at some frequency and the circuit will oscillate.

It is usual to use a short-circuited line, so that by moving the short along the line the circuit can be tuned.

Fig. 1(*d*)/XI:7 shows a type of circuit which may easily occur unintentionally owing to proximity of anode and grid conductors. This will oscillate when the reactance of the line is inductive and a 180° network of the type shown at (*c*) is constituted between anode and grid, in conjunction with the grid to ground capacities C_a and C_g.

Fig. 1(*e*)/XI:7 shows a circuit (supply circuits omitted) in which the coupling between grid and cathode is effected by tapping on to points on a transmission line rather less than half a wavelength apart. It is necessary to make the distance less than half a wave, because some phase shift takes place at the points of connection of anode and grid to the line. If the line itself is half a wavelength long or longer a certain amount of voltage step-down can be obtained from anode to grid by locating the anode at a voltage antinode and tapping the grid on to the line at the required distance from the anode. The length of the line will be in the neighbourhood of 2, 3, 4, etc., quarter wavelengths, and if an odd number of quarter wavelengths long should be open circuited at one end and short circuited at the other. If an even number of quarter wavelengths long the line should be open circuited or short circuited at both ends; for ease of adjustment the use of a short circuit is preferable.

The field of deliberate use of resonant line oscillators is for the generation of ultra-short waves, where they have merits from the standpoint of frequency stability. When the frequency is very high so that the length of line required is short, it is possible by suitable design to obtain circuit Q's ranging from 1,000 to 100,000, with values of 10,000 quite practicable under most circumstances. Terman states in *Radio Engineering* that if the line is provided with

temperature control the frequency stability of such resonant line oscillators is comparable with that of crystals. It is not clear that he is making the comparison on the basis of equal temperature variations and it is extremely unlikely that on this basis the resonant line approaches the performance of a crystal oscillator even approximately. While the reactance frequency slope of a shorted or open transmission line reaches infinity at quarter-wave, it drops away very rapidly and at quarter-wave $\pm 10\%$ the slope compares unfavourably with that of a crystal. Applications of resonant lines exist, however, at frequencies above 50–100 Mc/s, where the use of crystals and multipliers is impracticable or cumbersome.

8. Electron-Coupled Oscillator.

The chief value of this circuit is for the combination of more than one function in a single valve. It can be applied to practically

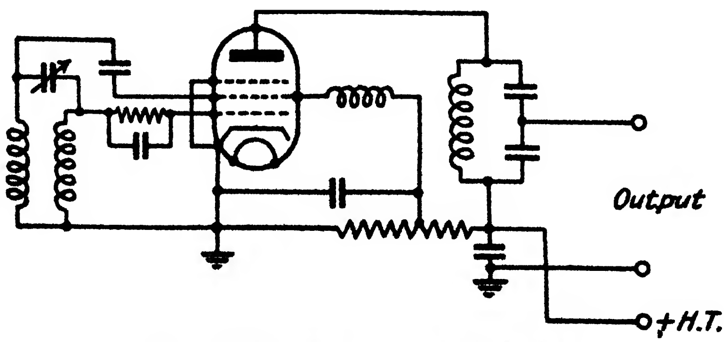


FIG. 1/XI:8.—Electron-Coupled Oscillator.

any multi-grid valve. The electron coupling does not constitute part of the loop circuit of the oscillator, but provides coupling from the oscillator to the output circuit. Fig. 1 shows a simple type of electron-coupled oscillator in which the cathode, control grid and screen grid of a pentode are operated as a triode oscillator with the screen grid acting as an ordinary anode. Part of the electrons leaving the cathode go to the screen and serve as " anode " current to the triode oscillator, while the remainder go to the anode proper and produce the power output by giving rise to a varying current through the impedance connected to the anode circuit.

Electron-coupled oscillators are used in radio receivers in order to provide the combination of a triode oscillator with a mixer valve which serves the function of beating an incoming signal with the local oscillator in a superheterodyne receiver. Fig. 1(c)/IX:13 shows

an octode valve arranged for this purpose, the incoming signal being applied to grid 4.

9. Beat Oscillators.

For audio-frequency measurements it is convenient to have available an oscillator which is continuously variable over the audio-frequency range from about 30 to 10,000 p:s. All straight oscillators as previously described, when constructed for use in the audio-frequency range, require the use of a number of inductances of different size in order to cover the range, while the condensers are of such size that they cannot be realized by continuously variable condensers but have to be constituted by condensers of graded size connected to the studs of a dial or dials. Such an oscillator is not continuously variable and as a result it can only be calibrated at a number of fixed frequencies. The oscillator can only provide frequencies between the calibration frequencies by a process of interpolation which is laborious, while the operation of the oscillator, even on the calibrated frequencies, is slow because of the number of settings which have to be changed.

It is a comparatively simple matter to construct an oscillator which will embrace the frequency range from 20 to 30 kc/s on a single condenser dial using one inductance only. If the output of this oscillator is modulated with a fixed-frequency oscillator located at 20 kc/s, the resulting products of intermodulation contain a term in the range of frequencies between zero and 10 kc/s. The method will be clearer after a discussion of the modulation process.

9.1. Intermodulation of Two Frequencies in a Square Law Modulator. In order to intermodulate or *beat* two frequencies

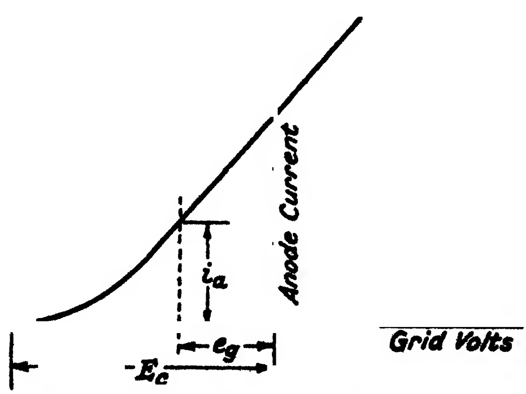


FIG. 1/XI:9.—Square Law Characteristic.

together the two frequencies are supplied to the grid of a valve which has approximately a square law characteristic. (A mixer may also be used: see XIII:8 and XIX:10.) That is to say, the curve relating plate current i_a and instantaneous grid bias e_g with a resistance equal to the working load in the anode circuit, is approximately of the form:

$$i_a = a(E_c - e_g)^2 \quad . \quad . \quad . \quad . \quad (1)$$

where E_c is the value of grid bias at which the anode current is reduced to zero and a is a constant depending on the valve, and anode load. Such a characteristic is indicated in Fig. 1.

Let E_b = the value of steady grid bias applied to the valve.
$e \sin \omega t$ = the variable frequency voltage applied to the grid of the valve, $\omega = 2\pi f$ (f is variable from 20 to 30 kc/s, for instance).
$e_0 \sin \omega_0 t$ = the fixed frequency applied to the grid of the valve, $\omega_0 = 2\pi f_0$(f_0 = 20 kc/s, for instance).

Then $\qquad e_g = E_b + e \sin \omega t + e_0 \sin \omega_0 t \quad . \quad . \quad (2)$

and $\qquad i_a = a(E_c - E_b - e \sin \omega t - e_0 \sin \omega_0 t)^2$

$$= a\Big[\underbrace{(E_c - E_b)^2}_{\text{D.C.}} + e^2 \sin^2 \omega t + e_0^2 \sin^2 \omega_0 t$$

$$+\underbrace{2(E_c - E_b)e \sin \omega t}_{\text{original frequency } f} - \underbrace{2(E_c - E_b)e_0 \sin \omega_0 t}_{\text{original frequency } f_0}$$

$$+2ee_0 \sin \omega t \sin \omega_0 t\Big] \quad . \quad . \quad . \quad . \quad (3)$$

Neglecting the D.C. and original frequency terms which will be eliminated by selective circuits:

$$i_a - (\text{D.C., etc.}) = a\Big[\underbrace{\tfrac{1}{2}e^2 + \tfrac{1}{2}e_0^2}_{\text{D.C.}} - \underbrace{e^2 \sin^2 \omega t}_{2f} - \underbrace{e_0^2 \sin^2 \omega_0 t}_{2f_0}$$

$$+\underbrace{ee_0 \sin (\omega + \omega_0)t}_{f+f_0} + \underbrace{ee_0 \sin (\omega - \omega_0)t}_{f-f_0}\Big] \quad . \quad . \quad (4)$$

The products of modulation are therefore: D.C., f, f_0, $2f$, $2f_0$, $f+f_0$ and $f-f_0$.

Considering only the difference term $f-f_0$, it is evident that as f varies from 20 to 30 kc/s the difference term varies from 0 to 10 kc/s. All the other terms except D.C. are equal to or greater than 20 kc/s.

Hence, by following the valve by a low pass filter with a cut-off just above 10 kc/s, any frequency in the range 0 to 10 kc/s can be obtained. In practice, series condensers and transformers

eliminate the D.C. component and restrict the lower end of the frequency range.

9.2. Frequency Stability of Beat Oscillators. Let the frequency stability of the component oscillators of respective frequencies f and f_0, be defined by fractions d and d_0 such that $\pm df$ is the number of cycles by which the variable frequency oscillator deviates from its indicated frequency as determined by its dial setting, and $\pm d_0 f_0$ is the deviation of the fixed frequency oscillator from its correct frequency.

Then the variable frequency has an actual frequency $f' = f \pm df$ and the fixed frequency has an actual frequency $f_0' = f_0 \pm df_0$. Hence the actual beat frequency

$$f_b' = f' - f_0' = f(1 \pm d) - f_0(1 \pm d_0)$$

The indicated beat frequency

$$f_b = f - f_0$$

Hence the deviation of the beat frequency from its indicated frequency

$$= f_b' - f_b = f(1 \pm d) - f_0(1 \pm d_0) - (f - f_0) = \pm fd \pm f_0 d_0 \quad . \quad (5)$$

and if $f = nf_0$

$$f_b' - f_b = \pm nf_0 d \pm f_0 d_0 = (\pm nd \mp d_0) f_0 \quad . \quad . \quad (6)$$

Taking the practical case where the errors subtract, the frequency error is:

$$f_b' - f_b = \pm(nd - d_0) f_0 \quad . \quad . \quad . \quad (7)$$

From this two important points emerge:

1. The value of f_0 should be kept as low as is consistent with adequate separation of the wanted frequency, $f_b = f - f_0$, from the unwanted frequencies.
2. Once f_0 has been chosen, the magnitude of the frequency error in cycles per second is independent of the output frequency of the oscillator. Hence the percentage error in the indicated output frequency is inversely proportional to the indicated frequency.

It follows that the long-period accuracy of a heterodyne or beat oscillator is considerably less than that of a straight oscillator. In practice, the error is reduced by introducing a method of rapid calibration of the oscillator. This is done by introducing a meter in the anode current circuit of the modulating valve and providing a means of varying the frequency of the "fixed" oscillator over a small range. When the beat frequency is in the neighbourhood

of a few cycles per second the meter-needle follows the beat frequencies and the position of *zero beat* (when the beat frequency is zero) is easily determined by swinging the condenser, controlling the frequency of the beat oscillator, with the condenser controlling the frequency of the variable oscillator set in the position of zero frequency. Calibration is then effected by setting the oscillator dial (the dial of the oscillator controlling the variable frequency) to the zero output (beat) frequency position and adjusting the condenser controlling the " fixed " frequency to zero beat. A beat-frequency oscillator usually consists of a fixed and variable oscillator, each contained with its separator valve in a separate screened compartment. The outputs of the separators are combined at the grid of a square law valve (or are fed to appropriate grids of a mixer valve) which is followed by one or more stages of amplification, a potentiometer being inserted at a convenient position to control the output level. The last stage, which may be a push-pull stage, feeds into a screened transformer with its secondary winding balanced to ground. The output impedance for audio-frequency work is usually 600 ohms and the output power between 10 and 100 milliwatts.

Apart from the stability, the accuracy of setting and reading a beat oscillator are not so high as in the case of a simple inductance capacity oscillator with decade sets of condensers controlling the frequency. The accuracy is, however, quite adequate for a large number of purposes.

10. Resistance–Capacity Oscillator.

Fig. 1 shows a type of oscillator in which the frequency determining elements are constituted by a resistance capacity network designed to introduce a phase shift of 180° at the frequency of oscillation. The phase shift round the oscillating loop is then adjusted to zero by introducing the appropriate number of commutations in the amplifier. In Fig. 1 the amplifier in the oscillating path is constituted by valves V_4, V_3, V_2, V_1 amplifying from right to left, and the frequency-determining network consists of C_3, C_4, C_9 and C_{12}, R_7, R_8, R_{10}, R_{20}, and R_{30}, terminated in R_1 and R_2. It is to be noted that this network consists of shunt condensers and series resistances so that increase of lag occurs with increase of frequency, until the frequency of 180° phase shift is reached where oscillation occurs.

Considering the couplings between valves provided by condensers C_2, C_6, C_8 and their associated anode and grid resistances, it is seen that with decrease of frequency an increase of lead is introduced

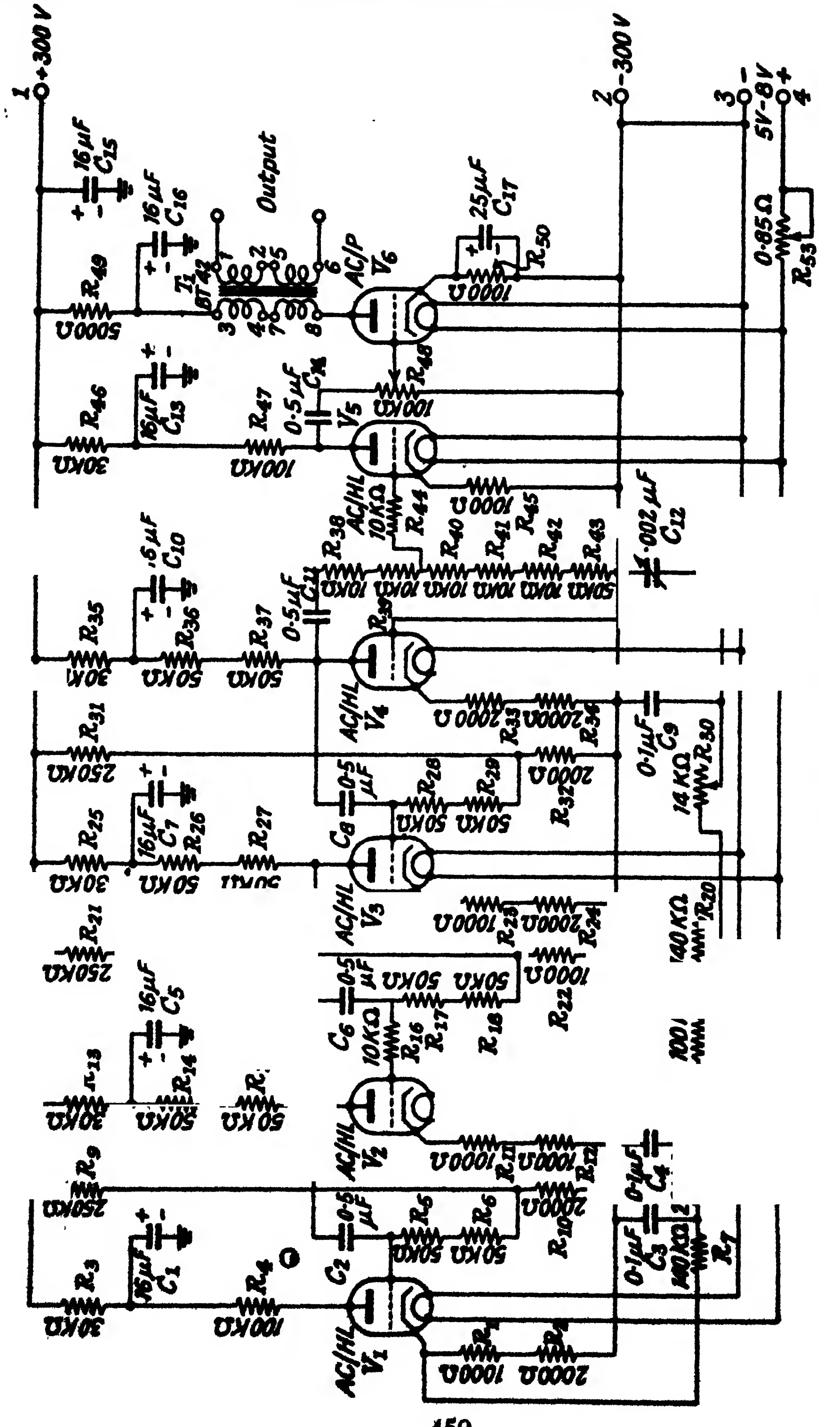


FIG. 1/XI:10.—Resistance Capacity Oscillator for 25 c/s.
(By courtesy of the B.B.C.)

and provision has to be made so that when 180° phase shift is reached there is no tendency to oscillate at a second (lower) frequency. This is prevented by staggering the cut-off frequencies of the coupling circuits between valves so that each occurs at as widely differing frequencies as possible. By so doing the rise of attenuation at the frequency of 180° lead is made sufficient to stabilize the circuit. See XXIII:6. Valves V_5 and V_6 constitute the output amplifier.

Such a type of circuit is useful because it enables a stable oscillator of very low frequency to be built from small cheap components, avoiding the use of large, heavy and expensive iron-cored coils which do not afford such a stable frequency of oscillation.

11. Freak Oscillations.

If a loop circuit exists with zero loop phase shift at two frequencies and a voltage amplification equal to or greater than unity at each frequency, one of three things may happen. If the two frequencies are close together one oscillation may predominate and suppress the other, or the oscillator may strike at either frequency at random. If the two frequencies are far apart the two oscillations may occur simultaneously with resultant intermodulation.

An occasional form of oscillation occurs in which as the oscillation builds up, the non-linearity of the circuit elements (probably the valves) causes the phase shift to change, and so makes the oscillating frequency change to a new frequency, at which the loop gain is smaller, so that the oscillation decays to the original amplitude and restores the original frequency. The result is a frequency wobble since the process is repeated periodically.

A type of parasitic oscillation is sometimes met in which an oscillation builds up so strongly that the grid current which flows builds up negative bias on the grids and the oscillation is suppressed until the negative charges on the grids are discharged when the oscillation builds up again. If the frequency of oscillation is an audio frequency in an audio-frequency amplifier, such an oscillation may sound like the clucking of a hen. If it is a supersonic frequency the oscillation may appear as an audio frequency or sub-audio frequency corresponding to the frequency of repetition of the build-up and decay. The remedy is to eliminate the feedback path: see XVII:5 and 6, and 9 to 12 inclusive.

In D.C. amplifiers and audio-frequency amplifiers with large coupling condensers, straight sub-audio-frequency oscillations may occur, due in particular to failure of H.T. and grid decoupling circuits at

low frequency; these are sometimes confused with the previous type of oscillation, and vice versa.

12. Parasitic Oscillations.

It is a truism to say that a parasitic oscillation occurs when, owing to an incidental circuit configuration, the conditions described above for the existence of an oscillator are fulfilled. In many cases, however, it is difficult to trace the loop path which has been accidentally established. Important examples of feedback loops which in practice are often very difficult to find, are described in XVII:9. In other cases the oscillator circuit conforms closely in form to one of the types of oscillator circuit described above in Figs. 2/XI:1 and 1/XI:7. Its existence may, however, be masked by the presence of other circuit elements, and neglect of some simple fact, such as the fact that a short straight piece of wire provides adequate inductance to constitute an oscillatory circuit in conjunction with a stray capacity. A circuit which appears to be earthed may, for instance, achieve a high potential above ground owing to the inductive reactance of the earth lead at high frequencies. The inductance of straight lengths of wire is given in Fig. 4/II:17.

For instance, from Fig. 4/II:17 the inductance of 12 ins. of 12 S.W.G. wire (dia. 0·104 in.) is $12 \times 0{\cdot}027\ \mu H = 0{\cdot}324\ \mu H$, and the reactance of this at 20 Mc/s ($\lambda = 15m$) is $0{\cdot}324 \times 6{\cdot}28 \times 20 = 40{\cdot}7\ \Omega$.

A further difficulty encountered is that a circuit may have several possible modes of oscillation corresponding to a number of incidental circuit configurations and it is often difficult to determine which incidental circuit is responsible.

12.1. Detection of Parasitic Oscillations. A parasitic oscillation may show itself in several ways. *In audio-frequency amplifiers* the oscillation may be audible. If of supersonic frequency it may drop the gain of the amplifier at the wanted frequencies or introduce distortion. If of sub-audio frequency it may be detected as a modulation of the output of the amplifier. It may give rise to an increase in the standing feed of class B amplifiers, and if of very large amplitude may increase or reduce the standing feed of class A amplifiers. In high-power amplifiers it may give rise to flashover. An incipient short-wave oscillation in an audio-frequency amplifier sometimes sounds like a loose contact. If any of these effects are present without any known cause an oscillation is to be suspected.

In radio-frequency amplifiers a common indication of oscillation is the presence of drive on a succeeding stage, this, however, is

usually of appreciable magnitude only when the parasitic frequency is near to the frequency to which the amplifier is tuned. Distortion of the modulated envelope is another symptom, observed by listening to the transmitter on a receiver, making sure that the receiver is not overloaded ; while the radiation of spurious carriers as sidebands of the main carrier is another effect. If the drive on a stage subject to one type of (ultra high frequency) oscillation is increased steadily and the anode feed of the stage is watched, a sudden kick in the anode feed may be observed at a certain amplitude of drive, due to the starting up of a parasitic oscillation. Low-frequency oscillations sometimes show up as a modulation of the carrier, while violent oscillations cause arcing and flashover. By bringing a small piece of metal, e.g. the hook on an earthing wand *with the earth lead completely removed*, close to a conductor at fairly high oscillating potential with regard to ground, arcs may be drawn from it. In certain cases of short-wave oscillations arcs as long as a foot may be drawn. Weaker oscillations can be detected by connecting one terminal of a neon lamp to the conductor. While it is nearly true to say that a source of oscillation which gives trouble in an audio-frequency circuit may also occur in a radio-frequency circuit, it is useful to list the types of trouble under the type of circuit to which they have special application.

12.2. Causes of Parasitic Oscillation—particularly affecting Audio-Frequency Amplifiers.

1. *Capacity Coupling* to a low-level point in the amplifier from a later high-level point. Remedy : screening.

2. *Magnetic Coupling* between the cores of audio-frequency transformers respectively in early and later stages. Remedy : rotate one or both transformers so that the coupling is a minimum and, if necessary, isolate one or both transformers from common mounting plate, if of steel or cast iron, by mounting on non-magnetic material about half an inch to an inch away from the plate.

3. *Common Impedance in High Tension Supply.* An example, of this is shown in Fig. 1. If the grid of V_2 is made positive its anode current increases, and owing to the increase of current through Z_g, the impedance looking back into the H.T. supply, the H.T. voltage drops so that the voltage on the anode of V_1 drops, and the grid of V_2 goes less positive. The anode current of V_2 drops and the anode goes more positive (owing to the amplification this increase in positive potential is greater than the direct drop in potential consequent on the fall in H.T. volts on the anode of V_1) so that the grid of V_3 goes more positive. In other words, the feedback e.m.f.

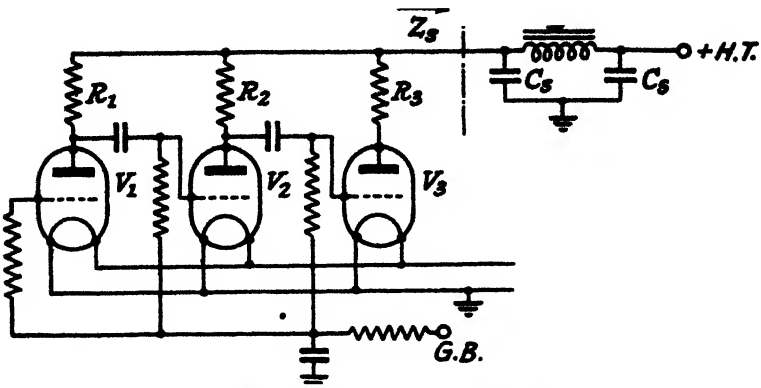


FIG. 1/XI:12.—Illustrating Common Impedance in H.T. Supply.

is in phase with the initial e.m.f. at any point in the circuit. This corresponds to zero phase shift round the loop, the condition for oscillation. (A more exact statement would include effect of all impedance magnitudes and angles.) The remedy is to increase the value of C_s so reducing Z_s, or to supply a separate H.T. filter for each stage or pair of stages. The feedback loop, consequent on an H.T. impedance common to two stages, only introduces negative feedback, unless there is phase shift in the coupling circuit between the stages. At the frequency at which 180° phase shift is reached the amplification then is normally negative so that oscillation cannot occur. Evidently no general rule can be established and each case must be weighed on its merits.

If it is economically undesirable to provide adequate filtering at very low frequencies, a compromise may often be effected by increasing the low-frequency cut-off of the amplifier by means of a series condenser or shunt inductance, or both, in the audio-frequency circuit.

4. *Unbalances in the Lines* connected to input and output of the amplifier. Remedy: balanced and screened transformers. See XVII:9. (Assuming that steps cannot be taken to balance the lines: evidently, if possible, this should be done.)

5. *Unscreened Lines*, connected to input and output of the amplifier, running close together. Trouble may be experienced even with screened pairs in high gain amplifiers. Remedy: screening and separation.

6. *A Common Earth Impedance* can introduce feedback in an unbalanced amplifier as indicated in Fig. 2. This shows a resistance-

coupled amplifier with an impedance Z_e from filament circuit to ground common to two stages, the input being provided by a transformer or other source of e.m.f. which applies an e.m.f. between the grid and the ground.

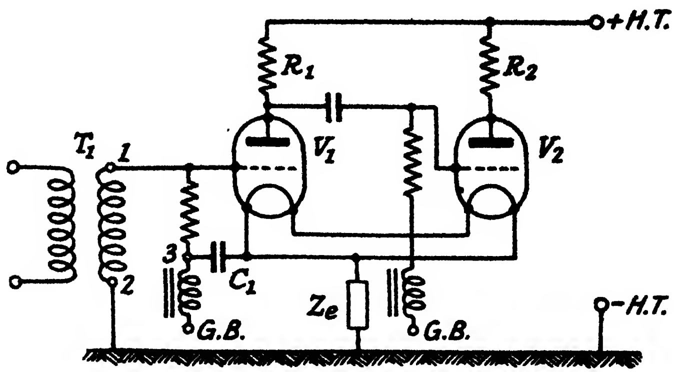


FIG. 2/XI:12.—Common Earth Impedance introducing Feedback in Unbalanced Type of Circuit.

If the grid of V_2 is made more positive the anode current of V_2 increases, and flowing through Z_e makes the filament circuit positive with regard to true ground. The current in V_1 is, therefore, reduced, because since its grid is tied to true ground through T_1, it becomes less positive with regard to its filament. The anode current through V_1 being reduced, as is the voltage drop in R_1, the anode of V_1 becomes more positive and so makes the grid of V_2 more positive. In other words, the feedback e.m.f. is in phase with the initial e.m.f. at any point in the circuit, which corresponds to zero phase shift round the loop, the condition for oscillation. Remedy : provide a low impedance path to ground, or decouple the input circuit of V_1 from ground by disconnecting terminal 2 on transformer T_1 from ground and connecting it to point 3, i.e. to the filament of V_1 (through C_1).

A case of a common earth impedance in a balanced and screened circuit is discussed in XVII:9.14.

7. *Parallel Grid and Anode Leads, or Parallel Cathode and Anode Leads*, when unscreened, may give rise to a resonant line short-wave oscillator. Remedy : insert stopper resistance in each lead as close as possible to each electrode. See 4 (*b*) in XI:12.3 for description of stopper resistances.

8. *Longitudinal Currents* in screens. Remedy : introduce break in screen, see XVII:9.14.

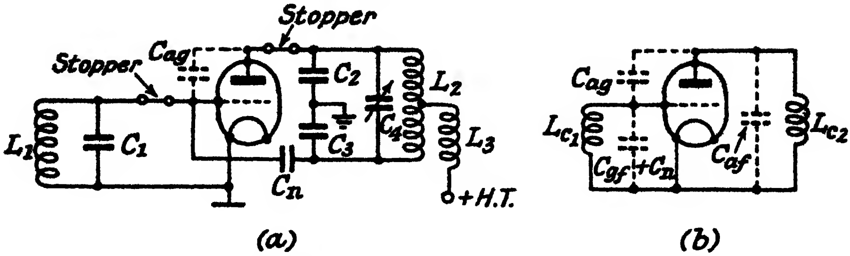


FIG. 3/XI:12.—Illustrating Method of Oscillation due to Inductance of Grid and Anode Leads.

(*a*) Practical Circuit; (*b*) Effective Circuit at High Frequencies.

Fig. 3 (*a*) shows a simple R.F. amplifier circuit. At very high frequencies this reduces to the circuit of Fig. 3 (*b*), which is a tuned-grid tuned-anode oscillator circuit with the grid and plate tuning capacities supplied by the grid-filament and anode-filament capacities of the valve. The inductances L_{c_1} and L_{c_2} are constituted by the inductances of the grid and anode leads to ground through capacities C_1 and C_2 respectively. Since the reactance of C_n is large *compared to* C_3 *and goes to ground through* C_3, C_n *is effectively* in parallel with the grid filament capacity C_{gf}, and is ineffective in supplying neutralization.

One remedy is to insert a small inductance in series with the anode of the valve, as close to the anode as possible, so as to tune the anode circuit to a different frequency from the plate circuit.

The stopper inductances on a medium-wave transmitter are usually constituted by between a dozen and half a dozen turns of about 14 S.W.G. wire wound in an air-spaced self-supporting solenoid, about 1 in. in diameter.

Experiment has shown that better suppression of parasitics is secured by tuning the anode circuit to a lower frequency than the grid circuit.

Alternately, a resistance may be inserted in series with the grid lead as close to the grid as possible, in order to damp out the parasitics: if, however, the grid is taking heavy grid current, it may be necessary to shunt the resistance with an inductance. A resistance shunted with an inductance may also be inserted in the anode lead, close to the anode. The exact measures adopted in practice are guided by experiment rather than by precept. The resistances inserted in grid and anode leads are called *stopper* resistances. The optimum value of these resistances must be found by trial, and a parasitic oscillation may sometimes be cut out by changing the value of stopper resistance. The value may vary from 200 ohms in a

high-power medium-wave transmitter to 10,000 ohms in a radio receiver.

The value of the inductance must be such that its reactance is high compared to the value of the stopper resistance at the parasitic frequency. Since the frequency of the parasitic oscillation is generally unknown the size of inductance must be found by trial, but usually between 5 and 10 turns of a solenoid of diameter large enough to clear the stopper resistance makes a suitable inductance. A neat arrangement is to locate the inductance coil round the stopper resistance.

As an alternative, the Q of the oscillatory circuit may be reduced by the use of heavy conductors, and in particular, conductors in the form of tape which have a low inductance ; it is evident that this expedient cannot be used where inductance is added to the circuit.

It may be remarked that transmitters should be, and usually are, designed so that condensers are connected directly to ground from grid and anode : as C_1 and C_2 in Fig. 3. Further, these condensers are *connected to points along the leads from* L_1 to grid and L_2 to anode, as close as possible to anode and grid respectively. The circuit stability is further improved by shunting a resistance, not shown in Fig. 3, from grid to ground which serves a threefold purpose : it loads the parasitic circuit, it masks the non-linearity of the grid impedance due to grid current, and it prevents dynatron oscillations due to negative grid current characteristics. As an example of such a resistance, see resistance R_1 in Fig. 1/X:31.

For the use of quarter-wave solenoids for reducing parasitic oscillations, reference should be made to the last two paragraphs of XVI:4.

(4) (*c*). *Parallel Oscillations in Push-Pull Circuits.* Fig. 4 shows a push-pull amplifier which at frequencies very much higher than the carrier frequency degrades to the oscillatory circuit of Fig. 3 (*b*), in which the valve now represents both valves of Fig. 4 in parallel and all capacities represent two symmetrically disposed corresponding capacities from Fig. 4 in parallel. The inductances L_{c_1} and L_{c_2} are constituted by the inductance from grid and anode to ground of the two grid leads and the two anode leads respectively. Remedy : stopper resistances and inductances.

At a considerably lower frequency than the carrier frequency, a parallel oscillatory circuit can be traced from Fig. 4 (*a*) as shown in Fig. 4 (*b*), where L_3 and L_4 are the inductances L_3 and L_4 in Fig. 4 (*a*), C_{ag} and C_{af} are the anode grid and anode filament capacities of one valve in Fig. 4 (*a*), the stray capacities being largely

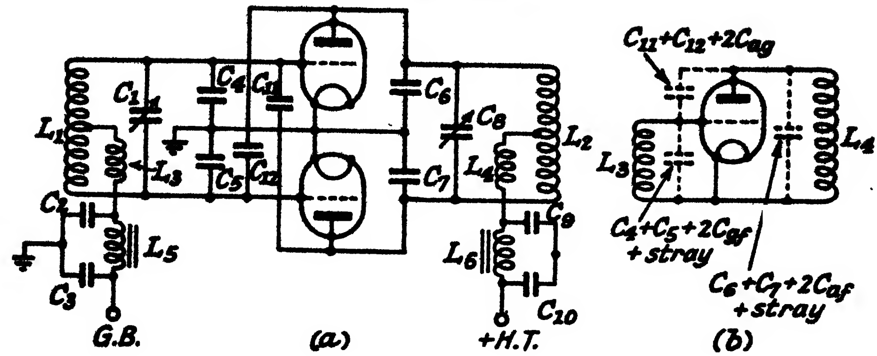


FIG. 4/XI:12.—(a) Neutralized Push-Pull Amplifier. (b) Parallel Low-Frequency Parasitic Circuit. (Parallel High-Frequency Parasitic Circuit is as Fig 3.)

constituted by the stray capacities of L_1, C_1, and C_8, L_2 to ground plus stray capacities of conductors to ground. This circuit is again a tuned-grid tuned-anode circuit and will only oscillate if anode and grid circuits tune to approximately the same frequency. Remedy: detune one of the circuits either by decreasing L_3 or increasing L_4. The practice of connecting a condenser from the top end of L_1 or L_4 to ground is undesirable since the tapping points on L_1 and L_4 are in general not central and the result would be to detune one side of the circuit.

4 (*d*). *Push-Pull Oscillations in Parallel Circuits.* In Fig. 5 is shown a circuit with two valves driven in parallel and feeding in parallel to an anode circuit. At high frequencies a push-pull circuit can be traced with conductor 1,1 constituting an inductance resonating with the grid filament capacities, and conductor 2,2 consti-

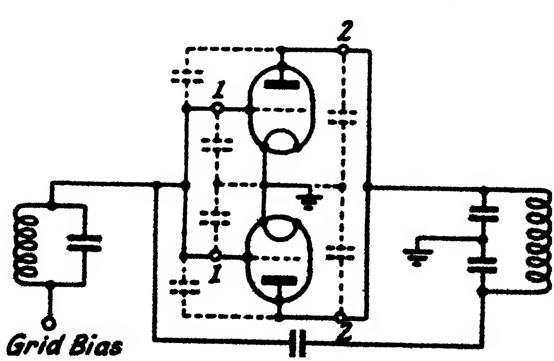


FIG. 5/XI:12.—Stray Capacities causing Push-Pull Oscillations in Parallel-Operated Valve Circuit.

tuting an inductance resonating with the anode filament capacities. Feedback is provided by the anode grid capacities so that this circuit constitutes a push-pull tuned-anode tuned-grid oscillator. These parasitics occur at a higher frequency than the high-frequency oscillators in 4 (*c*) and may become extremely severe giving rise to flashover and shutdown. Such oscillations may occur between any two valves or two groups of valves constituting part of a batch of valves in one-half of a push-pull circuit. Remedy: stoppers.

4 (*e*). *Dynatron Oscillations.* When a negative grid current slope gives rise to a negative resistance as in Fig. 1(*a*)/XI:5, any resonant circuit connected to the grid may oscillate.

Since, however, in class C amplifiers, when the valves are biased back to cut-off or below, no grid current flows until drive is applied, and the negative resistance does not become operative until drive is applied. The probability of the circuit constituted by L_1C_1 in Fig. 3(*a*)/XI:12 oscillating is therefore small, because such an oscillation would be quenched at a frequency very close to the frequency at which it tried to oscillate. The circuit constituted by L_{c_1} and C_{gf} in Fig. 3(*b*)/XI:12 will, however, oscillate, if at resonance it presents towards the grid a resistance as great as, or greater than, the grid negative resistance. More likely sources of oscillation are those due to the circuits constituted by L_3 and C_4 and C_8 in parallel or by L_5, C_2 and C_3 in Fig. 4 (*a*). The circuit due to L_3, C_4 and C_8 will probably oscillate at a long-wave frequency when the transmitter is driven, and will modulate the main carrier, with the result that two sidebands appear separated from the main carrier by a frequency distance equal to the frequency of parasitic oscillation. To what extent these are radiated depends on the selectivity of the following circuits and the parasitic frequency. If L_5, C_2 and C_3 constitute the oscillatory circuit the carrier will be modulated with a very low frequency.

Oscillations of this kind are normally prevented by the presence of shunt resistances from grid to filament, but when these have proved inadequate a diode has been connected between grid and filament with anode to grid and cathode to filament. The necessity for this expedient is, however, rare.

12.4. Location of Parasitic Oscillations. The location of a parasitic oscillation consists in the location of the loop or oscillatory circuit giving rise to the oscillation.

When an oscillation is observed in a circuit the first step is to find out which stages are affected. In the case of a low-power amplifier this can be done by monitoring at different points along

the circuit, by finding at what point in the circuit the normal drive is distorted, or which parts of the circuit suffer a loss of gain. In an A.F. amplifier monitoring can be done with high-impedance headphones, while in a low-power R.F. amplifier a valve voltmeter is necessary, or if a modulated input is used a radio receiver adjusted to low sensitivity can be used, if necessary in conjunction with a potentiometer. In the case of a high-power amplifier the oscillation can be detected by meter readings, by arcs drawn from the conductors to insulated pieces of metal and by the striking of a neon lamp held near to the circuit conductors. In the case of oscillations which only appear when drive is on or at a definite level of drive, the presence of the oscillation can be detected by slowly increasing the drive from zero and observing the circuit for any sudden change in conditions: meter flicks or audible clicks in the audio-frequency output or in the output of a receiver tuned to the transmitter frequency, if a transmitter is involved.

Certain types of oscillation occur in transmitters only when modulation is on. These can be observed by modulating the transmitter with tone, slowly increasing the modulation from zero, and observing for clicks, meter flicks, or sudden distortion of the tone. In the case of transmitters and high-power amplifiers, the later stages can be checked by removing the H.T. from each in turn, starting with the output stage and checking that the oscillation is still present. It may be more convenient to make later stages inoperative by making disconnections between stages, taking care not to remove the bias from any stage.

When the stage or stages involved in producing the oscillation are found the procedure varies according to the conditions observed. It is first necessary to establish

(a) Whether a stage or amplifier is oscillating internally or externally, i.e. by a feedback loop internal to the stage or amplifier, or by a feedback loop external to the stage, which may involve another stage or stages, or may involve only the input and output lines of the amplifier.

(b) If the loop is external to the amplifier, whether it involves other stages or whether it involves only the input and output lines of the amplifier.

(c) If another stage is involved, whether the feedback occurs from the output of the other stage or from a point internal to the stage, e.g. due to a common earth or H.T. impedance; also whether the loop enters the suspected stage internally or externally, i.e. by the input to the stage.

(*a*) is established by disconnecting the input and output lines of the stage close to the stage.

(*b*) is established by disconnecting preceding and following stages leaving the lines at input and output of the suspected stage connected.

(*c*) is established by disconnecting the input and output lines of the block of suspected stages leaving the connections between suspected stages, and then by shorting the input and output of the block of stages.

12.41. Inter-stage Oscillation. If the oscillation loop involves more than one stage the probabilities depend on whether the stages are contained in independent screens, whether these have independent grid and H.T. supplies or independent decoupling of the supplies; and on the run of the earth circuits.

If the oscillating stages are in one screen the physical layout of the circuit should be examined to see whether stray capacity exists between circuit elements or wiring in such a way as to introduce feedback from one stage to its input or from the output of a later stage to the input of an early stage. Sometimes capacity coupling, which is normally innocuous, is aggravated by a circuit fault such as the disconnection of a grid decoupling condenser, or an open circuit in a grid circuit. The possibility of magnetic coupling should also be considered: on low-power amplifiers mounted on steel panels and cast-iron racks magnetic coupling may even occur between amplifiers on different panels. The grid and H.T. decoupling should be examined and temporary measures may be taken to improve the decoupling to see whether this eliminates the oscillation.

If a common H.T. supply is used for the two stages, check that mutual impedance in the H.T. supply is not the source of feedback. This may be done either by shunting large condensers across the H.T. or by using separate sources of H.T. for each stage.

If the oscillation has not been traced the run of the earth circuits should be examined and drawn out in schematic form to see whether a mutual earth impedance is involved: a 10-ft. length of common earth lead is often sufficient to give rise to an oscillation.

12.42. Oscillations involving Input and Output Lines. If the oscillation involves no other stages, but only the input and output lines, check that the runs of the input and output lines are either far apart or are screened. In high-gain amplifiers it may be necessary to screen lines in addition to separating them, or to separate them in addition to screening them. With balanced input and out-

put lines, *as a temporary expedient*, an oscillation can sometimes be stopped by reversing either input or output leads. This is not satisfactory as a permanent measure unless the oscillation is far outside the band of wanted frequencies, since the presence of feedback tends to modify the gain frequency characteristic of the amplifier, and at frequencies where the gain is augmented amplitude distortion is increased, a particularly unsatisfactory condition in audio-frequency amplifiers. Finally, the balance of the input and output transformers should be checked: a simple way to do this in low-power audio-frequency amplifiers is to insert a balanced and screened 1 : 1 impedance ratio transformer (repeating coil), with the screen earthed, in series with the input and output in turn. This method would probably work satisfactorily at radio frequencies, but balanced screened transformers are not usually available at radio frequencies. The balance of a circuit can sometimes be improved by changing the point to which the input and output transformer screens are earthed.

12.43. Oscillations Internal to Stage. If the oscillation is internal to a stage the physical layout should be examined for electrostatic and electromagnetic feedback. In audio-frequency amplifiers feedback due to capacities external to the valve is the most probable cause of oscillation, while in radio-frequency amplifiers the most frequent cause of oscillation is due to feedback through the anode-grid capacity of the valve since care is usually taken in the design to avoid stray capacity feedback. If an oscillation occurs immediately after a circuit modification involving a change in physical layout has been made, then the possibility of stray capacity feedback should be considered.

12.44. Oscillation in R.F. Amplifiers. If the frequency of the oscillation can be identified, either by means of a radio receiver or a wavemeter, it may enable the corresponding circuit to be located immediately, by checking up the resonant frequencies of all circuits of which the constants are known.

If this is not possible, then it means that the frequency is a very high (or very low) frequency, beyond the range of the receiver, and if the oscillation has been located to a particular stage, stoppers should be inserted in anode and grid circuits. (If the frequency turns out to be a very low frequency, the stopper resistances will, of course, be ineffective.)

Broadly, oscillations at or near the carrier frequency may be identified by the fact that the closed circuit (anode tank circuit) R.F. ammeter shows a reading. Short-wave oscillations can be

identified because they persist when the anode tuning inductance is disconnected at both ends, and show no reading on the R.F. ammeter (which is in series with the anode tuning inductance). Long-wave oscillations, which usually occur only on parallel oscillations in push-pull circuits, can usually be identified because they are stopped by shorting any inductance in an analogous position to L_3 in Fig. 4(*a*)/XI:12.

If a push-pull oscillation in a push-pull amplifier is suspected, the presence or absence of this can be checked by placing a short from grid to grid or anode to anode of the push-pull stage. If this fails to remove the oscillation it is due to some other cause.

Another method of checking that a particular circuit is causing oscillation is to change the resonant frequency of the circuit by adding or subtracting capacity or inductance, and seeing if the frequency of the parasitic oscillation is changed. Provided due care is taken, certain oscillatory circuits can be cut out by shorting either the condenser or inductance of the circuit; if this removes the oscillation, the circuit in question is the source of oscillation, provided the short has not put any other possible parasitic circuit out of action.

Confusion is sometimes caused because, when a certain measure has eliminated one oscillation, another oscillation appears and masks the fact that the first oscillation has been eliminated. For this reason the conditions of the circuit when oscillating should be carefully noted: meter readings should be taken and the type of arc drawn from the conductors or the degree of illumination of a neon lamp should be carefully observed. If any change in oscillating conditions results from any circuit change the part of the circuit modified is suspect. The means of eliminating oscillations, once the oscillatory circuit is located, have been given above.

Generally speaking, when there is no indication of the type of oscillation existing, the only method of attack is to apply remedial measures to, or modify the characteristics of, each possible circuit in turn, and note whether the oscillation is suppressed or modified in character.

13. Stability Tests on Transmitters (see also XX:10.5).

There are a number of tests designed either to produce conditions close to the practical conditions of operation, or to produce alternative conditions under which any tendency to oscillate is likely to be shown up. The last test below simulates operating conditions, the others produce conditions favourable to oscillation.

1. *Increase of H.T. from Zero, with Zero Grid Bias or Very Low Grid Bias.* (Applicable to any stage.) With zero grid bias, or the minimum value of bias which can conveniently be applied, the H.T. is slowly increased from zero to the value at which the anode current corresponds to the rated dissipation of the valve. The anode-feed meter and grid-current meter are watched for flicks. This is the standard stability test in the B.B.C. and has the advantage that if violent oscillations occur they are limited in amplitude. This test is therefore a good one to use, whenever adjustment has been made to a stage, before proceeding to the other tests given below.

2. *Reduction of Grid Bias without Drive.* (Applicable to any stage.) With no drive and normal H.T. the grid bias of each stage is reduced in turn until the anode feed corresponding to the rated anode dissipation of the valves is reached. This is done steadily and the anode-feed meter and grid-current meter are watched for flicks. This test approximates very closely to working conditions, but not so closely as test No. 3.

3. *Variation of Drive and Modulation with Steady Tone.* (Applicable to a linear class B amplifier for modulated waves.) In this the drive is slowly increased from zero to full carrier value and all meters are watched for flicks. If no flicks occur, the transmitter is stable *up to carrier level.* To test the transmitter above carrier level: with normal carrier drive, tone modulation at about 1,000 cycles is applied, and the modulation slowly increased from zero to 100% modulation, grid and anode current meters being watched for flicks. The second part of this method can only be carried out if a continuously variable potentiometer is available in the audio-frequency circuit. If potentiometers with stops are used it is impossible to distinguish the jumps in current due to each step from current variation due to the inception of spurious oscillations. See also XX:10.5.

CHAPTER XII

DRIVE EQUIPMENT

1. Definition of Drive Equipment.

THIS is described in terms of B.B.C. practice.

Drive equipment includes all the apparatus involved in the generation of carrier frequency oscillations, the maintenance of these oscillations at their correct frequency within the tolerated limits of deviation, and their application to radio transmitters at the necessary level. The type of equipment varies with the requirements of frequency stability, with wavelength and with the particular means employed for checking and adjusting frequencies to their correct value. The requirements of frequency stability vary with the number of transmitters operating on the same wavelength, with their distances apart and with their service areas.

2. Cairo Requirements on Frequency Stability of Broadcasting Transmitters.

Where only one transmitter is working on a wavelength it is only necessary that the frequency stability shall conform to the requirements of the International Radiocommunication Conference at Atlantic city in 1947 as given in I:8.1.

In B.B.C. practice, in the lower frequency range above, the use of crystal oscillators enables carrier frequencies to be kept within a small fraction of ± 20 c/s, while in the short-wave band variable-frequency oscillators are in use with a frequency stability better than $\pm 0{\cdot}005\%$. These crystal oscillators are described in XI:4, while the variable frequency oscillators are described in XI:3.

3. Synchronized Operation of Transmitters on the same Wavelength.

When high-power transmitters modulated with the same programme are operated on the same wavelength at such distance apart that their service areas overlap, if the carriers are kept in exact synchronism, a fixed interference pattern is set up such that the two carriers vary in relative phase throughout the region of overlap, and while at certain points the carrier fields add, at others they subtract. If the fields are nearly of equal strength, at points where the carriers are in opposition an effective increase in percentage modulation results, accompanied by response distortion, since the sidebands are in random phase owing to random phase shifts occur-

ring in the transmitters and in the apparatus and lines feeding the transmitters. On high-percentage modulations serious amplitude distortion therefore results, corresponding to an effective over-modulation. At points where the residual carrier is thrown seriously out of its true phase relation with the sidebands, loss of amplitude in the detected wave, accompanied by non-linear distortion, results. Areas in which these effects occur of serious magnitude are called "mush" areas, and unless a transmitter is undermodulated, occur in localities where the relative field strength magnitudes are in the ratio of 3 or 4 to 1 or less.

When the two carrier frequencies are asynchronous, a moving interference pattern occurs, with the result that each receiver in the area experiences a series of complete cycles (of relative phase * relations between the two carriers) at a frequency corresponding to the beat frequency (equal to the difference in frequency) of the two carriers. When the beat frequency is fast the disturbing effect is increased.

Where overlap of service areas occurs it is usual to use high stability crystal oscillators so that each cycle of phase relations, i.e. the period of beat, is not less than about 20 seconds. This period was chosen because it is equal to the mean probable period of normal night fading in the medium-wave band. The oscillators used for this purpose have a long-term frequency stability, measured over a period of a month, equal to $\pm 2{\cdot}5/10^8$, and a short-term stability, measured over a period of several hours, equal to $\pm 1/10^8$.

If two transmitters, supposedly on the same frequency f, have a frequency stability of $\pm 2{\cdot}5/10^8$, and the two frequencies deviate in opposite directions to the extreme limit of the tolerance, the period of beat is $20 \times 10^8/f$. This can easily be seen since, if the nominal carrier frequency is 10^6 c/s, the frequency difference in such a case as the above is $2{\cdot}5/10^8 \times 10^6 \times 2 = 0{\cdot}05$ c/s. The period is therefore 20 seconds.

In the case of groups of small synchronized stations, where the service areas do not overlap, a frequency stability of $1/10^6$ is considered adequate and crystal oscillators are used with long-term stability which is better than $\pm 5/10^7$. These are checked and adjusted twice a month.

Synchronization by Direct Drive from Line Tone. The first synchronized transmitters in England were synchronized by

* The phase relation between two nearly identical frequencies is here defined for any instant of time as the angular distance between the nearest consecutive points, of zero amplitude and positive slope, on each wave.

direct drive from a tone generated by a valve-maintained tuning-fork at one station and distributed by line to all other transmitters in the group. At each transmitter in the group the tone was supplied to the input of a chain of multipliers which multiplied the line tone up to the carrier frequency and supplied it to the input of the transmitter R.F. drive chain.

When quiet lines were available this system was quite satisfactory, but it was found that the requirements on the line performance were much more rigid than those for lines used for any other purpose. Further, no satisfactory method of formulating these requirements could be found which enabled simple tests to be made on the lines to verify that they were satisfactory.

Often the lines were noisy, or subject to sudden variations in propagation time, and amplitude changes which introduced random effective phase changes. The result was to modulate the carrier, supplied to the transmitters, with various kinds of noise.

This system was therefore abandoned in favour of the use of crystal oscillators of high frequency-stability.

When these are used it is only necessary to check the frequency periodically by comparing the carrier frequency of a slave station with that of the master station, either transmitted through space to the slave or by means of a frequency divider chain, a land line and a frequency multiplier chain. In the B.B.C. all checking is now done by land line. The above system of periodical check is referred to in the B.B.C. as non-rigid synchronism. Rigid synchronism means that one carrier frequency is locked to another electrically.

4. The Beat Indicator.

The keystone of all frequency comparison methods is some visual form of beat indicator. In its simplest form this consists of a non-linear device which is supplied at its input with the two frequencies to be compared and which delivers at the output the beat (difference) frequency. This difference frequency is passed through a D.C. meter with a direct current of slightly larger amplitude than the beat-frequency amplitude. Since in general the beat is very slow, the fastest beat observed seldom exceeding one or two a second, the meter needle is able to follow the beat frequency. One of the frequencies is then adjusted until the needle of the meter remains stationary, or fluctuates about a mean position due to disturbances which are usually unavoidably applied to the detector (non-linear device) along with one of the compared frequencies. To smooth out the fluctuations, due to disturbances which are of too low

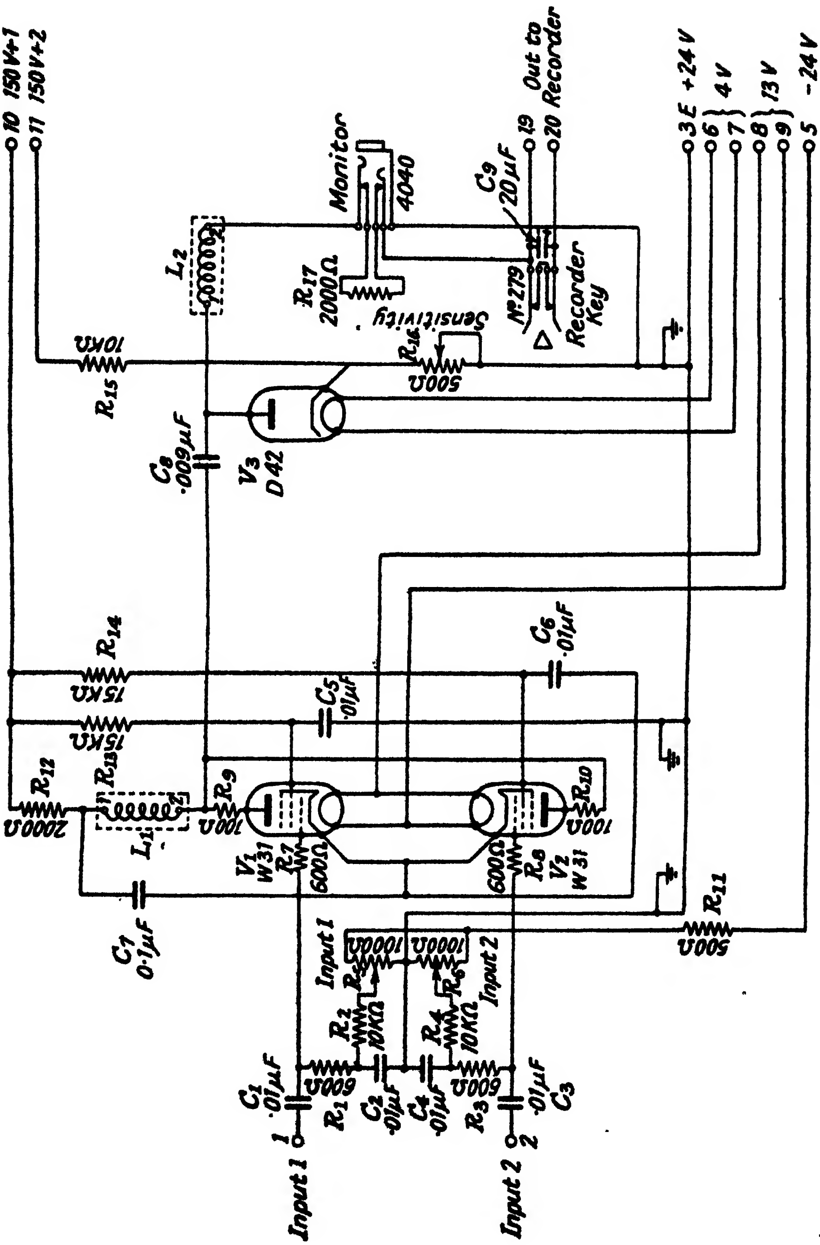


FIG. 1/XII:4.—Beat Indicator.
(By courtesy of the B.B.C.)

a frequency to be successfully dealt with by electrical filtering, a mechanical method of damping the meter movement is used. In one form of beat indicator in common use the meter is replaced by

a recording milliammeter which makes a permanent record of the beat. This has the advantage that it automatically counts the beats. The circuit of this beat indicator is shown in Fig. 1 in which each of the two incoming frequencies for comparison is fed into one of the two separating and amplifying pentodes with a common output into a diode detector which supplies the beat frequency plus direct current to the recorder. This beat indicator has now been replaced by the cathode-ray tube indicator described below, but is still in use for direct comparison of oscillator frequencies.

4.1. Cathode Ray Tube Indicator. The following arrangement has been found preferable to that using the beat indicator in that it is not so disturbed by phase modulation of the reference tone due to accidental effects in the land line.

The reference tone which is a submultiple of the master station radio frequency is made to generate a P.R.F. of the same frequency as the reference tone. The pulses so derived are applied to the Y plates of a C.R.T. having its time base locked to a submultiple of the slave radio frequency, this submultiple frequency being a multiple of the line reference frequency.

Synchronism between master and slave is then indicated by a stationary pulse image. If the time base deflection is from left to right, the effect of the slave station radio frequency being one c/s high would be to cause the pulse image to travel from left to right at a velocity of one-tenth of the sweep distance in one second. If the radio frequency were one c/s low, the pulse image would move from right to left at the same rate.

5. Methods of Frequency Checking and Adjustment in Synchronized Groups.

In all methods of frequency checking one station is made a master station, and the frequency of all remaining stations in the group, sometimes referred to as slave stations, is made equal to that of the master station. The master station is usually one of the normal stations of the group, but in special cases may be any station where a suitable frequency standard is available. The B.B.C. now use a single sub-standard oscillator in London as master so that all stations are slave stations.

5.1. Frequency Checking by Radio. In this case the master station radiates unmodulated carrier, for a short period, and the slave stations receive the master station on a receiver with a meter in the diode load, or in a special low impedance detector circuit. Each station also injects the carrier frequency output from its own

drive equipment into the frequency checking receiver, and adjusts its frequency until zero beat is obtained, as indicated by the current variation in the diode circuit which serves as a beat indicator.

Normally, frequency checking by radio can, however, be carried out only when the group as a whole is shut down. Further, in slave stations where there are other transmitters, reception of the master station may be difficult when the other transmitters are radiating. For these reasons frequency checking by line, which can be carried out at any time, even when the group is in normal service, is preferred.

By the use of directional reception at a site a few miles from each slave station it is sometimes possible to check frequency without shutting down the group of transmitters. The directional properties of the aerial are used to bring the received signals of the local slave and the master to comparable amplitudes. The beat frequency is passed back to the parent slave station by causing the beat frequency to modulate a local oscillator, usually generating a frequency of about 1,000 c/s. At the time of writing (March 1951), frequency checking by radio is obsolete in the B.B.C.

5.2. Frequency Checking by Line. In this case the master station is equipped with frequency dividers. These are devices which, when driven by the carrier frequency at their input, deliver at their output a submultiple of the carrier frequency locked to the carrier frequency. Locking implies that any given number N of cycles of carrier frequency observed at any time always give rise to N/n cycles of the output frequency, where n is the dividing factor of the divider and is an integer. The vector describing the carrier frequency rotates at n times the rate of the vector describing the submultiple frequency, and there is no slip between these vectors.

The division ratio which can be obtained in practice with a single divider which provides stable locking does not exceed 10. Reliable operation can, however, be obtained with any integral division ratio between 1 and 10. As the output frequency of the dividers has to pass over a land line, the final divider output frequency usually lies between 1,000 and 2,000 c/s, and for this reason it is necessary to use a number of dividers in tandem, the output of the first divider supplying the input to the next, and so on. For frequency checking purposes the line reference frequency is sent over a line, which is usually a music circuit, to each of the slave stations where it is applied to a chain of frequency multipliers which deliver at their output a frequency equal to the original (master station) carrier frequency locked to the line frequency. This is compared with the slave station carrier frequency in a beat indicator. On no account

must a carrier circuit be used as this may introduce frequency change. After multiplication, the received frequency is compared on a beat indicator with the frequency of the slave transmitter, and adjustment of the frequency of the slave transmitter drive is made until zero beat is obtained.

6. Frequency Dividers.

Three types of frequency divider are worthy of consideration: locked oscillators, multivibrators and modulator dividers.

6.1. Locked Oscillators. If an oscillator which naturally tends to oscillate at a frequency f has injected into its oscillating circuit a frequency f_2, approximately equal to $2f$ or a frequency f_3 approximately equal to $3f$, it is possible to pull the oscillator off its free frequency f and lock it at a frequency equal to $\frac{1}{2}f_2$ or $\frac{1}{3}f_3$. Such an oscillator can therefore be driven or locked by an input frequency 2 or 3 times its frequency of oscillation, which it can be made to deliver at a suitable output. Such a locked oscillator constitutes a frequency divider.

The locking (frequency) range of an oscillator using an inductance capacity oscillating circuit, and oscillating at a frequency determined by the loop phase shift, is comparatively small, and the division ratio obtainable under stable conditions of locking is not greater than two or three. For this reason, although locked oscillators have in the past been used as frequency dividers, their use has been superseded by what are called *relaxation oscillators* of a type known as *multivibrators*.

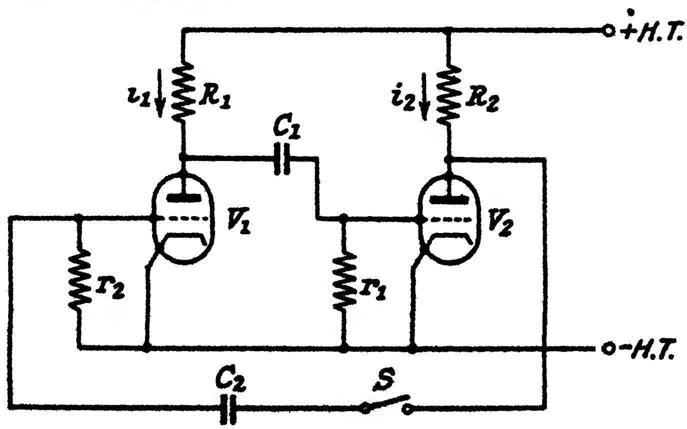


FIG. 1/XII:6.—Basic Multivibrator Circuit.

6.2. Multivibrators. The basic circuit of the original multivibrator is shown in Fig. 1. In such a circuit a relaxation oscillation

occurs giving rise to a wave form which is far from sinusoidal and is, therefore, rich in harmonics. It is convenient to regard the mode of

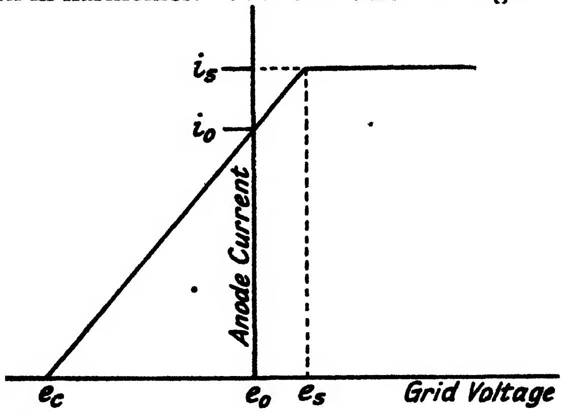


FIG. 2/XII:6.—Ideal Anode-Current Grid-Voltage Characteristic of Valve with Resistance Load.

oscillation as being entirely different from the modes of oscillation previously described, although an infinitely graded range of circuits can be constructed with performance characteristics intermediate between those of normal oscillation and relaxation oscillation.

The basic circuit is really superseded by the type of circuit discussed under XII:21 below, but an understanding of the basic circuit constitutes a convenient introduction to the later circuits.

For simplicity, consider the symmetrical case where the valves are identical and the anode resistances are equal, as are also the grid leaks and coupling condensers. Further, ideal valve characteristics will be assumed, as shown in Fig. 2.

Conventions.

e_c = the value of negative grid voltage at which anode current cut-off occurs.

i = the anode current of either valve.

i_o = the anode current of either valve at zero grid bias.

i_s = the value of anode current at saturation: determined by the external anode resistance and the value of H.T. supply voltage.

e_s = the value of grid volts at which the internal valve resistance becomes negligible compared to the external resistance in the anode circuit, i.e., the value of grid volts at which anode current saturation starts.

e = the value of grid bias of either valve, effective at any time.

When it is required to refer to quantities affecting one specific

valve, the suffixes 1 and 2 will be used, e.g.

e_{s_2} = the value of positive grid voltage on valve 2 which produces saturation.

A physical picture of the operation of the circuit can be obtained by considering the sequence of events when switch S is closed, the supply voltages being applied and C_2 having no charge.

At the instant of closing S, since C_2 has no charge, the potential at the anode of V_2 is applied to the grid of V_1. This potential will be much greater than e_{s_1} so that i_1 will jump from i_0 to i_s and the potential at the anode of V_1 will fall by an amount $(i_{s_1} - i_{0_1})R_1$, making the grid of V_2 negative by an equal amount. If $(i_{s_1} - i_{0_1})R_1$ is greater than e_{c_2}, and we will assume that it is, V_2 is put beyond cut-off and i_2 falls to zero. The anode voltage V_2 is therefore a maximum and C_2 now charges via R_2 in series with r_2 shunted by the grid-cathode resistance of V_1, which will be low compared with r_2 when the grid is positive to the cathode; simultaneously C_1 discharges via the anode-cathode path of V_1 and the grid leak r_1. As C_2 charges, the grid of V_1 becomes less positive: that is, e_1 falls. As soon as e_1 falls below e_{s_1}, any further charging of C_2, by increasing the anode potential of V_1, assists the discharging C_1 in raising the grid potential of V_2 above the cut-off point. As soon as e_2 rises above the cut-off value and i_2 starts to increase, the process accelerates so that i_2 jumps to i_{s_2} and i_1 falls to zero. That the process should accelerate when one anode current is rising and the other falling is readily seen, since an increase in i_2, say, causes a decrease in the anode potential of V_2, and thus in the grid potential of V_1; this decreases i_1 and increases the anode potential of V_1, which in turn increases the grid potential of V_2 and further assists in increasing i_2.

If at the time of this abrupt change the charging current of C_2 is small compared with i_{s_2}, there will be a fall in potential at the anode of V_2 almost equal to $i_{s_2}R_2$, and the grid potential of V_1 will be made negative by this amount. The anode current i_1 of V_1 therefore falls for some value between i_{0_1} and i_{s_1} to zero, and so the grid potential of V_2 is made positive by an amount greater than $i_{0_1}R_1$. C_1 now charges via R_1 and r_1 shunted by the grid-cathode path of V_2, which will be low in resistance compared with r_1, while C_2 discharges via the anode-cathode path of V_2 and the grid leak r_2. The charging and discharging of these condensers continues until the grid of V_1 is brought above the cut-off point, when the process again accelerates resulting in i_1 jumping to i_{s_1} and i_2 falling to zero.

It is thus seen that a cycle occurs consisting of two main parts (divided by transition states), one during which i_1 is zero and the

other during which i_s is zero. It is not possible accurately to calculate the frequency of oscillation, but an approximate formula can be obtained by neglecting the effect of the charging condenser and assuming that the duration of each part of the cycle is determined by the discharging condenser. This discharges via a valve anode-cathode path and a grid leak, and, at low frequencies with a high grid-leak resistance, the resistance of the anode-cathode path can be neglected in comparison with the grid-leak resistance. The grid potential decays from $-e_p = -(i_{s_1} - i_{0_1})R$ to $-e_c$ where $R = R_1 = R_2$, and so the time taken is

$$t = rC \log_e \frac{e_p}{e_c}$$

where

$$r = r_1 = r_2$$
$$e_p = (i_{s_1} - i_{0_1})R$$
$$R = R_1 = R_2.$$
$$C = \text{capacity in farads.}$$

In a symmetrical circuit the time for a whole cycle is twice this, and so the frequency is

$$f = \frac{1}{2rC \log_e \frac{e_p}{e_c}}$$

In an asymmetrical circuit the times for the two parts of the cycle must be calculated separately and the frequency is given by

$$f = \frac{1}{r_1 C_1 \log_e \frac{e_{p_1}}{e_{c_2}} + r_2 C_2 \log_e \frac{e_{p_2}}{e_{c_1}}}$$

At high frequencies, when the resistances have low values and the valves do not operate with a flat saturation current characteristic, it is not possible to give even an approximate formula. The chief value of the formula is to indicate that the time period of the circuit is controlled by the time constant of the condenser circuits and by the voltages e_p and e_c at which amplitude limitation is introduced.

When a flat saturation current characteristic is obtained and it is determined by the value of the anode load resistance, if $i_s >> i_0$, the frequency is substantially independent of the value of this resistance, because the saturation current varies inversely with load resistance so that the product $i_s R$ remains constant.

The basic circuit has the following disadvantages:

(1) There are two condenser-resistance circuits determining the frequency, and this complicates adjustments.

(2) There is no part of the circuit which does not contain the wave form, so that if two or more multivibrators are to be locked together, separator stages must be used.
(3) It is difficult to make the circuit oscillate above 100 kc/s.
(4) The circuit is sensitive to slight changes in supply voltages.

It was found by experiment that by introducing a common cathode resistance in the basic multivibrator circuit (no circuit for this is shown, but the position of R_c in Fig. 3 illustrates where a similar resistance should be placed in Fig. 1), a number of advantages are obtained.

(1) The loop gain is increased owing to positive feedback.
(2) The grid current is reduced.
(3) A more precisely determined value of limiting (saturation) current results; this is important when low anode resistances are used.
(4) The value of limiting current is reduced, which increases the frequency of oscillation for given coupling condensers and resistances.
(5) The frequency stability, with variation of H.T. is improved.
(6) If it is arranged that the locking voltage varies linearly with H.T., wide locking ranges can be obtained for variation of both supply voltage and frequency.

The improvement is sufficient to enable reliable frequency division to be carried out from frequencies of the order of one megacycle per second. A group of dividers was built on this principle to divide from 1,149 Kc/s to 1,122 c/s in four stages; the first two stages each divided by 4 and the second two by 8, giving an overall division ratio of 1,024.

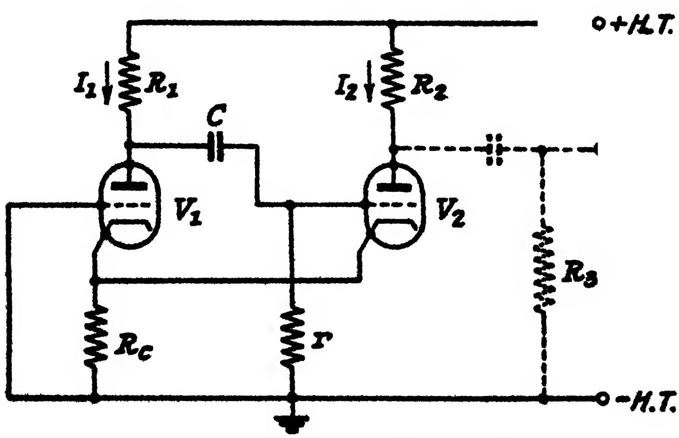


FIG. 3/XII:6.—New Type Multivibrator.

6.21. Single-Circuit Divider. (B.B.C. Patent No. 528,806.) Fig. 3 shows an improved type of multivibrator circuit due to

R. Calvert which has all the advantages obtained with the common cathode resistance in the basic circuit, and in addition the following :

(1) It is much simpler to adjust since it has only a single-frequency determining circuit.
(2) No separator stages are required.
(3) Reliable oscillation can be obtained at frequencies as high as one megacycle per second.
(4) It can be made to work with no grid current in either valve.
(5) When given a suitable locking voltage it can be made to work over extremely wide ranges of supply voltage and frequency. In a set of four dividers of this type dividing from 767 kc/s to 1,498 c/s the H.T. supply could be varied between 170 and 80 volts, or the frequency varied $\pm 18\%$, without the divider becoming unlocked.
(6) The new circuit gives locking to both pulses.

6.211. Method of Operation. For brevity, call the anodes of valves V_1 and V_2 in Fig. 3 A_1 and A_2, the grids G_1 and G_2, the cathodes K_1 and K_2, the anode currents I_1 and I_2 and the coupling condenser C as indicated.

The oscillating cycle may be divided into four periods, each of which has a starting condition, a transition time and an end condition, see Fig. 4. In two of the periods the transition time is so rapid that it is assumed to occupy zero time, while the lengths of the other periods are determined respectively by the times of charge and discharge of C. The full cycle consists of four alternate periods of short and long transition time, and in describing these it is convenient to begin the description of the oscillating cycle at the starting condition of one of the long periods.

Period 1.

(*a*) *Starting Condition :* A_1 has just gone to maximum positive and driven G_2 to maximum positive, I_2 is at saturation, K_1 and K_2 are at maximum positive due to I_2 flowing through R_c, $I_1 = 0$, A_2 to minimum positive.

(*b*) *Long Transition Time :* Controlled by charging of C: C charges through R_1 and r, and as it does so G_2 becomes less positive, I_2 falls, K_1 and K_2 become less positive until finally V_1 starts to take current ; i.e. I_1 starts to increase from zero.

(*c*) *End Condition :* I_1 starts to increase.

Period 2.

(*a*) *Starting Condition :* As at end of Period 1.

(*b*) *Short Transition Time* accelerated by positive regeneration.

Increase of I_1 drops anode volts on A_1 which drives G_2 negative which drops I_2 which makes K_1 and K_2 less positive which increases I_1 which drops anode volts on A_1 further, and so on until the end condition is reached.

(*c*) *End Condition :* I_1 at saturation value, voltage of A_1 at minimum positive, G_2 at maximum negative, $I_2 = 0$, voltage of A_2 at maximum positive, K_1 and K_2 at maximum positive.

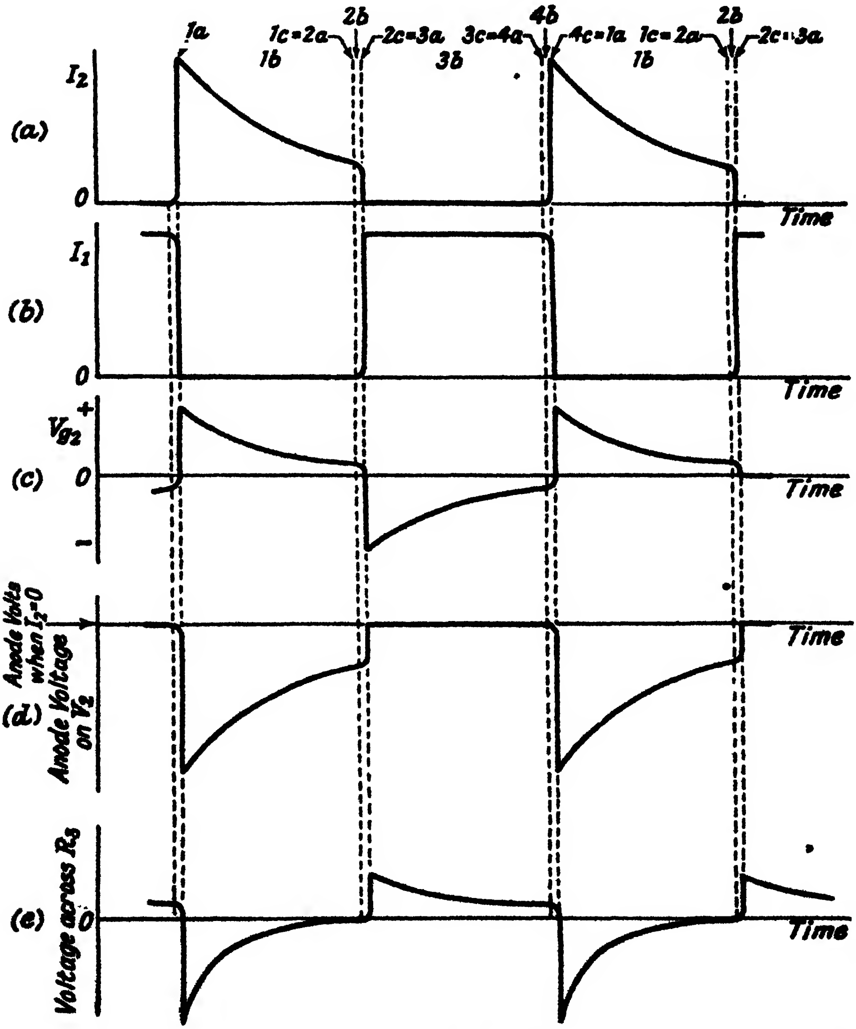


FIG. 4/XII:6.—(*a*) and (*b*) Anode Currents and (*c*) Grid Voltage of Valve V_2 in Fig. 3. (*d*) Anode Voltage of V_2 in Figs. 3 and 5. (*e*) Voltage across R_3 in Fig. 3.

Period 3.

(*a*) *Starting Condition :* As at end of Period 2. This, incidentally, is the inverse of the starting condition for Period 1.

(*b*) *Long Transition Time* controlled by discharging of C: C discharges through r and the anode to ground circuit of V_1, and as it does so G_2 becomes less negative, until I_2 starts to rise. During this transition time I_1 remains at maximum.

(*c*) *End Condition :* I_2 starts to increase.

Period 4.

(*a*) *Starting Condition :* As at end of Period 3.

(*b*) *Short Transition Time :* accelerated by positive regeneration. Increase of I_2 makes K_1 and K_2 more positive which drops I_1 which increases the positive volts on A_1 which makes G_2 positive which increases I_2 further, and so on until the end condition is reached.

(*c*) *End Condition :* As starting condition for Period 1.

In Fig. 4 the values of I_1 and I_2 are shown for one and a half cycles of oscillation, the conditions 1*a*, 1*b*, 1*c*, 2*a*, 2*b*, 2*c*, etc., being indicated. Also shown in Fig. 4 are the grid and anode voltages on valve V_2 and the output voltage across R_3, see Fig. 3.

Two important points are to be noticed : the grid of V_1 is always at ground potential, and the potential of the anode of V_2 is described by a wave form of the same shape as that of I_2 but reversed in sign. The first point means that the multivibrator can be driven with a locking voltage from any source without feeding back energy into the driving source. The second point means that the output from the anode of V_2 can be used to supply the next divider, without the necessity of a separator stage.

The most convenient way to couple the anode of V_2 to the following circuit is by means of a resistance capacity coupling as indicated in Fig. 3, the output voltage being taken from R_3. The anode voltage wave and the approximate general form of the voltage wave across R_3 are indicated in Fig. 4.

The form of the voltage wave across R_3 must evidently be such that the mean D.C. current is zero, in other words, the area enclosed between the curve and the axis of time must be the same each side of the axis. It is important to note that both positive and negative swings consist of a peak of voltage followed by a decay.

6.22. Locking of Multivibrators. The locking of multivibrators oscillating with one period, by oscillations having another period, depends on the fact that when the amplitude of the locking oscillation is suitably adjusted, only those impulses (given to the

multivibrator once every cycle of the locking oscillation) which occur at or near one of the short transition times are effective in controlling the change of currents in the multivibrator. If, for instance, a multivibrator is in condition 1*b* approaching 2*b*, a voltage pulse injected at a suitable point in the circuit (e.g. the grid of V_1 in Fig. 3) in such a sense as to aid the transition 2*b*, will cause condition 2*b* to occur just before it would have done if the multivibrator were oscillating freely at the same point of its cycle. A second requirement of locking is therefore that the sense of the locking pulse arriving just before a short transition time shall be such as to aid the impending transition.

A third requirement is that the free frequency of the unlocked divider should be some 10% lower than the frequency to which it is locked.

A fourth and rather obvious requirement is that the locking pulse should be sharply defined with regard to its location in time. For this reason sinusoidal oscillations are not satisfactory for locking, since the current rise and fall is too gradual. Where locking has to be effected from a source producing sinusoidal e.m.f.s, a square wave is generated by applying the sine-wave voltage to a valve, biased to cut-off and driven hard into anode limitation. Coupling from the anode of the limiting valve, or following amplifying valve, if required, is then made through a resistance capacity coupling similar to that in Fig. 3. This converts the square wave into a peaky wave of the form of the lower wave in Fig. 4, except that the form is symmetrical about the time axis. The resultant wave produces two locking pulses per cycle: one at the instant of rapid increase to a positive voltage, and one at the instant of rapid increase to a negative voltage.

The locking pulses are applied at the grid of V_1, Fig. 3: for instance, the grid of V_1 might conveniently be lead to ground through the coupling resistance of the driving source, e.g. R_2 in Fig. 3, when the locking wave is derived from another multivibrator.

If the grid of V_1 is driven positive I_1 rises and hence a positive pulse assists transition 2*b*, while a negative pulse assists transition 4*b*, see curve of I_1 in Fig. 4.

6.221. Locking Condition for Division by an Odd Number. Little has been said about the relative values of charge and discharge times of condenser *C*, Fig. 3. In Fig. 4 it has been assumed that these are such that the time interval from 1*a* to 2*c* (= 3*a*) is the same as the time interval from 3*a* to 4*c* (= 1*a*). This happens to be the required condition for division by an odd number.

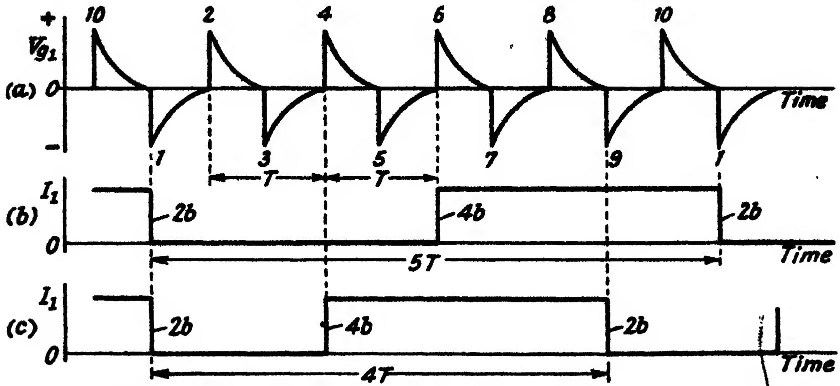


FIG. 5/XII:6.—Value of Locking Voltage V_{g_1} applied to the Grid of V_1, Fig. 3, and Value of I_1 for Division Ratios of 5 and 4.

Fig. 5 shows at (*a*) a locking wave form V_{g_1}, which is assumed to be applied to the grid of V_1 in Fig. 3. At (*b*) in Fig. 5 is shown I_1, the anode current through V_1 when the multivibrator is locked by a wave-form having a fundamental frequency equal to five times the fundamental frequency of the multivibrator.

It is evident that if the incoming negative pulses 1 initiate the transitions *2b*, then positive pulses 6 are in the right sense to initiate the transitions *4b*. Hence an odd number of (incoming) half-periods to each half-cycle of the multivibrator satisfies the requirement of correct sense of control during both transition periods *2b* and *4b*, when the two halves of the multivibrator cycle (i.e. the two long transition times) are equal.

It is evident that to preserve the correct sense of control the number of half-periods of the incoming frequency occurring in one half-cycle of the multivibrator cycle must always be odd.

6.222. Locking Condition for Division by an Even Number. If the two halves of the multivibrator cycle are equal, division by an even number demands an even number of incoming half-periods per multivibrator half-period: this condition will not produce locking, because the right sense of control does not occur at both transitions. The problem is resolved by making the two halves of the divider cycle unequal, as shown at (*c*) in Fig. 5, where the case of division by 4 is illustrated. In this case the shorter half-cycle $= 3\frac{T}{2}$ and the longer half-cycle $= 5\frac{T}{2}$, where T is the period of the incoming wave. In general, if n is the division ratio (equal to input frequency divided by output frequency), the

short half-cycle length $= (n - 1)\frac{T}{2}$ and the long half-cycle length $= (n+1)\frac{T}{2}$. The factor $(n+1)/(n-1)$ is then called the ***pulse width ratio.***

It may be remarked incidentally that the ideal locking waveform is one consisting of a succession of short-duration pulses, alternately positive and negative, separated by intervals of no current. Such a wave-form allows the multivibrator to settle down to its normal mode during the period immediately preceding each short-time transition period at which control or locking takes place. Any trailing edge on the pulse (an extreme case of such a trailing edge occurs in a square wave) may delay the operation of the divider and cause unreliable locking. The peaky wave form consequent on passing square waves or the output of a multivibrator through a resistance capacity circuit is therefore ideal for locking.

6.223. Adjustment of Relative Lengths of the Two Half-Periods of a Multivibrator. Since the pulse-width ratio of a divider varies with the division ratio it is evidently necessary to be able to control the relative times of charge and discharge of condenser C, Fig. 3. The condenser charges through R_1 and r in series and discharges through r and the anode-to-ground circuit of V_1. The time of discharge may be increased relative to the charge time by inserting a resistance directly in series with the anode circuit of V_1, but it can only be reduced by decreasing the effective anode-to-

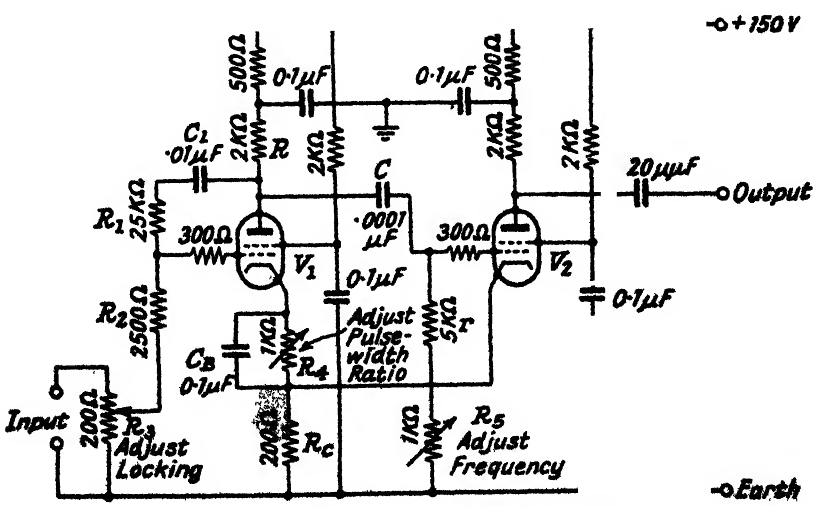


FIG. 6/XII:6.—Improved Multivibrator.

ground impedance of V_1. This may be done by applying negative feedback as shown in Fig. 6, where feedback is applied by the potentiometer made up of resistances R_1 and R_2, coupled to the anode of the valve through condenser C_1. The amplitude of the multivibrator wave-form fed back into the input circuit is very small, as can be seen by looking at the values of R_1, R_2 and R_3. When the multivibrator is driven from another multivibrator there is therefore no danger of locking back. This danger is also guarded against by the fact that the output valve of the preceding multivibrator is driven into limitation and its anode circuit is only coupled back to the oscillatory circuit through its cathode resistance.

Fig. 6 shows the complete circuit of the improved type of multivibrator. The variable resistance R_4 provides fine control of pulse width ratio, while the resistance R_5 provides fine control of frequency.

6.3. Modulator Dividers. Multivibrators, when used as dividers, have the disadvantage that, if the driving frequency fails, the multivibrator continues to oscillate at some frequency near to, but lower than its locked frequency. If therefore, as is sometimes the case, it is required to drive transmitters from a crystal-controlled oscillator at a submultiple of the crystal frequency, multivibrator type dividers are unsuitable, since if the crystal oscillator fails the transmitter may continue to radiate at a frequency equal to the free unlocked frequency of the divider. As this uncontrolled frequency is some 10% lower than the locked frequency, it is preferable that the drive should fail completely.

This requirement is satisfied by the class of modulator dividers described below. These have a further important advantage over multivibrator dividers, which is that they are not subject to a slip of 180° which may occur at the input frequency to any multivibrator.

These make use of a push-pull modulator as shown in Fig. 7, see XIII:10. The frequency to be divided is applied to the push-

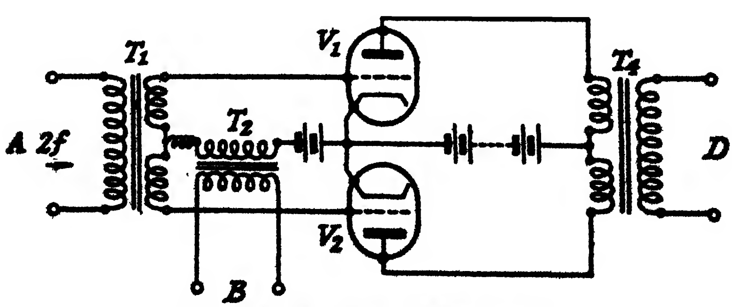


FIG. 7/XII:6.—Push-Pull Modulator.

pull input (the primary of T_1) and the output is taken from the push-pull output (the secondary of T_2).

If the frequency to be divided is called f_1, reference to condition 2 of the table on p. 539 shows that if a second frequency f_2 is applied at the parallel input no output at all appears in the push-pull output when frequency f_1 is removed, since sum and difference frequencies obviously cannot be present when only one frequency is present.

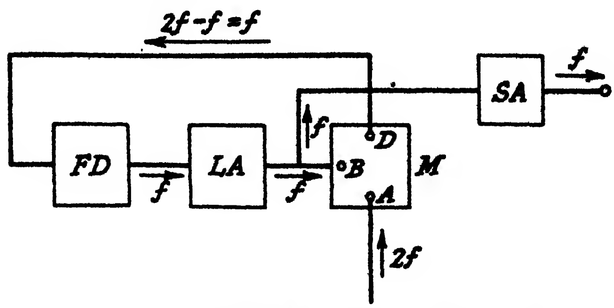


FIG. 8/XII:6.—Modulator Divider dividing by 2.

The simplest possible modulator divider circuit is shown schematically in Fig. 8. This is a divider by two. The input frequency $2f$ is applied at the push-pull input A of the modulator M, which has the effect of making the modulator M effectively pass a frequency f applied at input B since the difference frequency then appearing at output D is equal to $(2f-f)=f$. FD is a frequency determining circuit introducing a steep phase-shift-frequency characteristic into the circuit and LA is a limiting amplifier. When $2f$ is applied at A an oscillating loop exists through FD, LA and M, so that the circuit is free to oscillate at frequency f. It cannot oscillate at any other frequency, as the following argument shows: a circuit can only oscillate when there is unit amplification round the loop: in other words, if the loop is cut and the oscillating frequency is applied at any point of the loop so as to feed round the loop in the direction of amplification, the same frequency appears at all other points of the loop. If, for instance, the loop circuit is cut between M and FD, and a frequency f is applied to the input of FD, the same frequency appears at the output of M which satisfies the above requirement. If, however, some other frequency, $f+df$, is applied at the input of FD it traverses FD and is amplified in LA and applied at the input of M. At the output of M there now appears a new frequency of value $2f-(f+df)=f-df$. The

circuit will only oscillate therefore at frequency f. In designing modulator dividers the above line of argument should be used to check that the circuit is not free to oscillate at some frequency other than f: if a loop gain exists at frequency $f+df$ the circuit may oscillate at such a frequency.

The output of the circuit at frequency f may be taken away through a separating amplifier connected to the output of LA. It is to be noted that any power at frequency $2f$, which is present in the modulator output, due to unbalance in the modulator, is suppressed in FD and cannot pass through the modulator from A to B since these circuits are conjugate (i.e. effectively in bridge connection with regard to one another).

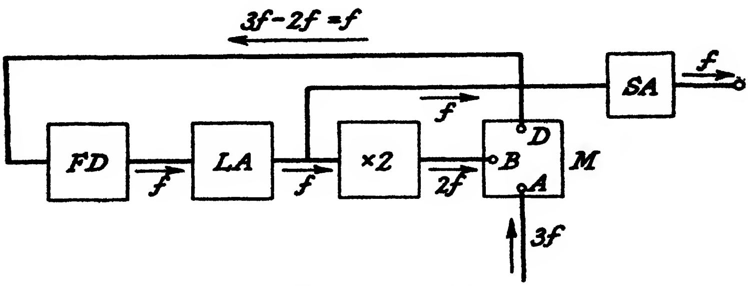


FIG. 9/XII:6.—Modulator Divider dividing by 3.

A divider by three is shown in Fig. 9. In this case a frequency multiplier by two is inserted after the limiting amplifier. Fig. 9 is otherwise self-explanatory. Similarly by using multipliers by 3, 4 and 5, etc., dividers by 4, 5 and 6, etc., can be constructed.

As this type of divider is more complicated, larger and more expensive than the multivibrator type, it is not used unless it is essential.

Fig. 10 shows the circuit of a practical modulator divider which is normally used for dividing by 3 or by 5. The frequency to be divided is applied at the input of transformer T_1 at the left of the circuit and so is applied to the double-balanced modulator, which is of the type described in XIII:10.1. The output of the modulator is applied through transformer T_2 to the input of valve V_1 which is driven into anode limitation and constitutes the limiting amplifier. The output of the limiting amplifier is applied to the grid of valve V_2, which has a tuned anode output circuit that selects the required output difference frequency, supplied by the modulator, and also

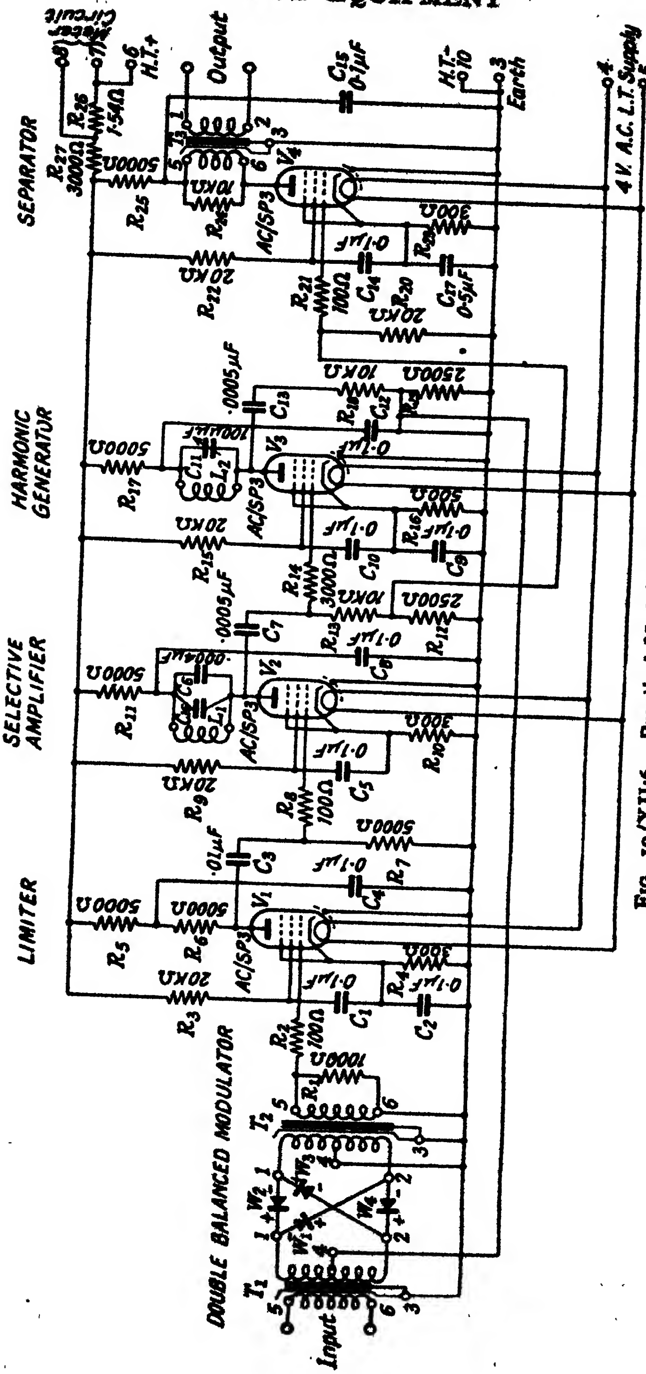


FIG. 10/XII:6.—Practical Modulator Divider.
(By courtesy of the B.B.C.)

constitutes the frequency determining circuit. It will be noted that in this circuit the position of the limiting amplifier and the frequency determining device have been reversed because the limiting amplifier is of a type which distorts the wave form applied to its grid, and the wave form must be purified before being supplied to the output circuit and to the input of the harmonic generator.

Two outputs are taken from the anode circuit of V_2, one to drive the harmonic generator, or frequency multiplier V_3, and one to supply the divided frequency to the grid of V_4, which is a straight class A amplifier valve delivering the divided frequency to the output through transformer T_3. Part of the output voltage of V_3 is taken from the centre of the potentiometer, constituted by R_{18} and R_{19}, to the mid-point of the secondary winding of transformer T_1, and so supplies the multiplied (output) frequency to the modulator. The operation of the circuit is now clear from its analogy with Figs. 8 and 9, remembering that the positions of *FD* and *LA* are interchanged.

If, for instance, it were required to supply drive to a transmitter on a carrier frequency of 200 kc/s having a frequency tolerance of $\pm 2{\cdot}5/10^6$, a GT-cut crystal of 100 kc/s and a frequency doubler could of course be used. Assume, however, that a high stability drive of 1,000 kc/s is available. A frequency divider dividing by five could be used.

The output frequency from the divider would therefore be $f = 200$ kc/s, while the input frequency would be $5f = 1{,}000$ kc/s. The frequency fed back to the modulator from the output of the frequency multiplier stage would therefore be $4f = 800$ kc/s and the anode circuit of the frequency multiplier stage V_3 would therefore be tuned to 800 kc/s. The anode circuit of V_2 would evidently be tuned to 200 kc/s.

In later models of this divider it is intended to replace the output transformer T_3 by a circuit tuned to the output frequency in order to provide further discrimination against unwanted frequencies introduced either by the modulator or by the limiter.

Since this section was first written a simplified type of modulator divider using a single valve has been introduced in the B.B.C. (B.B.C. Provisional Patent No. 20026/49).

This type of divider has completely displaced the multivibrator type of divider, since when the input fails it gives no output so that there is no danger of a reference tone of incorrect frequency being transmitted to a slave station.

7. Frequency Multipliers.

Multivibrators may be used as frequency multipliers, but are unsatisfactory for the following reasons.

There are two possible methods of use : the multivibrator may be run at a multiple of the locking voltage frequency, or the multivibrator may be locked on its fundamental frequency and the required harmonic selected from its output by means of a tuned circuit.

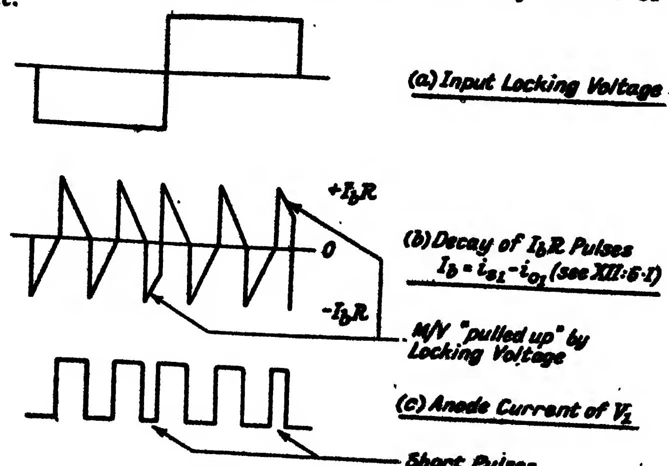


FIG. 1/XII:7.—Multivibrator Wave Forms.

The difficulty with the first method is that the multivibrator receives only one locking pulse for every n current pulses, where n is the multiplying factor, and during the rest of the succeeding multivibrator pulses it runs free. This condition is illustrated in Fig. 1 for a multiplier by 5.

The locking voltage is shown at (*a*), as a square-topped symmetrical wave (the argument is easily extended to any other form of locking voltages). The decay of the pulses of the multivibrator is shown in (*b*). It has been assumed that the multivibrator is running slightly slow and is pulled up every 2½ cycles by the vertical strokes of the locking voltage. The current pulses are shown in (*c*), from which it will be seen that every time the multivibrator is "pulled up" by a locking stroke the corresponding current pulse is reduced in width. The only condition under which the current pulses are all of the same width, therefore, is when the "free" frequency of the multivibrator is an exact multiple of the frequency of the locking voltage. If, however, the multivibrator "free" frequency is not an exact multiple of the locking frequency, as in

practice it will not be, the locking pulses to the next multivibrator are not spaced equally in time, and it is not hard to see that when one has gone but a little way up a chain of such multipliers a random element can creep in, pulses being missed entirely or extra pulses added. In other words, the locking range is extremely narrow for both circuit elements and supply voltage variations.

The second method, while it is free of the above difficulty, since each multivibrator is given a locking voltage at its own fundamental frequency, is not well suited to multiplication by even numbers. This is because, for stable operation when so locked, the multivibrator pulse-width ratio should be the same as that of the locking voltage; but the locking voltage is obtained from a tuned circuit and therefore it is approximately sinusoidal, so that the multivibrator pulse-width ratio is approximately 1:1, and a square wave with a 1:1 pulse width ratio does not contain any even harmonics.

It would appear, therefore, that there is no point in using locked multivibrators as frequency multipliers. If each multivibrator is locked on a harmonic of the input frequency the locking range is of necessity extremely small; while if each is locked to the same frequency as its locking voltage it serves only to produce a square wave at this frequency and requires two valves. A close approximation to a square wave can be produced more simply by one valve.

7.1. Harmonic Generator Multiplier. The simplest form of harmonic generator circuit is constituted by a half-wave or full-

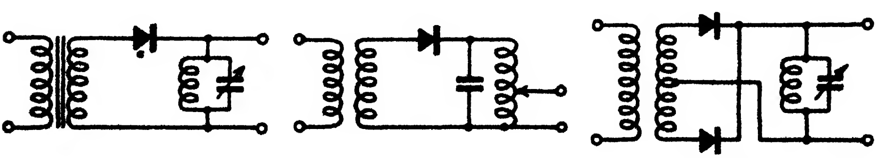


FIG. 2/XII:7.—Simple Harmonic Generator Circuits.

wave rectifier; typical circuits are shown in Fig. 2. The circuits at (*a*) and (*b*) are half-wave rectifier circuits suitable for working into following circuits respectively of high and low impedance. Provided the reverse resistance of the rectifier is high compared with the impedance of the tuned circuit at the required frequency the voltage across the tuned circuit is of the form in Fig. 1(*b*)/VIII:1, and is therefore suitable for multiplying by even numbers. The circuit at (*c*) is a full-wave rectifier circuit and produces a wave form as in Fig. 1(*c*)/VIII:1 and is therefore also suitable for multiplying by even numbers.

The damping on these circuits is, however, high, so that they are therefore only suitable for use as frequency doublers, and their use is comparatively rare. The effect of damping is discussed below. The tuned circuits in these circuits would therefore be constructed to resonate at twice the input frequency.

7.2. Harmonic Generators for Multiplying by Odd Numbers. If a valve biased back to cut-off is driven with a voltage of amplitude equal to several times the grid bias necessary to take the valve to cut-off, and if a high resistance is inserted in series with the grid, the resulting anode current gives an approximation to a square-topped wave. As soon as the grid is driven sufficiently positive to cause grid current to begin to flow, limitation occurs, while the negative half-cycle is eliminated completely since the valve is biased to cut off.

The harmonic component frequencies of a square wave are given in Fig. 3(*c*)/VIII:1 and consist of the fundamental frequency plus odd harmonics. If a tuned circuit or band-pass filter is placed in the anode circuit of the valve adjusted to resonate at 3 or 5 times the input frequency, such an arrangement can be used for multiplying by 3 or 5. Higher multiplication factors are difficult to obtain because of the damping of the circuit.

If a square wave of current flows through a parallel tuned circuit, the circuit receives an impulse at the beginning and end of each half-period. Between these times the circuit oscillates at its natural frequency, and the amplitude decays until the next impulse is received. The importance of the decay is reduced if the multiplier is followed by any form of limiting device, but if no limiting device is used each multiplier output will be modulated by a wave form corresponding to the decay curve of its tuned circuits, starting twice per cycle of the input wave.

Fig. 3 shows a type of harmonic generator which contains both the principles described above. A full-wave rectifier provides a frequency doubler working into the amplifier valve V_1. A band-pass filter in the anode of V_1 tuned to twice the input frequency supplies the doubled frequency to the grid of V_2 through a limiting resistance of 100,000 Ω. V_2 constitutes a multiplier by 5 in conjunction with the band-pass filter in its output circuit tuned to 5 times its input frequency. The whole circuit therefore multiplies by ten. Such harmonic generators have the advantage of automatically introducing limiting.

7.3. Harmonic Generators for Multiplying by Even Numbers. Figs. 6, 7 and 8/VIII:1 give the harmonics (expressed

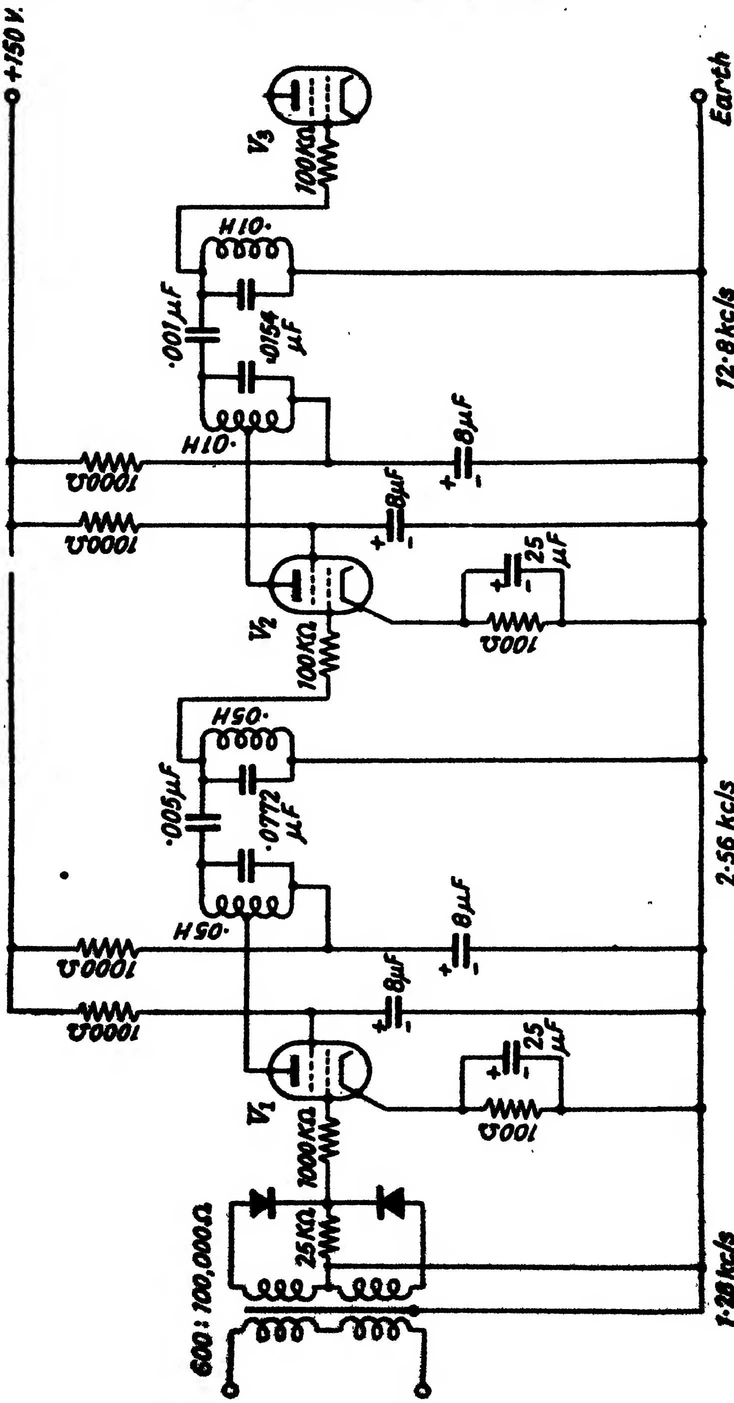


FIG. 3/XII:7.—Harmonic Generator Multiplier. (Values given for Input Frequency of 1·28 kc/s.)
(By courtesy of the B.B.C.)

as a percentage of the peak current) present in the anode current of valves, respectively of linear and square law anode current grid voltage characteristics, driven by a sinusoidal input voltage, and biased to different angles of current flow. While, in the case of the second harmonic, little advantage, if any, is to be obtained by reducing the angle of current flow below 180°, in the case of the higher even harmonics considerable advantage is *sometimes* to be obtained by adjusting the angle of current flow below 180°.

Harmonic generators having multiplication factors of 2 and 4 are therefore made by biasing a valve back to the point at which the amplitude of the required harmonic is greatest when driven with a sine-wave input voltage of the maximum amplitude which does not cause the valve to take grid current.

No limitation occurs, and if limitation is required it must be introduced separately. As before, tuned circuits, or band-pass filters are connected in the anode circuit and tuned to the harmonic it is required to select.

7.4. Limitation on Multiplication Factor. As the multiplication factor is increased the efficiency of the multiplier is reduced since the amplitudes of the higher order harmonics decrease rapidly. This, however, does not constitute the real limitation, which is an experimental one determined by considerations of stability. When a chain of multipliers is operated to give a high multiplication ratio, it is found that reliable locking is not always obtained when the multiplication per stage exceeds 4 for even multipliers and 5 for odd multipliers.

The reason for this is that the tuned circuit in the anode of each multiplier valve oscillates at its natural frequency during periods of zero anode current change, and the oscillation set up in the tuned circuit of stage 1 of a multiplier chain during one period of current change is not necessarily exactly in phase with the oscillation set up by the previous period of current change, which is still persistent in the circuit. Further, if the multiplication factor of each stage is n, the number of cycles executed by the anode circuit of stage 1 during a period of zero anode current change is not exactly $\frac{n}{2}$, assuming the period of zero current change to be half full period. Suppose it is $\left(\frac{n}{2}+\frac{1}{m}\right)$ cycles and that the input frequency to stage 1 is f. If the second stage is an ideal multiplier by n its anode circuit

should carry out $\frac{n^2}{2}$ oscillations at frequency n^2f during half a cycle of f the input frequency to the first stage, but actually it executes $\left(\frac{n^2}{2}+\frac{n}{m}\right)$ cycles. The error is therefore $\frac{n}{m}$ cycles and is unimportant, provided the error does not approach half a cycle at frequency n^2f. If it does, there is a possibility of oscillation being stopped because each train of $\left(\frac{n^2}{2}+\frac{n}{m}\right)$ cycles at frequency n^2f then starts up in reverse phase to the previous oscillation. It is to be noticed that the error in the following stages is cumulative, being respectively $\frac{n^2}{m}, \frac{n^3}{m}$, etc., in the 3rd, 4th and following stages. There is therefore a danger that in later stages the new oscillation may be started up in correct phase one whole cycle later. In other words, a multiplier chain designed for a multiplication ratio of 100 may introduce an extra cycle in the last stage and multiply by 101. This has been observed, as have also intermediate conditions in which the output frequency represents no stable multiple of the input frequency.

If f is the input frequency and f_0 is the natural frequency of the anode circuit of the first stage the value of m may be determined as follows. The time of a half-period is $\frac{1}{2f}$, and during this time the anode circuit, oscillating at its natural frequency, executes $\frac{f_0}{2f}$ oscillations instead of $\frac{n}{2}$. The number of cycles in error is therefore:

$$\frac{f_0}{2f}-\frac{n}{2}=\frac{1}{m}$$

If $f_0 = nf$, $\frac{1}{m} = 0$; if $f_0 > nf$, $\frac{1}{m}$ is positive; and if $f_0 < nf$, $\frac{1}{m}$ is negative.

The value and sign of m therefore depends on the accuracy of tuning of the anode circuit of stage 1. The above argument has assumed every stage ideal except the first stage, but in practice the same effect occurs in each stage. The importance of accurate tuning in the earlier stages is however evidently greater than that of later

stages because of the cumulative nature of the effect. Fortunately it is found that, once a multiplier is stabilized, it remains stable, and the selective circuits in the anode are therefore installed with fixed tuning determined during the initial adjustment of the multipliers.

The selective circuits in the anodes of the multiplying stage, of which examples appear in Fig. 3, are not true band-pass structures. The first inductance nearest the anode is chosen to give an optimum compromise between high Q (to give low decrement) and high anode impedance, and the second coil is usually made equal to it. The circuit is tuned by alternative adjustment of both tuning condensers (replaced by a decade condenser box) to optimum output and maximum stability, after which fixed values of capacity corresponding to the values so found, are inserted. The series condenser between the two tuned circuits is made small to prevent the first tuned circuit from surrendering its energy too quickly to the following tuned circuit, while the decrement of the whole circuit is further reduced by increasing the limiting resistance in series with the grid of the following valve.

It is therefore evident that, when harmonics are generated by a wave form having zero rate of current change during any appreciable fraction of a period, a considerable amount of art is necessary to ensure reliable operation.

It appears probable that, in the future design of harmonic generators, harmonics will be generated by the use of valve characteristics having curvatures of 2nd, 3rd, 4th or 5th order, etc., throughout the range of amplitudes over which they are driven. In other words, the valves will not reach zero anode current at any part of the cycle. In this case the valve can effectively be replaced by a number of generators having e.m.f.s respectively equal to each of the frequencies generated in the anode circuit. Under such conditions the oscillation set up in the anode circuit is a forced oscillation of constant amplitude and frequency equal to an exact multiple of the input frequency. The selective circuits can then be constituted by normal band-pass filters terminated, for instance, in their mid-band image impedance. With such a design no critical adjustments are necessary and it is quite impossible for the multiplier to go out of lock. The multiplication per stage is in practice limited only by the requirement that there shall be no cumulative loss of level through the chain, and even this limit may be exceeded if the insertion of straight amplifying valves is an economic proposition.

8. Vernier Scales.

The accuracy of setting of dials assumes great importance in radio-frequency work, where a dial may control the frequency of an oscillation supplying the drive to a transmitter. The limitations of physical dimensions impose a restriction on the size of scale which can be embodied in a particular piece of apparatus, and the consequence is that the accuracy of many pieces of apparatus is greater than that of the unaided eye reading a simple scale.

To increase the accuracy of the eye in reading, instead of using a pointer mark in relative motion to a scale, a small subsidiary scale, called a vernier scale, is associated with the pointer mark.

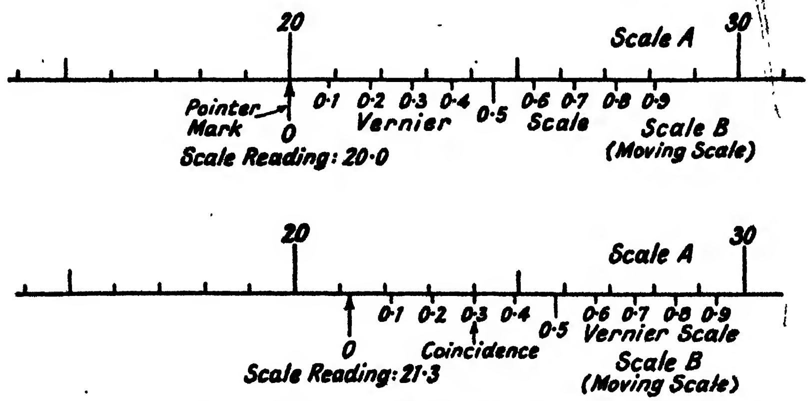


FIG. 1/XII:8.—Forward-reading Vernier.

In the Forward-Reading Vernier shown in Fig. 1 each of the vernier divisions is equal to nine-tenths of the small-scale divisions on the main scale A. As a consequence, every time scale B is moved a tenth of a division to the right with regard to scale A, from the position shown in the upper diagram of Fig. 1, it brings into coincidence with a division mark on scale A, a division mark on scale B removed one to the right from the mark at which coincidence previously occurred. Hence by counting along from the zero mark on the vernier scale (the arrow) to the vernier division mark at which coincidence occurs, it is possible to determine how many tenths of a division scale B has moved from the position at which coincidence occurred at the zero mark of the scale. The method of reading the scale will now be obvious : the scale reading in the upper position is 20·0 and in the lower position 20·3.

In the Backward-Reading Vernier shown in Fig. 2, each of the vernier divisions is equal to eleven-tenths of the small-scale divisions

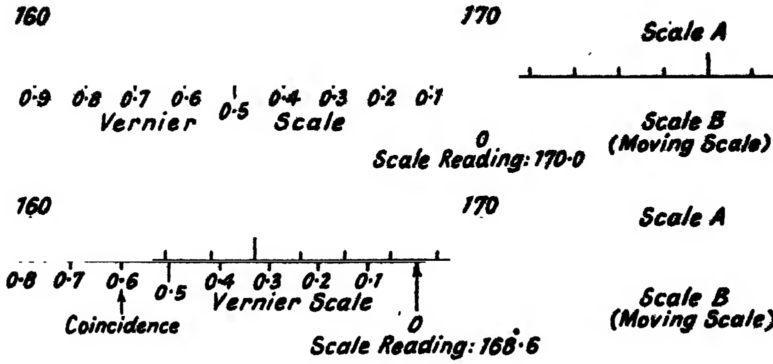


FIG. 2/XII:8.—Backward-reading Vernier.

on the main scale A. The consequences are identical in effect with those in the case of the forward reading vernier except that scale divisions are now counted backwards instead of forward. The scale reading in the upper position of Fig. 2 is 170 and in the lower position 168·6.

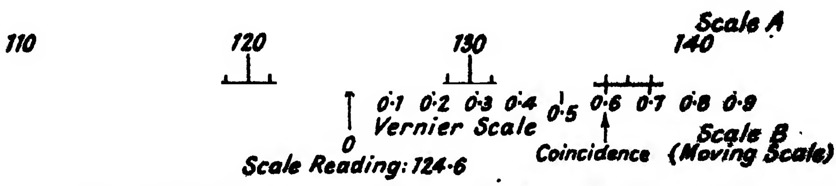


FIG. 3/XII:8.—Forward-reading Vernier with Double Spacing.

A little consideration will show that provided the vernier scale divisions are equal to an integral number of main scale divisions minus a tenth of a division, the vernier scale divisions may be increased in size relative to the main scale divisions. Fig. 3 shows a forward-reading vernier scale in which the vernier divisions are equal to twice the main scale divisions less a tenth.

In practice, the vernier scale divisions are usually unmarked, but the meaning of the divisions can always be determined rapidly by inspection. If the zero of the vernier is set opposite a main scale division, so constituting one coincidence, by counting along the vernier scale until the next coincidence is found, the number of vernier scale divisions corresponding to a main scale division can be found.

CHAPTER XIII

MODULATORS AND MODULATION

1. Different Types of Modulation.

MODULATION is the process by which a carrier frequency is modified so as to carry intelligence. The most important practical form this modification takes is a variation of *amplitude* of the carrier wave in accordance with the instantaneous value of the voltage which describes, and is initially derived from, the intelligence to be conveyed. The intelligence to be conveyed may consist of speech or music, television, telegraph or facsimile (photograph) transmission pulses. For simplicity in discussion the modulating voltages will be considered to be at audio frequency. Unless otherwise specified, the process of modulation will always be considered to be that of *amplitude modulation.*

Instead of varying the amplitude of the carrier wave it is also possible to vary the frequency or the phase of the carrier wave in accordance with the wave form of the intelligence to be transmitted, giving rise respectively to *frequency modulation* and *phase modulation.* While certain advantages are claimed for frequency modulation, from the point of view of noise reduction, it has not come into use in Europe, although a number of transmitters using frequency modulation have been built in America. Other types of modulation of considerable promise are pulse-width modulation and pulse-position (in time) modulation.

Pulse code modulation is a system in which amplitude is quantized. That is to say the amplitude range of a speech wave, say (which for simplicity may be assumed to be held at constant volume by some form of A.V.C.), is divided into a number of discrete and equal intervals so that it is possible to define a number of discrete amplitudes. Time is also divided into a number of discrete intervals and during each interval of time the mean amplitude is measured and a pulse code is transmitted to convey the amplitude. The advantage claimed for such a system is that it affords a high signal-to-noise ratio, and, in common with other pulse modulation systems in which channelling is carried out on a time sharing basis, it is free from intermodulation between channels.

Amplitude modulation is usually carried out with valves, but it may also be achieved with any non-linear circuit element such as copper-oxide rectifiers, while another method of modulation consists in varying a circuit element so as to vary the efficiency of

transmission of the carrier frequency and so its amplitude. Copper-oxide rectifiers constitute the only alternative to valves which is of practical importance.

Valve modulators may be divided broadly into two types:

(1) *Anode or Plate Modulators*, in which the modulating voltage (of wave form corresponding to the intelligence to be transmitted) is applied to the anode of the valve in which modulation takes place. The carrier voltage is then applied to the grid of the modulated amplifier, except in the case of diode modulators in which the carrier is also applied to the anode of the diode.

(2) *Grid Modulators* in which the modulating voltage as well as the carrier voltage is applied to the grid of the valve in which the modulation takes place.

A certain inconsistency in terminology exists because, while the term " modulator " is used by some authorities to describe the valve or, more particularly, the circuit device in which modulation takes place, other authorities use the term " modulator " to describe the audio-frequency or other amplifier supplying the modulating voltages to the valve in which modulation takes place, which is then called the modulated amplifier. To avoid confusion in the discussion, in this section only, the following conventions will be observed where relevant.

Modulator = Device producing modulation. In anode modulators this includes modulating and modulated amplifying valves. In grid-modulated amplifiers it includes only the valves in which modulation takes place. In both cases the grid input and output circuits are included.

Mod-ing Amp. = Modulating Amplifier = Amplifier applying signal wave form to modulated amplifier.

Mod-ed Amp. = Modulated Amplifier = Amplifier in which modulation of the carrier wave takes place.

There are three important types of plate or anode modulator and another worthy of note, while there are three important types of grid modulator. These are summarized in Table I (p. 500).

With the exception of the diode modulator all anode-modulated amplifiers consist of a class C amplifier driven hard into anode limitation by carrier-frequency drive applied to its grid, and driven on its anode with the audio-frequency modulating voltage. The latter effectively varies the instantaneous value of the voltage effective on the anode, and since the carrier-frequency voltage output of a class C amplifier driven hard into anode limitation is proportional to the anode voltage, the resultant output wave is modulated by the audio frequency and has an envelope of the same shape as

TABLE I

Type of Modulator	Limitation on use at High Power	Modulated Amplifier		Approximate No-mod. Efficiencies of Complete Modulator as defined
		Carrier Input to	Modulating Voltage Input to	
1. Heising or Choke Mod. (Class A Mod.)	Low efficiency	Grid	Anode	25%
2. Class B Mod.	None	Grid	Anode	65%
3. Series Mod.	Low efficiency	Grid	Anode	25%
4. Diode Mod.	High carrier-drive power	Anode	Anode	—
5. Class C Grid Mod.	Poor linearity, low efficiency. Per cent mod. < 100%	Grid	Grid	Below 35%
6. Anode Bend Grid Mod.	Low efficiency and low percentage modulation	Grid	Grid	Very low
7. Suppressor Grid-Class C Mod. (using Pentode)	Low efficiency and low power rating of available valves. Percentage modulation $\not>$ 80%	Control grid	Suppressor grid	Below 35%

that of the modulating wave. When the peak value of the modulating voltage applied to the anode of the modulated amplifier is equal to the applied steady H.T. volts effective on the anode, 100% modulation occurs. If the audio swing exceeds the steady H.T. voltage the anode of the modulated amplifier is driven negative and overmodulation occurs: during the time that the anode voltage is negative there is no output from the modulated amplifier and the envelope is no longer the same shape as the modulating wave form. Fig. 9/XX:10 shows at (*a*) a 100% modulated wave, at (*b*) a wave modulated less than 100%, and at (*c*) an overmodulated wave, the degree of overmodulation being very much larger than should ever be observed in practice. The modulating wave form in each of these cases is sinusoidal.

Let the valves of the modulated amplifier be designated by V_2. It is to be noted that if a steady R.F. drive is applied to the grid of V_2, it must be of sufficient amplitude to drive V_2 into anode limitation at the moment of 100% peak modulation, i.e. at the instant when the modulating voltage is at the peak of its positive half-cycle and equal to the H.T. voltage. At this instant the anode volts effective on V_2 is twice the steady H.T. volts, and the grid swing necessary to drive V_2 into anode limitation is *approximately* 1·5 times the grid drive necessary to achieve this in the carrier (unmodulated)

condition. This means that the power-handling capacity of the carrier-drive valve driving V_2 must be about 2·25 times the value necessary to provide adequate drive in the carrier condition. A reduction in the size of the drive valve can be secured if this is designed, so that with its normal H.T. it can provide a drive large enough (or just a little larger) to drive V_2 into anode limitation in the carrier condition, and is then anode modulated, so that at 100% peak mod. its voltage output is increased by about 50%. This measure is not worth while in small modulators, but is sometimes

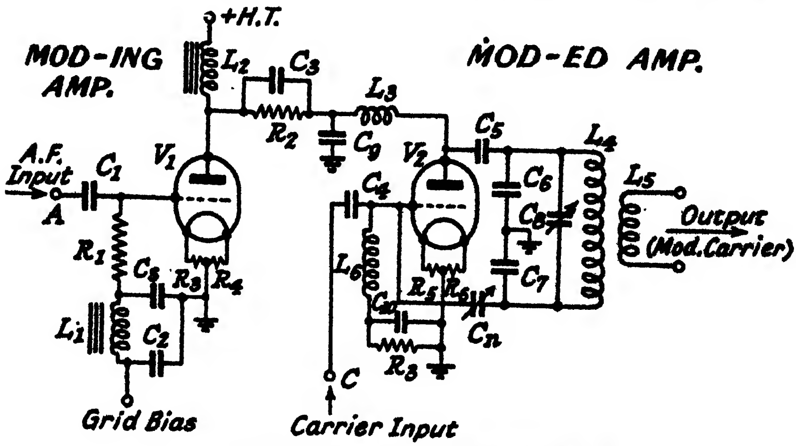


FIG. 1/XIII:2.—Heising or Choke Modulator.

used in high-power modulators where the resultant economy is worth having. See XIII:4 for more exact statement.

2. Heising or Choke Modulation.

Fig. 1 shows a circuit of a typical Heising or Choke Modulator. V_1 is an A.F. (mod-ing) amplifier valve, choke coupled by means of the choke L_2 and condenser C_3 to the anode of V_2, a class C R.F. amplifier valve driven hard into anode limitation by a carrier-frequency voltage applied at C. The R.F. anode peak volts of V_2, which is called the mod-ed amplifier, are therefore substantially equal to its instantaneous (H.T.) anode voltage, which varies in accordance with the audio-frequency output of V_1. The R.F. output from the mod-ed amplifier is therefore modulated in accordance with the wave form delivered by the mod-ing amp. Anode current for V_2 is supplied through the dropping resistance R_2, and the bias for V_2 is obtained from its grid current flowing through R_3. *It is essential therefore that the anode volts should not be applied to the circuit*

until the carrier drive has been applied at C. L_3 and C_9 constitute an R.F. filter to prevent the R.F. voltages, generated between the anode of V_2 and ground, from dissipating power in the anode circuit of V_1. C_5 is a blocking condenser. C_6 and C_7 are part of the tuning or tank circuit condenser and are earthed at their common point to provide a voltage in reverse phase to provide feedback for capacity neutralization. C_n is the neut condenser. C_8 provides variable tuning. L_4 is the anode-tuning inductance and L_5 the output inductance magnetically coupled to L_4. L_1, C_2 and C_6 are grid smoothing for V_1, while the grid bias for V_1 is fed through R_1. L_6 is an R.F. choke.

L_2 is called the *modulation choke.* R_3, R_4, R_5 and R_6 are called *centre-pointing resistances*, and are inserted to equalize the emission of the filament along its length. The value of each resistance R_3, R_4, etc., is about 5 times the resistance of the associated filament when hot.

It is to be noted that the presence of R_2 enables the use of a higher anode voltage on the mod-ing amp. (V_1) than the mod-ed amp. (V_2). This is of importance from the point of view of obtaining adequate audio-frequency power for modulation purposes and for preserving linearity. The coupling condenser C_3 would be unnecessary if, as in the original Heising modulation circuit, equal anode voltages were used on both mod-ing and mod-ed amplifiers.

It will be noted that condenser C_9 is in shunt across the audio-frequency circuit, and in long-wave circuits when the carrier frequency may be only 20 times the highest audio frequency to be transmitted, with a value of C_9 large enough to provide adequate filtering, its reactance at high audio frequencies may be such as to degrade the audio-frequency response curve. When this occurs a smaller value of C_9 can be used by inserting in series with it an inductance resonant with C_9 at the carrier frequency.

The original Heising circuit did not use grid-current bias, but external bias was provided for the mod-ed amp. by means of a machine and smoothing circuits or a battery: this arrangement is still used by some designers. Experiments have shown, however, that an improvement in linearity and hence in envelope wave form is obtained by arranging that at least part of the bias is provided by grid current flowing through a grid resistance such as R_2 (see XIII:4). On the other hand, by providing part of the bias externally, V_2 is protected against excessive anode current, if by any chance anode volts are applied before R.F. drive. In some modulators therefore the lower end of R_2, instead of returning to the filament

earth of V_2, is taken to the smoothing circuit of a grid-bias source. The time constant of the circuit constituted by R_2 and C_{10} must be less than the period of f_a, the highest audio frequency to be transmitted: $R_2 \times C_{10}$ must not exceed about a half of $1/f_a$.

A circuit similar to that of Fig. 1 is used in the low-power modulators in the A units of the B.B.C. medium-wave "Regional" transmitters at Brookman's Park, Moorside Edge, Westerglen and Washford.

This circuit has two limitations. First, the mod-ed amp. must constitute a suitable load impedance to face the anode of the mod-ing amp., and secondly, the mod-ing amp. must always operate in class A.

These disadvantages are removed by the insertion of an impedance-matching transformer between the mod-ed and mod-ing amplifiers. Since, when such a transformer is used, the mod-ing amp. is almost invariably operated in class B, in order to increase the efficiency, it is not necessary to consider the class A operation of the resultant circuit. The mod-ed amplifier is of course always a class C amplifier.

3. Class B Modulation.

Fig. 1 shows a typical class B modulator circuit using push-pull in both mod-ing and mod-ed amplifiers.

The class B mod-ing amplifier is constituted by the two valves V_1 and V_2 driven in push-pull and feeding into the modulation transformer T in their anode circuits. The grids of V_1 and V_2 are loaded with resistances R_1 and R_2, which may in a large amplifier be as low as 200 to 300 ohms, in order to mask the non-linearity of the grid input circuit due to flow of grid current. In order to avoid passing the anode current of the mod-ed amp., constituted by valves V_3 and V_4, through the secondary of the modulation transformer, anode current for the mod-ed amp. is supplied through the modulation chokes L_m which are connected to +H.T. The audio-frequency drive to the mod-ed amp. is applied through condensers C_1 and C_2. In some transmitters condensers C_1 and C_2 are replaced by a condenser C_5, shown dotted, in the earth lead of the secondary winding. Condensers C_3 and C_4 and chokes L_1 and L_2 constitute a filter preventing R.F. currents from the anode of the mod-ed amp. from flowing into the modulation transformer. The inductances L_g are the grid chokes, which should preferably be quarter-wave chokes, and resistance R_g supplies grid-current bias.

External bias is applied at the terminal marked G.B.

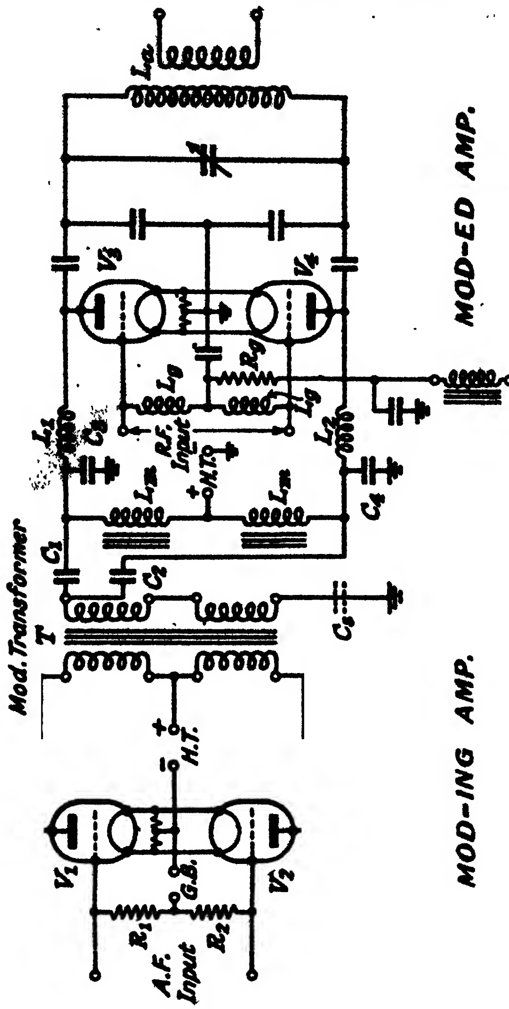


FIG. 1/XI 1:3.—Typical Class B Modulator Circuit.

In some recent transmitters only one reactor L_m and only one condenser C_1 is used. Audio drive and H.T. are then applied through a single circuit, as L_1C_3 to the mid-point of the anode tuning inductance L_a.

It may be remarked in passing that it appears that in the circuit of Fig. 1, the outer ends of the two inductances L_m are always at the same potential, and may therefore be joined together without any change in the performance of the circuit. In such case the two inductances L_m may be replaced by one inductance and the two condensers C_1 and C_2 by one condenser without further circuit modification.

There is no reason why in general this modification should not be made.

3.1. Temporary Conversion of Class B Modulator to Class A Operation. In the event of failure of the modulation

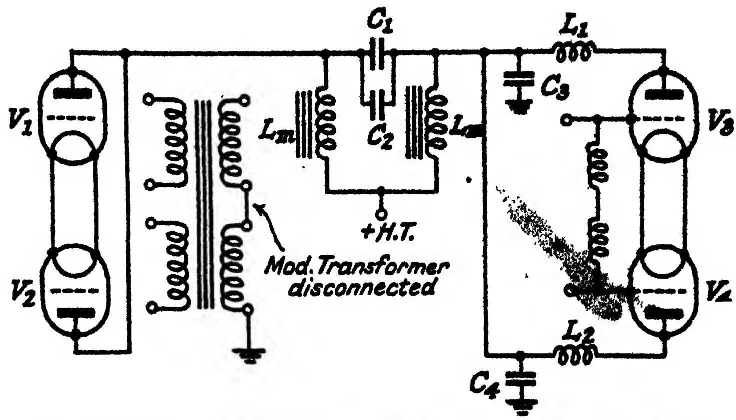


FIG. 2/XIII:3.—Class B Modulator arranged for Class A Operation.

transformer and the spare provided, a class B modulated transmitter may be adjusted to class A operation by modifying the circuit as shown in Fig. 2 and adjusting the bias on the mod-ing amplifier so that it operates in class A, due regard being paid to the current rating of the chokes, L_m.

The value of the H.T. is unchanged, and since the mod-ing amplifier in class A cannot give such a high output as when operated in class B (the power output is approximately reduced to a half), it is only possible to obtain a peak modulation just under 70%. This results in a reduction in level, and therefore in speech-to-noise ratio at the receiver, of 3 db. This is not a large amount, and this arrangement provides a very useful emergency condition.

3.2. Series Modulation. In the simplest case of a series modulator the H.T. on a normal modulated amplifier is varied in accordance with the modulating voltage by inserting in series between the H.T. supply and the modulated amplifier a valve anode circuit which constitutes a resistance varying under the influence of modulating voltages applied to its grid. This valve constitutes the mod-ing amplifier.

The feeds (mean anode currents) of the mod-ed and mod-ing amplifiers are the same, since the anode current of the mod-ed amplifier flows through the mod-ing amp., and vice versa. The

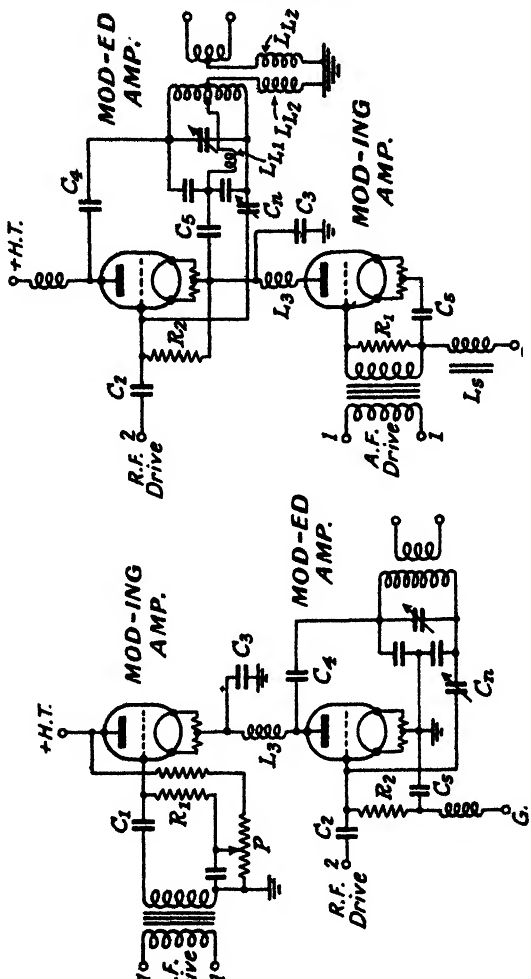


FIG. 3/X

mod-ing amplifier is operated in class A, so that this does not constitute a high efficiency system.

Fig. 3 shows two alternative arrangements of a series modulator. In the circuit at (*a*) the mod-ed amp. valve has its cathode at ground potential while the mod-ing amp. has its cathode at the same potential as the anode of the mod-ed amp. In the circuit at (*b*) the mod-ing amp. valve has its cathode at ground potential while the mod-ed amp. valve has its cathode at the same potential as the anode of the mod-ing amp. In each case a specially insulated source of supply with a low capacity to ground has to be provided to heat the cathode which is at high potential to ground.

Although only one valve is shown in each amplifier, each amplifier may consist of any convenient number of valves. But, whereas the mod-ed amp. may have its valves connected in push-pull, the mod-ing amp. valves must be connected in parallel.

Both circuits of Fig. 3 are self-explanatory. In each case the A.F. drive, or the modulating voltage, is applied at terminals 1,1, while the R.F. drive is applied between point 2 and ground. Inductance L_3 and condenser C_3 constitute an R.F. filter to keep R.F. off the mod-ing valve. L_2 and C_2 provide grid-bias smoothing. A point of interest is the method of biasing the mod-ing amp. valve, in case

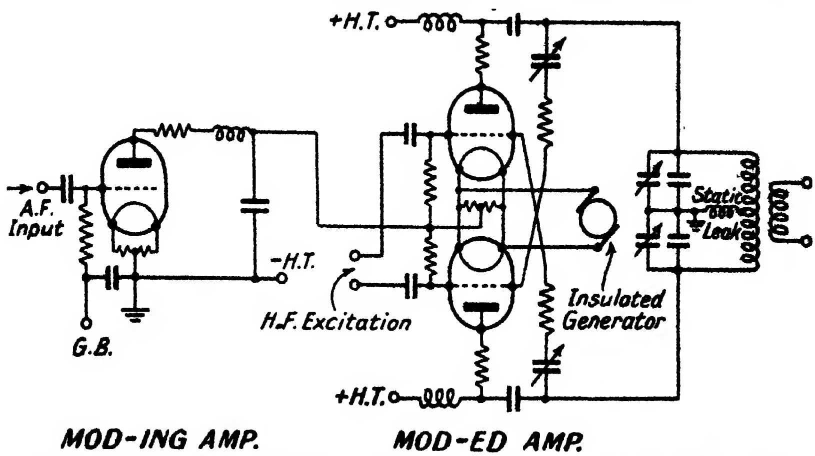


FIG. 4/XIII:3.—Series Modulator in B.B.C. Medium Frequency 150-kW. Transmitter at Droitwich.

(*a*), by means of the potentiometer P; other methods of providing bias may however be used. The output circuit of the mod-ed amp. is of conventional type. In case (*b*) condensers C_4 and C_5, in conjunction with the inductance leak L_{L_1} prevent high D.C. voltage across the tank condensers; while the inductive leaks L_{L_2} prevent any high D.C. voltage occurring between the output coupling coils and tie the D.C. potential of these elements to ground.

It will be noticed that the sum of the H.T. voltages effective between anode and cathode of mod-ing amp. and between anode and cathode of mod-ed amp. is equal to the total H.T. voltage available. To obtain linear modulation it is usually necessary to arrange that the H.T. voltage effective on the mod-ing amp. is nearly twice that on the mod-ed amp. This means that the necessary H.T. available must be nearly three times that required on the mod-ed amp. Such a requirement practically excludes the use of series modulation in

the final stage of a transmitter, if it were not already excluded by the low efficiency of series modulators. Apart from television and other broad band circuits the future field of use of series modulators is not yet evident; applications will, however, probably appear, particularly in view of the low envelope-phase shift introduced by a series modulator and its resultant utility when envelope feedback is used. See XXIII:8.

3.3. Design of a Series Modulator. The steps in the design of a series modulator are as follows:

(1) Design the mod-ed amp. to deliver the required output power and to operate on just over a third of E the available H.T. Call the value of H.T. on the mod-ed amp. E_d. If the total driven feed of the mod-ed amp. is $\bar{\imath}$ the audio-frequency impedance presented towards the anode circuit of the mod-ing amp. is $R_d = E_d/\bar{\imath}$. The value of H.T. available for use on the mod-ing amp. is then $E_m = E - E_d$.

(2) Choose a valve for use in the mod-ing amp. which is capable of operating on a value of H.T. equal to E_m.

(3) On the valve field of the chosen valve draw a class A load line such that an anode-voltage swing of $\pm E_d$ can be obtained in the linear region of the valve field. This load line must be drawn through a point corresponding to an anode voltage E_m and a value of anode current i_m which brings the anode dissipation within the permitted anode dissipation for the valve. If linearity cannot be obtained with an anode-voltage swing of $\pm E_d$, then either the voltage on the mod-ed amp. must be reduced and so the value of E_m increased, or else another type of valve must be used.

This step is only a preliminary one to establish that the valve is suitable.

(4) The number of valves n to be used in the mod-ing amp. is then given by the nearest integer *greater* than $\bar{\imath}/i_m$. Remember that $\bar{\imath}$ is the total current taken by the mod-ed amp. which is equal to the total current taken by the mod-ing amp. The current taken by any one valve in the mod-ing amp. is therefore $\bar{\imath}/n$. The impedance facing each valve in the mod-ing amp. is $R_m = nR_d$. R_m is therefore the slope of the load line for each valve in the mod-ing amp.

(5) Draw a load line on the valve field of the valve chosen for the mod-ing amp. through the point: anode volts $= E_m$, anode current $= \bar{\imath}/n$, at a slope R_m, and check that for an

anode-voltage swing of $\pm E_a$ the load line traverses a linear region of the valve field.

(6) The required grid bias on the mod-ing amp. is then given by the bias which gives an anode current of i/n with an anode voltage equal to E_m.

4. Principle of a Modulated Amplifier.

As indicated in the section on Heising modulation, the operation of a modulated amplifier depends broadly on the fact that the R.F. output from a class C amplifier driven hard into anode limitation is substantially proportional to the instantaneous value of the H.T. volts. Hence, when the instantaneous value of the H.T. volts is varied in accordance with any (modulating) wave the R.F. output is modulated so as to have an envelope of the same form as the modulating wave.

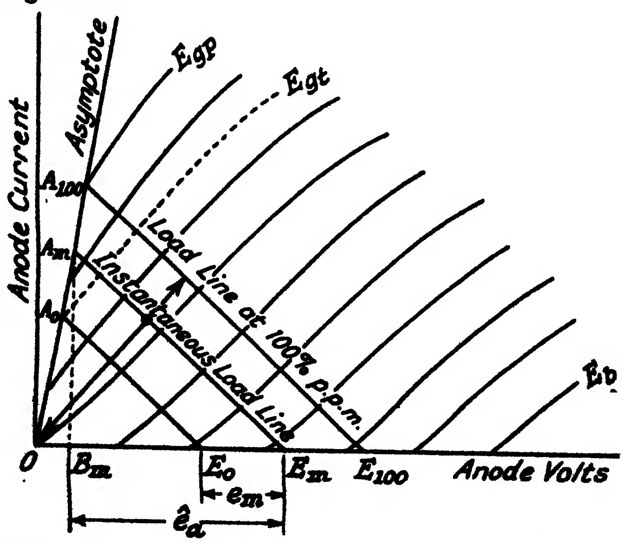


FIG. 1/XIII:4.—Valve Field illustrating Operation of Modulated Amplifier.

In X:27 it is shown that in a hard-driven class C amplifier the minimum positive excursion of the anode is approximately determined on the valve field by the projection on the axis of anode volts of the point of intersection of the normal load line with the asymptote. The anode peak volts are then equal to the difference between the value of the applied H.T. volts and the minimum positive excursion of the anode.

This is illustrated, for the case of an anode-modulated amplifier, in Fig. 1, which shows a field of valve characteristics for a valve in a modulated amplifier.

E_0 is the value of steady H.T. and E_0A_0 is the load line in the carrier condition. The grid bias is E_b and the grid is driven with such an R.F. drive that the maximum positive excursion of the grid is E_{gp}.

e_m is the instantaneous value of the modulating voltage effective on the anode so that the instantaneous value of the anode volts $E_m = E_0+e_m$. The corresponding instantaneous load line is E_mA_m, which during the modulation cycle oscillates about the mean position E_0A_0 with an amplitude proportional to the peak value of e_m. When the peak value of e_m is equal to E_0, 100% modulation occurs and during each modulation cycle E_mA_m travels from E_0A_0 to $E_{100}A_{100}$, back through E_0A_0 to disappear in the origin O and then emerges again to return to E_0A_0.

Corresponding to the position in which the instantaneous load line is shown, the minimum positive excursion of the anode is B_m and the anode peak volts with the load line in this position are given by $\hat{e}_a = E_m - B_m$. The position of B_m is determined by the projection on the axis of the anode volts of the intersection of E_mA_m with the asymptote.

Assuming that E_mA_m is always parallel to E_0A_0, it is evident that $\hat{e}_a$ is proportional to the length of E_mA_m, which is proportional to the length of the line OE_m, that is to E_0+e_m. Hence $\hat{e}_a$ is proportional to E_0+e_m, which is the requirement for linear modulation.

In practice, the load line tends to change its slope throughout the modulation cycle because its slope is determined by the angle of current flow at the threshold of anode limitation, and this evidently changes throughout the modulation cycle. This tendency is substantially offset by the varying degree to which the modulated amplifier is driven into anode limitation during the modulation cycle. In the absence of other sources of non-linearity, some increase in linearity may therefore be expected from any step by which the percentage change in the angle of current flow over the modulation cycle is reduced. This may be achieved by increasing the grid drive; this incidentally increases also the degree of overdrive. If this results in too high a value of grid dissipation or peak anode current, the grid bias may be increased, which will reduce the maximum positive excursion of the grid (say to its original value) and so the peak grid current and the grid dissipation: it will also reduce the angle of current flow, and the degree of overdrive.

More explicitly, if linearity is increased by increasing the grid drive with grid bias unaltered, the angle of current flow in the carrier condition will be increased, the power output will be slightly

increased and the efficiency reduced. If the grid drive and grid bias are increased so that the maximum positive excursion of the grid is kept constant, the angle of current flow in the carrier condition will be reduced, the efficiency will be slightly increased and the power output will be slightly reduced. In general, the grid drive should be the maximum permitted by grid dissipation and peak anode current, so that it is only possible to increase linearity by reducing the angle of current flow. *In cases below where it is stated that linearity may be increased by reducing the angle of current flow, it is this step which is intended.*

Three other effects occur which contribute to non-linearity. The first is due to the fact that the minimum positive excursion of the anode is not exactly determined by the intersection of the load line and the asymptote. As the modulating voltage reduces the anode potential towards zero, the degree to which the valve is driven into limitation increases, and the minimum positive excursion of the anode deviates below the asymptote limit in varying degree as described in X:27. The second is due to the fact that the asymptote is not a straight line: see X:27.1.

The third is due to the fact that, when the modulating voltage swings the anode of the modulated amplifier towards low values of anode voltage, the anode-current wave form departs widely from that of a sinusoid and approaches more closely to that of a square wave. Under this condition the value of $f = \frac{i_f}{i_p}$ is greater than that for a sinusoid. This effect appears to dominate all other effects in that the residual distortion is of the same kind as the distortion due to this effect: the output from the modulated amplifier is too great at low levels of anode volts. This residual distortion can be reduced by any means which effectively makes the angle of current flow increase faster as the modulation cycle approaches its positive peak. This may be done by the use of grid current bias as described below.

By deriving part of the grid bias of a modulated amplifier from a resistance in the grid circuit, through which the (smoothed) grid current flows, provided the time constant of the smoothing circuit is less than (about half) the period of the highest modulating frequency to be reproduced, the grid bias is effectively varied at modulation frequency. The varying grid bias follows the modulating frequency, while the radio frequency grid drive remains constant. As the H.T. increases, the grid current falls owing to the anode robbing the grid of electrons, the grid bias falls and the angle of

current flow then increases by an amount greater than that due to the increase of H.T. alone.

It is found in practice that it is possible to choose such a ratio between the fixed bias and the bias provided by grid current that substantially linear modulation results.

It is also found that when a modulated amplifier has been adjusted for linear modulation, if the H.T. *is varied from zero to twice its steady value, the anode current is proportional to the* H.T. *throughout this range of variation.* This provides the justification for the usual assumption that a modulated amplifier presents a resistance load towards the modulating amplifier. See CV, however.

Since, with any steady value of H.T. volts, the peak anode volts and therefore the peak value of the fundamental frequency component of anode current (i_f) is proportional to H.T. volts, and since the mean anode current ($\bar{i}$) is proportional to H.T., it follows that, when the H.T. is varied by modulation, the instantaneous value of $u = i_f/i_a$ is constant throughout the modulation cycle, where $i_a = \bar{i}+i_m$ and i_m is the instantaneous value of current supplied by the modulating amplifier.

When varying modulation is applied, very small variations of driven feed are observed; these are, however, too small to affect the practical value of the above statements.

A method of effecting an economy in the size of valve driving the grids of the mod-ed amp. consists in anode modulating the valve driving the mod-ed amp. This reduces the size of valve required in the drive amplifier because the anode dissipation of the drive amplifier is unchanged by modulation. If the driving stage is modulated 100% the output voltage from the driving stage at 100% p.p.m. is twice that in the carrier condition, and its power output is four times, so that this apparently leads to a possible reduction of anode dissipation of four times. In practice, however, the ratio between the grid voltage necessary to drive the mod-ed amp. valves into anode limitation in the carrier (no mod.) condition and that necessary to drive them into anode limitation at 100% peak positive modulation, is usually not greater than about 1·5. Further, if the driver stage were modulated 100% this would give too small a value of drive on the mod-ed amp. at the bottom of the modulation cycle, and it is therefore usual to modulate the driving stage about 50% leading to an increase in mod-ed amp. grid drive at 100% p.p.m. of about 1·5 times, and a reduction of the anode rating of the driving stage to $\frac{1}{1{\cdot}5^2}$, or rather less than half.

The question is sometimes asked why an anode-modulated amplifier is not operated as a class A amplifier. The first and most important answer is that the efficiency would be low, and the second answer is that it could not be a class A amplifier as defined: the angle of current flow could not be 360°. This is because the process of modulation in an anode-modulated amplifier is a process of limitation, which would be effective in cutting the tops of both positive and negative halves of the carrier wave.

Value of $f = \hat{i}_f/i_p$ in a Modulated Amplifier. It will be appreciated that the technique of drawing load lines defining peak current, and of deriving the value of the impedance facing the anode from the slope of this line and the value of f, is applicable only to class C amplifiers which are not driven into anode limitation. This technique is therefore only applicable to a hard-driven modulated amplifier, if at some point in the modulation cycle the amplifier is not driven into anode limitation. Evidently the only point in the cycle where it is possible for the amplifier to be operating without being driven into limitation is at 100% p.p.m.

As a design measure it is therefore convenient to make two assumptions.

These are:

1. That at 100% p.p.m. the amplifier is just driven up to the threshold of anode limitation.

2. That at 100% p.p.m. the amplifier is biased as a class B amplifier, so that the angle of current flow is 180°.

Evidently under these conditions the load line at 100% p.p.m. is capable of being drawn in terms of the technique already described, and the value of f is 0·5. The impedance facing the anode is therefore determined by drawing the load line through the point of intersection of the maximum permissible peak anode current (don't forget to allow for grid current in calculating total emission) and the asymptote, unless a lower value of power output is required than this method provides.

In conformation with the two assumptions above, the initial adjustment of a modulated amplifier should be made so that as far as possible the amplifier is on the threshold of upward modulation with a small margin, so that the angle of current flow at 100% p.p.m. is as close as possible to 180°. See XIII:4.1.

Determination of Peak Anode Current in a Modulated Amplifier.—(*To determine peak emission don't forget to add peak grid current to peak anode current.*)

The following rule of thumb is in common use: From the angle

of current flow in the carrier condition, find the value of g in the carrier condition. The peak anode current in the carrier condition is then given by dividing the anode feed by this value of g. The next step is to obtain the peak anode current at 100% p.p.m. by doubling the peak anode current in the carrier condition.

On the above basis, if the angle of current flow in the carrier condition is 140°, the value of g is 0·25 and the peak anode current at 100% p.p.m. is 8 times the anode feed. The last is sometimes used as an alternative rule of thumb. In other words, an additional assumption is made: that the angle of current flow in the carrier condition is 140°.

It will be evident that while these rules have done valiant service they are not entirely fundamental, since on the one hand the angle of current flow at 100% p.p.m. is not the same as that in the carrier condition, and on the other, the amount to which the amplifier is driven into anode limitation differs between the carrier condition and 100% p.p.m.

A safe rule for finding the peak anode current at 100% *p.p.m. is that it is obtained by finding, on the valve field, the intersection with the asymptote, of the characteristic corresponding to the maximum positive excursion of the grid at* 100% *p.p.m. and then reading off the corresponding anode current.* Fig. 1/X:27 shows the valve field of a 4030C valve working along the normal load line *EP* with a grid bias of − 500 volts, a grid drive of 1,700 volts and an H.T. of 12,000 volts. The maximum positive excursion of the grid is then 1,700 − 500 = +1,200 volts and the intersection of the E_g = +1,200 volts characteristic with the asymptote occurs approximately opposite 21 amps. anode current. This is the peak anode current as determined by this method. (The true peak anode current is read off opposite P_2 and is 18·5 amps. The error in this case is exaggerated by the angle of approach of characteristic and asymptote, but it is always on the safe side and is usually smaller.)

In the case of grid-current bias, the determination of the maximum positive excursion of the grid at 100% p.p.m. involves a measurement of the fall in the grid bias at 100% p.p.m. from the bias in the carrier condition. This can best be measured by means of a cathode-ray oscillograph with its plates connected directly (i.e. with no series condensers) across the grid-current bias resistance, or across part of it. An R.F. filter consisting of a series inductance and a shunt condenser may be necessary between the C.R.O. and the bias resistance. This filter should be such as to have no effect at the frequency of the value used for modulating the transmitter

during the measurement. The sense of the connection should be noted so as to distinguish between the rise in bias voltage during the negative modulation cycle, and the fall in voltage during the positive modulation cycle. The latter is, of course, the quantity to be measured. The maximum positive excursion of the grid at 100% p.p.m. is then equal to: carrier condition grid bias minus fall in bias at 100% p.p.m. plus peak grid drive.

Value of $u = \frac{\hat{i}_f}{\hat{i}}$ in a Modulated Amplifier. *The value of* u is required in the carrier condition in order to determine the efficiency, but since the value of u is constant during the modulation cycle, it may evidently be determined at any convenient point in the cycle. Since the curves on Fig. 1/X:22 correspond to sinusoids, the best point in the modulation cycle to take is evidently that at which the wave form approaches most closely to a sinusoid; this is evidently at 100% peak positive modulation where the valve is driven least into anode limitation. The value of u should therefore be obtained by entering in Fig. 1/X:22 the angle of current flow at 100% p.p.m. When fixed bias is used this is easily calculated from the operating conditions at 100% p.p.m. which are easily determined; when grid-current bias is used the angle of current flow at 100% p.p.m. can only be determined simply when the mod-ed amp. is lined up to the threshold of upward modulation, when the angle of current flow is known to be 180° at 100% p.p.m. When the mod-ed amp. is lined up to a smaller angle of current flow at 100% p.p.m. this angle must either be guessed from the angle of flow in the carrier condition or else determined by measuring the grid-bias reduction at 100% p.p.m. as described above.

When more accurate measurements of the efficiency and power output are required these may be obtained by direct measurement, as described in X:33. Where such measurements are made there is no point in determining the value of u.

Relation between Peak Anode Current and Peak Emission. It is important to remember that the peak emission taken from the valve is greater than the peak anode current by the sum of the currents taken by the other electrodes such as the grid and the screen. The limiting value of peak emission is usually reckoned at 0·9 of the total emission; but it is recommended that a factor of safety be introduced by making the actual peak emission equal to not more than 0·8 of the total emission, i.e. equal to 0·9 of the permissible peak emission.

4.1. Upward and Downward Modulation in Modulated Amplifiers. It has been pointed out that in a properly adjusted modulated amplifier, the anode feed is proportional to the value of H.T. volts. The same proportionality holds when the H.T. is varied at modulation frequencies. (See, however, CV.) In VIII:2 it is explained that the mean value of programme voltages is zero: the time integral of all positive voltages is equal to the time integral of all negative voltages. It follows that the observed anode feed of a properly adjusted modulated amplifier is substantially independent of modulation. The application of modulation voltages does not cause any variation of the anode feed meter because the increase in current during the positive half modulation cycle is balanced by the decrease in current during the negative half-modulation cycle.

In practice, a very small variation of anode current occurs which represents a normal variation, and any departure from this condition constitutes upward or downward modulation.

Upward modulation is constituted by a rise of anode feed with increase of modulation, and *downward modulation* by a fall in the anode feed with increase of modulation. According to the sense of the normal variation these changes may add to or subtract from the normal variation.

Upward modulation may be caused by non-linearity in the valve at the bottom end of the load line, but in a properly designed modulator the upward modulation is not large and in any particular case may represent a normal condition. It may also be caused by inadequate grid bias, in which case the fall of grid volts at positive peak modulation may result in the valve being *biased* above cut-off during a fraction of the positive modulation half-cycle.

Downward modulation is the more usual accompaniment of a faulty adjustment. In order of probability this may be due to:

1. Inadequate grid drive. In this case the modulated amplifier may not be driven into anode limitation at positive peak modulation and the increase in current during the positive part of the modulation cycle is less in effect than the fall in current during the negative part of the modulation cycle.
2. Too low a value of impedance facing the anode of the modulated amplifier, consequent, for instance, on mistuning or maladjustment of the aerial coupling circuits. In this case, although the grid drive may be normal, it is inadequate to drive the modulated amplifier into anode limitation, and the same effects occur as in (1), with resultant fall of mean

anode feed. Even if the grid drive is adequate to drive the modulated amplifier into anode limitation, when the anode impedance is too low, non-linearity at the top end of the load line may give rise to downward modulation.

3. Cathode Limitation. This may be remedied (*a*) by replacing any valve with low emission, by restoring filament voltage to normal if it has fallen below, or if it is being operated below, marked volts : (*b*) by increasing the angle of current flow by decreasing the fixed bias and/or the grid-current bias, if this can be done without introducing upward modulation : (*c*) by increasing the impedance facing the anode. This constitutes a last resort as it involves a reduction of power output.

4.2. Initial Adjustment of a Modulated Amplifier. A modulated amplifier must have :

1. Sufficient external steady grid bias (i.e. independent of grid current) to protect the valves if the grid drive fails.
2. Sufficient grid-current bias so that in conjunction with the steady bias it prevents the angle of current flow at 100% peak positive modulation from exceeding 180°. An angle of current flow greater than 180° is not in itself serious, but what is serious is the resultant non-linearity, and, to a lesser extent, the reduction of efficiency The indication of this undesirable condition is upward modulation.
3. Sufficient grid drive to ensure that the amplifier is driven into anode limitation at 100% peak modulation. The indications of inadequate drive are downward modulation and non-linearity.
4. As large an angle of current flow as possible when in the carrier condition. This is not essential but is desirable to obtain the maximum power output. (Non-linearity in both the cases above refers to envelope non-linearity which can be observed either on a cathode-ray oscillograph or on an audio-frequency harmonic measuring set.) It may, however, be necessary to reduce the angle of current flow to improve linearity.

A modulated amplifier must *not* have :

1. An excessive grid dissipation. This sets a limit to the permissible grid drive, which however is usually not reached. See X:38 for calculation of grid dissipation.

2. An excessive peak anode current. This means that the output circuit must be adjusted to an impedance which will make the valves operate along the chosen load line.

The procedure for adjustment of a modulated amplifier immediately following and the procedure described in XIII:4.3 are both designed to adjust the angle of current flow at 100% p.p.m. to 180°. This is, firstly, because this condition can be realized by a definite indication (the threshold of upward modulation), and secondly, because it represents the condition of maximum power output. The angle of current flow in the carrier condition is left to take care of itself.

If, having adjusted the circuit to an angle of current flow of 180° at 100% p.p.m., a lower angle of current flow is required either to improve linearity or to increase the efficiency in the carrier condition or to decrease the power output, this is effected by increasing the grid bias and grid drive, and increasing the anode impedance so that with the new value of grid drive the anode peak volts are the same in the carrier condition as they were with the initially found adjustments: i.e. with 180° angle of current flow at 100% p.p.m.

It will be appreciated that the procedures described below are by no means unique; many other methods may be used, and these methods may be varied according to circumstances.

The first procedure is as follows:

1. On the valve field for the valves in use in the modulated amplifier draw the load line at 100% peak positive modulation, that is, with twice the H.T. volts effective on the anode, and note the peak current i_{p_s} when the valve is driven to the threshold of anode limitation with twice normal H.T. (This must not exceed 0·8 × the total emission of the valve minus peak grid current.)

   Note also the bias necessary to take the valve to cut-off with normal H.T. and with twice normal H.T., and call these E_c and E_{c_s} respectively. Note also the voltage up to which the grid must peak to make the valve just reach anode limitation at 100% peak positive modulation with the chosen load line: call this voltage E_{gp}.

2. If Z_L is the slope of the load line, the impedance which must face each valve is $Z_e = \dfrac{Z_L}{f}$, where f corresponds to an angle of current flow of 180°, i.e. $f = 0{\cdot}5$. Hence $Z_e = 2Z_L$.

According to the arrangement and the number of valves the required total circuit impedance can now be determined. The circuit should be set up to this impedance. This can be done either by measurement on an R.F. bridge or by using the amplifier as a bridge, driving it with a known grid drive $\hat{e}_g$ and 180° angle of current flow and observing the driven feed $\bar{\imath}$. The peak current is then $i_p = \pi\bar{\imath}$ and the peak anode volts $\hat{e}_a$ can be either measured (which is best) or determined by plotting (on the valve field) the peak current on the anode-voltage anode-current characteristic corresponding to the maximum positive excursion of the grid. The projection of this point on the axis of anode volts gives the limit of excursion of the anode volts and so the anode peak volts. Then $Z_L = \hat{e}_a/i_p$ and $Z_c = 2Z_L$ = the impedance facing one valve.

3. Apply a value of external bias sufficient to prevent the valve passing a damaging value of anode current in the event of failure of grid drive.
4. Determine the grid bias E_{b_2} necessary to take the valve to cut-off with twice normal H.T. This is determined directly from the valve characteristics.

   Determine also the value of grid drive $\hat{e}_g$ necessary, with twice normal H.T. and a load line Z_L, to make the amplifier drive up to E_{gp}, the threshold of anode limitation. E_{gp} = the characteristic passing through the point of intersection of the "asymptote" (see X:27) and the load line drawn with slope corresponding to Z_L, through the point: anode volts = twice H.T. and anode current = zero. The value of grid drive is given by $\hat{e}_g = E_{gp} - E_{b_2}$.

   The working value of drive is then $\hat{e}_g$. Subsequent adjustments in the values of $\hat{e}_g$ may be necessary as part of step No. 7 below.

   (If the mod. amp. is to be operated with all fixed bias, then E_{b_2} is the working value of grid bias and the angle of current flow in the carrier condition is given by equation (6)/X:25. The value of $\hat{e}_a$ in this equation may either be read off from the valve characteristics or calculated from equation (2)/X:34. Steps 5 and 6 are then omitted.)
5. Determine the grid bias E_b necessary with normal H.T. to make the amplifier operate at an angle of current flow of 120° with a value of grid drive equal to $\hat{e}_g$ as determined

in step 4. (120° is chosen as the most probable angle of flow in a mod. amp. with grid current bias.)

From equation (5)/X:25,

$$E_b = E_c - (\hat{e}_g - \hat{e}_a/\mu)\cos\theta/2$$
$$= E_c - 0{\cdot}5(\hat{e}_g - \hat{e}_a/\mu)$$

($\hat{e}_a$ is determined as in the note in brackets at the end of step 4.) E_b is then the working value of grid bias in the carrier (no mod.) condition.

6. Set up the circuit with an impedance Z_e facing the anodes, a value of fixed grid bias sufficient to protect the valve and a value of grid resistance as below. Apply normal H.T. and *gradually* increase the grid drive up to $\hat{e}_g$ as determined in step 4, simultaneously adjusting the grid current bias resistance, so that, when $\hat{e}_g$ is reached, the bias due to grid current builds the total grid bias up to the value E_b as determined in step 5.

   To prevent the occurrence of excessive grid dissipation during this process it is best to start from too high a value of grid-current bias resistance and bring the grid drive straight up to $\hat{e}_g$, subsequently reducing the resistance until the total bias equals E_b. This method is also simpler, but since the value of resistance is initially unknown it is not always possible to be certain of starting with too high a resistance.

   Evidently many variations of this method are possible provided care is taken to avoid excessive grid dissipation.

7. Apply sinusoidal anode modulation of low depth and observe the modulated output on a cathode-ray oscillograph, preferably using the trapezium diagram. (See XX:10.4.)

As part of the last step the modulation depth should be increased up to 100% or until envelope distortion appears.

If the crests of the envelope are flat, try increasing the grid drive. If this does not remove the distortion it may (although this is unlikely) be due to too high a peak current, in which case the anode impedance must be increased. If the grid drive is too low, downward modulation is usually present. To be consistent with the initial design assumptions the grid drive should be adjusted to a value just greater than that at which downward modulation and distortion disappear.

If upward modulation occurs at 100% mod. the grid bias should be increased until it just disappears. Normally, to ensure that an

angle of current flow of 180° is obtained at 100% p.p.m., the grid bias should be reduced until upward modulation occurs at 100% p.p.m. and then increased until it just disappears. If downward modulation accompanied by distortion at 100% peak mod. occurs, the grid drive should be increased.

If the troughs of the envelope are flat it is probably due to "bottom bending" and can only be remedied by a large increase in anode impedance with a corresponding decrease in power output. If a trapezium diagram is obtained with concave sides (which is unlikely) it may mean that the grid-current bias is introducing over-correction and an improvement may be obtained by increasing the fixed bias and readjusting the grid-current resistance as in section 6.

If in a modulated amplifier with all fixed bias a trapezium diagram with convex sides is obtained, an improvement may be obtained by introducing grid current bias with a reduced value of fixed bias.

The method of adjusting a modulated amplifier with all fixed bias is a comparatively simple matter and can be easily derived from the above more generally applicable method.

It must always be remembered that all envelope distortion is not due to the mod-ed amplifier. A certain amount occurs in the mod-ing amplifier and it is assumed that the latter has been tested when working into a dummy load equal to the impedance presented by the mod-ed amp. to the mod-ing amp. If the mod-ed amp. is distortionless the mod-ing amp. distortion will still appear in the modulated envelope and must be allowed for. Further, it is not a practical proposition to strive for very low distortion at very high percentage modulation. In the case of high-power modulation, if the R.M.S. audio-frequency harmonics do not exceed 4% of the amplitude of the modulating frequency at 95% modulation, this should be regarded as satisfactory; this meets the C.C.I. requirements.

The above process has adjusted the mod-ed amp. so that the angle of current flow at 100% peak positive modulation is just under 180° If the linearity is still not adequate, an improvement in linearity can sometimes be obtained by increasing the grid bias and grid drive. This results in a lower angle of current flow in the carrier condition, which can be determined, and a lower angle of current flow at 100% p.p.m. which, when grid-current bias is used, cannot be determined. Unless the anode load impedance is increased, the peak current will therefore be increased and the valve will no longer work along its original load line. If a peak voltmeter is available, the anode load impedance should be increased until the

anode peak volts are unchanged by the increase of grid bias, when the grid drive is increased until anode limitation occurs.

If no peak voltmeter is available no accurate criterion for the increase of anode load impedance exists, unless fixed grid bias is used, in which case the angle of current flow at 100% p.p.m. can be calculated and the ratio by which *f* is changed can be determined. This is also the reciprocal of the ratio by which the anode load impedance must be increased. Where grid current bias is used the best approximation that can be made is to increase the load impedance in the reciprocal of the ratio of the apparent change of *f* in the carrier condition, obtained by entering the angles of current flow in the carrier condition in Fig. 1/X:22. This gives fictitious values of *f*, since anode limitation is serious in the carrier condition, and for this reason, if non-linearity is observed after such a change, it must be remembered that it may be due to too high a value of peak current and that it may be removable by increasing the anode impedance.

It often happens that it is necessary to adjust a modulated amplifier when peak voltmeters are not available for measuring the peak grid volts and the peak anode volts. In this case the above procedure cannot be followed, and the procedure to be used varies according to what indications are available. With no indications at all except the audio-frequency output of a receiver tuned to the transmitter, it is only possible to adjust the grid bias and grid drive until upward and downward modulation are absent, and distortion, as judged by ear, is absent. If an aerial ammeter is available and the normal current is known, the impedance facing the anodes of the valves in the modulated amplifier may be adjusted until the aerial current is normal with the maximum safe grid drive. The maximum safe grid drive can only be determined if the normal grid current is known, but it usually is known.

If a cathode-ray oscillograph is available, it provides a better criterion of distortion than a receiver, since it gives some indication of the type, and therefore the probable cause, of distortion.

If the normal driven feed of the valve is known, the anode impedance can be adjusted to give normal driven feed with normal grid current indicating normal grid drive. This adjusts the anode impedance to normal, and is of course, a standard procedure.

4.3. Simple Procedure for adjusting a Modulated Amplifier. The procedure given above for the initial adjustment of a modulated amplifier constitutes a fundamental method which leads to a known adjustment by a series of positive steps. Further, a

knowledge of these steps is extremely useful as an aid to the adjustment of modulated amplifiers when all the aids required for the full method are not available. As already indicated, however, peak voltmeters are not always available, and a method will now be described which makes use of the indications which are available, or should be available on every transmitter.

This method may also be said to constitute an initial method in that it can be used to adjust a modulated amplifier for which no settings are known.

The indications which are normally available are the anode current and a field of anode-voltage anode-current characteristics for the valves in the modulated amplifier. A cathode-ray oscillograph is desirable, but not essential. The steps in the adjustment are then as follows:

(1) Bias the valves to cut-off with normal H.T. by means of external grid bias (i.e. no grid-current bias. The amplifier is then working in class B).

(2) Find what value of grid drive is necessary to drive the valves into anode limitation with normal H.T. with the anode circuit in tune, and the output coupling adjusted to what is believed to constitute an approximation to the proper value. This means that the impedance facing the anodes of the valves is adjusted to what is believed to be the correct value. The method of determining the *carrier condition limiting drive*, as this value of grid drive will be called, is to increase the grid drive gradually until further increase of grid drive causes no increase in carrier output, as indicated on the aerial or feeder meters, or no increase in anode current. *The limiting value of grid drive is then just below the value at which the limitation is noticeable.*

Because of the uncertainty in the value of anode impedance, grid drive should be increased slowly, and the anode feed watched to see that the rated anode dissipation of the valve is not exceeded by the product of anode volts and anode feed, less the estimated R.F. power output.

(3) With the carrier condition limiting drive, and other conditions as in (2), note the driven feed. The peak anode current per valve is then equal to the driven feed multiplied by π and divided by the number of valves.

(4) Plot the peak anode current on the asymptote of the valve characteristics: i.e. plot the point corresponding to the intersection of the asymptote with the horizontal drawn through the peak anode current. Draw a line from this point to the point: anode

volts = H.T. effective on valve anodes, anode current = 0. This line may be regarded as the carrier condition load line, and the impedance facing the anode of each valve is equal to twice the impedance corresponding to the slope of this line.

(5) The load line drawn above is a good approximation to the normal load line for the case where the amplifier is lined up to work at 180° angle of current flow at 100% positive peak modulation. The working peak anode volts can therefore be obtained from it by projecting the intersection of the peak anode current and asymptote on to the axis of anode volts. The distance between the resultant point on the axis of anode volts and the H.T. is equal to the peak anode volts.

(6) Calculate the output power as a quarter of the product of the peak anode volts and peak anode current with the limiting value of grid drive. Calculate the efficiency on the assumption of 180° angle of current flow, using the value of anode peak volts derived above. Decide whether it is necessary to increase the efficiency (by increasing the impedance facing the anodes) or to increase the power output (by reducing the impedance facing the anodes), assuming this to be possible. Make the appropriate adjustment of anode output coupling to modify the anode impedance in the required direction.

(7) Repeat steps (2) to (6) until the required impedance, corresponding to the optimum compromise between power output and efficiency, is obtained. Check that the peak anode current at 100% p.p.m. is not excessive. For this purpose draw a line through the point: anode current = 0, anode volts = twice H.T. parallel to the carrier condition load line, and read off the peak anode current at the point where this line cuts the asymptote. Don't forget to allow for grid current when reckoning total emission. A generous factor of safety should be introduced to allow for over drive: i.e. the grid drive may be adjusted above the value which just reaches the threshold of anode limitation at 100% p.p.m.

(8) Apply sufficient external bias to protect the valves and insert a high value of grid-current bias resistance. Apply about 1·5 times the value of the carrier condition limiting drive and gradually bring up the modulation.

(9) Reduce the grid-current bias resistance until upward modulation appears and then increase it until upward modulation just disappears. This adjusts to 180° angle of current flow at 100% p.p.m.

(10) Observe the detected output by ear or the output wave form on a cathode-ray oscillograph if available and adjust the grid drive until it is just greater than the value at which distortion disappears,

or the value at which downward modulation disappears, whichever is the greatest.

(11) Repeat (9) and (10) until grid bias and grid drive are just on the safe side of the values at which upward and downward modulation and distortion respectively occur.

(12) If distortion is still present, an improvement in linearity may be obtained by increasing the grid-current bias resistance so that the angle of current flow is reduced. If this is done, it will be necessary to increase the grid drive. Also, of course, the power output will be less than the value determined above, while the peak anode current will be greater. In most cases the increase in peak anode current will not cause the total emission to be exceeded, but if it does, non-linearity will occur and the impedance facing the anodes will have to be increased, with a consequent further reduction of power output.

At 100% p.p.m. the slope of the normal load line is $Z_L = fZ_e = 0{\cdot}5Z_e$ for 180° angle of current flow at 100% p.p.m., where Z_e is the impedance facing the anode of one valve as measured on an R.F. bridge. The anode peak volts at 100% p.p.m. are given by projecting on to the axis of anode volts the intersection of this load line with the asymptote. The anode peak volts in the carrier condition may be obtained by projecting on to the axis of anode volts the intersection with the asymptote of the load line drawn as specified in (4) above.

4.4. Required Power-Handling Capacity of Modulating Amplifier in Terms of Unmodulated Carrier Power Output of Modulated Amplifier. It is usually considered that adequate modulating power-handling capacity is provided if, with a sinusoidal modulating wave, the modulating amplifier is able to modulate the carrier output from the modulated amplifier to a depth of 100% without exceeding the limits of tolerable distortion. The calculation below gives the required sinusoidal power, which the modulating amplifier must provide, in terms of the unmodulated power output of the modulated amplifier.

The conventions used are the same as those at the beginning of Chapter X; all quantities apply to the modulated amplifier except where otherwise stated.

$$P_o = \frac{e_a i_1}{2} = \frac{EZ_L}{R_a+Z_L}\cdot\frac{i_1}{2} \text{ (see equation (2)/X:34)}$$

The D.C. power input to the modulated amplifier is EI.

At 100% modulation the peak value of the modulating voltage

is E and the peak value of the current supplied by the modulating amplifier is $\hat{\imath}$. Assuming a sinusoidal modulating wave, the modulating frequency power supplied by the modulating amplifier is

$$P_m = \frac{E\hat{\imath}}{2}$$

$$\frac{P_m}{P_0} = E\hat{\imath} \times \frac{R_a+Z_L}{EZ_L\hat{\imath}_f}$$

$$\therefore P_m = \frac{1}{u}\cdot\frac{R_a+Z_L}{Z_L}P_0$$

And since R_a, the slope of the asymptote, is very small compared with Z_L,

$$= \frac{P_0}{u}, \text{ very nearly.}$$

In a modulated amplifier the angle of current flow will probably lie somewhere near 120°, so that the value of u is somewhere near 1·8. The modulating power required is therefore about 55%, say 60% of the unmodulated carrier power.

5. Diode Modulator.

This form of modulator deserves much more attention than it has received since it is one of the most faithful forms of modulator

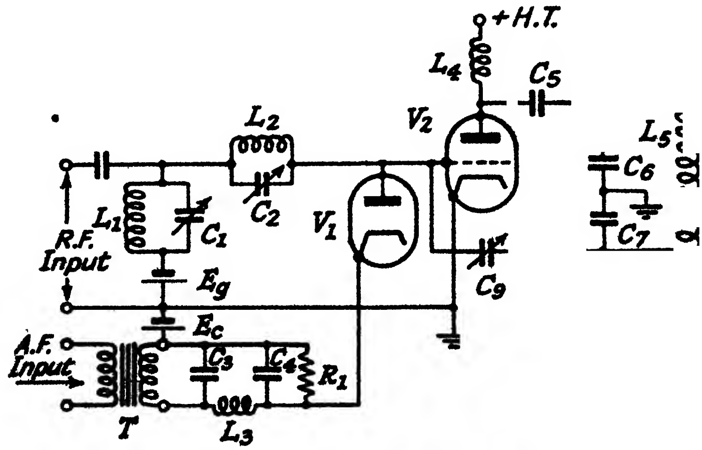


FIG. 1/XIII:5.—Diode Modulator.

known: it is capable of 100% modulation with an amplitude of individual envelope harmonics under 1%: and it is very simple. A typical circuit is shown in Fig. 1. L_1C_1 constitute the anode tuning circuit of the R.F. driving valve, not shown, which drives

the grid of V_2 through the rejector circuit L_2, C_2 tuned to the driving carrier frequency. V_2 is a normal class B amplifier biased to cut-off by bias source marked E_g. L_4 is the anode choke of V_2, while L_5, L_6, L_7, C_5, C_6 and C_7 constitute a normal anode output circuit, C_6 being the neut. condenser. V_1 is a diode, in the cathode circuit of which is resistance R_1 driven with audio frequency supplied by transformer T through the low-pass filter, C_3, L_3, C_4 which is inserted to prevent carrier frequency voltage from affecting the audio-frequency source, and also to present a low impedance in shunt across R_1 at the carrier frequency. The cathode of V_1 is maintained at a potential E_c above ground by means of the battery marked E_c. Although the battery is shown in such a sense as to make E_c positive, E_c may be negative if required. The impedance of L_2, C_2 is adjusted by varying the L to C ratio so that it presents an impedance high compared to the impedance of V_1 when conducting, but not high compared to the input reactance presented by the reactance of the anode cathode capacity of V_1 and the grid cathode capacity of V_2. A good compromise value is to make the impedance of L_2, C_2 equal to the combined input reactance of the two valves.

By means of E_g and E_c the anode of V_1 is maintained at a mean negative potential E_a relative to its cathode given algebraically by

$$E_a = -E_g \pm E_c \quad . \quad . \quad . \quad . \qquad (1)$$

If E_c is positive it must be less than E_g, so that E_a is always negative.

During negative half-cycles of the carrier frequency the anode of V_1 and the grid of V_2 are driven more negative, so that no current flows through V_1, and, since V_2 is biased to cut-off by E_g, there is no output from V_2. During positive half-cycles the anode potential of V_1, and the grid of V_2 follow the instantaneous value of the carrier voltage until the anode potential of V_1 has risen by an amount E_a when the anode of V_1 reaches the same potential as its cathode. Further rise in potential of the anode of V_1 is substantially prevented because, as soon as the anode goes further positive, anode current flows, and the carrier drive is highly attenuated by the potentiometer constituted by the series rejector circuit L_2, C_2, and the shunt diode circuit. The amplitude of carrier drive on the grid of V_2 therefore is substantially equal to the value of E_a. When an audio-frequency input is applied through T, the potential of the cathode of V_1 is varied at audio frequency and normal modulation of the grid drive on V_2 results, since this is equivalent to varying the value of E_a at audio frequency. If the peak value of the audio-frequency voltage effective across the resistance R_1 is E_m, when $E_m = E_a$, as defined by equation (1), 100% modulation occurs: the

carrier drive on V_2 is varied from zero to twice E_a. It is important to note that below 100% modulation no audio-frequency current can pass through the diode V_1 into the R.F. circuits, since the cathode is always positive with regard to the anode. Even if during short periods of overmodulation the diode becomes conducting, the path from the anode of V_1 to ground through L_2 and L_1 is of such low impedance that substantially no change in bias of V_2 can occur.

Provided adequate grid drive is provided to meet the 100% peak mod. condition the drive effective on the grid of V_2 is independent of the amplitude of carrier drive supplied across L_1 and C_1, and dependent only on the instantaneous value of E_a as controlled by the values of E_g, E_c and the audio-frequency source. Further, by making the amplitude of carrier drive across V_1, as measured by the negative peaks, large compared to $2E_a$, the percentage of change in angle of current flow through V_1 can be made small, and so a high degree of linearity can be obtained.

6. Class C Grid Modulator.

The circuit of a grid modulator is shown in Fig. 2 and explained at the end of this section.

A grid modulator is a normal class C amplifier in which the bias is effectively varied at audio frequency so that the amplitude of grid swing effective above the cut-off voltage of the valve is varied at audio frequency. The valve is heavily biased and driven on its grid with both the carrier frequency and the modulating audio frequency.

Fig. 1 shows a curve of anode current against grid voltage plotted for a class C amplifier valve. E_{gp} is the maximum value of positive grid swing permitted from normal considerations of linearity and peak current applicable to a class C amplifier. E_s is the change of grid voltage from E_{gp} to E_c the cut-off bias = the maximum permissible value of the effective grid peak volts. E_b is the value of steady grid bias and $\hat{e}_g$ is the peak value of the carrier frequency grid drive.

Then $$E_s = E_{gp} - E_c \quad . \quad . \quad . \quad . \quad (1)$$

$\hat{e}_g$ is adjusted so that its magnitude is given by

$$\hat{e}_g = \tfrac{1}{2}E_s = E_c - E_b \quad . \quad . \quad . \quad (2)$$

Hence $$\hat{e}_g = E_c - E_b + \tfrac{1}{2}E_s \quad . \quad . \quad . \quad (3)$$

In the absence of audio-frequency drive therefore the peak volts effective on the grid, i.e. the amount the grid is swung above the cut-off voltage, is $\tfrac{1}{2}E_s$. This will therefore give an unmodulated carrier-

frequency output of amplitude corresponding to a grid drive of $\frac{1}{2}E_s$ above cut-off *at an angle of current flow considerably less than* 180°.

If, now, an audio-frequency voltage of amplitude $\hat{e}_{ag} = \frac{1}{2}E_s$ is

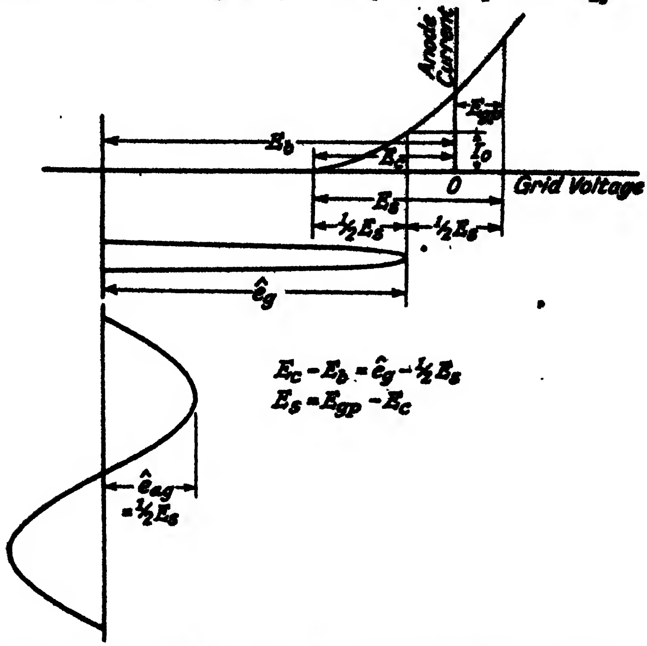


FIG. 1/XIII:6.—Design of Class C Grid-Modulated Amplifier.

also applied to the grid, the bias is effectively varied at audio frequency from $E_b + \frac{1}{2}E_s$ to $E_b - \frac{1}{2}E_s$. The carrier frequency peak volts effective on the grid above cut-off are therefore swung from zero to E_s, the envelope of the peak volts following exactly the audio-frequency drive.

The linearity of the arrangement is not very good, for the following reasons.

In general, anode-current grid-voltage curves are not straight lines, but give greater increases of anode current for a given change of grid drive at high grid drives than at low grid drives, so introducing non-linearity. Inspection of Fig. 1 will make this clear. A second contributory cause of non-linearity, which is additive to the first, is due to the angle of current flow, which progressively increases as the modulation cycle proceeds from 100% negative peak modulation (at the negative peak of the audio-frequency modulating wave) to 100% positive peak modulation. The effect is, however, to some extent self-compensating, since, although the com-

ponent of fundamental frequency contained in a series of current pulses corresponding to a low angle of flow is less than that in a high angle of flow (expressed as a percentage of the peak current), the peak current increases by very nearly the necessary amount to effect complete compensation. If the valve had zero internal impedance complete compensation would take place.

The net result is that the degree of linearity is represented fairly closely by the anode-current grid-voltage characteristic of the valve drawn from points taken along a load line Z_L corresponding to Z_e, the real impedance facing the anode circuit and the value of current flow θ at 100% peak modulation,

$$\text{i.e.}\quad Z_L = f_\theta Z_e$$

when f_θ is the value of f read from Fig. 1/X:22 corresponding to angle θ.

The method of design of a grid-modulated amplifier is therefore as follows. The optimum load line is drawn on the field of anode-voltage anode-current characteristics for the modulated amplifier valve. This gives the value of $\hat{\imath}_p$, the peak current at maximum output (100% peak positive mod.), and Z_L, the slope of the load line, also E_{gp} and E_c and hence E_s. It also gives the value of $\hat{e}_a$, the peak anode volts at 100% peak positive mod.

If the required carrier power is P, at 100% peak mod. the instantaneous power is $4P$ and the angle of current flow θ can be determined from the value of f by reference to Fig. 1/X:22, where f is determined by

$$4P = \tfrac{1}{2} f \hat{\imath}_p^2 Z_L, \quad \text{(see equation (1)/X:23)}$$

$$\therefore f = \frac{8P}{\hat{\imath}_p^2 Z_L} \qquad \qquad (4)$$

(If the value of f given by (4) is greater than 0·5, the valve is incapable of supplying the required power. Either a new load line must be drawn or a larger valve or more valves must be used. Once the value of f is found the value of θ, the angle of current flow, can be read from Fig. 1/X:22.)

Then $\left(\hat{e}_g - \frac{\hat{e}_a}{\mu}\right)\cos\frac{\theta}{2} = \hat{e}_g - E_s$ (see equation (5)/X:25)

$$\hat{e}_g = \frac{E_s - \frac{\hat{e}_a}{\mu}\cos\frac{\theta}{2}}{1 - \cos\frac{\theta}{2}} \qquad \qquad (5)$$

and

$$E_b = E_c - \hat{e}_g + \frac{1}{2}E_s \qquad \qquad (6)$$

It is a truism to say that the linearity of the modulator may be improved by arranging that the curved part of the valve characteristic is not used. Linearity may therefore be improved by accepting a percentage modulation less than 100 and so avoiding the use of the bottom part of the anode-current grid-voltage curve where the departure from linearity is greatest. For this purpose the value of $\hat{e}_g$ is increased so that the carrier level is increased. The value of $\hat{e}_{ag}$ is then reduced so that the value of E_{gp} is unaltered. This means that the maximum possible percentage modulation is less than 100. The output wave may then be restored to 100% modulation by overbiasing in following amplifiers, which would normally be operated as class B amplifiers. This is really only delaying the problem to another stage, which may, however, by good fortune have better linearity near cut-off.

A little consideration will show that no simple push-pull circuit will eliminate envelope non-linearity in the case of a grid modulator. By performing a normal modulation in a single-sided modulator, and then inverting the envelope by the addition of a carrier 180° out of phase with the modulated carrier, the resultant wave, if added to a wave produced in a second single-sided modulator fed with audio frequency 180° out of phase with the first modulator, will give cancellation of second-order harmonics in the envelope. This is hardly worth doing except at low power, where better types of modulator are available.

Fig. 2 shows a circuit of a grid modulator.

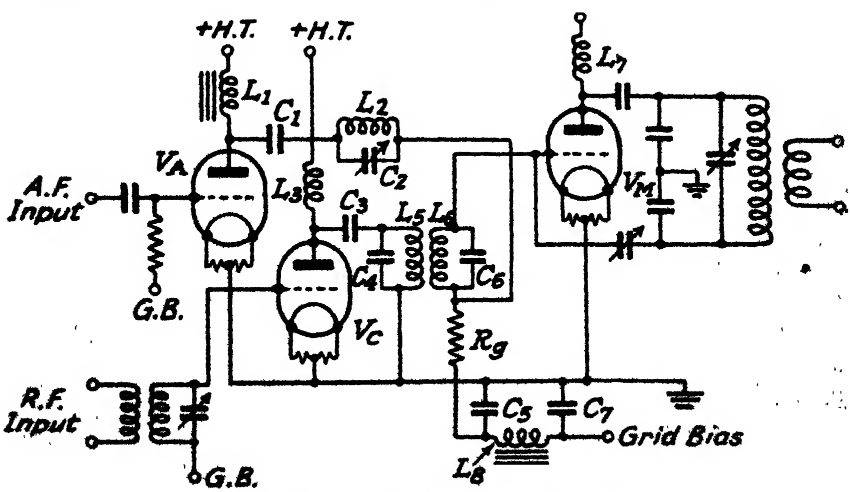


FIG. 2/XIII:6.—Grid Modulated Amplifier.

Audio frequency is supplied to the grid of V_A, which is choke fed through L_1 and supplies audio frequency through C_1, L_2 and L_3 to the grid of V_M, the modulated amplifier. R_g is inserted to provide a suitable audio-frequency load for V_A. Carrier frequency is supplied to the grid of V_C, which is fed through the high-frequency choke L_4 and supplies the carrier frequency through the blocking condenser C_3 and the coupled circuit C_4, L_5, L_6, C_5 to the grid of V_M. The rejector circuit L_3, C_2, is tuned to the carrier frequency and prevents the carrier-frequency energy from being dissipated in the anode circuit of V_A. V_M is shown with a conventional output circuit.

7. Anode Bend Modulator.

This is used in low-power modulators where the question of efficiency is unimportant. If in the circuit of Fig. 2/XIII:6 V_M is a valve with a square-law anode-current grid-voltage characteristic it can be made to operate as an anode-bend modulator by suitably adjusting the values of bias and drive. The bias E_b is adjusted to a value approximately equal to $\frac{1}{2}E_c$ (see Fig. 1). In the valves which are not true square-law valves some advantage may be gained

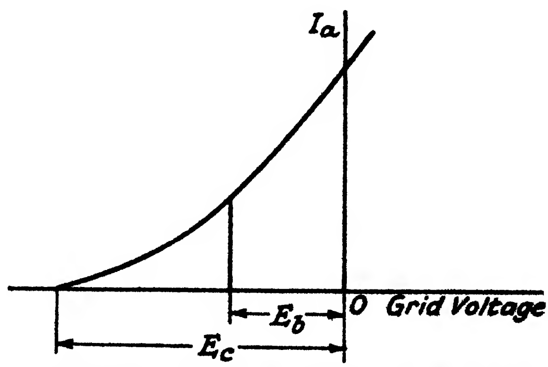


Fig. 1/XIII:7.—Curve of Anode Bend Modulator.

by using a higher bias and smaller values of drive. The carrier frequency peak drive $\hat{e}_g$ and the audio-frequency peak drive $\hat{e}_{ag}$ are adjusted so that their sum $\hat{e}_g+\hat{e}_{ag}$ is rather less than $\frac{1}{2}E_c$. The anode current is therefore never reduced to zero.

For simplicity put $\hat{e}_{ag} = A$ and $\hat{e}_g = C$ and let

$$A \sin at = \text{the audio-frequency drive}$$
$$C \sin ct = \text{the carrier-frequency drive.}$$

Suppose that the anode-voltage anode-current characteristic is given by

$$i_a = g(e - E_c)^2 \quad . \quad . \quad . \quad . \quad (1)$$

where e = the instantaneous value of the grid voltage and g = a constant dependent on the valve characteristic (this has no relation to g, the mean to peak ratio in class C amplifiers).

Under the conditions of operation described:

$$e = E_b + A \sin at + C \sin ct \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$i_a = g(E_b - E_c + A \sin at + C \sin ct)^2 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$= g[(E_b - E_c)^2 + A^2 \sin^2 at + C^2 \sin^2 ct + 2(E_b - E_c)A \sin at$$
$$+ 2(E_b - E_c)C \sin ct + 2AC \sin at \sin ct] \quad . \quad . \quad . \quad (4)$$

$$= g[(E_b - E_c)^2 + \tfrac{1}{2}A^2 - \tfrac{1}{2}A^2 \cos 2at + \tfrac{1}{2}C^2 - \tfrac{1}{2}C^2 \cos 2ct$$
$$+ 2(E_b - E_c)A \sin at + 2(E_b - E_c)C \sin ct$$
$$+ AC \cos (c - a)t - AC \cos (c + a)t] \quad . \quad . \quad . \quad . \quad (5)$$

Considering each term in detail:

$g[(E_b - E\ )^2 + \tfrac{1}{2}A^2 + \tfrac{1}{2}C^2]$ is the amplitude of the direct current, i.e. the mean anode feed as read on a D.C. meter.

$-\tfrac{1}{2}gA^2 \cos 2at$ is a current of twice the modulating audio frequency and amplitude $\tfrac{1}{2}gA^2$.

$-\tfrac{1}{2}gC^2 \cos 2ct$ is a current of twice carrier frequency and amplitude $\tfrac{1}{2}gC^2$.

$2g(E_b - E_c)A \sin at$ is a current of frequency equal to the modulating audio frequency and amplitude $2g(E_b - E_c)$.

In practice, if a selective circuit is used in the anode of the valve, designed to accept the frequencies corresponding to the modulated carrier frequency and suppress other frequencies, these currents will be ineffective in producing output. It may be remarked also that since the types of selective circuit normally used do not present the same impedance to wanted and unwanted frequencies, the relative amplitudes of the above unwanted currents and the wanted currents represented by the remaining terms will be different from those represented in equation (5). This is however a minor point of no importance.

The remaining terms constitute the modulated wave:

$2g(E_b - E_c)C \sin ct$ is a carrier-frequency current of amplitude $2g(E_b - E_c)C$.

$gAC \cos (c - a)t$ is a current of frequency equal to the difference between carrier and audio frequency and is therefore the *lower sideband frequency.*

$gAC \cos (c + a)t$ is a current of frequency equal to the sum of carrier and audio frequency and is therefore the *upper sideband frequency.*

The depth of modulation m is given by the sum of the amplitudes of the sideband frequencies divided by the carrier frequency amplitude.

$$\therefore\ m = \frac{2_g AC}{2_g(E_b - E_c)C} = \frac{A}{E_b - E_c} \qquad (6)$$

It will now be clear why the percentage modulation must be low. Assume an ideal square-law valve characteristic so that E_b should be made equal to $\frac{1}{2}E_c$ for maximum output (considering the case where the grid cannot be run positive on account of grid current; it cannot be driven below cut-off). For maximum efficiency of modulation the sideband amplitude gAC must be a maximum and in the ideal case $A+C = \hat{e}_g + \hat{e}_{ag} = \frac{1}{2}E_c = E_b$.

The maximum value of the product of two quantities, of which the sum is constant, occurs when they are equal, so that gAC is a maximum when

$$A = C = \tfrac{1}{2}E_b = \tfrac{1}{4}E_c = \tfrac{1}{2}(E_b - E_c) \qquad . \qquad . \qquad (7)$$

Whence from (6) inserting the value of A from (7) the maximum value of m is 50%. A little consideration will show that the same answer will be obtained for the case when the grid can be run positive, provided the bias adjusts the valve to the centre of the available grid base. By sacrificing the sideband amplitude by reducing $\hat{e}_g$ and increasing $\hat{e}_{ag}$ the percentage modulation may be increased, but it can only reach 100% in the limit when the sideband amplitude has been reduced to zero. In practice, owing to power supply variations, still lower values of modulation than 50% have to be accepted.

The efficiency of this type of modulator, expressed by ratio of output carrier power to anode input power, is very low.

The anode bend modulator is representative of a class of modulators which all operate on the principle of producing intermodulation between a carrier frequency and a modulating or band of modulating frequencies. While equation (1) is formulated above to express the relation between the grid voltage and anode current in a valve, if E_c is made equal to zero, it may equally well be used

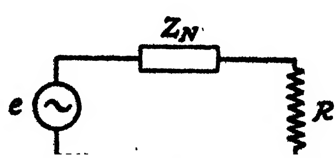


FIG. 2/XIII:7.—Non-Linear Circuit Element in Series with a Resistance.

to represent the relation between the impressed voltage e and the current i_a flowing in the simple circuit of Fig. 2.

If e is then defined by equation (2), the currents flowing through R in Fig. 2 are defined by equation (5) with $E_c = 0$. For low values of carrier frequency, a suitable type of non-linear circuit element for insertion in place of Z_N in Fig. 2 is provided by a copper-oxide rectifier.

8. Suppressor-Grid Class C Modulator.

This is a class C edition of the principle of intermodulation used in a mixer valve in a radio receiver, obtained by adjusting the bias of a pentode control grid to cut-off or below. It depends on the fact that the amplification factor of a pentode from control grid to anode is directly proportional to the potential of the suppressor grid, over a large range of voltages. Unfortunately this proportionality does not extend down to zero voltage (i.e. the suppressor-grid potential equal to that of the cathode : the amplification constant from control grid to anode is not equal to zero in this condition), and as a consequence a suppressor-grid modulator does not give 100% modulation. It is, however, possible to obtain a percentage modulation as high as about 80% with a high degree of linearity, i.e. R.M.S. envelope harmonic percentage below 5% of the fundamental modulating frequency.

Assume that the amplification factor from control grid to anode is given by

$$\mu = qe_s \quad . \quad . \quad . \quad . \quad (1)$$

where e_s is the instantaneous voltage of the control grid and q is a factor depending on the valve.

If a steady voltage $C \sin ct$ is applied to the control grid, the anode-voltage swing

$$e_a = \mu C \sin ct = qe_s C \sin ct \quad . \quad . \quad . \quad (2)$$

If now a steady voltage E_s plus a sinusoidal voltage $A \sin at$ is applied to the suppressor grid, i.e. if

$$e_s = E_s + A \sin at \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\begin{aligned} \text{Then } e_a &= q(E_s + A \sin at)C \sin ct \\ &= qE_sC \sin ct + qAC \sin at \sin ct \\ &= qE_sC \sin ct + \tfrac{1}{2}qAC \cos (c-a)t - \tfrac{1}{2}qAC \cos (c+a)t \quad . \quad (4) \end{aligned}$$

This is a normally modulated wave of depth of modulation

$$m = \frac{qAC}{qE_sC} = \frac{A}{E_s} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The argument above applies equally to class A operation, such,

for instance, as occurs in a mixer valve in a radio receiver. No attempt has been made to relate the above parameters to the characteristics of a representative valve since in practice the operating conditions can best be obtained empirically. The amount of work necessary naturally depends on what information has been supplied by the valve manufacturer, who should in any case be consulted, if any of the electrodes of a valve is to be operated over a range of voltages outside those specified by the manufacturer. In the case of pentodes supplied for suppressor-grid modulation, full operating conditions are often specified, but when this is not the case the procedure is as follows:

A load line is drawn on the valve field corresponding to the value of *screen* volts used, which represents the optimum compromise between the requirements of power output, efficiency and linearity. The angle of current flow varies throughout the modulation cycle, and further, the angle of current flow with the value of suppressor volts, for which the valve field has been drawn, depends on that value of voltage. It is therefore not possible to determine this angle of current flow initially without an unnecessary amount of labour. It is therefore assumed to be 120°. The impedance facing the anode is therefore determined for the preliminary essay.

If the valve has a tungsten filament, a check is then made that the peak current at the upper limit of the linear range of the load line does not exceed the permissible peak current for the valve. If the valve has an oxide-coated filament, the mean current corresponding to half the maximum peak current and an angle of current flow of 180° must be calculated, and this must not exceed the rated current for the valve. Also the anode dissipation, assuming the same mean current, should be checked against the rated anode dissipation of the valve.

The valve is then set up as a class C amplifier with this value of anode impedance, and the control grid drive and bias set to give a maximum positive excursion of the grid, which will give a peak current equal to half the peak current at the upper end of the linear range of the load line, and an angle of current flow of 120°. The suppressor grid bias and screen grid bias have the values for which the valve field is drawn.

A low value of modulating voltage is then applied and the modulated output is viewed on a cathode-ray oscillograph, using the trapezium diagram (see XX:10.4). The modulating voltage is increased until distortion appears. If this distortion is in the troughs, the suppressor grid bias is adjusted to make the suppressor grid more

positive; if the peaks of the envelope are distorted, the suppressor grid bias is made more negative. If, during this process, upward modulation occurs, the control grid bias is increased and the control grid drive increased to keep the peak positive excursion of the control grid constant.

The maximum percentage modulation without distortion is then determined, as well as the power output in the carrier condition, i.e. with the suppressor and control grid biases and control grid drives at their last determined values and with no modulating voltage.

The process is then repeated with higher and lower values of the maximum positive excursion of the control grid. The condition which gives the optimum compromise between carrier power output and modulation depth is then selected, and the angle of current flow in this condition is determined. The impedance facing the anodes is then adjusted to make the valve work along the chosen load line with this angle of current flow, and the procedure, to determine the suppressor grid bias in the carrier condition, the carrier power output and the maximum linear modulation, is repeated, with the finally chosen values of control grid drive and bias.

The above method requires the use of grid and anode peak voltmeters, an R.F. bridge and a cathode-ray oscillograph. It is quite impossible to set up a suppressor-grid modulator to optimum conditions without an anode peak voltmeter and a cathode-ray oscillograph, but the grid peak voltmeter and R.F. bridge may be dispensed with by suitable makeshift. The valve may be used to measure the impedance facing the anode by biasing it as a class B amplifier and measuring the driven feed and the anode peak volts. The impedance facing the anode is then given by the anode peak volts divided by 1·57 times the driven feed.

An approximation to the maximum positive excursion of the grid may be determined when the anode load impedance is known. The anode peak volts divided by the anode load impedance gives i_f, and $u = i_f$ divided by the driven feed. Entering the value of u in Fig. 1/X:22 gives the angle of current flow and also the value of g, the mean to peak ratio. Hence dividing the driven feed by g gives the peak anode current. Plotting the peak anode current on the temporary load line (drawn for the value of anode load and the value of f, read from Fig. 1/X:22, corresponding to the angle of current flow) gives the maximum positive excursion of the grid.

It is evident that the above measurements must be made with the values of screen-grid and suppressor-grid volts for which the valve field is drawn.

The suppressor-grid modulator is so simple that no circuit need be drawn to represent the general case. The pentode is provided with a straight output circuit without neutralization; an audio-frequency circuit with appropriate bias is connected to the suppressor grid, and a radio-frequency circuit with appropriate bias is connected to the control grid.

The advantages of the suppressor-grid modulator are as follows:

1. Low audio-frequency power required, since the suppressor grid draws no grid current.
2. High linearity of modulation over the range of percentage modulation for which the modulator is effective, which corresponds to a maximum modulation of about 80%.
3. The audio-frequency and radio-frequency circuits are completely separate and no limitation of the audio-frequency range occurs consequent on the introduction of selective circuits for separating modulation-frequency and carrier-frequency currents. This is very important in long-wave circuits.
4. No capacity-neutralizing circuit is necessary.

The efficiency in the carrier condition is about 35% since the anode-voltage swing is then only half of the swing at 100% peak modulation.

8.1. Anode Modulation of Pentode (Suppressor-Grid Modulator). It will have been realized from the above discussion that simple suppressor-grid modulators are not driven into anode limitation and only approach anode limitation at 100% peak modulation. By applying anode modulation as well as grid modulation, a suppressor-grid modulator may be operated at high efficiency, but in this case only advantage No. 4 remains. In such cases it is usually necessary to modulate the screen-grid volts with the same voltage that is applied to the suppressor grid. See XIII:10.2.

9. Conversion to 100% Modulation.

A wave which is less than 100% modulated may be converted to 100% modulation by subtraction of a carrier-frequency voltage, in other words, by adding carrier frequency 180° out of phase. The elements of a circuit for doing this are shown in Fig. 1.

V_1 is a suppressor-grid modulator, and V_2 is the carrier-suppressor amplifier. Both valves are pentodes with the carrier frequency applied in push-pull to their control grids and taken off in parallel from their anodes. Resistance R_4 in conjunction with the parallel-tuned circuit L_1, C_1, provides a simple means of adjusting the carrier-

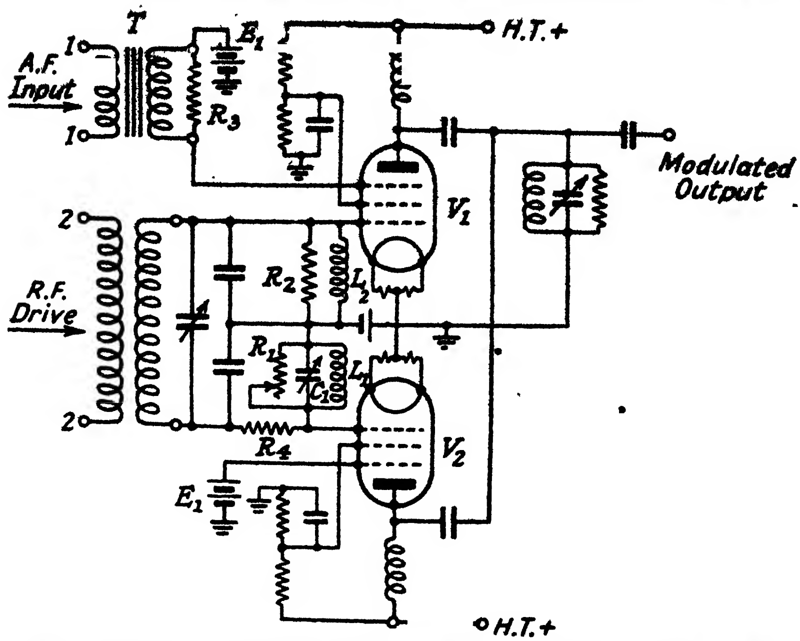


FIG. 1/XIII:9.—Suppressor-Grid Modulator with Carrier Suppression.

frequency phase in the anode circuits to exactly 180°. Resistances R_1 and R_2 are inserted to mask the effect of grid current in the control grids, R_2 being shunted with a choke L_2 to conduct the grid current to ground. Resistance R_1 is made variable to provide a means of adjusting the amplitude of the carrier out of phase current in the anode circuit. It is not advisable to use the suppressor grid of V_2 for adjusting the gain of V_2 since this has the effect of changing the angle of current flow in V_2 which should be the same as that of V_1 in the absence of modulation. The suppressor grid of V_2 is therefore biased through suitable decoupling circuits not shown, from the same source (E_1) as V_1.

The balance of carrier in the output circuit is accomplished by viewing the output wave on a cathode-ray oscillograph, with the maximum audio-frequency input to transformer T that is permissible from considerations of linearity, and a magnitude of carrier drive on V_1 determined as described above. R_1 and C_1 are then adjusted alternately until the amplitude of carrier in the extreme bottom of the trough of the modulation cycle is zero.

10. Push-Pull or Balanced Modulators.

Fig. 1/XIII:10 shows schematically at (*a*) a push-pull modulator

using anode-bend modulation and at (*b*) a push-pull modulator using suppressor-grid modulation. In each case T_1 is a push-pull input transformer with the two halves of its secondary winding balanced. T_4 is a push-pull output transformer with the two halves of the primary winding balanced; T_2 is an input transformer feeding in parallel to both valves, while T_3 is an output transformer usually omitted. In practice, if T_3 is used, in which case T_4 would probably be omitted, the direct anode current would be by-passed by means of a choke and a condenser.

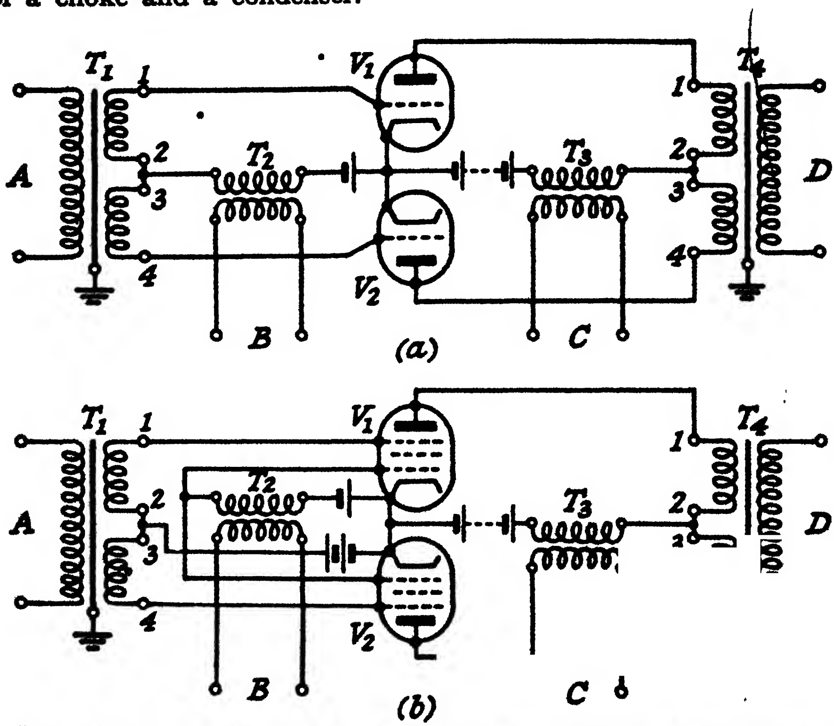


FIG. 1/XIII:10.—Push-Pull Modulator: (*a*) using Anode Bend Modulation, (*b*) using Suppressor-Grid Modulation.

The sense of the balanced windings is indicated by the terminal numbering. Most transformers are wound and numbered to conform to the convention that connecting an odd terminal of one winding to the even terminal of another winding connects the windings in series aiding.

The advantage of a push-pull or balanced modulator is the possibility it introduces of separating certain of the modulation products from one another. For instance, when the sideband frequencies only are required, a balanced modulator is used.

Assuming the balance of transformers and valves to be ideal (in practice this balance is usually only sufficiently good to attenuate the unwanted frequencies by about 20 db.), the table below summarizes the frequencies which are effective in the two output circuits for the four possible combinations of inputs.

Condition	In at *A*. Push-Pull Input	In at *B*. Parallel Input	Out at *C*. Parallel Output	Out at *D*. Push-Pull Output
1	f_1, f_2	—	$2f_1$, $2f_2$ f_1-f_2 f_1+f_2	f_1, f_2
2	f_1	f_2	$2f_1$, $2f_2$ f_2	f_1 f_1-f_2 f_1+f_2
3	f_2	f_1	$2f_1$, $2f_2$ f_1	f_2 f_1-f_2 f_1+f_2
4	—	f_1, f_2	$2f_1$, $2f_2$ f_1, f_2 f_1-f_2 f_1+f_2	0

A similar Table appeared in the *Proceedings of the I.R.E.* for June, 1925, in an article by R. A. Heising on Single Sideband.

The input at *A* may be called the push-pull input and the input at *B* may be called the parallel input. A similar convention will be applied to the outputs. The parallel input and output are sometimes called the conjugate input and output.

10.1. Double Balanced Modulator. This provides a still greater frequency selectivity, it being possible to obtain, for instance, sideband frequencies in one output unaccompanied by either of the original frequencies. One arrangement of a double balanced modulator designed to achieve the above selection is indicated in Fig. 2.

Two identical balanced modulators are supplied with frequency f_1 in relative phase opposition (i.e. differentially) at their push-pull input circuits and with f_2 in relative phase opposition in their conjugate input circuits. Their outputs are connected in series aiding. In the absence of drive from f_2, f_1 appears in opposite phase in each output circuit and so cancels. When f_2 is applied to the conjugate circuit it does not appear in the output circuit of either modulator,

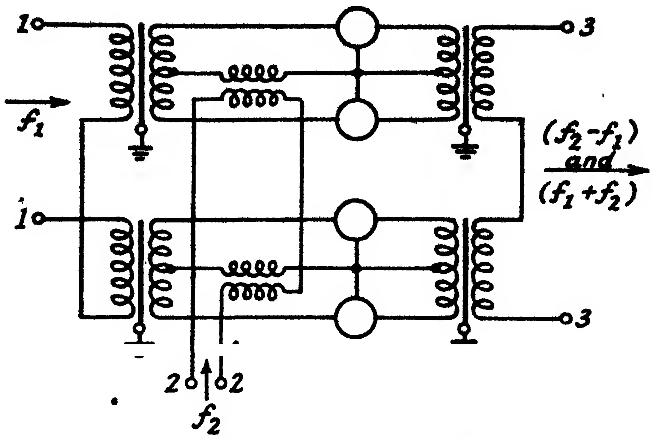


FIG. 2/XIII:10.—Schematic of Double Balanced Modulator.

but being supplied differentially the difference frequencies in the output circuits add. This may be seen more clearly by considering f_2 to be a lower frequency than f_1. In this case, during the positive half-cycle of f_2, one modulator has more gain for f_1 than the other, so that f_1 appears in the output with an envelope of the form of the wave shape of f_2. During the negative half-cycle the position is reversed and f_1 appears in the output reversed in phase and again with an envelope of the form of the wave shape of f_2. But such wave forms correspond to the presence of sum and difference frequencies only, i.e. f_1-f_2 and f_1+f_2.

Fig. 3 shows a simple practical form of double balanced modulator

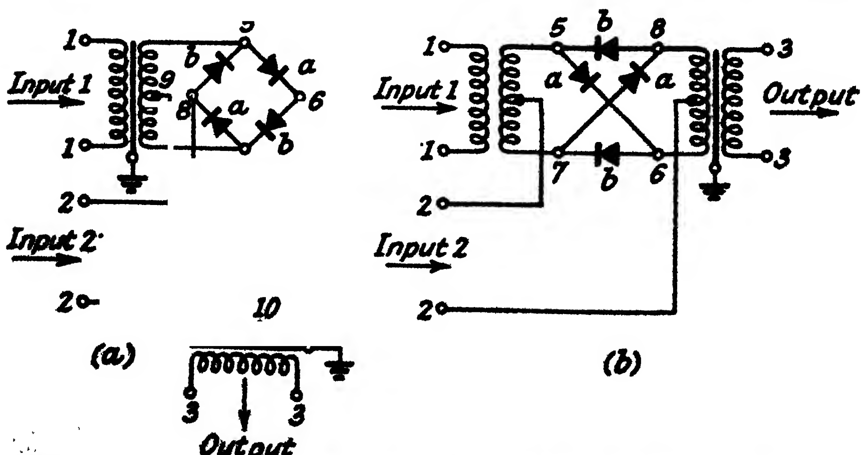


FIG. 3/XIII:10.—Simple Practical Double Balanced Modulator: The Ring Modulator.

using copper-oxide rectifiers ; this is sometimes called the *ring modulator*. The circuits at (*a*) and (*b*) are identical, but most people will find the arrangement at (*a*) most easy to understand. The transformers at input 1 and at the output of the modulator have respectively the secondary and primary winding tapped ; in each case the halves of the winding are balanced for inductance and for admittance to the ground and screen.

The operation of this modulator depends on the fact that the copper-oxide rectifier is a device which offers a very high resistance to the flow of currents passing through it in one direction—the non-pass direction—and a very much lower resistance to the flow of currents passing through it in the other direction—the pass direction. Further, the resistance in the pass direction falls as the voltage across the rectifier increases. The result is that the effect of passing a current through the rectifier in the pass direction is to increase the conductance of the rectifier as the voltage across it increases.

If a frequency f_1 is applied at input 1 it is evident that, if the rectifiers *a* and *b* have identical characteristics, the four rectifiers constitute a balanced bridge and no output can therefore appear at terminals 3,3.

Also if a frequency f_2 is applied at input 2, no output can appear at terminals 3,3, because the primary winding of the output transformer and the secondary winding of the input transformer are both balanced. When the two frequencies f_1 and f_2 are applied respectively and simultaneously at input 1 and input 2, if the rectifiers are such that their conductance is proportional to the voltage across them, the frequencies appearing in the output consist of f_1+f_2 and f_1-f_2. This may be seen more clearly by considering f_2 to be a lower frequency than f_1. In this case if the positive half-cycle of f_2 is considered to occur when terminal 9 is made positive with regard to terminal 10, during this half-cycle rectifiers *b* are made non-conducting while rectifiers *a* have their conductance increased in proportion to the current driven through them by the source of voltage supplying f_2 to terminals 2,2. The result is that during this half-cycle frequency f_1 appears in the output in one sense (or phase) with instantaneous envelope amplitudes cor esponding to the instantaneous amplitudes of f_2. Similarly, during the next (negative) half-cycle of f_2, rectifiers *a* are non-conducting and rectifiers *b* conducting, and during this half-cycle frequency f_1 appears in the output in reverse sense (or phase) with instantaneous envelope amplitudes corresponding to the instantaneous amplitudes of f_2. The process

is repeated in subsequent cycles of f_2. The resulting wave form corresponds to the presence of sum and different frequencies only, i.e. f_1+f_2 and f_1-f_2.

The form of rectifier characteristic required to make the conductance proportional to voltage amplitude is a square-law characteristic, that is, one in which the current through the rectifier is proportional to the square of the voltage effective across the rectifier. This may be demonstrated very simply as follows:

A square-law characteristic may be represented by $i = kv^2$
where i = the current through the rectifier
v = the voltage across the rectifier
and k = a constant.

Hence the conductance $= \frac{i}{v} = kv$, that is, the conductance is proportional to the applied voltage. Few rectifiers have such a characteristic, although by the addition of series resistance the rectifier characteristic may be made to approximate more closely to a square-law characteristic. The result is that, in practice, frequencies other than sum and difference frequencies are produced, of value and magnitude depending on the forms of the rectifier characteristic. Whether or not these are serious depends on their location with regard to the wanted frequencies.

It will be appreciated that the above argument is developed on the assumption that the impedance of all sources, effective as generators applying voltage across the rectifiers, is low compared to the lowest value of rectifier resistance.

10.2. Use of Tetrodes in Modulated Amplifiers. When a tetrode is used in an anode modulated amplifier, since the screen grid has a large control of the anode current, it is necessary to modulate the screen-grid volts as well as the anode volts. These are modulated to the same percentage depth as the anode volts. That is to say, the modulating voltage swing on the screen grid is arranged so that it is always the same percentage of the screen-grid steady voltage as is the anode-voltage swing in relation to the steady anode volts. See XIII:8.1.

11. Spectrum of Modulated Waves.

The spectrum of a modulated wave is a representation of the amplitude of each frequency component. The form of the spectrum depends on the carrier frequency, the spectrum of the modulating wave and the type of modulation.

11.1. Amplitude-Modulated Wave. The analytical expres-

sions for an amplitude-modulated wave for the case where the modulating frequency is a sine wave may be derived by inspection* as follows. Consider the modulation of a direct current of amplitude I_0 constituted by the addition of a sine wave $mI_0 \sin at$. The direct current is shown at (*a*) in Fig. 1 and the modulated wave at (*b*). It is evident that the amplitude of the modulated wave at (*b*) at any time t is given by

$$= I_0 + mI_0 \sin at = I_0(1 + m \sin at) \quad . \quad . \quad (1)$$

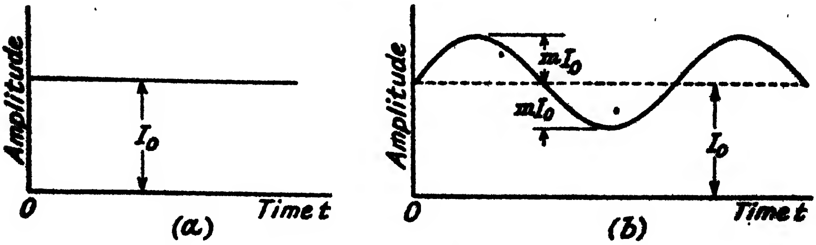


FIG. 1/XIII:11.—Modulation of a Direct Current by a Sine Wave.

Further, if m becomes greater than unity the current becomes negative, and although in this case there is no physical reason why this should not be permitted, we are still free to choose any convention we like for percentage modulation and to say that when $m = 1$ the modulation is 100%, and when $m = 0{\cdot}5$ the modulation is 50%, and so on. Since $50\% = \frac{50}{100} = 0{\cdot}5$, m is therefore defined as the depth of modulation.

The direct current can be called the carrier current: it would, for instance, "carry" the modulating frequency through a one-way device such as a rectifier: this has no further value as an analogy but serves to introduce the idea of a carrier current.

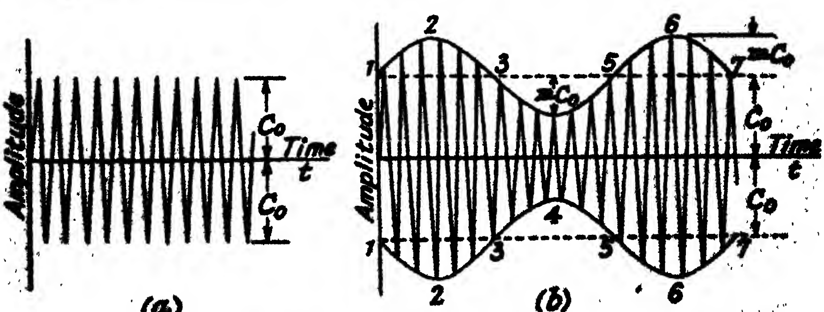


FIG. 2/XIII:11.—Modulation of a Carrier Frequency Wave by a Sine Wave.

If the carrier current is not a steady current but an alternating current, as shown at (*a*) in Fig. 2, of instantaneous amplitude $c = C_0 \sin ct$, the resultant modulated wave is shown at (*b*). It is not very hard to see that the instantaneous amplitude of this wave is obtained by replacing I_0 in equation (1) by $C_0 \sin ct$ and is:

$$i = C_0 \sin ct + mC_0 \sin ct \sin at = C_0 \sin ct(1 + m \sin at) \quad . \quad (2)$$

Considering the first form of this expression and expanding the second term

$$i = C_0 \sin ct + \tfrac{1}{2}mC_0 \cos (c - a)t - \tfrac{1}{2}mC_0 \cos (c + a)t$$

If f_0 is the carrier frequency and f_a is the modulating frequency

$$c = 2\pi f_0 \text{ and } a = 2\pi f_a$$

$$\text{Therefore } i = C_0 \sin 2\pi f_0 t + \tfrac{1}{2}mC_0 \cos 2\pi(f_0 - f_a)t$$
$$- \tfrac{1}{2}mC_0 \cos 2\pi(f_0 + f_a)t \quad (3)$$

The modulated wave therefore consists of a carrier frequency of amplitude C_0 *independent of the depth of modulation* and two other frequencies of equal amplitude $\frac{1}{2}mC_0$ which are therefore directly proportional in amplitude to the depth of modulation. These are called *sideband frequencies*. The lower sideband frequency is of frequency $f_0 - f_a$ and the upper sideband frequency is of frequency $f_0 + f_a$.

Where the modulating wave is complex, consisting of a number of frequencies, each frequency in the modulating wave gives rise to two sideband frequencies. An audio-frequency spectrum extending, for instance, from 0 to 10 kc/s, when used to modulate a carrier wave, gives rise to a modulated wave with a spectrum extending from 10 kc/s below the carrier frequency to 10 kc/s above the carrier frequency.

The total effective percentage modulation is determined by the vector sum of the peak values of all the sideband frequencies divided by the carrier amplitude. This is important to remember, since the presence of a certain percentage modulation at one frequency determines the percentage modulation permissible at another frequency. Normally the percentage modulation is taken care of automatically by observation of the peak volts amplitude of the modulating wave in relation to the H.T. volts on the modulated amplifier (assuming anode modulation). Occasions arise, however, where it is necessary to consider the individual contributions of different frequencies or different parts of the modulation spectrum.

From examination of (3), when $t = 0$ the vectors describing the carrier and the two sidebands are in the relative positions shown in Fig. 3 (*a*), the carrier frequency vector rotating f_0 times per second

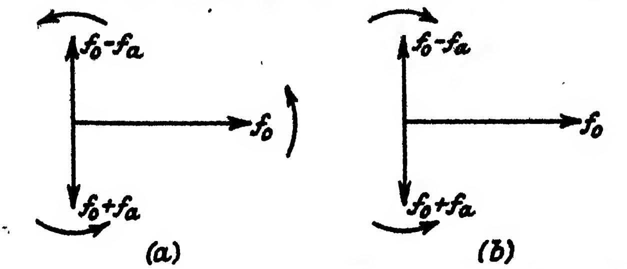


FIG. 3/XIII:11.—Relation of Vectors describing Carrier and Sidebands in normally Modulated Wave.

in the direction of the arrow and the upper and lower sidebands rotating in the same direction respectively f_0+f_a and f_0-f_a times per second. For the purposes of considering subsequent *relative* positions of the vectors it is permissible and convenient to subtract f_0 from the rates of rotation of all three vectors. This gives rise to the condition shown in Fig. 3 (*b*), where the carrier vector is fixed while the sideband vectors rotate in opposite directions at frequency f_a in such fashion that they always occupy symmetrically opposite positions except when they are coincident which happens twice per revolution. In the position shown at (*b*) the two sideband vectors cancel and this position corresponds to instant 1 on Fig. 2 (*b*). A quarter of a cycle of f_a later the two vectors are both horizontal, pointing to the right and adding to the carrier vector. This corresponds to instant 2 in Fig. 2 (*b*). A quarter of a cycle of f_a later the vectors are in the configuration of Fig. 3 (*b*) with their positions interchanged; this corresponds to instant 3. A quarter of a cycle later they are coincident and subtracting from the carrier; this corresponds to instant 4. The configuration at instant 5 is a repeat of instant 1.

From examination of equation (2) or from consideration of Figs. 2 and 3 it is evident that the sidebands *alone* combine to produce a wave of carrier frequency varying in amplitude with a period of amplitude variation half the period of f_m. During one half-cycle of f_a (from $t=0$ to $t=2$ in Fig. 2 (*b*)) the "carrier frequency" produced by the sidebands adds to the carrier frequency and during the next half-cycle period of f_a it subtracts from the carrier frequency. Examination of the form of equation (2) and Fig. 2 (*b*) shows that the envelope of the wave corresponding to the two sideband frequencies acting alone is a series of half-sine waves, the phase of the varying amplitude "carrier frequency" so constituted being reversed at the end of each half-period.

FIG. 4/XIII:11.—Envelope and "Phase" of Composite Wave due to Side-bands alone.

The envelope of such a wave is shown in Fig. 4, with the "sign" (phase) of the carrier frequency as indicated.

11.2. Phase-Modulated Wave. This is a wave in which the *phase* of a carrier wave is varied (relative to an arbitrary standard of phase presumed to be communicated to the receiver by some means so far unspecified but evidently devisable) in accordance with the amplitude of the modulating wave. Except in the Chireix system, where an approximation to a phase-modulated wave is used as an intermediate step in producing a normal amplitude-modulated wave, phase modulation has not been used. It constitutes, however, an easy method of approach to frequency modulation, and in fact contributes considerably to a proper understanding of frequency modulation.

If the convention for percentage modulation enunciated with regard to equation (1) were preserved, the instantaneous phase of a phase-modulated wave with a sinusoidal modulating frequency would be given by

$$\theta = \theta_s + m\theta_s \sin at \qquad . \quad . \quad . \quad (4)$$

where θ_s = the phase of the carrier frequency in the absence of modulation.

m = the percentage modulation.

$a = 2\pi f_a$, where f_a is the modulating frequency.

In this case with 100% modulation ($m = 1$), at the negative peak of the modulation the carrier phase would instantaneously be reduced to zero (with reference to the assumed arbitrary standard of phase). While such a form of wave might have a useful application, in general the peak modulation may be greater or less than 100% in terms of the above convention: the phase swing at the highest peak of the modulation cycle is not θ_s but some other value θ' which bears no relation to θ_s and may be greater or less than θ_s. When this phase swing occurs it may subsequently be convenient to say that 100% modulation occurs and to define any swing θ'', less than θ', by the relation $\theta'' = m\theta'$, where m is the percentage

modulation. For the present, however, it is simpler to determine the frequency spectrum in terms of the amplitude of phase swing or *modulation index* θ'', and so to consider the equation:

$$\theta = \theta_0 + \theta'' \sin at \quad . \quad . \quad . \quad . \quad (5)$$

In this case if the peak amplitude of the modulated carrier current is I, the instantaneous amplitude is given by:

$$i = I \sin (ct + \theta_0 + \theta'' \sin at) \quad . \quad . \quad . \quad (6)$$

where $c = 2\pi f_0$ and f_0 is the carrier frequency.

This is the fundamental equation for a phase-modulated wave, but for purposes of relating it to a frequency-modulated wave it will be modified as follows. The quantity $\theta_d = \theta'' \sin at$ describes the instantaneous value of the deviation of the phase angle from θ_0. The rate of change of θ_d is $a\theta'' \cos at$, and, while it may appear cumbersome, it is still permissible to express the value of θ_d by its rate of change multiplied by time,

i.e. by
$$\theta_d = ta\theta'' \cos at \quad . \quad . \quad . \quad . \quad (7)$$

Substitute (7) in (6):

whence
$$i = I \sin (ct + \theta_0 + ta\theta'' \cos at) \quad . \quad . \quad . \quad (8)$$
$$= I \sin [(c + a\theta'' \cos at)t + \theta_0] \quad . \quad . \quad . \quad (9)$$

θ_0 is arbitrary and by choice of the origin for t can be made equal to zero. In this case equation (9) can be rewritten as:

$$i = I \sin (c + a\theta'' \cos at)t \quad . \quad . \quad . \quad (10)$$

Substituting $c = 2\pi f_0$ and $a = 2\pi f_a$

$$i = I \sin 2\pi(f_0 + f_a\theta'' \cos at)t \quad . \quad . \quad . \quad (11)$$

By putting $f_a\theta'' = f''$

$$i = I \sin 2\pi(f_0 + f'' \sin at)t \quad . \quad . \quad . \quad (11a)$$

Any of the equations (6), and (9) to (11*a*) correspond to the same amplitude of component frequencies in the spectrum, the only difference between them being constituted by the phase of the component frequencies.

11.3. Frequency-Modulated Wave. This is a wave in which the *frequency* of a carrier wave is varied in accordance with the amplitude of a modulating wave. In this case, if the instantaneous value of the carrier frequency is given by:

$$f = f_0 + f'' \cos at \quad . \quad . \quad . \quad . \quad (12)$$

and if the peak amplitude of the modulated carrier current is I the instantaneous amplitude is given by:

$$i = I \sin 2\pi(f_0 + f'' \cos at)t \quad . \quad . \quad . \quad (13)$$

which is identical in form with equation (11*a*).

By analogy with the derivation of equation (11) from equation (6) with $\theta'_0 = 0$, it is evident that equation (13) describes a wave identical with that described by:

$$i = I \sin\left(ct + \frac{f''}{f_a} \sin at\right) \quad . \quad . \quad . \quad (14)$$

which is identical in form with equation (6) when $\theta_0 = 0$.

For the case of modulation by a sinusoidal wave only it follows from equations (6) and (14) that both phase modulation and frequency modulation have a spectrum corresponding to the equation

$$i = I \sin (ct + M \sin at) \quad . \quad . \quad . \quad (15)$$

where M is called the *modulation index*, following the proposal of Van der Pol,

$c = 2\pi f_0$,

f_0 = the carrier frequency in the absence of modulation,

$a = 2\pi f_a$

f_a = the modulating frequency.

For phase modulation $M = \theta''$

For frequency modulation $M = \dfrac{f''}{f_a}$

where f'' is the amplitude of frequency swing.

In the case of frequency modulation therefore the modulation index is inversely proportional to the modulating frequency.

11.4. Modulation by a Band of Frequencies.

11.41. Amplitude Modulation. Consider a band of modulating frequencies

$$A_1 \sin a_1t + A_2 \sin a_2t + A_3 \sin a_3t + \ldots$$

and suppose that these are used to produce amplitude modulation of a carrier wave to a depth of modulation m and that each frequency acting alone modulates the wave to a depth of modulation respectively equal to m_1, m_2, m_3, etc.

Then $\qquad m = m_1 + m_2 + m + \ldots$

Further, if A_1, A_2, A_3, etc., are the amplitudes of the modulating frequencies at any chosen point in the circuit, let $m = kA$ where $A = A_1 + A_2 + A_3 + \ldots$

Then $\qquad m_1 = kA_1$, $m_2 = kA_2$, $m_3 = kA_3$, etc.

In other words, the modulation contributed by any modulating frequency acting alone is proportional to the amplitude of that frequency.

In the general case, of course, with a band of modulating frequencies

$$A_1 \sin (a_1t+\phi_1)+A_2 \sin (a_2t+\phi_2)+A_3 \sin (a_3t+\phi_3)+ \ldots$$

the depth of modulation may be represented by

$$m = \hat{m}_1 \sin (a_1t+\phi_1)+\hat{m}_2 \sin (a_2t+\phi_2)+\hat{m}_3 \sin (a_3t+\phi_3)+ \ldots$$

where $m_1 = kA_1$, $m_2 = kA_2$, $m_3 = kA_3$, etc.

The same argument applies in the cases of phase and frequency modulation. This rather elementary analysis is not repeated in the two following cases because the main object of sections 11.41, 11.42 and 11.43 is to make clear the particular characteristics of each type of modulation as described in the three lines in italics in each section.

11.42. Phase Modulation. In this case, suppose that the same band of modulating frequencies produces a phase swing $\theta'' = kA = M$, and that each modulating frequency acting alone produces a phase swing respectively equal to $\theta_1'' = M_1$, $\theta_2'' = M_2$, $\theta_3'' = M_3$, etc.

$$\begin{aligned} \text{Then} \quad \theta'' &= \theta_1''+\theta_2''+\theta_3''+ \ldots \\ \text{or} \quad M &= M_1+M_2+M_3+ \ldots \\ \textit{and} \quad \theta_1'' &= M_1 = kA_1,\ \theta_2'' = M_2 = kA_2,\ \theta_3'' = M_3 = kA_3, \text{ etc.} \end{aligned}$$

In this case the modulation index constituted by each modulating frequency acting alone is proportional to the relative amplitude of the modulating frequency.

11.43. Frequency Modulation. Suppose that the same band of frequencies produces a frequency swing $f'' = kA$ and that each modulating frequency acting alone produces a frequency swing respectively equal to f_1'', f_2'', f_3'', etc.

$$\begin{aligned} \text{Then} \quad f'' &= f_1''+f_2''+f_3''+ \ldots \\ \text{and} \quad f_1'' &= kA_1,\ f_2'' = kA_2,\ f_3'' = kA_3, \text{ etc.} \\ \text{if} \quad a_1 &= 2\pi f_{a_1},\ a_2 = 2\pi f_{a_2},\ a_3 = 2\pi f_{a_3}, \text{ etc.} \end{aligned}$$

the modulation indices corresponding to each frequency are:

$$M_1 = \frac{f_1''}{f_{a_1}} = \frac{kA_1}{f_{a_1}},\ M_2 = \frac{f_2''}{f_{a_2}} = \frac{kA_2}{f_{a_2}},\ M_3 = \frac{f_3''}{f_{a_3}} = \frac{kA_3}{f_{a_3}}, \text{ etc.}$$

The modulation index contributed by each modulating frequency is therefore proportional to the relative amplitude of the modulating frequency and inversely proportional to its frequency.

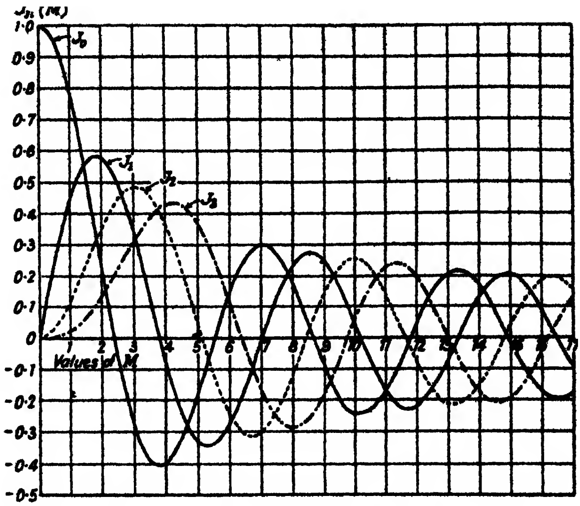


FIG. 5/XIII:11.—Bessel Functions, $J_n(M)$, of Integral Order. $n = 0, 1, 2, 3$.
(By courtesy of Dr. B. Hague and the Physical Society.)

A frequency-modulated wave is therefore equivalent to a phase-modulated wave in which the relative amplitude of the frequencies in the modulating wave have been modified so that the ratios of the modified to unmodified frequency components are inversely proportional to frequency. E. H. Armstrong has used the principle for producing a frequency-modulated wave. The audio frequency is passed through a network with a response curve such that the output voltages are inversely proportional to frequency. The resultant wave form is used to phase modulate a low frequency which is then multiplied up to the required carrier frequency.

11.5. Numerical Determination of Spectra of Frequency-Modulated and Phase-Modulated Waves. The wave form

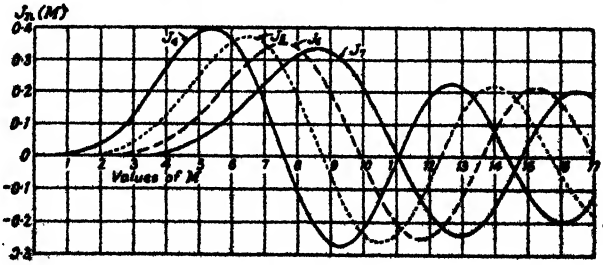


FIG. 6/XIII:11.—Bessel Functions, $J_n(M)$, of Integral Order. $n = 4, 5, 6, 7$.
(By courtesy of Dr. B. Hague and the Physical Society.)

corresponding to equation (15) contains a carrier frequency and multiple sidebands spaced symmetrically each side of the carrier, separated by intervals equal to f_a.

The amplitudes of these frequencies are:

Sideband Number	*Frequencies*	*Amplitude*
—	f_0	$J_0(M)I$
1	$f_0+f_a,\ f_0-f_a$	$J_1(M)I$
2	$f_0+2f_a,\ f_0-2f_a$	$J_2(M)I$
3	$f_0+3f_a,\ f_0-3f_a$	$J_3(M)I$

I is the unmodulated carrier amplitude and $J_0(M)$, $J_1(M)$, $J_2(M)$, etc., are functions of M defined by the curves of Figs. 5, 6 and 7. The fact that these are called Bessel functions does not make the curves any more difficult to read. Sidebands 4 are of magnitude $J4(M)$, and so on.

Example. A frequency-modulated wave resulting from modulation by an audio-frequency wave $f_a = 5{,}880$ c/s has a total frequency swing of 100 kc/s, i.e. $f'' = 50$ kc/s. If this wave produces an unmodulated field of 1 millivolt per meter at a certain point, when radiated, what is the strength of carrier and sidebands at that point when the wave is modulated?

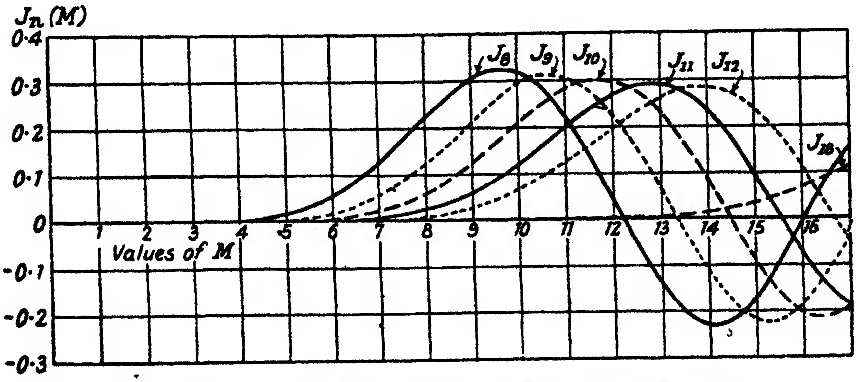


FIG. 7/XIII:11.—Bessel Functions, $J_n(M)$, of Integral Order. n = 8, 9, 10, 11, 12 and 18.

(By courtesy of Dr. B. Hague and the Physical Society.)

A. $M = 50/5{\cdot}88 = 8{\cdot}5$. Hence, disregarding their signs, the amplitudes of $J_0(M)$ to $J_{12}(M)$ are respectively 0·03, 0·27, 0·03, 0·27, 0·21, 0·07, 0·28, 0·32, 0·27, 0·17, 0·08, 0·04, 0·02. When modulation is present the amplitude of the carrier is therefore $30\mu V/m$ and the amplitudes of the sidebands in $\mu V/m$, assuming the numbering in the table above, are: (1) 2770; (2) 30; (3) 270; (4) 210; (5) 70, and so on.

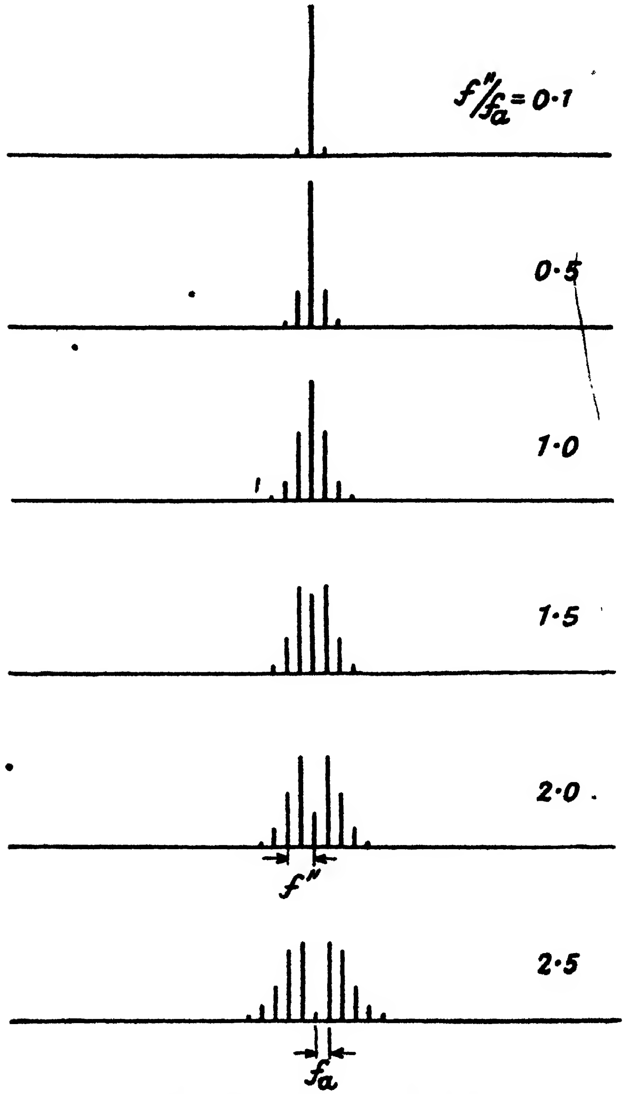


FIG. 8/XIII:11.—Spectra of Frequency-Modulated Waves for different Values of $M = \frac{f''}{f_a}$: f_a constant and f'' varied.

(By courtesy of Dr. Balth van der Pol and the Editor of the *Journal of the American Institute of Radio Engineers*.)

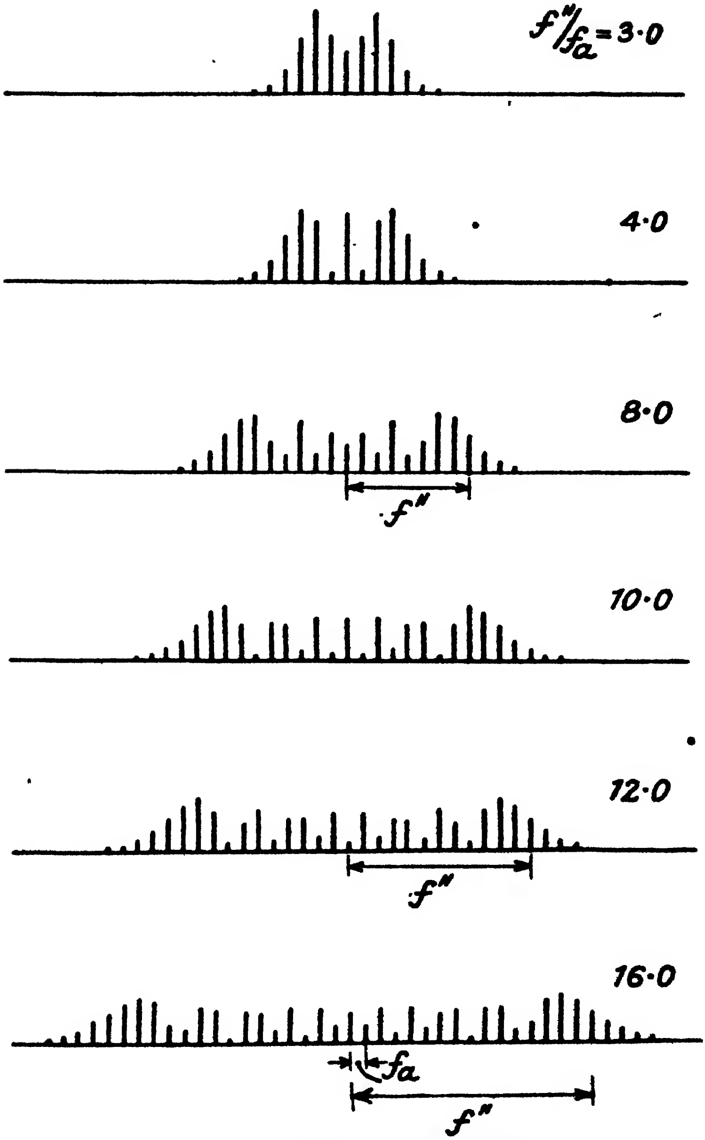


FIG. 8/XIII:11 (*continued*).

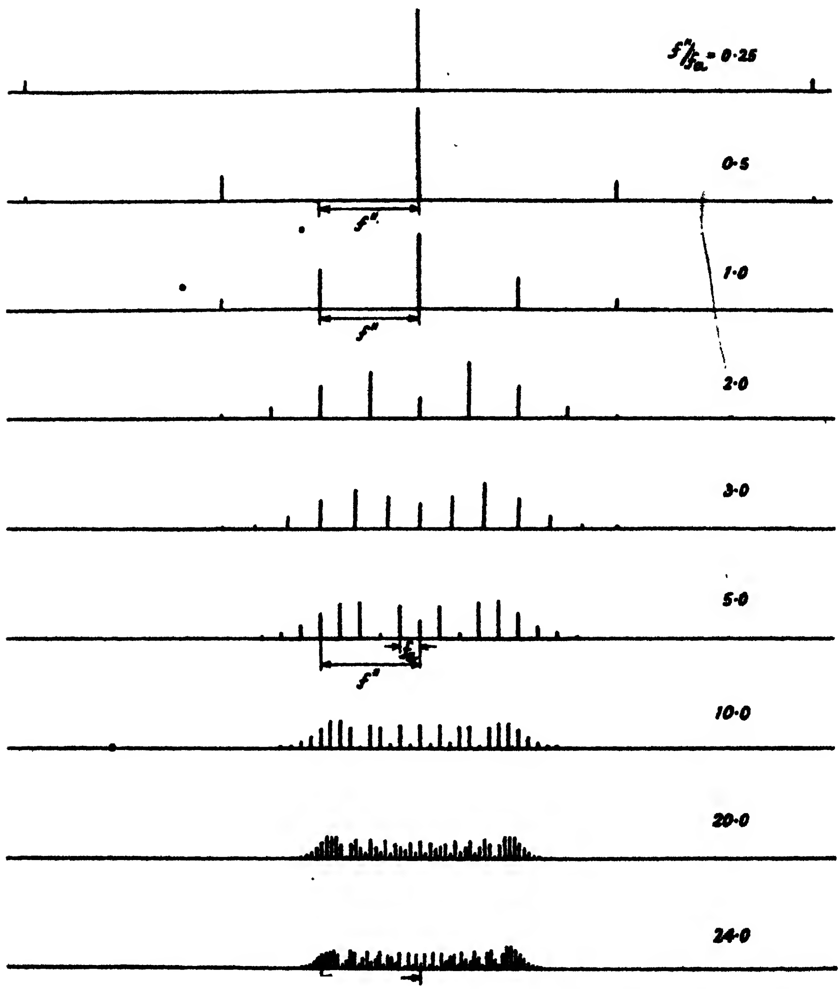


Fig. 9/XIII:11.—Spectra of Frequency-Modulated Waves for different Values of $M = \frac{f''}{f_a}$: f'' constant and f_a varied.

(By courtesy of Dr. Balth van der Pol and the Editor of the *Journal of the American Institute of Radio Engineers*.)

The spectrum resulting from modulation by a band of frequencies is obtained by calculating the modulation index $M_n = f''_n/f_{an}$ for each modulating frequency and calculating the amplitudes of the carrier and sidebands for ***each*** modulating frequency ***inserting the signs of*** $J_0(M_n)$ to $J_{12}(M_n)$. See XII:11.43 for determination of the values of M_n: i.e. M_1, M_2, etc.

The amplitude of the carrier is then given by the ***algebraic*** series:

Carrier amplitude $= I[J_0(M_1) + J_0(M_2) + J_0(M_3) + \ldots]$

In general the sideband frequencies are all different, but where two sideband frequencies are the same their amplitudes must be added algebraically.

For calculating phase-modulation spectra the procedure is identical, but in this case the values of $M_1 = \theta''_1$, $M_2 = \theta''_2$, etc., are directly proportional to the amplitudes of the modulating frequencies and are determined as indicated in XIII:11.42 above.

11.6. Frequency Range occupied by a Frequency-Modulated or a Phase-Modulated Wave. The relative amplitudes of carrier and sidebands have been worked out by Van der Pol for a number of representative values of M and are shown in Figs. 8 and 9. The frequency scales have been related to the frequency-modulation case in such a way that Fig. 8 represents the case of f_a constant and f'' varying, while Fig. 9 represents the case of f'' constant and f_a varying.

It is evident from both of these figures that provided the value of M is greater than 5 (as, for example, in the case of a frequency swing of ± 50 kc/s in a system transmitting up to 10 kc/s in the audio-frequency range), the part of the spectrum *of useful amplitude* is not greatly in excess of the range of carrier swing. For the case of $M = 5$, the frequency range is about 1·6 times the range of carrier swing: if the amplitude of carrier swing is 50 kc/s and $M = 5$, the frequency range occupied is about

$$2 \times 50 \times 1{\cdot}6 = 160 \text{ kc/s}$$

The table below gives the approximate width of spectrum in terms of $S = 2f''$

M	*Band Width*
0·5	$4S = 8f''$
1·0	$2S = 4f''$
2·0	$2S = 4f''$
3·0	$2S = 4f''$
5·0	$1{\cdot}6S = 3{\cdot}2f''$
10	$1{\cdot}4S = 2{\cdot}8f''$
20	$1{\cdot}2S = 2{\cdot}4f''$
24	$1{\cdot}2S = 2{\cdot}4f''$

(It will have been noted that throughout this discussion the swing f'' has been taken as equal to the amplitude of swing, just as I in the expression $I \sin \omega t$ is the amplitude of current and is referred to as the current.)

With a swing of $f'' = \pm 50$ kc/s and a highest modulating frequency of $f_a = 10$ kc/s the minimum value of $M = \frac{f''}{f_a}$ is 5, so that the required frequency range is $3{\cdot}2f'' = 160$ kc/s.

11.7. Reduction of Noise Level at the Receiver in a Frequency-Modulation System. Since the amplitude of the carrier in a frequency-modulation system is constant it is possible, after R.F. amplification, to insert in the receiver a limiter which is adjusted to so low a level as to suppress the tops of the carrier frequency peaks. According to claims which have been made it appears that this limitation can be caused to such an extent that the carrier is limited to a half of its free amplitude. In the receiver this is followed by a circuit called a *discriminator* in which the response, as measured by *ratio* : output voltage over input voltage, is proportional to frequency. This produces a wave which, in addition to being frequency modulated, is also linearly amplitude modulated. This wave is then detected on a linear detector such as is provided by a normal diode circuit.

Under these conditions a frequency-modulated system with a value of f'' equal to the top frequency in the audio-frequency range has a speech-to-noise ratio $\sqrt{3}$ times (on a voltage ratio basis) that of the speech-to-noise ratio of an amplitude-modulated wave system transmitting the same audio-frequency band width: an improvement of 4·5 db.: see XIII:11.73 for proof. As the amplitude of carrier swing is increased the noise ratio is improved in direct proportion to the amplitude of carrier swing: for proof see XIII:11.72. A frequency-modulated system passing an audio-frequency range up to 10 kc/s with an amplitude of carrier swing of ± 50 kc/s has therefore a speech-to-noise ratio $5 \times \sqrt{3}$ times (18·7 db.) better than an amplitude-modulated system passing the same range. The above supposes that the noise level is low compared to the carrier frequency.

In frequency-modulation systems it is also customary to use *pre-emphasis* of the high frequencies by means of an equalizer with a loss dropping at high frequencies in accordance with the fall in the envelope of the spectra of all classes of programme. This gives

a further gain in speech-to-noise ratio variously stated to be between 5 and 9 db.

E. H. Armstrong's very instructive explanation * of the action of the limiter is given in XIII:11.71 and 11.72 below.

11.71. Behaviour of the Limiter. This was determined experimentally by Armstrong. He says:

(Assume signal level above peak noise level). Two effects will occur (i.e. in the limiter). One of the effects will be to suppress in the output circuit of the limiter all components of the disturbing currents which are in phase with or opposite in phase to, the carrier. The other effect will be to permit the passage of all components of the disturbing currents which are in quadrature with the carrier.

Both the above effects are brought about by a curious process which takes place in the limiter. Each component within the band creates an image lying on the opposite side of the carrier frequency, whose frequency difference from the carrier is equal to the frequency difference between that current and the original components. The relative phase of the original current in question, the carrier and the image current is that of phase modulation—that is, at the instant when the original component and the image are in phase with each other the carrier will be in quadrature with them both, and at the instant that the carrier is in phase with one of these two frequencies it will be out of phase with the other.

He states that on detection the amplitude of the image frequency subtracts from that of the initial noise frequency, the final resulting disturbance being proportional to the difference between the amplitudes of noise and image frequency reaching the detector.

11.72. Demonstration that the Effective Noise Level in the Receiver is inversely proportional to the Amplitude of Carrier Frequency Swing. Assume that in the receiver the carrier frequency has been beaten down to a mean frequency of 400 kc/s and has a 100% modulation amplitude of swing of ± 10 kc/s. A series resonant circuit of high Q inserted in a constant current circuit, such as the anode circuit of a pentode, and tuned to 390 kc/s, may have a reactance characteristic as shown at MN in Fig. 10, and the same curve defines the relative response of the discriminator

* "A Method of Reducing Disturbances in Radio Signalling by a System of Frequency Modulation," *I.R.E. Proc.*, May, 1936, p. 689.

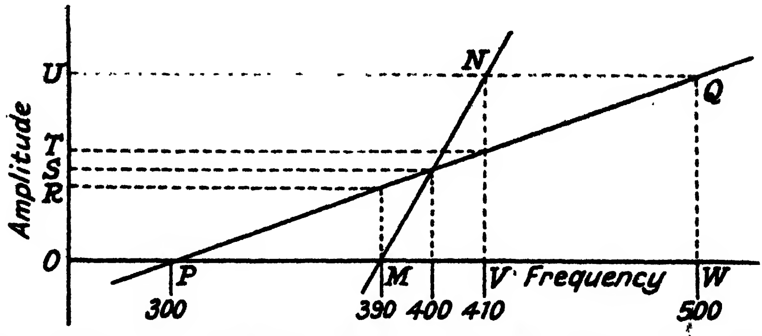


FIG. 10/XIII:11.—Illustrating Noise Amplitudes at Output of Circuit for Producing Amplitude Modulation. (Discriminator Circuit.)

(By courtesy of Dr. E. H. Armstrong and the American Institute of Radio Engineers.)

which is used for converting frequency modulation into amplitude modulation.

With such a circuit, noise frequencies and image frequencies occurring at 390 kc/s are reduced to zero, while noise frequencies and image frequencies at 410 kc/s are increased in amplitude, with regard to the midswing carrier level, by a factor of 2, and these frequencies are fully effective in contributing to noise, since the image frequencies are reduced to zero.

The circuit noise at these frequencies is therefore directly *proportional* to OU, the response at the maximum noise frequency, minus zero, the height of the corresponding image frequency, i.e. to OU. The circuit noise at other frequencies is evidently less and proportional to their distance from the carrier. The total integrated noise is however *proportional* to OU.

Now consider what happens when a selective circuit of characteristic PQ is used, which is supplied with carrier frequency swing of ± 100 kc/s for 100% modulation.

It is now evident that, while the midswing carrier level is unaffected, the difference between the amplitudes of interfering frequencies 10 kc/s away from carrier and their image frequencies is reduced from OU to RT: the total circuit noise is proportional to $OT - OR = RT$.

By the geometry of the figure it is evident that

$$\frac{OU}{PW} = \frac{RT}{MV}$$

$$\therefore \frac{RT}{OU} = \frac{MV}{PW} = \frac{20 \text{ kc/s}}{200 \text{ kc/s}} = \frac{1}{10}$$

That is, the noise voltage effective in the 200-kilocycle band is one-tenth of the noise voltage effective in the 20-kilocycle band.

Noise voltage is therefore inversely proportional to carrier swing: speech-to-noise ratio on a voltage basis is directly proportional to carrier swing. Doubling the carrier swing halves the noise and doubles the speech-to-noise ratio, both on a voltage basis.

11.73. Comparison of Speech-to-Noise Ratios in Amplitude-Modulated and Frequency-Modulated Systems passing the same Band Width.

Case I: Amplitude of Carrier Swing equal to Half the Band Width: Equal Carrier Powers.

A uniform noise spectrum is assumed, and the received amplitudes of unmodulated carrier are assumed to be the same in each case.

f_0 = the carrier frequency in the amplitude-modulated system = the mean position of the carrier in the frequency-modulated system.

b = the band width in the amplitude-modulated system.

$\pm\frac{1}{2}b$ = the amplitude of carrier swing in the frequency-modulated system.

f_1 = the lower limit of carrier swing = the lower limit of the amplitude-modulated wave sidebands.

f_2 = the upper limit of carrier swing = the upper limit of the amplitude-modulated wave sidebands.

Then $f_1 = f_0 - \frac{b}{2}$ and $f_2 = f_0 + \frac{b}{2}$.

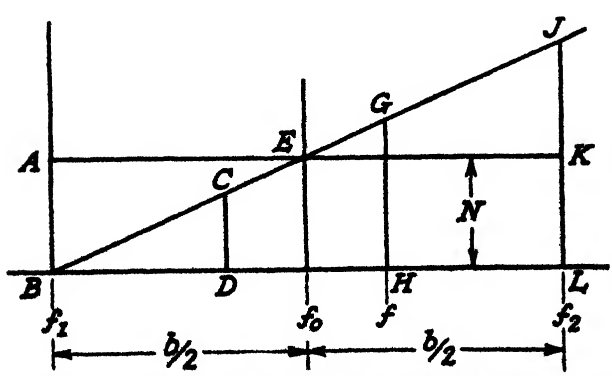


FIG. 11/XIII:11.—Initial Noise Spectra in Amplitude-Modulated and Frequency-Modulated System.

Referring to Fig. 11, if N is the noise voltage per c/s, then the line AK represents the noise level in the amplitude-modulated system in volts per c/s and the line BJ the noise level in the frequency-modulated system, subject to qualifications given below. The slope of BJ is determined by the slope of the attenuation of the discriminator network following the limiter.

f_0 is the carrier frequency and f_1 and f_2 the limits of the band as specified above.

Amplitude-Modulated System. Assuming a circuit resistance R at the point where the noise voltage is N volts per cycle, the total noise power in the amplitude-modulated system is given by :

$$W_a = \frac{1}{R}\int_{-b/2}^{+b/2} N^2 df = \frac{N^2 b}{R}$$

Frequency-Modulated System. Consider a noise frequency f of amplitude GH. Its effective amplitude in contributing to noise is

$$A = GH - CD$$

where CD is the amplitude of its image frequency equidistant from the carrier f_0 and on the other side of f_0.

Evidently

$$GH = \frac{2N}{b}(f - f_1) \text{ and } CD = \frac{2N}{b}BD = \frac{2N}{b}HL = \frac{2N}{b}(f_2 - f)$$

$$\therefore\ A = GH - CD = \frac{2N}{b}(2f - f_1 - f_2) = \frac{4N}{b}(f - f_0) = \frac{4N}{b}f_s$$

where $f_s = f - f_0$.

The power in the frequency range f_1 to f_2

$$= W_f = \frac{16N^2}{b^2R}\int_{-b/2}^{+b/2} f_s^2 df_s = \frac{16N^2}{b^2R}\left[\frac{1}{3}f_s^3\right]_{-b/2}^{+b/2} = \frac{16N^2}{3b^2R}\left[\frac{b^3}{8}+\frac{b^3}{8}\right] = \frac{4N^2b}{3R}$$

This must be reduced by a factor 2 since half the noise frequencies (those components in phase or anti-phase with the carrier) are suppressed in the limiter. A further reduction by a factor 2 must be introduced, since the image frequencies are created by absorbing half the energy of the remaining noise frequencies.

This gives $$W_f = \frac{N^2b}{3R}$$

Whence $$\frac{W_f}{W_a} = \frac{1}{3}$$

That is, the noise power effective in the frequency-modulated system is only 1/3 the noise power effective in the amplitude-

modulated system of equal band width. The speech-to-noise ratio in the frequency-modulated case is therefore $\sqrt{3} = 1{\cdot}732$ times that in the amplitude-modulated case, on a voltage ratio basis; i.e. 4½ db. very nearly. If pre-emphasis is used, a further improvement in signal-to-noise ratio between 5 and 9 db. must be added.

Case II: Amplitude of Carrier Swing greater than Half the Band Width: Equal Power Cost at Transmitters.

In this case the improvement in the signal-to-noise ratio of the frequency-modulation case over the amplitude-modulation case is

$$\left(4{\cdot}5+20\log_{10}\frac{2f''}{b}+P\right)\text{ decibels}$$

where f'' = the amplitude of carrier swing.

b = the band width of the signal assumed the same for each type of modulation.

P = 5 to 9 db. (due to pre-emphasis).

The above assumes that the received amplitudes of unmodulated carrier are the same in each case, which for equal transmission paths assumes that the radiated carrier power is the same in each case. The practical case of interest is that the power expenditure in radiating the unmodulated carrier should be the same in each case. In the frequency modulation case the most probable figure for the efficiency of the final stage is little in excess of 75%, and may be less. In the amplitude modulation case, if any of the forms of high efficiency transmitter discussed in XIV:9 and 10 are used, the efficiency of the final stage or its equivalent is not less than 65%.

The difference between the two cases is represented by less than half a decibel.

The conclusions relating to economy and signal-to-noise ratio can now be worked out by the reader for himself. He should not forget maximum power taken from H.T. power supply, see XIV:9, and requirements on H.T. smoothing, see XIV:12.

11.8. False Phase Modulation. The statement is sometimes met that a phase-modulated wave is equivalent to an amplitude-modulated wave with the carrier shifted in phase by 90°. A phase-modulated wave is represented by:

$$i = I\sin(ct+\theta''\sin at) \qquad . \quad . \quad . \quad (16)$$

This results from putting θ_0 = zero in equation (6)

$$\therefore\ i = I\sin ct\cos(\theta''\sin at)+I\cos ct\sin(\theta''\sin at)$$

If θ'' is very small, $\theta''\sin at$ tends to zero, $\cos(\theta''\sin at)$ tends to

unity, and $\sin(\theta'' \sin at)$ tends to $\theta'' \sin at$.

$$\therefore\ i = I \sin ct + I\theta'' \sin at \cos ct \quad . \quad . \quad (17)$$

An amplitude-modulated wave may be represented by:

$$i = I \cos ct(1 + m \sin at)$$
$$I \cos ct + Im \sin at \cos ct \quad . \quad . \quad . \quad (18)$$

If m, the percentage modulation in equation (18), equals θ'', the phase swing in radians in equation (17), the equations differ only in the phase of the carrier which is represented by a sine function in equation (17) and by a cosine function in equation (18), and therefore differs by 90° in the two cases.

Hence an approximation to a phase-modulated wave with a low modulation index can be obtained by amplitude modulation in a balanced modulator so as to obtain sidebands free from carrier, and then restoring a carrier large in amplitude compared to the sidebands, with 90° shift in phase compared to the original carrier. The usefulness of such a procedure has yet to be demonstrated, and the only object the above explanation appears to serve at the present time is to provide an understanding of the somewhat vague statements which sometimes are found along the lines of the above introductory paragraph. See, however, XIV:5.

12. Heising Modulation Formulae.

These relate to an amplitude-modulated wave, ***and apply only to modulation with a sine wave.***

Conventions.

I_0 = R.M.S. aerial or feeder current in carrier (no modulation) condition.
I_m = R.M.S. aerial or feeder current on modulation.
R = aerial series resistance or feeder (resistive) impedance.
P_0 = aerial or feeder power in carrier condition.
P_m = aerial or feeder power in modulation.
m = modulation depth.

Then
$$P_0 = I_0^2 R \quad . \quad . \quad . \quad . \quad (1)$$

and P_m = Power due to carrier + Power corresponding to each sideband

$$= I_0^2 R + \left(\frac{m}{2} I_0\right)^2 R + \left(\frac{m}{2} I_0\right)^2 R$$
$$= I_0^2 R\left(1 + \frac{m^2}{2}\right) = I_m^2 R \quad . \quad . \quad . \quad . \quad (2)$$

Increase in Aerial and Feeder Power from No-mod. to Mod. Condition. From (1) and (2)

$$\frac{P_m}{P_0} = 1+\frac{m^2}{2} \qquad \therefore P_m = \left(1+\frac{m^2}{2}\right)P_0 \quad . \quad . \quad (3)$$

Equation (3) gives the mean aerial power and should not be confused with the relation

$\hat{P}_m = (1+m)^2P_0 =$ the instantaneous value of the power at peak positive modulation.

Increase in Aerial and Feeder Current from No-Mod. to Mod. Condition. From (2)

$$I_m^2 = \left(1+\frac{m^2}{2}\right)I_0^2 \quad . \quad .. \quad . \quad . \quad (4)$$

$$\therefore I_m = I_0\sqrt{1+\frac{m^2}{2}} \quad . \quad . \quad . \quad (5)$$

Determination of Modulation from Measurement of I_0 and I_m. From (4)

$$\frac{m^2}{2}I_0^2 = I_m^2 - I_0^2$$

$$m = \sqrt{2\left(\frac{I_m^2}{I_0^2} - 1\right)} \quad . \quad . \quad . \quad (6)$$

While equation (6) is entirely rigid, in practice, owing to errors in reading R.F. ammeters, due sometimes to the distortion of the scale at the low reading end, and also because of the sensitivity of the formula to errors in the value of I_m/I_0, this formula does not provide a reliable indication of modulation depth. In the past, however, it has often been the only means of determining percentage modulation, and, where no other means are available, it is still perforce in use. The formula is more accurate at high modulations, and for this reason, if, for instance, it is required to line up a transmitter to 40% modulation ($m = 0{\cdot}4$), it is preferable to adjust the transmitter to 80% modulation ($m = 0{\cdot}8$) by adjusting the modulation until I_m reaches its corresponding value, and then to reduce the modulation by 6 db. by means of any suitably situated calibrated potentiometer.

This will be made clearer by inserting the values of $m = 0{\cdot}4$ and $m = 0{\cdot}8$ in equation (5). The resulting values of I_m are respectively $I_m = 1{\cdot}04I_0$ and $I_m = 1{\cdot}15I_0$. It follows that, in the 40% modulation case, if a 2% error is made in reading I_0 and I_m and if the errors add, the percentage modulation is subject to an error of plus

40% or minus 100%! Corresponding errors in the case of 80% modulation have a much less serious effect, but even in this case the permissible margin of error is small.

It has been suggested by E. Green (Bibliography, M.11) that distortion of the envelope of the modulated wave form can lead to serious errors when this formula is used to indicate modulation depth, and for this reason there has been a prejudice against the use of this formula at high percentage modulations, where distortion is greatest. Unless carrier float occurs, this is not the case as the following line of argument shows. I am indebted to Mr. Green for helping me to present this effect in its proper perspective.

Consider the case where the envelope of a modulated wave consists of a sine wave due to a modulating frequency of amplitude M corresponding to a true percentage modulation m, and a harmonic frequency of amplitude pM giving rise to sidebands of amplitude p times the amplitude of the sidebands due to the modulating frequency. The observed value of the R.M.S. aerial or feeder current I_{m_0} on modulation is given by:

$$I_{m_0}R = I_0^2R+\frac{m^2}{2}I_0^2R+\frac{(pm)^2}{2}I_0^2R$$

$$\therefore\ \frac{I_{m_0}^2}{I_0} = 1+\frac{m^2}{2}+\frac{(pm)^2}{2}$$

The value of modulation given by equation (6) is then

$$m_c = \sqrt{2\left(\frac{I_{m_0}^2}{I_0^2}-1\right)} = \sqrt{m^2+(pm)^2}$$
$$= m\sqrt{1+p^2}$$

The quantity $\sqrt{1+p^2}$ therefore represents the error ratio by which the observed value of modulation m_0 must be divided to give the true value of modulation m. It will be appreciated that a value of $p = 0{\cdot}05$ corresponds to a 5% harmonic (i.e. a harmonic of amplitude equal to 5% of the modulating frequency), $p = 0{\cdot}1$ to a 10% harmonic, and so on. The value of the error ratio is given below for various values of p.

p	*Harmonic*	$\sqrt{1+p^2}$
0·01	1%	1·00005
0·05	5%	1·00124
0·1	10%	1·005
0·15	15%	1·01125
0·2	20%	1·02
0·3	30%	1·044

The error is evidently quite negligible up to 15% distortion.

A more general treatment may be developed to show that the above error ratios are effective when p represents the R.M.S. value of the distortion products expressed as a fraction of the amplitude of the R.M.S. value of the modulation frequency. This covers the case where the distortion products consist of more than one frequency.

Carrier Float. If the mean level of the carrier wave changes when modulation is applied, the carrier is said to float. It is evident that any attempt to determine the percentage modulation by measuring the carrier level before modulation is applied and relating it to the R.M.S. current when modulation is applied must lead to a wrong value of modulation.

Carrier float occurs when the carrier wave is effectively modulated by a voltage which has a steady component. This steady ("D.C.") component may be introduced in the modulated amplifier itself or, in the case of series or choke modulation, may be introduced in the modulating amplifier. It cannot be applied to the modulated amplifier in modulating systems in which modulation is applied through a condenser or through a transformer.

In general, carrier float occurs when the mean value of the increase of carrier amplitude during the positive half-modulation cycle is not equal to the mean value of the decrease of carrier amplitude during the negative half-modulation cycle. It is evident therefore that carrier float may be introduced in any high-frequency amplifier amplifying a modulated wave.

The Heising formula in equation (6) may be used to give the correct level of modulation when carrier float occurs, by entering, instead of I_0, the value of carrier which occurs on modulation which will be called the float level of the carrier and will be represented by I'_0. The value of I'_0 may be found very simply by means of an ordinary diode rectifier to which the unmodulated carrier and the modulated carrier are applied in turn. The ratio I'_0 to I_0 is then equal to the ratio of the rectified currents through the diode as read on a D.C. meter in series with the diode. When this correction is applied, the resulting value of modulation will be accurate within 1% up to 15% distortion.

CHAPTER XIV

TRANSMITTER TYPES

1. Trend of Development.

THE evolution of transmitter types has been animated by a drive towards systems which give high economy and high linearity.

In one of the earliest types of transmitter, modulation was effected in the final stage by shunting the output circuit of a high-frequency amplifier with the anode-cathode circuit of a valve, driven in its grid circuit with audio frequency so as to vary the shunt resistance across the circuit. Such an arrangement was poor from all points of view: the efficiency, the linearity and the maximum percentage modulation obtainable were all low. The Heising system of modulation represented a big advance in all these respects. In the original Heising circuit the dropping resistance R_2 in Fig. 1/XIII:2 and coupling condenser C_3 were omitted so that the anode volts on both modulating and modulated amplifiers were the same. The introduction of R_2 and C_3 contributed appreciably to linearity, since the mod-ing amp. can so be operated with a higher anode voltage than the mod-ed amp.

2. Low-Power Modulation.

The efficiency of the Heising modulation system, in which the modulating amplifier was operated as a class A amplifier, was about three-quarters that of the final high-frequency amplifier used for amplifying the modulated wave in a low-power modulation system. On this account an advantage was obtained from the point of view of efficiency by using low-power modulation. Also, at the time transmitter powers began to increase above 10 kW (unmodulated carrier power in the aerial), the technique of making large audio-frequency chokes and transformers was not far advanced, and the possibilities of obtaining reasonable quality from class B audio-frequency amplifiers were not fully realized. As a consequence of this the first high-power transmitters were built with low-power modulation. The modulated wave was then amplified in a chain of linear amplifiers, the last amplifier of which was called a power amplifier and supplied radio-frequency power to the aerial. The B.B.C. "Regional" Transmitters at Brookman's Park, Moorside Edge, Westerglen and Washford are examples of low-power modu-

lation, operating with an aerial carrier power of about 50 kW. For description of Heising class A modulation see XIII:2.

3. Series Modulation.

In an attempt to eliminate the chain of linear high-frequency amplifiers, which are necessarily inefficient, and were then considered comparatively difficult to stabilize, the system of series modulation at high power was introduced. On account of the necessity of operating two valve-anode circuits in series, high-voltage requirements prevented such a modulator constituting the output stage of a transmitter, and transmitters were built with a series modulated amplifier constituting the penultimate stage, the final stage being a normal high frequency power amplifier. The B.B.C. medium frequency transmitters at Droitwich (150 kW), Burghead and Lisnagarvey (100 kW) are examples of this type. For description of series modulation see XIII:3.2 and 3.3.

4. Class B Modulation.

When the design and construction of high-power audio-frequency transformers and chokes became a practical possibility, high-power class B audio-frequency amplifiers were developed and used to modulate the final stage of a high-power transmitter. Modulators designed in this way are called class B modulators. The total anode efficiency of the final stage in the carrier (unmodulated) condition, including both modulating and modulated amplifiers, is about twice that of a linear high-frequency amplifier amplifying a modulated wave.

Apart from one or two small Doherty-type transmitters, class B transmitters represent the latest and most efficient form of transmitter in use in the British Isles. The B.B.C. medium-wave transmitters at Stagshaw and Start Point and the short-wave transmitters at Daventry are examples of this type of transmitter. For description of class B modulation see XIII:3 and X:9.

5. The Chireix System.

This is one of the most outstanding examples of a lead in technique, since the Chireix system was in operation in France before class B modulation was proposed. It has an efficiency in practice which approaches class B very closely and appears to have a potential efficiency higher than class B.

The Chireix system operates by supplying the load from two valves or banks of valves driven with R.F. grid voltages in *phase and amplitude* relationship dependent on the amplitude of the audio-

frequency voltage in such a way that a normally modulated wave is produced in the load, while the anode-voltage swings are high throughout the whole modulation cycle so that high efficiency results. The following explanation is due to R. L. Fortescue.

A number of methods can be used for deriving the pseudo phase-modulated waves on the grids of the output valves, but the simplest one to consider is probably that in which a normally modulated carrier wave is added in quadrature to an unmodulated carrier wave.

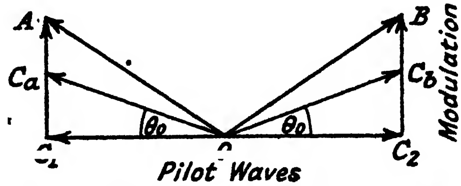


FIG. 1/XIV:5.—Relation of Pilot Wave and Modulated Wave in Chireix System. (By the courtesy of R. L. Fortescue and the B.B.C.)

Referring to the vector diagram in Fig. 1, the unmodulated carriers or pilot waves OC_1 and OC_2 have the modulated carriers C_1C_a and C_2C_b added in quadrature. Evidently at 100% modulation C_1C_a and C_2C_b vary from 0 respectively to C_1A and C_2B, twice their amplitudes when unmodulated. The resultant waves are represented by vectors which travel from OC_1 to OA and OC_2 to OB and back again throughout the modulation cycle. It is evident that a form of phase modulation is produced, in combination with some modulation of amplitude.

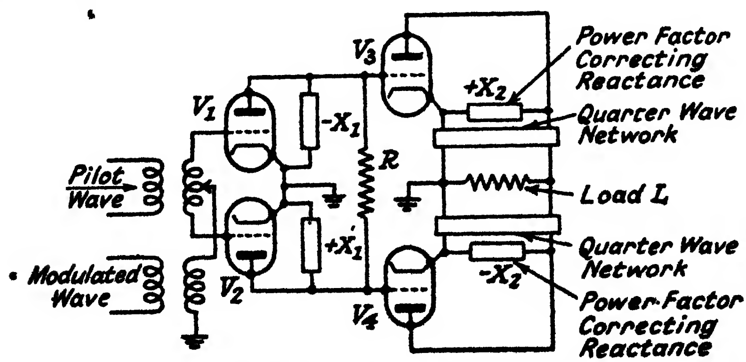


FIG. 2/XIV:5.—Theoretical Chireix Circuit. (By the courtesy of R. L. Fortescue and the B.B.C.)

In the circuit of Fig. 2 the unmodulated pilot carrier of amplitude proportional to $OC_1 = OC_2$ is supplied to the grids of V_1 and V_2 in phase opposition (i.e. in push-pull) and a modulated wave in

quadrature, or carrier wave proportional to $C_1C_a = C_2C_b$ is supplied, to the grids in parallel. The drives on V_1 will be called OC_1 and C_1A and the drives on V_2 will be called OC_2 and C_2B. Without any further consideration it is evident that, assuming similar phase shift characteristics in each amplifier chain, the carriers OC_1 and OC_2 cancel one another in the load L; while the modulated carriers C_1C_a and C_2C_b add in the load L, so that a normally modulated wave is built up in the load L. The function of the quarter-wave networks is to preserve high impedance at the anodes of V_3 and V_4 when the currents driven through the load by the resultants of the two sets of quadrature waves approach phase opposition.

The function of the apparently redundant carriers OC_1 and OC_2 is to maintain high anode swings on the output valves and therefore high efficiency throughout the modulation cycle. In combination with the quarter-wave networks they serve as impedance transforming devices which have a ratio varying throughout the modulation cycle, so that varying power can be supplied at approximately constant voltage.

In practice, in order to reduce the variation of the amplitude of the resultants, the vectors C_1C_a and C_2C_b are not added at right angles, but are phased so that in Fig. 1 they should be drawn tilted towards one another. Similar arguments to the above still apply, however, since cancellation of the pilot waves still occurs owing to the opposed phase shifts introduced in the amplifier chains by $+X_2$ and $-X_2$; see paragraph immediately following.

Since in the unmodulated condition the resultant carrier vectors OC_a and OC_b each depart from phase opposition by some small angle θ_0, the impedance facing the anode of each output valve will be reactive, which would lead to low efficiency if it were not corrected. For this purpose the correcting reactances $+X_2$ and $-X_2$ are provided, equal in magnitude and opposite in sign. These ensure unity power factor in the carrier condition only.

During the modulation cycle the reactance facing the output valves varies with the result that, owing to the fact that the internal impedances of the output valves are not zero, a phase displacement occurs between the grid and anode voltages which varies throughout the modulation cycle and so introduces non-linearity.

For this reason, a load R, see Fig. 2, is connected between anode and anode of the penultimate stage, so loading the pilot wave circuit. Correcting reactances X_1 and $-X_1$ are provided as in the case of the output circuit, and, provided that proper relationships are maintained between R and L, the load impedance and the internal

impedances of the valves, a phase deviation between the grids and anodes of the penultimate stage occurs, which, to a first order of approximation, is equal and opposite to that occurring in the output stage.

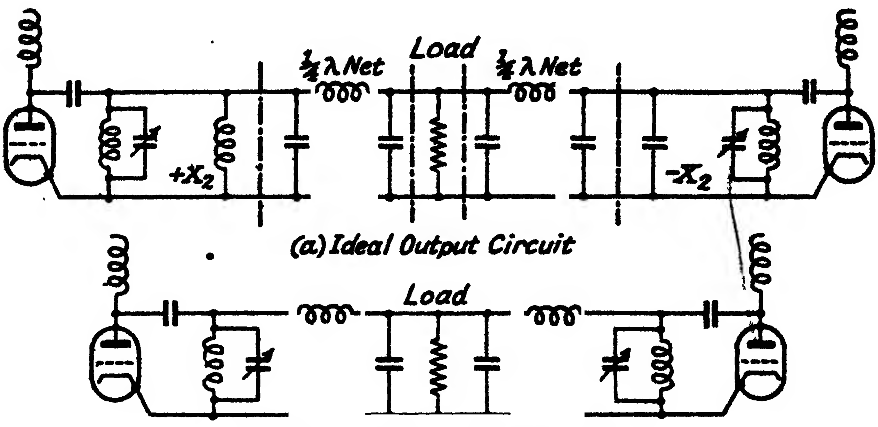


FIG. 3/XIV:5.—Output Circuit in its Ideal Form, and when Reduced to a Minimal Network for Practical Purposes.

(By the courtesy of R. L. Fortescue and the B.B.C.)

The full ideal output circuit of the Chireix system is shown in Fig. 3 (*a*) and the practical circuit, which results from combining certain shunt elements, is shown in Fig. 3 (*b*).

An improved form of Chireix operating at high power (450 kW unmodulated carrier) has been built which uses amplitude modulation to develop the negative modulation envelope, and phase swing as described above to develop the positive modulation envelope.

6. The Doherty System.

Referring to Fig. 1, in this system the load is supplied from two valves which may be termed respectively the carrier valve and

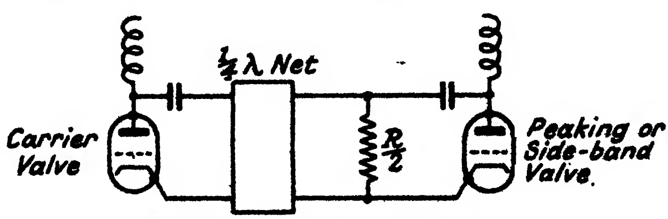


FIG. 1/XIV:6.—Doherty and Fortescue Systems.

the peaking or increment valve. Both valves are driven with a normally modulated wave with the voltages on the grids in quad-

rature, so that the voltages in the load are in phase. In the absence of modulation the carrier valve, which is driven into anode limitation and/or grid limitation in the carrier condition, is alone operative and delivers carrier power to the load $R/2$ through the quarter-wave network N. The value of the load resistance is made equal to $R/2$, where R is the optimum impedance for both valves to work into in the peak 100% modulation condition. The characteristic impedance of the quarter-wave network is made equal to R, so that in the carrier condition the carrier valve is faced with an impedance $2R$.

The peaking valve is biased back so that in the carrier condition this valve is only just beginning to take current.

During the negative modulation cycle the carrier valve supplies a current envelope to the load corresponding to normal modulation, while during the positive modulation cycle, owing to anode limitation, it supplies substantially constant current to the load, equal to the carrier load current. During the positive modulation cycle the peaking valve supplies to the load such increments of current that the envelope of the load currents corresponds to a normally modulated wave. In this way a normally modulated wave is built up in the load.

The following consideration may make the processes during the positive modulation cycle clearer:

A quarter-wave network has the property that the current output is given by the input voltage divided by its characteristic impedance, see XVI:4, *regardless of the terminating impedance*. If constant voltage is maintained across the input of the quarter-wave network, then constant current is delivered to the load. Since the carrier valve is driven into limitation during the positive modulation cycle, the anode-voltage swing, which is the voltage across the input of the quarter-wave network, is substantially constant, so that constant current is supplied to the load by the quarter-wave network. Substantially constant voltage is supplied to the input of the quarter-wave network because a valve driven into anode limitation has a very low effective internal impedance; see X:34. In the positive part of the modulation cycle the peaking valve takes current, and therefore causes current to flow through it during the negative halves of the R.F. cycles, i.e. when the carrier valve is applying negative voltages to the top of the load, or is causing current to flow up the load (electrons to flow down it). The fact that the peaking valve takes current under these conditions causes its anode voltage to fall, and since it is coupled to the load through

a condenser it also applies negative voltages to the top of the load, giving rise to currents which add to those supplied by the carrier valve.

The circuit is set up so that at 100% peak modulation both valves are supplying equal current to the load; hence the peaking valve and the quarter-wave network are each faced by an impedance equal to R. The carrier valve therefore delivers twice as much current to the quarter-wave network at 100% peak modulation as it does in the carrier condition, and since its anode-voltage swing is maintained constant it is supplying twice the power to the quarter-wave network and hence to the load.

In the 100% peak modulation condition, therefore, the load is receiving twice carrier power from each valve, which means that the load power is four times carrier power, which is the normal requirement for a 100% modulated wave.

If the carrier valve is driven into anode limitation in the carrier condition, it is evident that there is envelope distortion immediately below carrier level: linearity is impaired during the negative modulation cycle. Using grid limitation evades this difficulty. A certain amount of non-linearity at the ends of the negative modulation cycle can be offset by making the peaking valve start to take current at values of grid drive slightly below carrier level. In practice this is done, and evidently a certain amount of art is involved in matching the inevitable bottom bend of the peaking valve to the limitation curve of the carrier valve.

It is evident that during the positive modulation cycle, if the grid swing on the carrier valve were maintained constant at the carrier level, the anode swing on the carrier valve would drop (because the impedance looking into the quarter-wave network falls) by an amount determined by the effective internal impedance of the carrier valve. Provided the effective internal impedance were constant, as it is very closely, this drop would not introduce non-linearity, and theoretically could be offset by adjusting conditions on the peaking valve so that it delivered more than its fair share of power at the 100% peak modulation condition. Art again steps in, however, since the grid drive on the carrier valve tends to rise during the positive modulation cycle, and this may more than offset the fall in the impedance facing the anode circuit so that some form of grid limitation becomes necessary. A simple way of doing this is by taking advantage of the grid current of the carrier valve and increasing the impedance facing its grid circuit.

Although in the above description it has been assumed for sim-

plicity that both output valves require to work into the same impedance in the 100% peak modulation condition, in practice since the requirements on linearity on the two valves are different, an increase in linearity in the peak modulation condition (or an increase in power output for a given linearity) can be obtained at the expense of efficiency at high modulation depths by arranging that at 100% peak modulation the peaking valve is working into a lower impedance than the carrier valve. This results in the quarter-wave network having an impedance which differs from twice the load impedance.

It is evident that since the mean modulation is less than 20%, and modulation depths above 50% occur for a small fraction of the total time that carrier is radiated, efficiency in the carrier condition is of paramount importance and low efficiency (of the order here involved) at high modulation depths is of very much smaller importance. The mean percentage modulation is usually considered to be between 15 and 20%.

The modulated wave for supplying both carrier and peaking valve is obtained from any suitable type of modulator, the circuit being split at any later convenient point so as to provide drives to the two output valves capable of independent adjustment of amplitude. In order that the voltages supplied to the load from each output valve shall be in phase it is necessary to insert a second quarter-wave network either in series with the carrier or peaking valve. It is usual to insert the quarter-wave network in the input to the carrier valve, since, when this is done, the occurrence of grid current in the carrier valve, during the positive modulation peak, increases the impedance at the input to the second quarter-wave network and so effects a reduction of voltage applied to the grid of the carrier valve. This gives a measure of protection to the grid of the carrier valve, reducing over heating due to excessive grid dissipation. Owing to the low angle of current flow in the peaking valve (due to it being biased back), it is necessary to drive this valve with such a peak voltage that there is a danger of this valve failing from too high a grid dissipation. In practice it is this valve which usually introduces difficulty on this account.

In practice, in a Doherty amplifier initially designed to operate on anode limitation, it is usually found that it is necessary to introduce grid limitation (by increasing the impedance facing the carrier valve grids) to effect a good envelope marriage with the peaking valve. It may therefore be preferable to design so that grid limitation occurs before anode limitation is approached (see XIV:6.21); this also avoids the non-linear region of the valve field.

6.1. Improved Doherty System (B.B.C. Patents 532,204 and 524, 588). Successful experiments have been made with an improved type of Doherty system designed to give improved linearity and to reduce the disadvantage due to excessive grid dissipation in the peaking valve. In this system, which is otherwise identical with the Doherty system as described above, the peaking valve is biased as a normal class C high-frequency amplifier and driven with a wave form corresponding to the sidebands during the positive half of the modulation cycle, and of zero amplitude during the negative modulation cycle. Such a wave form is very simply produced, by means, for instance, of a class B modulator with zero H.T. on the modulated amplifier, or by means of a suitably biased class C grid modulator, or suppressor-grid modulator.

This system appears to constitute a definite advance on the original Doherty system, and on the Fortescue system; see XIV:7. The advance on the Doherty system is due to the fact that the peaking valve in the improved system is always operated with an angle of current flow of 180° leading to higher envelope linearity and lower grid dissipation. The advance on the Fortescue system is due to the fact that the increase in power input with modulation is less.

6.11. Design of a Doherty Amplifier (Improved System). This method of design is due to D. B. Weigall. It will be described in terms of an amplifier using one valve for the carrier valve and one valve for the peaking valve: the conversion to *n* valves in each case is accomplished by dividing the load impedance and the quarter-wave line impedance by *n*. The angle of current flow will be assumed to be 180° at all times, as is the case in the improved system.

This method of design leads to slightly different values of quarter-wave network and load impedance, but is essential if the carrier valve is to be driven substantially into anode limitation in the carrier condition. This is because, if the original method of design is followed, both valves work into the same load line at 100% p.p.m. and both deliver the same power: hence, if the carrier valve is driven into anode limitation in the carrier condition, the peaking valve is driven into anode limitation at 100% p.p.m. and envelope distortion results because the valve is non-linear in the region of anode limitation. An alternative way of avoiding this difficulty, which also gives improved linearity, is to arrange that the grid limitation in the carrier valve operates at a lower value of grid drive than anode limitation. Evidently this method leads to slightly lower values of power output and efficiency than the Weigall method.

Carrier Valve. Draw, on the valve field for the valve to be used, the load line representing the optimum compromise between *efficiency* and *power output* when the valve is driven into grid and/or anode limitation, on the assumption that this represents the carrier condition on the carrier valve. Draw a second line at twice the slope (representing the condition on the carrier valve at 100% p.p.m.) and from the intersection of this line with the asymptote of valve characteristics draw a line parallel to the axis of anode current to meet the first line at a point P. Point P then determines the following quantities relating to the carrier valve in the carrier condition: the grid and anode peak volts, the peak anode current, the power output, the efficiency and the impedance facing the anode which will be called R_{ac}.

Peaking Valve. Draw, on the valve field, the load line representing the optimum compromise between *linearity* and *efficiency* for a power output equal to twice the power output from the carrier valve in the carrier condition. This then determines for the peaking valve the same quantities that were found immediately above for the carrier valve, but now at 100% p.p.m. R_L, the required load impedance (indicated by $R/2$ in Fig. 1), is then equal to half the impedance facing the anode of the peaking valve at 100% p.p.m.

The impedance of the quarter-wave network is then equal to $\sqrt{R_L R_{ac}}$.

The grid drive on the peaking valve in the carrier condition is obtained by plotting on the valve field the point: anode current = 0, anode volts = H.T. − peak load volts in carrier condition. The grid drive is then equal to the grid volts marked against the (anode-current anode-voltage) characteristic passing through this point minus the grid bias. This grid drive is provided in the improved system by an unmodulated carrier frequency voltage supplied to the grid of the peaking valve, by any convenient means, independently of the sidebands (which are supplied during the positive half of the modulation cycle only).

The above procedure determines the load impedance and the impedance of the quarter-wave network exactly and the grid drives approximately. In practice, the drives on the carrier and peaking valves in the carrier condition are adjusted to provide a proper marriage of the two envelope curves introduced by the limitation of the carrier valve and the "starting up" of the peaking valve. This means that the carrier valve drive is adjusted just below the value at which limitation occurs, while the peaking valve drive is adjusted so that in the carrier condition it supplies

a small amount of power to the load and therefore takes a small feed.

6.2. Grid-Modulated Doherty Amplifier. This is an obvious modification of the Doherty system, in which the carrier and peaking valves are each grid-modulated amplifiers of the type described in XIII:6. That is to say, each valve, instead of being driven with a modulated wave, is driven with a carrier wave, and with an audio-frequency wave constituting the modulating wave. Both valves are so biased and supplied with such R.F. drive that in the carrier condition the carrier valve supplies substantially all the power to the load. They are driven with such amplitudes of audio or modulating frequency that during the negative modulation cycle the load voltage falls in accordance with the instantaneous amplitude of the modulating voltage, while, during the positive modulation cycle, the instantaneous bias on the peaking valve is decreased so that the positive excursion of the grid is increased in such a way that the output of the peaking valve varies in somewhat the same way as it does in the case where a normally modulated drive is used. The case is not directly comparable because the angle of current flow on both valves is varying throughout the modulation cycle.

In practice, further complications occur for various reasons. For instance, the variation of grid current throughout the modulation cycle will produce appreciable change of grid bias unless the resistance of the grid-bias source is negligible. Also, the variation of the angle of current flow, in addition to its effect in varying the load lines of the valves, introduces appreciable distortion during the negative modulation cycle where its effect adds to the distortion due to the curvature of valve characteristics : this makes it necessary to introduce feedback to reduce distortion.

At the start of the positive modulation cycle, the low angle of current flow of the peaking valve, combined with the valve curvature, gives rise to increments of load current which are too low, if the contribution of the carrier valve is represented by constant current supplied to the load (and equally by constant voltage at the input to the quarter-wave network). This merely means that in the adjustment of the carrier and peaking valve voltages in the carrier condition, the carrier valve drive is reduced rather more below the voltage at which limitation occurs than in the case of the types of Doherty amplifier described above.

Since the carrier valve is not driven into limitation in the carrier condition, the peaking valve is not driven into limitation at 100% p.p.m. even if the quarter-wave line is made equal to twice the load

impedance. The load line of the peaking valve at 100% p.p.m. is further and advantageously reduced in length, and so removed from the region where distortion occurs, by two other effects. The first effect is due to the increase in the angle of current flow at 100% p.p.m., which reduces the peak anode current. The second effect is that, since the contribution of the carrier valve to the load current is represented at 100% p.p.m. by a current in excess of that at carrier level (since limitation does not occur until a value of current slightly above carrier level), the contribution of the peaking valve to load current at 100% p.p.m. is less than the load current in the carrier condition, instead of being equal to that current. This also reduces the peak anode current. The power contributed by the peaking valve at 100% p.p.m. is therefore less than twice carrier power, while the power contributed by the carrier valve at 100% p.p.m. is greater than twice carrier power. These effects are normally of negligible size, however.

6.21. Design of Grid-Modulated Doherty Amplifier. The design of a grid-modulated Doherty amplifier contains rather more art than the design of the improved Doherty amplifier, which is a matter of quite reasonable precision. This is largely owing to the possibility of choosing the grid bias so as to adjust the angle of current flow to obtain an optimum compromise between the requirements of linearity and power output. The small margin between the carrier condition grid drive and the grid drive at which limitation occurs also may introduce an element of uncertainty which, in practice, means merely that the final adjustments may cause the amplifier to work under conditions slightly removed from the assumed conditions. This is due to the low angle of current flow when the peaking valve begins to operate.

Design must therefore proceed by assuming a value of grid bias, designing and setting up the circuit, and then adjusting the grid bias and drives for optimum linearity. At any stage in the proceedings, if the grid bias differs widely from the value assumed in the original design, it may be desirable to carry out a re-design based on the new grid bias. In the discussion below it will be assumed that the grid bias is the same on both peaking and carrier valves, because this is a simple (but by no means an essential) condition, and also because this condition obtains in certain transmitters in use.

Initially, it is necessary to consider only the circuit of Fig. 1 and to imagine that each valve is supplied with means for bias and for driving the grid with carrier frequency, and with audio (modu-

lating) frequency of any required amplitude, the amplitude of each audio-frequency and each carrier-frequency drive being capable of independent adjustment. Independent adjustment is not necessarily a practical condition: it has evidently both advantages and disadvantages.

Design will now be carried out for a transmitter to deliver 1 kW of unmodulated carrier power, using two W.E. Co. 357A valves as carrier valves, and two identical valves for the peaking valves. The grid bias will be assumed to be 440 volts, because this is the value used in the W.E. Co. 443A transmitter. The load line chosen as an initial assumption would, in practice, represent the designer's estimate of the optimum compromise between the requirements of linearity efficiency and power output, with an eye on the 100% p.p.m. peak current and carrier condition anode feed of the carrier valve: these are respectively limited by the peak emission and anode dissipation of the 357A valve. In the present case it is most instructive to admit that the load line has been chosen as the best estimate of the original designer's choice of load when designing the 443A transmitter. The conditions obtained therefore represent the operating conditions of this transmitter, within the normal limits of accuracy.

Fig. 2 shows the valve field of the 357A valve, with the carrier-valve load line for the carrier condition (determined as above), the value of H.T. used being 3,500 volts.

Conventions as at the beginning of Chapter X. Also:

p.p.m. = peak positive mod.
p.n.m. = peak negative mod.
$E_{gp}(0, C)$ = max. pos. excursion of carrier valve grid: no mod.
$E_{gp}(100, C)$ = max. pos. excursion of carrier valve grid: 100% p.n.m.
$E_{gp}(100, P)$ = max. pos. excursion of peaking valve grid: 100% p.p.m.
$E_{gp}(0, P)$ = max. pos. excursion of peaking valve grid: no mod.
$\hat{e}_{gmc}$ = peak modulating voltage on the grids of the carrier valves.
$\hat{e}_{gmp}$ = peak modulating voltage on the grids of the peaking valves.
$\hat{e}_{gc}$ = peak grid drive on carrier valves.
$\hat{e}_{gp}$ = peak grid drive on peaking valves.
R = characteristic impedance of quarter-wave network.

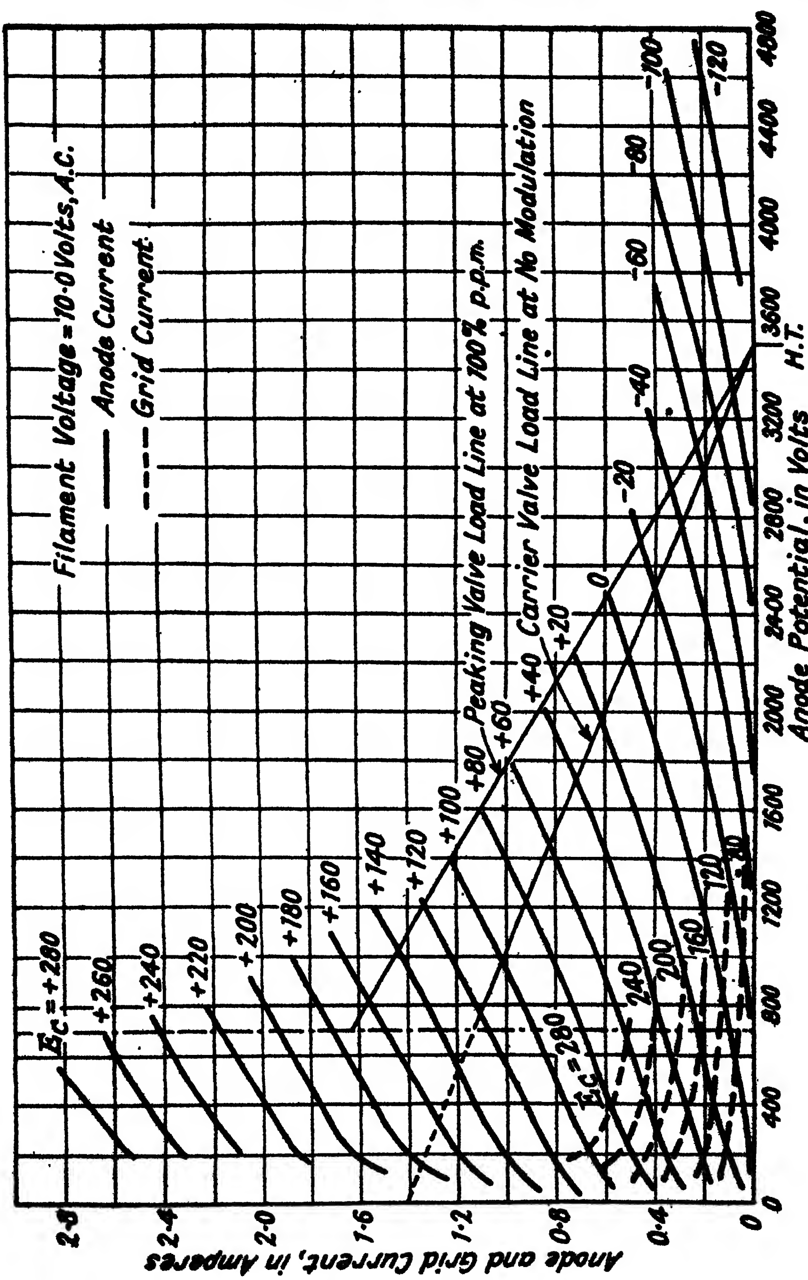


FIG. 2/XIV:6.—Field of W.E. Co. 357A Valve.
(By courtesy of the Western Electric Company.)

The only information available is the H.T. volts, the load line in the carrier condition, the grid bias, the required carrier power, and the cut-off bias which may be read from Fig. 2 : $E_c = -110$ volts.

Information Obtained from Considering the Carrier Condition. *Carrier Valves.* From the carrier condition may be obtained the impedance facing the carrier-valve anodes in the carrier condition ($= 2R$), $\hat{e}_{gc}$, $\hat{e}_{gmc}$, and the peak anode volts and peak anode current of the carrier valves in the carrier condition. The angle of current flow will be found incidentally.

Choose any point on the load line where the anode volts and anode current can be read easily, such as anode volts = 1,000, anode current = 1·0.

Then

$$Z_L = \frac{\hat{e}_a}{\hat{i}_p} = \frac{3{,}500 - 1{,}000}{1\cdot 0} = 2{,}500$$

$$\hat{i}_p = \frac{\hat{e}_a}{2{,}500} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In the carrier condition each carrier valve delivers 500 watts, so that

$$\tfrac{1}{2} f \hat{i}_p \hat{e}_a = 500 \quad . \quad . \quad . \quad . \quad (2)$$

From (1) and (2)

$$\hat{e}_a = \sqrt{\frac{2{,}500{,}000}{f}} = \frac{1{,}580}{\sqrt{f}} \quad . \quad . \quad . \quad . \quad (3)$$

Also, from (5)/X:25, the angle of current flow

$$\theta = 2\cos^{-1}\frac{E_c - E_b}{\hat{e}_{gc} - \dfrac{\hat{e}_a}{\mu}} = 2\cos^{-1}\frac{440 - 110}{\hat{e}_{gc} - \dfrac{\hat{e}_a}{30}} \quad . \quad . \quad (4)$$

(μ is determined in the region: anode current = 0, anode volts change from 3,600 to 1,800. Then grid volts change from -120 to -60: $\mu = 1{,}800/60 = 30$.)

It is now necessary to find what values of $\hat{e}_a$, f, $\hat{e}_{gc}$ and θ fit simultaneously equations (3) and (4), and the load line of Fig. 2.

Try $\theta = 90°$, so that, from Fig. 1/X:22,

$$f = 0\cdot 31,\ \hat{e}_a = 1{,}580/\sqrt{0\cdot 31} = 2{,}840.$$

Plot the point: anode current = 0, anode volts = 3,500 − 2,840 on Fig. 2 and draw a vertical through this point. At the intersection of this vertical with the load line read off $E_{gp}(0, C) = +127$ and $\hat{i}_p = 1\cdot 14$. Hence

$$\hat{e}_{gc} = 127 + 440 = 567.$$

Check: $\theta = 2\cos^{-1} 330/(567 - 2{,}840/30) = 89°$. This is as close as the curves on Fig. 2 can be read.

Hence, in the carrier condition:

$\theta = 90°$, $f = 0{\cdot}31$, $u = 1{\cdot}884$, $g = 0{\cdot}1625$ *
$\hat{\imath}_p = 1{\cdot}14$, $\hat{\imath} = g\hat{\imath}_p = 0{\cdot}185$, $\hat{\imath}_f = f\hat{\imath}_p = 0{\cdot}354$
$\hat{e}_a = 2{,}840$, $E_{gp}(0, C) = +127$

$$Z_c = \frac{\hat{e}_a}{\hat{\imath}_f} = \frac{2{,}840}{0{\cdot}354} = 8{,}000\ \Omega = \text{impedance facing one valve.}$$

The load peak current $= 2 \times 2\hat{\imath}_f = 1{\cdot}416$.
The load peak volts $= \frac{1}{2}\hat{e}_a = 1{,}420$.

Hence, impedance looking into the quarter-wave network in the carrier condition = the impedance facing two valves = 4,000 Ω, the characteristic impedance of the quarter-wave network is 2,000 ohms and the load impedance is 1,000 ohms very nearly. Since the modulating voltage must reduce the positive excursion of the grid bias to the cut-off bias at 100% peak negative mod:

$$\hat{e}_{gmc} = E_{gp}(0, C) - E_c = 127 + 110 = 237 \text{ volts}$$

$$\eta = \frac{1}{2}\frac{\hat{e}_a}{E}u = \frac{1}{2} \times \frac{2{,}840}{3{,}500} \times 1{\cdot}884 = 78{\cdot}5\% = \text{the efficiency of}$$

the carrier valves in the carrier condition.

Check. The power output from each carrier valve

$$= \tfrac{1}{2}\hat{e}_a\hat{\imath}_f = \tfrac{1}{2} \times 2{,}840 \times 0{\cdot}354 = 502 \text{ watts.}$$

Peaking Valves. The value of $E_{gp}(0, P)$ is read off from Fig. 2 as the limit of positive grid excursion necessary to make the peaking valve just begin to take current when the load peak volts and hence the peak anode volts on the peaking valve are equal to their value in the carrier condition, that is $= \frac{1}{2} \times 2{,}840 = 1{,}420$. The minimum anode volts are then $3{,}500 - 1{,}420 = 2{,}080$, and hence, from Fig. 2,

$$E_{gp}(0, P) = -60$$
$$\hat{e}_{gp} = E_{gp}(0, P) - E_b = 440 - 60 = 380$$

Information Obtained from Considering the 100% p.p.m. Condition. The anode peak volts of the carrier valve are assumed to remain constant during the positive modulation half-cycle. This desirable condition is finally achieved by adjustment of the grid limitation to provide a proper marriage of the peaking valve and grid limitation characteristics, observing the modulated envelope on a cathode-ray oscillograph. Since the impedance facing the carrier-valve anodes falls as the positive modulation approaches peak

* During checking it was found that more accurate values of u and g are: $u = 1{\cdot}880$; $g = 0{\cdot}1650$. These errors, which were introduced by misreading Fig. 1/X : 22, are, however, quite negligible.

modulation from the carrier condition, it is necessary that grid limitation should only set in gradually, so that the positive excursion of the grid rises appreciably above the value at carrier level, but only enough to maintain constant the anode volts and not enough to cause them to rise.

Since the anode peak volts of the carrier valves are constant during the positive modulation half-cycle, the input voltage to and the output current from the quarter-wave line are constant. The load current supplied by the carrier valves is therefore constant at carrier level, which in the present case is 1·416 peak amps.

Peaking Valve. At 100% p.p.m. the peaking valves therefore supply 1·416 peak amps. to the load, and the anode peak volts are equal to the load peak volts = 2 × 1·416 × 1,000 = 2,840 very nearly.

The minimum positive excursion of the anode is 3,500 − 2,840 = 660, as for the carrier valve in the carrier condition.

The load line for the peaking valve at 100% p.p.m. therefore terminates on the vertical through the point: anode volts = 660, and at a point on this vertical determined by the fact that the peak anode current for one valve

$$= \hat{i}_p = \frac{i_f}{f} = \frac{1\cdot416}{2f} = \frac{0\cdot708}{f} \qquad . \quad . \quad . \quad (5)$$

The angle of current flow is

$$\theta = 2\cos^{-1}\frac{E_c - (E_b - \hat{e}_{gmp})}{\hat{e}_{gp} \qquad \hat{e}_a}$$

But $E_{gp}(100, P) - E_b = \hat{e}_{gmp} + \hat{e}_{gp}$

$$\therefore\ \hat{e}_{gmp} = E_{gp}(100, P) \quad E_b - \hat{e}_{gp} \qquad . \quad . \quad (6)$$

$$\theta = 2\cos^{-1}\frac{E_c + \hat{e}_{gp} - E_{gp}(100, P)}{\hat{e}_{m} \quad \frac{\hat{e}_a}{\mu}} = \frac{110 + 380 - E_{gp}(100, P)}{380 \quad \frac{2{,}840}{30}}$$

$$= 2\cos^{-1}\frac{270 \quad E_{gp}(100, P)}{285} \qquad (7)$$

It is now necessary to find values of all the unknowns in equations (5) and (7) which satisfy these equations and also satisfy the condition for the termination of the load line evolved above.

Try $E_{gp}(100, P)$ +170, so that $\theta = 2\cos^{-1}\frac{100}{285}$ 139° 8′, $f = 0\cdot435$, $\hat{i}_p = 0\cdot708/0\cdot435 = 1\cdot625$, which, when plotted on Fig. 2 on the ordinate through the point: anode volts = 660; gives

$E_{gp}(100, P) = 175$ so that the chosen value of $E_{gp}(100, P)$ is too low.

Try $E_{gp}(100, P) = +174$, so that $\theta = 2\cos^{-1}\dfrac{96}{285} = 140° \ 40'$, $f = 0{\cdot}4375$, $i_p = 0{\cdot}708/0{\cdot}4375 = 1{\cdot}62$, and $E_{gp}(100, P) = 174$ as nearly as the accuracy of Fig. 2 permits. These are therefore the final values.

The corresponding load line has been drawn on Fig. 2.

The required value of the modulating voltage on the peaking valve can now be found. This is given by equation (6) and is:

$$\hat{e}_{gmp} = 174 + 440 - 380 = 234$$

Since $\hat{e}_{gmc} = 237$ it is evident that the required values of modulating voltage are the same on both carrier and peaking valves (it is evident that the accuracy of the valve chart and the design method is not accurate enough to specify quantities more closely than this). This constitutes an important advantage of grid limitation over anode limitation, since it simplifies the circuit and eliminates one adjustment.

Adjustment of Load Impedance and Quarter-Wave Network. The impedance of the load is best adjusted on an R.F. bridge and the quarter-wave line then adjusted until its input impedance, when terminated in the load, is four times the load impedance.

When a bridge is not available, the load impedance and the input impedance of the quarter-wave network may be measured as described in X:26.3 if a peak voltmeter is available.

Another method consists in arranging that the peaking valves alone are connected to the load, and adjusting the value of the load impedance until a value of efficiency is obtained which corresponds to the operating adjustments used and to the required load impedance. An example of how to calculate the efficiency of the peaking valves, when driven as a normal class C amplifier under specified conditions, is given below. It follows that in the case for which calculation is made below, if the efficiency is adjusted to 58·3% by varying the load impedance, the load impedance is adjusted to 1,000 ohms within the limits of accuracy of the method.

Similarly, by calculating the efficiency of the carrier valves, when working without the peaking valves, into the input of the quarter-wave line terminated only in the correct load impedance, the quarter-wave line may be adjusted to its correct value. This is evidently done by adjusting the load impedance to its correct value, connecting it to the quarter-wave line, and adjusting the impedance of the quarter-wave line until the observed efficiency of

the carrier valves is equal to the efficiency calculated for the operating conditions and the required value of quarter-wave line impedance.

If possible, the initial adjustment of a transmitter should be made on a bridge and the efficiency observed when the load impedance is correct. This serves as a criterion for future adjustments.

The method of calculating the efficiency under given operating conditions is given below.

Efficiency of Peaking Valves when Working with a Grid Bias 440 *Volts,* H.T. = 1,750 *Volts, and Supplying* 600 *Watts of Unmodulated Carrier, into a Load of* 1,000 *ohms.*

The above may be taken to represent a chosen condition for the transmitter just considered.

Then

$$\frac{e_a}{2 \times 1{,}000} = 600 \quad \therefore \ \hat{e}_a = 1{,}095$$

For one valve

$$\tfrac{1}{2} f i_p \hat{e}_a \quad 300$$

$$\therefore \ f i_p \quad \frac{600}{1{,}095} \quad 0{\cdot}548$$

Since from Fig. 2 with H.T. = 1,750, $E_c = -55$:

$$\theta = 2\cos^{-1}\frac{440 - 55}{\hat{e}_{gp} - \frac{1{,}095}{30}} = \frac{385}{E_{gp} - E_b - 36{\cdot}5} = \frac{385}{E_{gp} + 403{\cdot}5}$$

Try $\theta = 95°$, $f = 0{\cdot}325$, $i_p = 0{\cdot}548/0{\cdot}325 = 1{\cdot}7$

From Fig. 2, plotting point: anode volts = 1,750 − 1,095, $i_p = 1{\cdot}7$: $E_{gp} = +180$. Check: $\theta = 2\cos^{-1}(385/583{\cdot}5) = 97°\,20'$.

Try $\theta = 97°$, $f = 0{\cdot}328$, $i_p = 1{\cdot}67$, $E_{gp} = +178$

Check: $\theta = 2\cos^{-1}(385/581{\cdot}5) = 97°$.

Hence, $\theta = 97°$, $u = 1{\cdot}86$

$$\eta = \frac{1}{2} \times \frac{1{,}095}{1{,}750} \times 1{\cdot}86 = 58{\cdot}3\%$$

6.22. Example of a Practical Circuit. The circuit used in a practical transmitter is shown in Fig. 3.

The audio-frequency drive is applied at the input of T_1 and is amplified by means of valves V_3 and V_4, which are each equipped with parallel feedback paths from anode to grid, by means of C_3,R_3 and C_4,R_4. The anode of V_4 is capacity coupled to the top of the audio-frequency choke L_3, which supplies bias and audio-frequency drive to the two grid-modulated amplifiers V_1 and V_2.

The carrier-frequency drive is applied to L_1, coupled to L_2, which

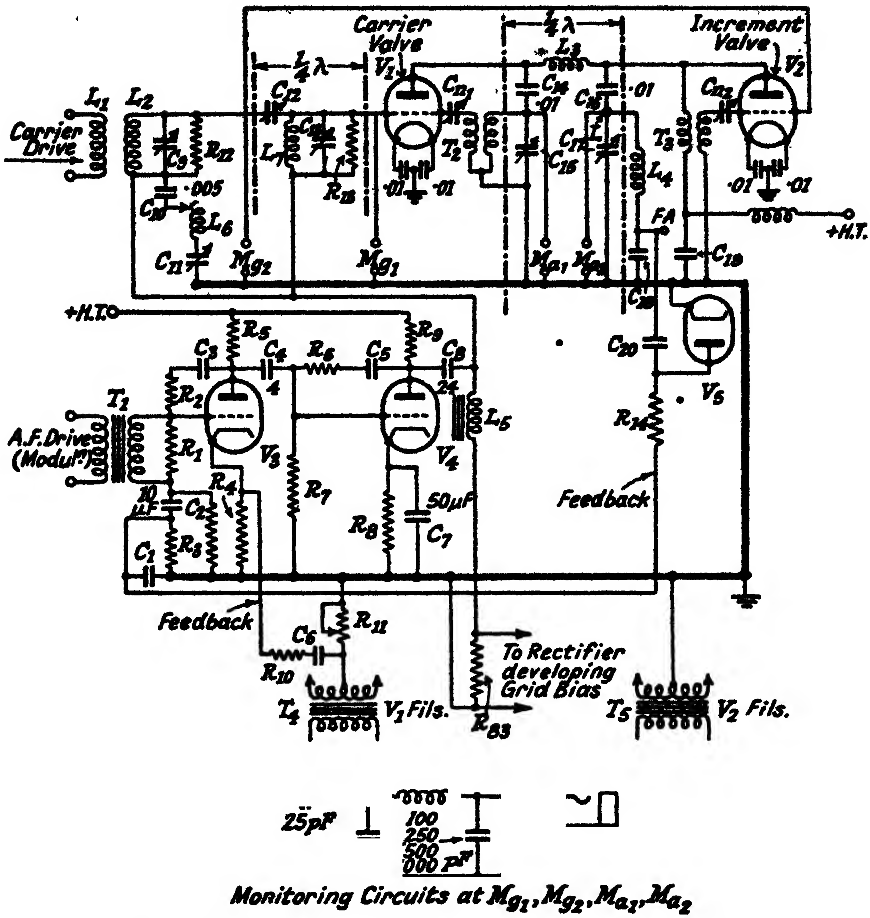


FIG. 3/XIV:6.—Typical Grid-Modulated Doherty Amplifier.

is tuned by C_9 and loaded by R_{12}. The bottom end of L_2 is provided with a radio-frequency earth through the tuned circuit C_{10}, L_6, C_{11}. The grid of V_2, the peaking or incremental valve, is driven directly from the top of L_2, while the grid of V_1, the carrier valve, is driven through the quarter-wave network constituted by C_{12}, C_{13}, L_7, and R_{13}. This quarter-wave network serves several functions: it provides a phase shift of 90° lead to offset the 90° lag introduced by the quarter-wave network in the output of V_1; it presents an input impedance which rises as the grid impedance of V_1 falls with increase of grid drive and so compensates to some extent for the simultaneous fall in impedance of the grid input of V_2; further, it introduces increasing attenuation to the grid drive as the grid drive

increases during the positive modulation half-cycle. It is evident that this attenuation may be made to supplement or replace the effect of anode limitation: this is evidently desirable only if by so doing linearity is increased—any attempt to operate the carrier valve (in the carrier condition) below anode limitation results in a lowering of efficiency. Still another function served by the elements of the quarter-wave network is to provide a means of adjusting the magnitude of the drive on V_1: this can be done by changing the value of C_{12} or C_{13} and then adjusting the unchanged condenser until the required phase relations between the grid drives of V_1 and V_2 are restored. Normally a voltage step-up is provided so that the carrier-valve grid drive is greater than the peaking-valve grid drive.

Since the lower ends of L_2 and L_7 are connected to the top of L_8, the grids of valves V_1 and V_2 are each driven with equal amplitudes of audio frequency. By adjustment of the input quarter-wave network, the grid drive on V_1, the carrier valve, is adjusted to be greater than the drive on V_2, the incremental valve, so that, in the carrier condition, V_2 delivers very little power to the load at L. Ideally V_2 should deliver no power in the carrier condition, but it is found that, owing to the curvature of valve characteristics, some improvement in linearity results by making V_2 give a small contribution in the carrier condition. A little consideration will show that if there were no R.F. drive on V_2 it would absorb power from the load, while, by adjusting the drive on V_2 just above the value at which it takes no power, it can be made to deliver a small amount of power to the load.

The quarter-wave network in the anode circuit of V_2 is constituted by the components between the chain-dotted lines. The aerial feeder circuits are connected at FA, and inductance L_4 and capacity C_{16} constitute an L matching network presenting the required load impedance at L.

Transformers T_2 and T_3 supply capacity-neutralizing voltages to the neutralizing condensers C_{n1} and C_{n2}.

Monitoring circuits as shown at the bottom of Fig. 3 are connected in the grid and anode circuits of valves V_1 and V_2 at the points M_{g1}, M_{g2}, M_{a1} and M_{a2}. These are used for connecting the plates of a cathode-ray oscillograph so that the relative phase of the voltages, at any two points so equipped, may be compared.

In addition to the internal feedback paths in the audio-frequency drive amplifier, there are two paths provided for feedback of the envelope of the modulated wave constituting the output of valves V_1 and V_2.

One path is constituted by R_{10} and C_8, and feeds back into the cathode resistance of V_3 the voltage developed across R_{11} in the filament circuit of V_1 the carrier valve. This provides feedback through the whole modulation cycle: the anode current of V_1 increases during the positive modulation half-cycle owing to the decrease in anode impedance as the peaking valve comes into action, and decreases during the negative modulation half-cycle owing to the decrease in maximum positive grid excursion (of the carrier valve).

The other feedback path is constituted by the rectifier valve V_5, and resistance R_{14}, and also feeds back a voltage having the same form as the whole modulation envelope into the resistance R_8 in series with the grid input circuit of valve V_3.

There are several possible ways of setting up the circuit of Fig. 3; the following method serves to illustrate the principles involved.

Each valve is neuted cold with no H.T. on it and the anode of the other valve earthed. Indicating devices such as a cathode-ray oscillograph, are connected in turn at M_{a1} and at M_{a2}, and after the tuning condensers C_{16} and C_{17} have been adjusted to give maximum indication, the neut condensers C_{n1} and C_{n2} are adjusted for minimum indication, with normal grid drive.

The load impedance facing the peaking valve is then adjusted to its correct value. This may either be done by measuring with an R.F. bridge the parallel resistance between the plate of the peaking valve and ground, adjusting L_4 until the required impedance is obtained, or by setting up the peaking valve to a predetermined value of efficiency as already described. The load impedance facing the carrier valve is then adjusted; this involves adjusting the quarter-wave network to its correct value. If an R.F. bridge is available the inductance of L_3 is adjusted so that its reactance is equal to the required characteristic impedance of the quarter-wave network. The tuning condensers C_{16} and C_{17} are then adjusted in turn so that the near end impedance of the quarter-wave network, with the far end shorted, is a maximum.

If an R.F. bridge is not available L_3 is adjusted so that with the tuning condensers at their correct positions, the efficiency of the carrier valve, when driving the load through the quarter-wave network, is equal to a predetermined value for prescribed conditions of operation. For each setting of L_3 the correct positions of the tuning condensers are obtained by disconnecting the anode of each valve in turn and driving the other valve with the end of the quarter-wave network remote from it shorted to ground through C_{14} or C_{15}. The

tuning condensers are adjusted to minimum anode current, which means that the impedance looking into the quarter-wave network is a maximum : the impedance looking into a lossless quarter-wave network with the far end shorted is infinity. The settings of these condensers so obtained should be noted as they are the final settings, and the condensers should now be left on these settings.

With low grid drive and working grid bias, so that the peaking valve contributes no output, connect the X plates of a cathode-ray oscillograph to M_{g2} and the Y plates to M_{a2}, so comparing the phase of the voltage applied at the input of the first quarter-wave network in the grid circuit of the carrier valve with the phase of the voltage delivered by the carrier valve to the load. These two voltages should be in phase opposition and condenser C_{12} should be adjusted until the figure on the cathode-ray oscillograph is a straight line. This phasing operation should be repeated every time any adjustment is made to the quarter-wave network in the grid circuit of the carrier valve.

With working bias, bring up the grid drive on the carrier valve to its working value and read the anode current on the peaking valve. At this stage it is sufficient to adjust it to about a seventh of the anode current of the carrier valve. If the peaking-valve current is too great or too small it is necessary to adjust the relative amplitudes of drive on carrier and peaking valves. This may be done by adjustment of C_{13} in the quarter-wave network, after which the circuit must be re-phased by adjustment of C_{12}.

Modulation may now be applied, with R_{14} disconnected to remove the overall feedback circuit, in order that distortion shall be more easily seen. C_{12} is then adjusted until the R.F. amplitude in the troughs of the modulated wave is a minimum as observed on a cathode-ray oscillograph connected at M_{a2}. If the envelope is unsymmetrical about the carrier level so that the increase of amplitude during each positive half-cycle of modulation is greater than the fall of amplitude during each negative half-cycle of modulation, or vice versa, the relative amplitudes of modulating voltage on carrier and peaking valve must be changed. This is achieved by adjustment of resistance R_{11} which controls the envelope voltage fed back from the carrier valve.

If the marriage of the peaking valve output with the carrier valve output is unsatisfactory it will show up as an irregularity in the modulation envelope in the neighbourhood of carrier level. For observing this, the trapezium diagram is best (see XX:10.4). When this occurs it is necessary to adjust the relative amplitudes of R.F.

grid drive on carrier and peaking valve by adjustment of the grid input quarter-wave network.

The overall feedback circuit may then be restored by reconnecting R_{14}, after which the transmitter is ready for service.

6.23. Representation of Voltages and Currents in a Grid-Modulated Doherty Amplifier. Fig. 4 shows the general form of the voltages and currents in a grid-modulated Doherty amplifier in which the carrier amplitude has been adjusted below the value at which limitation occurs. The result is that at the beginning of the positive half-modulation cycle the anode volts of the carrier valve and the output current from the quarter-wave network rise by a fraction Δ times the carrier amplitude.

If the grid voltages are ignored, Fig. 4 also represents the performance of a straight Doherty amplifier as described at the beginning of XIV:6.

In practice Δ is never quite zero because it is always necessary to adjust the carrier a little below limitation, so that the increase in the carrier valve contribution to the load current compensates for the fact that the peaking valve doesn't start up quite fast enough. Δ is the amount of the margin and in practice is fixed by adjustment of the carrier level (in conjunction with adjustments of the impedance of the stage driving the carrier grid), so that a proper envelope marriage is obtained between the contributions to the load of carrier and peaking valves.

The grid voltages in Fig. 4 are not to scale, but the other voltages and currents are shown respectively in terms of $\hat{e}_0$, the carrier condition load peak volts and $\hat{\imath}_0$ the carrier condition load peak current. The symbol conventions for voltages are otherwise as given in XIV:6.21.

The instantaneous values of voltage and current are shown as full lines, and the envelopes as dotted lines.

Two conditions are shown: the carrier condition, and the condition of 100% modulation with a sinusoidal modulating wave. The modulating waves appear at (A) and (B) as the sinusoidal base line about which the R.F. grid drives vary.

The carrier valve anode voltage shown at the top of division (C) rises slightly during the positive modulation half-cycle (from its value at carrier level) until anode limitation is reached; the amount by which it rises is represented for convenience by the factor $(1 + \Delta)$. It then falls with the modulation during the negative modulation half-cycle.

The output current from the quarter-wave network shown at

the bottom of (*D*) is equal to the input voltage to the network divided by Z_0, the characteristic impedance of the quarter-wave network. It is therefore of the same form as the carrier valve anode voltage which is supplied to the input of the quarter-wave line.

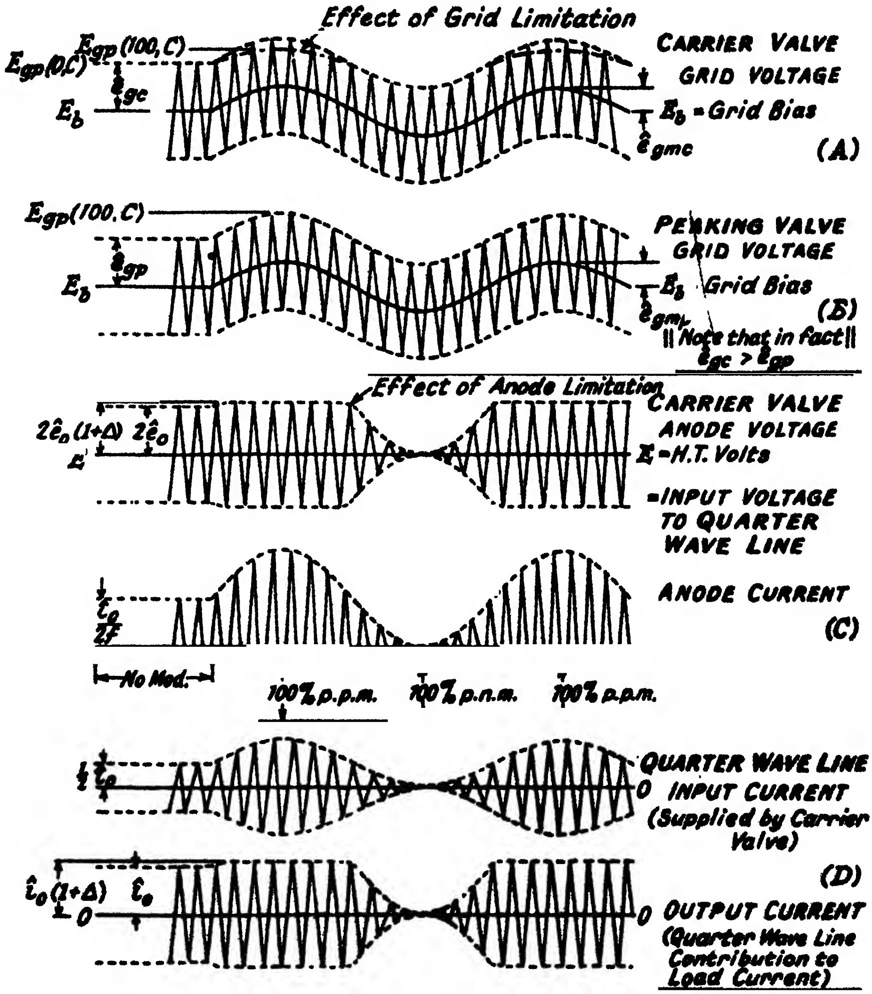


FIG. 4/XIV:6.—Voltages and Currents in Grid-Modulated Doherty Amplifier. (By courtesy of Mr. D. R. Cowie.)

Since the input voltage to the quarter-wave line (= the anode voltage of the carrier valve) is constant during the bulk of the positive modulation half-cycle, the output current from the quarter-wave network is constant during the same time.

The current contributions of the peaking valve to the load are

shown at (E), and it is evident that these, adding to the output current from the quarter-wave network, shown at the bottom of (D), must give rise to the load-current wave form shown at (G). The load volts must therefore be of the form shown at (H).

Since the output voltage of the quarter-wave network = the load voltage, and since the output voltage of a quarter-wave network = $Z_0 \times$ input current to the network, it follows that the input current to the network must be of the form shown at the top of (D).

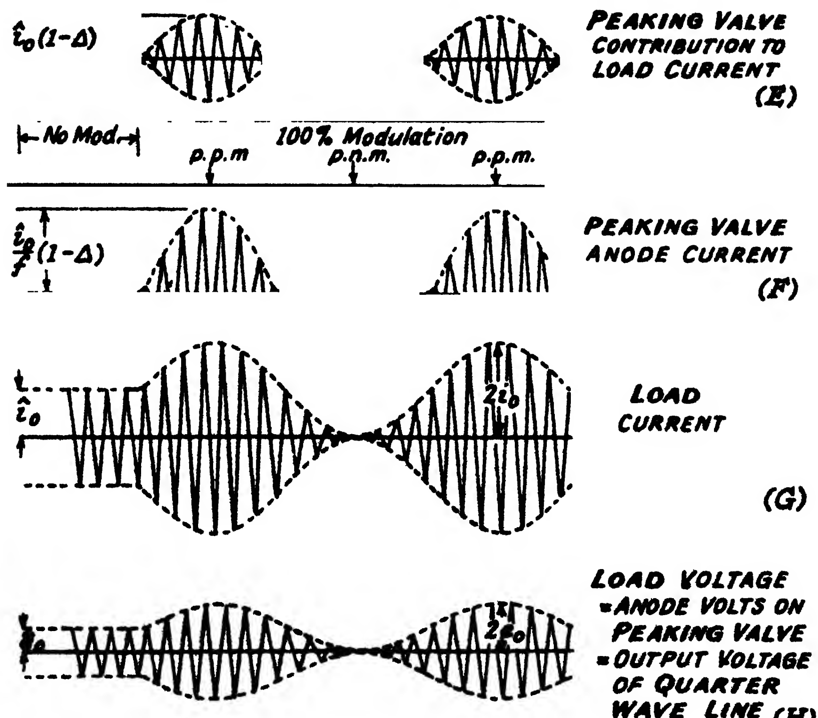


FIG. 4/XIV:6 (*continued*).—Voltages and Currents in Grid-Modulated Doherty Amplifier.

The factor f is the peak fundamental (component of anode current) to peak (anode current) ratio determined by the angle of current flow, and amplitudes involving this factor cannot be shown to scale since it is a variable. It will immediately be appreciated that these curves are only approximate since in practice they are modified by variations in the angle of current flow. Their value in giving a picture of the working of the amplifier is, however, little impaired by this.

The grid voltages are not to scale and the R.F. grid drives $\hat{e}_{gc}$ and $\hat{e}_{gp}$ are shown equal whereas $\hat{e}_{gc}$ is greater than $\hat{e}_{gp}$.

The derivation of the factor $(1 - \Delta)$ and the reason for its appearance is evident from the derivation of the factor $(1 + \Delta)$ given above.

The very small anode current taken by the peaking valve in the carrier condition and the resultant contribution of the peaking valve to the load current have been omitted from (F) and (E) respectively.

7. The Fortescue System (B.B.C. Patents 504,196 and 520,075).

This system may be regarded as a solution of the old problem of transmitting the carrier from one amplifier operated at high efficiency and the sidebands from another, the outputs from the two amplifiers being combined in the load. Previously it has been considered that a bridge is necessary for the purpose of combining the two outputs, so introducing a loss of 3 db. which, of course, offsets the advantage obtained. Alternatively the Fortescue system may be regarded as a modification of the Doherty system.

In the Fortescue system the output circuit is the same as that of Fig. 1/XIV:6. One valve, which will be called the carrier valve, is driven hard with unmodulated carrier, and supplies power to the load through a quarter-wave network, and one valve (replacing the peaking valve) called the sideband valve has its anode connected directly to the load and supplies the sidebands to the load. Impedances are adjusted so that approximately the same relations hold as in the case of the Doherty system. The sideband valve is biased as a normal high-frequency amplifier, and to prevent it taking current in the carrier condition, when the carrier valve is causing a voltage swing to occur across the load and therefore across the anode of the sideband valve, a carrier voltage is supplied to the grid of the sideband valve in phase opposition with the drive on its anode produced by the carrier drive on the grid of the carrier valve, and of such amplitude that in the ideal case (rectilinear valve characteristics in the sideband valve) the sideband valve just takes no current in the carrier condition.

Under these circumstances the operation of the system may be regarded as follows. Assuming the carrier valve to have zero internal impedance (see X:34), the carrier valve supplies constant voltage to the input of the quarter-wave network and therefore constant current to the load. The sideband valve supplies a current to the load which adds or subtracts from the carrier current and varies linearly in amplitude in accordance with the envelope of the

sideband voltages applied to the grid. The current through the load therefore consists of carrier plus sideband.

It is found in practice that owing to the inevitable non-linearity in the sideband-valve near cut-off an increase in linearity is obtained by arranging the static bias on the sideband valve, so that it takes a small anode current; the resultant loss in efficiency is then slightly offset by the second step, also favouring linearity, of adjusting the carrier bias so that in the carrier condition the sideband valve supplies a small R.F. current to the load.

The Fortescue system has the following advantages over the Doherty system:

1. Increased efficiency of the output stage in the carrier condition due to the possibility of driving the carrier valve hard so that it operates at high efficiency. Comparative figures are about 60% for the Doherty system and 65% for the Fortescue system.
2. Increased efficiency of the penultimate stage.
3. Ease of adjustment.

The linearity of the two systems appears to be substantially the same. Harmonics in both systems reach a maximum of 3% to 4%.

8. The Floating Carrier System.

This is a means of increasing the overall efficiency of a modulated amplifier by operating with a low value of H.T. for low values of the modulating frequency voltages. When the audio-frequency modulating voltages reach a predetermined value, which may be about half their maximum value (the maximum value corresponding to 100% modulation in a normal transmitter), means operated by the modulating voltage increase the H.T. in such a way that adequate H.T. is always available, full H.T. being reached when the modulating voltage reaches a value corresponding to 100% modulation.

Since the modulated amplifier is hard driven the result is that the carrier floats from a low value in the unmodulated condition to about twice that value at full modulation. At full modulation the increased carrier is modulated 100%.

The system has a minor disadvantage owing to the action of the A.V.C. in receivers which keeps the mean output carrier-level constant. The result is that if the carrier is set to float to twice its normal value a volume compression of about 6 db. occurs. The reports on the effect of this are not consistent, but it appears to be noticeable, and in transmitters which are fed through audio-

frequency limiters, adjusted so that they also introduce compression, the extra compression appears likely to be serious.

Fig. 1 shows the floating carrier circuit in one of the B.B.C. short-wave transmitters at Daventry. The four CAM5 valves marked V_3 constitute the modulator, in series with the valves V_4, not shown,

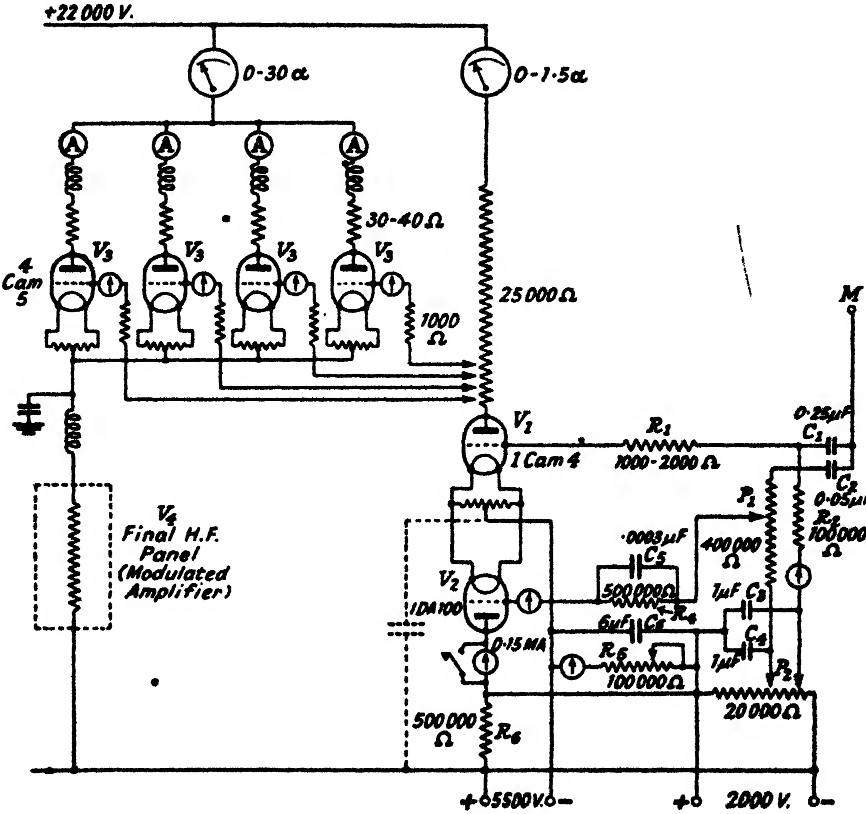


FIG. 1/XIV:8.—Floating Carrier Circuit.
(By courtesy of Marconi's Wireless Telegraph Company and the B.B.C.)

which constitute the modulated amplifier. In other words, series modulation is used. Valve V_1 serves to feed the modulating voltages to the grids of the valves V_3 and also to apply a smoothed change in potential (derived from the modulating voltages) to the grids of V_3, which, by changing the resistance of V_3, changes the H.T. on the modulated amplifier V_4 and so provides the carrier float.

The modulating voltages are applied at M and are fed through blocking condenser C_1 and stopper resistance R_1 to V_1 which amplifies them in the normal way and drives the modulated valves V_3.

The modulating voltages are also fed through C_2, potentiometer P_1 and the equalizing device C_3 and R_4 to the grid of V_2 which is biased so that its anode resistance begins to be lowered only when the modulating voltages reach a predetermined fraction of their maximum value which is about 50%. The object of the equalizing device is to secure an approximately equal drive of V_2 at all audio frequencies, that is, to offset the shunting effect of the input capacity of V_2 across the high impedance potentiometer P_1.

When the resistance of V_2 is lowered its anode becomes less positive, so that the whole of potentiometer P_2 becomes more negative. Since V_1 is biased from P_2, and since its filament is held at a constant potential (5,500 volts below earth) the grid of V_2 then goes more negative with regard to its filament. Owing to the smoothing action of condensers C_3, C_4 and C_5, the change of potential of the grid of V_1 follows only the envelope of the audio-frequency modulating voltages. The result is that as the modulating voltages increase above the level at which V_2 begins to pass current, the anode of V_1 becomes more positive. The resistance of valves V_2 is therefore reduced and the H.T. on the modulated amplifier rises.

It will of course be evident that many other arrangements could be devised to produce the same effect.

At the present time it does not appear as if there is any great future for floating carrier since other competitive systems are more likely to hold the field. It has, however, been described because it is in use at the present time.

9. Efficiencies of Output Stages of Different Systems.

These are derived along the lines developed by L. F. Gaudernack in the *I.R.E. Proceedings* for August 1938. The efficiency is expressed in terms of the anode power input to the last stage at each value of percentage modulation, for a transmitter supplying 100 kW of unmodulated carrier to the aerial.

9.1. Class A High-Power Modulation. This represents the case of Heising and Series modulation.

Conventions.

P_c = unmodulated carrier power = 100 kW.

P = total anode input power.

E = H.T. volts on modulated amplifier.

η_c = modulated amplifier efficiency.

η_m = modulating amplifier efficiency.

η_{100} = modulating amplifier efficiency at 100% modulation.

I = D.C. current taken by modulated amplifier in carrier condition.
m = percentage modulation.
R_m = resistance presented to modulating amplifier by modulated amplifier.
P_{ing} = anode power taken by modulating amplifier.
P_{ed} = anode power taken by modulated amplifier.

$$P = P_{ing} + P_{ed}$$

Then
$$EI = \frac{P_0}{\eta_c} \qquad I = \frac{P_0}{E\eta_c}$$

$$R_m = \frac{E}{I} = \frac{E^2\eta_c}{P_0}$$

$$\therefore\ P_{ing} = \frac{m^2E^2}{2R_m\eta_m} = \frac{m^2P_0}{2\eta_c\eta_m}$$

also

$$P_{ed} = \frac{P_0}{\eta_c}$$

$$= \frac{P_0}{\eta_c}\left(1 + \frac{m^2}{2\eta_m}\right)$$

But
$$\eta_m = m^2\eta_{100}$$

$$\therefore\ P = \frac{P_0}{\eta_c}\left(1 + \frac{1}{2\eta_{100}}\right) \quad . \quad . \quad . \quad (1)$$

Putting $\eta_c = 0{\cdot}7$ and $\eta_{100} = 0{\cdot}3$, which are fair representative values for the efficiencies of a modulated amplifier and a class A amplifier respectively, for 100 kW carrier power, i.e. $P_0 = 100$:

$$P = \frac{100}{0{\cdot}7}\left(1 + \frac{1}{0{\cdot}6}\right) = 382 \text{ kW constant,}$$

i.e. independent of modulation depth . (2)

9.2. Class B High-Power Modulation. Observations at one of the B.B.C. transmitters using this system have given the following approximate figures for a particular set of adjustments :

Carrier power output from modulated amplifier = 88 kW.
Power consumption in carrier condition :

Modulating Amplifier	18·75 kW	(due to standing feed)
Modulated Amplifier	118 kW	
Total	136·75 kW	

At 100% modulation the power consumption by the modulating amplifier is 91 kW, an increase of 91 − 18·75 = 72·25 kW. Assum-

ing linear characteristics for the modulating valves, the power consumption of the mod-ing amp. can be written $18{\cdot}75 + 72{\cdot}25m$ kW where m is the percentage modulation.

The power for the whole transmitter is therefore

$$136{\cdot}75 + 72{\cdot}25m$$

For a 100 kW carrier, evidently

$$P = \frac{100}{88}(136{\cdot}75 + 72{\cdot}25m)$$

$$= 155 + 82m \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The equivalent theoretical expression is developed by Gaudernack in the *I.R.E. Proceedings* for August 1938, p. 388 ; but the numerical values chosen by him are different.

9.3. Low-Power Modulation. In this case the output stage is a high-frequency amplifier which has a peak efficiency in practice of about 68%, and an efficiency in the carrier condition of 34%. Further, the power input does not vary with modulation.

The anode input power for 100 kW carrier power is therefore

$$P = \frac{P_0}{0{\cdot}34} \quad \frac{100}{0{\cdot}34} = 295 \text{ kW} \qquad (4)$$

9.4. The Chireix System. If it were theoretically possible to keep both tubes in class C operation with constant high efficiency η for all load amplitudes, the input power would depend on the degree of modulation and would be :

$$= \frac{P_0}{\eta}\left(1 + \frac{m^2}{2}\right) \qquad (5)$$

As has been stated, the impedance facing the anodes becomes increasingly reactive as the modulation cycle proceeds in either direction away from the carrier level. This results in a drop in efficiency. For a carrier power of 100 kW Chireix gives the following figures for different modulation depths :

$m =$	0	0·3	0·6	0·8	1·0	
P	167	174	212	246	294 kW	. . . (6)
	(133)	(139)	(169)	(197)	(235)	(6a)

These figures are based on a valve efficiency of 75%, and coupling circuit efficiency of only 80% giving an overall efficiency of 60%.

The figures in brackets give the corresponding figures, assuming 100% efficiency for the coupling circuits, and therefore compare directly with the other power consumptions calculated here. It is not at all clear why, for the zero modulation condition, Chireix

obtained such low efficiencies in his coupling circuits and, in default of evidence to show that his circuits are necessarily less efficient than normal circuits, it appears that, in the unmodulated condition, this system is capable of being made more efficient than any of the others.

9.5. The Doherty System.

Conventions.

V_L = value of envelope of load voltage.
V_c = carrier level load voltage.
m = modulation depth.
$\omega = 2\pi f$ where f = modulating frequency.
I_c = current delivered to the quarter-wave line by the carrier valve.
Z_0 = characteristic impedance of the quarter-wave line.
i_c = D.C. input to carrier valve at any point in the modulation cycle.
i_p = D.C. input to the peaking valve at any point in the modulation cycle.
P_0 = unmodulated carrier power = 100 kW.
P = total anode input power.
η_c = efficiency of output stage in carrier (no-mod.) condition.
η_p = efficiency of the peaking valve at 100% peak modulation.

Assuming linearity and sinusoidal modulation, since the voltage at the load end of the quarter-wave line is of the form

$$V_L = V_c(1 + m \sin \omega t) \qquad (7)$$

the current supplied to the quarter-wave line by the carrier valve is

$$I_c = \frac{V_c}{Z_0}(1 + m \sin \omega t) \qquad (8)$$

It follows that the D.C. input to the carrier valve is of the form

$$i_c = 1 + m \sin \omega t \qquad (9)$$

where the D.C. current input in the carrier condition is set equal to unity.

Again, since the current supplied by the carrier valve to the load is constant, and the powers supplied to the load at 100% peak modulation are equal, the D.C. input to the peaking valve during the time that the peaking valve is delivering power is of the form

$$i_p = 2m \sin \omega t \qquad (10)$$

where the D.C. current input at 100% peak modulation has been made equal to 2 to accord with equation (9) above. Equation (10)

evidently defines i_p for $\pi > \omega t > 0$ while for $2\pi > \omega t > \pi$ evidently $i_p = 0$.

The mean values of the two currents over a complete modulation cycle are, respectively,

$$\bar{i}_p = \frac{\bar{i}}{\pi} m$$

So that the input power corresponding to a value of percentage modulation m is :

$$P = \frac{P_0}{\eta_c}.1 + \frac{P_0 . 2m}{\pi\eta_p} \quad . \quad . \quad . \quad . \quad (11)$$

While the carrier valve is operated at approximately 70% efficiency in the carrier condition, the standing feed to the peaking valve reduces η_c, the effective efficiency of the output stage to about 63%, while in practice owing to the high peak current consequent on the low angle of current flow, to obtain a high output from the peaking valve with reasonable linearity, a voltage utilization of about 70% only may be obtainable. This is evidently a variable figure, and by tolerating a lower power output from any particular valve set-up a higher voltage utilization may be obtained. At 100% peak modulation the angle of current flow in the peaking valve is about 120°, which corresponds to a value of $u = i_f/\bar{i} = 1{\cdot}79$. This gives $\eta_p = \frac{1}{2} \times 0{\cdot}7 \times 1{\cdot}79 = 67\%$, say 65%.

Inserting these values of η_c and η_p in (11)

$$P = 159 + 98m \text{ kW} \quad . \quad . \quad . \quad (12)$$

Since the angle of current flow in the peaking valve varies during the positive modulation cycle from substantially zero at zero modulation to 120° at 100% peak modulation, the power input between 0 and 100% modulation is rather less than is given by equation (12). The deviation is, however, unimportant.

It is to be noted that the above argument proves rigidly the correctness of the first of the assumptions made by Gaudernack on p. 989 of the above-mentioned article.

The second alternative assumption based on constant efficiency during the modulation cycle is manifestly false, since the efficiency of the peaking valve evidently varies widely during the half of the modulation cycle when it is operative.

9.51. Improved Doherty System. The general equation for the power input to the two output valves of this system is the same as (11).

Since the angle of current flow in the peaking valve at 100%

positive peak modulation is 180°, the efficiency at 100% positive peak modulation with a voltage utilization of 0·9 is

$$\tfrac{1}{2} \times 0{\cdot}9 \times 1{\cdot}57 = 70\%.$$

Owing to the lower value of peak current (than in the straight Doherty system) it is possible to operate the peaking valve at a higher value of voltage utilization than in the Doherty system proper. A figure of 65% will however be assumed for η_p because sufficient experimental evidence is not available to demonstrate a definite advantage in this respect in favour of the improved system. The carrier level conditions are the same as in the Doherty system proper and a figure of 63% will again be taken as the most representative value.

Hence
$$P = 159+98m \text{ kW} \qquad . \quad . \quad . \quad (13)$$
which is the same as for straight Doherty.

9.6. The Fortescue System.

Conventions.

P_0 = unmodulated carrier power = 100 kW.
η_c = anode efficiency of carrier valve.
η_s = anode efficiency of sideband valve at 100% peak modulation.
P_c = input power to carrier valve.
P_s = input power to sideband valve.
$P = P_c+P_s$.
m = percentage modulation.

$P_c = \dfrac{P_0}{\eta_c}$ independent of modulation.

For the sideband valve the maximum input power $= 2P_0/\eta_s$ and the mean input power $= \dfrac{2}{\pi}m.\dfrac{2P_0}{\eta_s}$

$$\therefore\ P = \frac{P_0}{\eta_c}+\frac{4P_0}{\eta_s}.\frac{m}{\pi} \qquad . \quad . \quad . \quad (14)$$

In practice, owing to the necessity for making the sideband valve take a small current in the carrier condition, the efficiency of the whole output stage in the carrier condition is only about 65%, although the carrier valve may be operating at 75% efficiency or higher.

This is conveniently represented by making $\eta_c = 65\%$ in the above formula, while in practice to obtain a high output with

reasonable linearity η_s becomes also about 65%. Putting these values into equation (14), for a 100 kW carrier power

$$P = 154 + 196m \text{ kW} \quad . \quad . \quad . \quad (15)$$

10. Comparison of Different Systems.

While it is not sufficient to consider only the anode power consumption of the final stages of transmitters (see next section), since the greatest power consumption takes place here, it is useful to use the anode power consumption of the last stage to give a first basis of comparison.

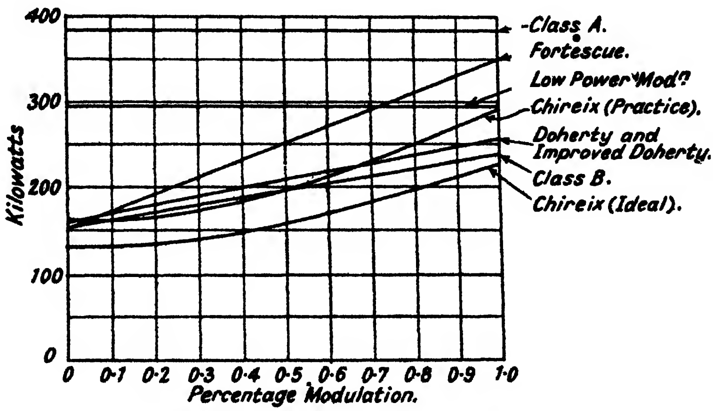


FIG. 1/XIV:10.—Anode Power Input to Last Stage, for Different Systems, for Carrier Power of 100 kW.

The final formulae (relations (2), (3), (4), (6), (6a), (12), (13), and (15)) for the above types of transmitter are plotted in Fig. 1 and give, for modulation percentages from 0 to 100%, the anode input power to the final stages of transmitters supplying 100 kW of unmodulated carrier to the aerial. In the case of class B operation the figures above include the power supplied to both modulating and modulated amplifier.

Systems such as Chireix and Fortescue, which do not require linearity on output stages driven hard with carrier, score an extra and appreciable advantage over similar systems. For instance, in the present study the Fortescue system uses 154 kW on the output stage in the carrier system, and the Doherty system uses 159 kW, a comparatively small difference. The penultimate stage of the Doherty system, however, assuming reasonable probabilities in design, consumes 17 kW, while that of the Fortescue system only consumes 1·5 kW, so that the total consumptions are 176 kW for

the Doherty compared with 155·5 for the Fortescue; the Fortescue system only consumes 88% of the power of the Doherty system in the carrier condition. The Fortescue system, however, has an appreciably higher increase of power consumption with modulation depth than have the other systems, but since the mean modulation depth is normally below 20%, the maximum demand is probably not more than 12% greater than straight Doherty and about 14% greater than a class B system. On the other hand, the rating of the H.T. supply generators in order to supply the anode power at 100% peak modulation is some 40% greater in the case of Fortescue than in the case of Doherty or class B, and this is an appreciable consideration even when the reduced power in the penultimate stage is taken into account.

Fig. 1 shows that there is very little to choose between the two types of Doherty and class B from the point of view of anode consumption. On account of the difficulties of adjustment which have been experienced when the Chireix system has been set up, except under the auspices of Chireix himself, it appears that one of these systems is the most practical proposition at the present time. Of these, the choice appears to be between the improved Doherty system and class B.

The only advantage which the Doherty system offers in comparison with class B is economy on first cost since the Doherty system does not require the costly audio-frequency modulating transformers and chokes, together with spares, which have to be provided.

If a new standard type of transmitter is contemplated in the near future, the improved Doherty system appears as a strong competitor since it has all the advantages of the Doherty system while avoiding the high grid dissipation in the peaking valve, while it has all the advantages of the Fortescue system (except the high efficiency of the penultimate carrier stage) without the high rise in power on modulation.

Heising and series modulation have both the low efficiency of class A modulation and are therefore only suitable for use in low-power modulated transmitters which are also ruled out on the grounds of efficiency.

Floating carrier is not a strong competitor since it has not yet been successfully applied to a transmitter which is class B modulated in the final stage. Under these circumstances under practical conditions of operation, the saving of power with floating carrier is no greater than that occurring with class B and Doherty, while it has,

of course, the disadvantage that it modifies the volume range of the programme.

Audio-Frequency Harmonics. In general it is true to say that low-power modulated systems, high-power class A modulated systems and Chireix have such low harmonic content that they can be operated without feedback. Doherty, Fortescue and class B all have harmonic contents which lie in the neighbourhood of 4% to 6% without feedback. They must therefore be operated with 6 to 10 db. of feedback (i.e. harmonic reduction due to feedback).

Chireix, incidentally, appears to be very much in advance of his time from every point of view except ease of adjustment: he was first in the field with high efficiency systems, and he chose the type which was theoretically the most efficient and which has the lowest harmonic content of any high-efficiency system without feedback.

11. Observed Overall Efficiencies of Transmitters.

The overall efficiency of a transmitter is the ratio of the unmodulated carrier power to the total power required to operate the transmitter. The latter comprises such items as H.T., filaments and grid-bias supplies to all stages, interlock circuits, and all valve-cooling auxiliaries such as water-pumps and air-blowers, but excludes such items as heating, lighting and ventilation and domestic services in the transmitter building.

The overall efficiency therefore depends not only on the final stage efficiency but also on the general economy of design of the transmitter.

Table I shows a summary of the various supplies for a number of different types of transmitters together with their overall efficiencies for the carrier power at which the observations were taken.

The overall efficiency is obtained by dividing the carrier power by the total transmitter consumption.

It is evident that, in the case of transmitters operating below their rated carrier power, the overall efficiency shown is less than the maximum.

In the case of certain class B high-power modulated transmitters, a figure is given for the percentage increase in power input to the transmitter (i.e. mod-ing plus mod-ed amps.) from zero modulation to 100% modulation. This can only be compared with the figure of 55% (corresponding to the increase from 155 to 240 kW shown on Fig. 1/XIV:10) in the case of transmitters operating on their full rated output.

TABLE I

Power Analysis for Different Transmitter Types

Abbreviations :

H.P.M. = High-power Modulated
L.P.M. = Low-power Modulated
F.S. = Final Stage
Fil. = Filaments
E.H.T. = E.H.T. Consumption
B = Class B Modulated
S = Series Modulated
P.S. = Penultimate Stage
Aux. 1 = Auxiliary H.T.s, Grid Bias and Interlock Circuits
Aux. 2 = Valve-cooling Auxiliaries

Type	Wavelength	Actual Carrier Power kW	Transmitter Consumption					Overall Efficiency	E.H.T. Power Consumption 100% Mod. kW	Rise % in E.H.T. Power Consumption at 100% Mod.
			Fil. kW	E.H.T. kW	Aux. 1 kW	Aux. 2 kW	Total kW			
1. L.P.M.	Medium	50	39	212	3·8	2·3	257·1	19·4%	(212)	Nil
2. S in P.S.	Long	214	138·8	730	18·9	23·2	910·9	23·5%	(732)	Nil
3. S in P.S.	Medium	50	34	225	14·5	14·5	288	17·4%	(225)	Nil
4. B in F.S.	Medium	50	29	131	9·5	22·5	192	26%	192	46·5%
5. B in F.S.	Medium	100	68	208	19	34	329	30·5%	323	55%
6. B in F.S.	Medium	106	47	174	4	7	232	46%	256	47%
7. B in F.S.	Short, 16, 19 and 25 m.	70	70	162·5	8·1	19·3	259·9	26·9%	236	45%
8. B in F.S.	Short 25 and 31 m.	79	92·5	175	12·6	38·5	318·6	24·5%	—	—

12. H.T. Smoothing Circuits.

The H.T. for transmitters is generated either by machines or by mercury-arc rectifiers. In either case there is an appreciable ripple voltage at the output of the generator, and, in the case of mercury-arc rectifiers, a large ripple voltage, although this is considerably reduced by the use of six-phase rectifiers.

It is therefore necessary to insert between the generator and the transmitter some form of low-pass filter with an effective cut-off frequency below the lowest ripple frequency. Although the main ripple frequency of a six-phase rectifier is at 300 c/s, appreciable ripple occurs at frequencies as low as 50 c/s, and the effective cut-off of the smoothing filter should therefore lie below 50 c/s.

The C.C.I.R. requirements state that the modulation of transmitters by noise frequencies shall be 54 db. below 100% modulation. The H.T. smoothing must therefore ensure that this condition is met as far as H.T. ripple is concerned. Since no requirements of terminal impedance exist (it is in fact an advantage to have reflection loss at each end of the filter), it is usual to determine the size of the series inductances and shunt condensers composing the filter by the cheapest values which give the required cut-off frequency, and impedance. The presence of the smoothing filter modifies the impedance presented by the H.T. source towards the transmitter, and it is necessary to ensure that the impedance looking back into the filter is not too high. The main H.T. of a transmitter usually supplies only the high-powered stages of the transmitter, and it is necessary to consider in turn the effect of impedance in the H.T. supply on each type of transmitter.

In the case of low-power modulated transmitters the final stages are linear class B high-frequency amplifiers, and the only requirement on the smoothing-circuit impedance is that it shall not present too large an opposition to the flow of audio frequencies, otherwise the effective value of the H.T. will vary throughout the modulation cycle with resultant distortion and lowering of output. This is usually effected by making the last condenser of the smoothing circuit sufficiently large. Usually a condenser of a few microfarads is adequate.

In Heising modulation (which generally does not constitute one of the later stages of a transmitter) the H.T. current does not vary appreciably and there is no need to take any special precautions, provided the degree of smoothing is adequate.

Series modulation must have approximately the same value of

smoothing circuit impedance as a linear class B high-frequency amplifier. The same remark applies to the Chireix system.

Class B modulation is dealt with below as this is a special case. The Doherty system, the improved Doherty system and the Fortescue system require values of smoothing circuit impedance commensurate with class B modulation systems.

12.1. Class B System. Most of the causes of distortion in class B amplifiers have already been dealt with. It is, however, useful to summarize these as a preliminary to considering the question of H.T. smoothing. This is done below, together with the remedies.

Grid Current. Since class B amplifiers are usually driven into positive grid, large grid currents usually flow, of magnitude which increases rapidly near the peak positive grid excursion, in such a way as to give rise to serious non-linearity. The grid must therefore be driven from a low-impedance source. If it were possible to design intervalve transformers with very small leakage reactance, it would be possible to achieve this low impedance by having a suitable step-down ratio from the anode of the previous valve. In practice, the presence of leakage reactance makes it desirable to supplement this step by loading the grid with a resistance equal to about a fifth of the lowest resistance to which the grid falls at maximum peak positive grid volts.

Load Line. The valve must operate along its optimum load line, which usually represents a compromise between the requirements of linearity and power output.

Reactance Component of Load Impedance. The load line is located on the assumption of a pure resistance load. In practice the load is reactive, due partly to the leakage reactance of the output transformer, and partly to the inevitable capacities which occur across the load proper, either due to stray capacities, or, in the case of modulators, to the filters which are inserted to prevent R.F. voltages from the modulated amplifier from dissipating power in the modulator valves. This reactance, if of appreciable magnitude, can introduce serious distortion in class B amplifiers. The leakage reactance of the output transformer, as well as all shunt capacities, must therefore be kept as low as possible.

Grid Bias. The grid bias must evidently be adjusted so that the best possible marriage occurs between the characteristics of the valves on each side of the push-pull. As a corollary, the effective values of H.T. and grid bias must be kept constant at the values used in the initial adjustment of the marriage. If the H.T. is subject

to variation it will sometimes be found that, by making the grid bias vary with the H.T. (by deriving it from the same source) the bad effects of variation in H.T. can be reduced.

H.T. Smoothing. Since, in a class B amplifier, the effect of the grid drive is such that positive parts of the driving wave form make the H.T. supply currents to the valves on one side of the push-pull, while negative parts of the driving wave form make the H.T. supply currents to the valves on the other side of the push-pull, both positive and negative parts of the driving wave form cause the H.T. to supply current in the same sense. Further, this current, during each period of positive or negative drive, has a form identical with that of the driving wave, and equally important, the mean current increases with the amplitude of drive. The H.T. supply circuit therefore must offer a low impedance to all audio frequencies, and also to frequencies corresponding to the rate at which the envelope of the driving wave increases in amplitude. In the case of speech, the rate of build-up of the envelope depends on the frequency at which syllables follow one another—the syllabic frequency, which has an upper limit at about 10 to 15 cycles per second.

If the H.T. smoothing circuit offers appreciable impedance at audio frequencies, non-linear distortion results : sum and difference frequencies (combination tones) and harmonics of all frequencies in the driving wave appear in the output. If the H.T. smoothing circuit offers appreciable impedance at syllabic frequencies, the mean value of H.T. effective on the amplifier or transmitter falls as the programme level rises, so that the possible output power is reduced.

The H.T. smoothing must therefore offer a low impedance to all frequencies below about 17 cycles per second and a low impedance to all frequencies above the lowest audio frequency to be transmitted.

It may be argued with considerable justification that the lowest audio frequency traversing the H.T. circuit is twice the lowest audio frequency transmitted. In the case of a sine wave drive, the wave form traversing the H.T. circuit is a series of half-sine waves, as shown in Fig. 1(*c*)/VIII:1. The lowest frequency in this wave is twice the frequency of the original driving frequency. This argument is probably correct and the criterion given below for the size of the final smoothing condenser is based on this assumption. The statement above therefore errs on the side of safety.

Fig. 1 shows the form of the most common type of H.T. smoothing circuit. This may be designed as a low-pass filter of conventional type, but a greater economy in design can be obtained by a rather more empirical method.

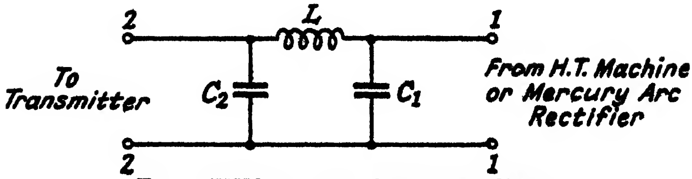


FIG. 1/XIV:12.—H.T. Smoothing Circuit.

In a large class B amplifier condenser C_2 may be of the order of 20 or 30 microfarads; its exact value has in the past been determined by reducing it until the harmonics of the lowest audio frequency transmitted are of tolerable value. It is usually found that, if C_2 is adjusted to satisfy this condition, combination tones (sum and difference frequencies, etc.) are also of tolerable magnitude; this is, however, by no means a foregone conclusion, and tests should be made to verify that such is the case. These tests may conveniently be made by driving the amplifier (or transmitter) with two equal amplitude frequencies of such amplitude as to fully load the amplifier (or transmitter) and of frequency difference equal to the lowest frequency difference which is likely to occur between two frequencies which are present simultaneously. The amplitudes of the resultant sum and difference frequencies should then be measured.

It will be realized that the value of C_2 depends not only on the tolerable values of harmonics, but also on the values of the associated circuit elements: in particular the valve-anode impedance and the load impedance presented by the output transformer towards the valves.

With a single frequency driving a class B amplifier, the effect of a finite impedance looking back into terminals 2,2 in Fig. 1 is the production of odd harmonics of any single frequency driving the input of the class B amplifier. The approximate magnitudes of these harmonics, expressed as a percentage of the fundamental driving frequency, are:

Third harmonic:	$27F$
Fifth harmonic:	$16F$
Seventh harmonic:	$12F$
Ninth harmonic:	$9F$

where
$$F = \frac{Z_s}{R_s + R_a} \quad . \quad . \quad . \quad . \quad (1)$$

Z_s = the impedance looking back into the smoothing circuit, e.g. into 2,2 at twice the driving frequency.

R_0 = the internal impedance of the valve = the mean slope of the anode-voltage anode-current characteristics in the region of the valve field traversed by the load line (drawn at slope R_a).

R_a = the impedance facing one valve.

It is hardly necessary to point out that; if

m = the number of valves on one side of the push-pull.

$n^2:1$ = the impedance ratio of the output transformer from whole primary to whole secondary.

R = the load impedance connected across the secondary of the transformer.

then
$$R_a = \frac{mn^2R}{4} \quad . \quad . \quad . \quad . \quad (2)$$

These formulae only apply for high levels of drive. At low levels where the valves operate largely in class A, or where valve curvature is important, they are not applicable.

Example. $C_2 = 30\ \mu F$, $R_a = 800\ \Omega$ and $R_0 = 1{,}600\ \Omega$ (this value of R_0 represents the case of a 4030B valve (see Fig. 2/X:11), while $R_a = 800\ \Omega$ is the value of impedance facing the anodes of these valves in the class B transmitters in use at Stagshaw and Start Point). The reactance of C_2 at frequency $2f$, where f is the driving frequency, is
$$\frac{5{,}300}{2f} = \frac{2{,}650}{f} \text{ ohms}$$

Hence
$$F = \frac{2{,}650}{2{,}400f} = \frac{1{\cdot}14}{f} .$$

The third harmonic is therefore $\frac{27 \times 1{\cdot}14}{f} = \frac{30{\cdot}78}{f}$ per cent of the output at the driving frequency; if $f = 30$ c/s this is just over 1% which agrees well with the mean of the values observed in practice; if $f = 60$ c/s, the percentage of third harmonic is just over 0·5%.

Evidently the method of using the above formulae is to select the permissible level of percentage harmonic, preferably the third harmonic, at the lowest frequency to be transmitted and then find the value of Z_2 and so the value of C_2.

Suppose, for instance, that the permissible level of the third harmonic of 30 c/s is 1%, then $27F = 1$ or $F = \frac{1}{27}$,

and
$$Z_2/(R_0+R_a) = \frac{1}{27},$$

so that the reactance of C_2 at 60 c/s $= (R_0+R_a)/27$ and hence

$$C_2 = \frac{27 \times 10^6}{2\pi . 60(R_0+R_a)}$$

so that if, for instance, $R_0=16{,}000$ and $R_a=800$, then $C_2=34{\cdot}2\ \mu$F.

The above formulae should only be used as a preliminary guide to the size of C_2 to be used in initial experiments, the final size being determined as the result of experiment.

The value of L is adjusted so that the impedance looking into terminals 2,2 goes to a maximum at about 20 cycles per second. Since the impedance of the rectifier or machine connected to 1,1 is usually low compared with the reactance of C_1, a fairly good approximation for the value of L is to make it resonate with C_1 at 20 c/s. The value of C_1 is determined largely by the requirements of efficient rectification and suppression of harmonics of the power frequency. It may be considerably smaller than C_2.

At high voltages the cost of the smoothing condensers is considerable, and it is therefore important not only to establish that C_2 is as large as it should be, but also to make sure that it is no bigger than it need be.

It will be appreciated that the effect of too small a value of C_2 is to introduce distortion of (single) *low* driving frequencies, the percentage harmonics becoming smaller as the frequency is increased. This point is important to remember because in most class B amplifiers it is found that on single-frequency tests the harmonics start off at a low value and rise to a high value as the frequency is increased from the lowest frequency to the highest frequency in the transmitted range. This rise is due to the increase of transformer leakage reactance with frequency. Sometimes a small fall in harmonics results as the frequency is first increased from a low value before the effect of the transformer reactance begins to take effect. This point is mentioned so that confusion will not be caused by the fact that the total harmonics increase with the increase of frequency. In practice the curve plotted between harmonics (of single frequencies) and driving frequency is further complicated by the distortion introduced by the other sources of non-linearity enumerated above, and it will sometimes be found, for instance, that an increase of one form of non-linearity produces a reduction in one or more of the harmonics, because the sense (phase) of the harmonics produced by the different sources of non-linearity is opposed. It is, however, quite impossible to lay down broad principles for dealing with all the conditions which may occur.

CHAPTER XV

OPERATION AND MAINTENANCE OF TRANSMITTERS

(In this chapter the term "Modulator" refers to the Modulating Amplifier as in established B.B.C. practice.)

1. Starting Up a Transmitter.

THE procedure below applies generally to large transmitters, with small variations in special cases. In the case of transmitters without air and/or water cooling, the order of starting up still applies, omitting the appropriate paragraphs.

1. Connect the incoming power supply to the station bus-bars or appropriate bus-bars. In stations generating their own power, this involves starting engines and machines, but in most stations it is only necessary to make the appropriate circuit breakers. This step applies no power to the transmitter.

2. Start the water-cooling system. This involves starting the pumps and turning on the cooling water through the jackets surrounding valve anodes and filament seals. *Check all water-flow meters.*

3. Start the air-cooling system. This involves starting air-blowers and turning on the general air-blast to valve, including grid seal and filament seal air cooling. *Check all air-flow meters.*

4. Apply filament volts to early stages. *Check all filament voltages.*

5. *See that all transmitter doors are shut. Make all mechanical and electrical interlocks.*

6. Bring up filament volts slowly on final stages. *Check all filament volts.*

7. Switch on crystal drive and *check that adequate drive is provided* by reading the grid current on the first valve in the transmitter R.F. drive chain, or other means if provided.

8. Apply grid bias to all stages. *Check all grid-bias voltages.*

9. Apply full (auxiliary) H.T. to early stages. *Check value of H.T. and all anode feeds, grid currents and closed (tank) circuit currents.*

10. Apply a low value of (main) H.T. to final stage. *Check anode feeds; grid currents and closed-circuit currents.*

11. Retune if necessary. Retuning of the output circuit of the

final R.F. output amplifier may be necessary due to weather changes: adjustment of the output coupling may also be necessary. Retuning of the whole transmitter is necessary every time the transmitter changes wave. In the case of the B.B.C. short-wave transmitters, wave changing may happen several times a day.

Each stage may be tuned to minimum anode current, to maximum closed-circuit current or to maximum grid current on the following stage. The condition of minimum anode current is usually much more sharply defined than the other two conditions and should be used as a criterion when possible.

12. Come up slowly to full H.T. on final stages. *Check anode feeds, grid currents and closed-circuit currents.*

13. Adjust modulation meter, where provided, for carrier level.

14. Apply audio-frequency, and line-up transmitter for modulation; see XV:4.

1.1. Putting a New Valve into Service. When starting up after putting a new high-voltage valve of type specified in IX:18.3 into service, or a valve (of the same type) which has been out of service for more than 7 days, the anode voltage on this valve should be brought up slowly a thousand volts at a time, and should not exceed 5,000 volts for the first ten minutes. It should then be increased gradually to full voltage over a period of not less than 30 minutes. See IX:18.3 for full procedure, list of valves affected and permissible departures from this procedure.

Immediately after starting up after making circuit modifications or restoring the normal circuits after any kind of test, *except on class B transmitters*, an overmodulation test must be made. This is done by bringing up the programme input level so that the programme meter peaks up to 7, modulating the transmitter 100%; the input is then increased about 6 db. and the transmitter modulated in this condition for about 3 minutes, to check that no flashover occurs.

2. Shutting Down a Transmitter.

In the normal course of events shutting down takes place in the reverse order. The main H.T., auxiliary H.T., grid-bias and filament volts are run down and switched off in turn, followed by switching off the drive and shutting down of the air and water-cooling system. In the case of big valves such as CAT14 and 4030C, except in an emergency, the cooling water is left circulating for a few minutes after removing the power supplies from the valve. Finally, the machines, if any, are shut down and the power supplies

are removed from the appropriate bus-bars. The engines, where these are used, are then shut down.

3. Shutting Down in Extreme Emergency.

In an extreme emergency either the auxiliary or the main H.T. or both may be removed instantaneously. This is called tripping the H.T. voltage. Most transmitters are provided with a switch for tripping the H.T.

The drive should not be removed from a transmitter with H.T. on, unless it has been previously ascertained (by bringing the transmitter slowly up to full H.T. without drive) that no abnormal values of feed result by so doing. Provided, however, this has been done, the drive may be removed at any time to suppress arcing, or for any other purpose.

4. Line-up of Transmitter for Modulation.

The process of lining up the transmitter consists in adjusting the level of a tone (transmitted to the transmitter by line with a definite reference level at the sending end) incoming to the transmitter so that a prescribed value (usually about 40%) of modulation results. The procedure is the same whatever type of programme meter is used, except for the sensitivity adjustment of the programme meter. The programme meter is bridged across some point of the audio-frequency input circuit to the transmitter, and is graduated in seven steps of 4 db. marked from 0 to 7.

4.1. Old-Type Programme Meter. This type of programme meter effectively measures the mean value of the line level. Experience has shown that the effective ratio of peak level to the mean level as observed on this instrument is 8 db. higher in the case of programme than in the case of line-up tone.

The consequence is that, (since the difference between 40% mod. and 100% mod. is 8 db.) if when the line-up tone has been adjusted to modulate the transmitter 40%, the sensitivity of the P.M. is adjusted to make it peak up to 7, programme which peaks the P.M. up to 7 will modulate the transmitter 100% Since the P.M. observed by the control operator is adjusted in the same way, and since the control operator makes his P.M. peak up to 7 on modulation, under the above conditions the transmitter is modulated 100%.

Procedure: Line up transmitter to 40% mod. on line-up tone and adjust P.M. sensitivity to give a deflection up to 7.

In certain cases where it is required to modulate a transmitter

less or more heavily, the initial line-up modulation is changed from 40% (e.g. to 35% or 45%) and the P.M. setting remains unchanged.

4.2. Peak Programme Meter. (For circuit description see XX:14.) This type of programme meter measures the peak voltage of both programme and line-up tone and therefore introduces no discrimination between the two measurements.

To unify procedure an artificial discrimination of 8 db. is introduced by adjusting the sensitivity of the P.M. so that when the transmitter is modulated 40% by line-up tone the P.M. shows a deflection of 5. If the programme volume is now adjusted to swing the P.M. up to 7 the transmitter will be modulated 100%.

Procedure: Line up transmitter to 40% mod. on line-up tone and adjust P.M. sensitivity to give a deflection up to 5.

As before, the figure of 40% mod. may be varied as required.

5. Running Adjustments.

Certain adjustments are necessary from time to time while the transmitter is running, particularly in the first half-hour after starting up, in order to maintain the correct operating conditions.

If any of the supply voltages (H.T., grid bias or filament) change they should be readjusted to their correct values. Continual small variations of short duration, due, for instance, to variations of the supply mains voltage, cannot however be avoided by adjustment.

If the anode feed or grid current of any valve varies, the grid bias and H.T. of that stage or the preceding stage respectively should be checked. If the grid current of a stage changes and the preceding stage has the correct H.T. and grid bias, the values of grid current along the chain should be inspected to see if the R.F. drive has altered.

If the feed or output peak volts of the power amplifier in a low-power modulated transmitter vary, the main H.T. and the grid bias should be checked. If the variation is considerable this may indicate a condition of mistune or a fault in the aerial coupling circuits. Small but appreciable variations may be due to a change in weather conditions necessitating adjustment of the output coupling.

During the operation of a transmitter a periodical check on the operating conditions of each stage should be made.

6. Installation of Spare Valves.

If a valve fails the H.T. and grid-bias voltage should be removed from the stage concerned and from any stage supplying drive to the stage affected. All supplies should be removed from the

enclosure containing the faulty valve, and in certain transmitters this may mean shutting down the transmitter.

Failure of drive in an early drive stage may result in a rise in H.T. on subsequent stages, and if this occurs the H.T. on these stages should be reduced until the fault is rectified.

It is important to note that XV:1.1 *applies to the installation of spare valves.*

7. Changing Wave.

The means employed for changing wavelength differs considerably in different transmitters. In any one transmitter any of the following means may be employed in different stages:

1. Change of taps on inductances, and/or removal, and replacement of fixed condensers, e.g. the B.B.C. 50 kW "Regional" Transmitters at Brookman's Park, Washford, Westerglen and Moorside Edge. Adjustment of variable condensers.
2. Complete replacement of inductances as well as change of condensers as in (1).
3. Switching in of complete new tuning units, built into the transmitter.
4. Removal of tuning unit assembly, which is contained in a wheeled truck, and replacement by another.
5. Replacement of a stage tuned to one frequency by a stage tuned to another frequency.

The B.B.C. has examples of all four last-named in its short-wave transmitters at Daventry.

Whichever method is used, the same basic principle must be observed: *The reactances of all components must remain substantially unchanged by the change of component and change of wave.* If, therefore, the values of inductance and capacity are known for any one wavelength λ_1 corresponding to a frequency f_1, the values of the component for a new wavelength λ_2 corresponding to a frequency f_2 are obtained by multiplying the values of both inductance and capacity by $\frac{\lambda_2}{\lambda_1} = \frac{f_1}{f_2}$, where f_1 and f_2 are respectively the frequencies corresponding to λ_1 and λ_2. Allowance must of course be made for stray capacities.

The required values of mutual coupling remain unchanged, but in practice couplings must be adjusted to their correct values as part of the process of tuning.

In the case of routine wave changes which are made continually in accordance with the requirements of propagation, components

and variable condensers are set to marked values, and the transmitter is nearly in tune before it is run up. Tuning must, however, be carried out in this case, but is usually a very simple matter.

In the case where a transmitter is brought up for the first time, after the components have been adjusted to their new approximate values as determined by the principles above, considerably more care must be observed. Each stage must first be neutrodyned. The transmitter should be brought up on the lowest value of H.T. at which tuning can be carried out. This is because, when the anode circuit of a valve is out of tune, it may present a very low impedance towards the valve, with the consequence that, first, the valve will have a high anode current; secondly, it will deliver very little power to the output circuit so that nearly all the (increased) power supplied to the valve is dissipated in the valve anode and may give rise to valve failure; thirdly, it may oscillate.

The operation of tuning should be carried out starting at the lowest power stage and working towards the output. As each stage is brought into tune with low H.T. the H.T. may be progressively increased, retuning at each increase, until finally the stage is in tune with full H.T. At this stage the closed-circuit current should be examined to see how far it has deviated from its previous value. If the deviation is greater than about 15% the L to C ratio of the circuit should be adjusted: if the current is too high the inductance should be increased and the capacity reduced, and vice versa. An exception occurs in the case of short-wave transmitters, where, for instance, it may be necessary to tolerate a circulating current on 15 metres which is twice that on 40 metres, owing to the impossibility of reducing circuit capacities. If the circulating current is too high, overheating of the coils and connections (particularly the latter) may take place, while if it is too low, difficulty may be experienced in obtaining proper coupling in cases where the inductance is mutually coupled to another inductance supplying the following stage or the load. A high closed-circuit current may also cause the rating of any mica condensers in circuit to be exceeded.

The closed-circuit current is also important because it determines the kVA/kW ratio of the circuit which may be calculated very simply from the formula

$$\text{kVA/kW ratio} = \frac{\text{Anode peak volts} \times \text{Circulating current}}{\text{Power input to anode circuit} \times \text{Anode efficiency}}$$

In the absence of any other criterion for determining the correct value of circulating current it should be adjusted to give a kVA/kW

ratio of 5. In short-wave circuits it may, however, be necessary to tolerate a considerably higher value: e.g. 15 or slightly more.

The operation of tuning should include the adjustment of any mutual couplings to give the correct value of anode impedance facing each valve as determined by the peak volts and anode efficiency. In other words, any output coupling of a stage should be adjusted until the stage gives the normal efficiency when driven to give the required anode peak volts, or required grid volts on the following stage. The practical indication of this is that normal anode current is obtained when the valve is driven to give normal anode peak volts or normal output power. The condition of optimum coupling should be approached from the condition of minimum coupling, the process of tuning and adjustment of coupling being as described in VII:14.4.

When first applying power to the modulated amplifier of a class B modulated transmitter, after a wave change, or any other major circuit change, to guard against damage to the chokes and modulation transformer by parasitic oscillations, these should be shorted until the modulated amplifier has been stabilized or proved to be stable.

8. Reducing Power Output of a Transmitter.

It is sometimes required to reduce the power output of a transmitter. This may be achieved in a number of ways; the particular method used depends on the magnitude of the power reduction, the type of transmitter, whether the reduction is temporary or permanent and the time available to make the change.

The power output may be reduced:

1. By reducing the number of valves in the output stage: for permanent reduction, or for temporary changes where facilities exist.
2. By reducing the coupling between the output stage and the load, e.g. aerial feeder: for temporary or small reductions on large transmitters and permanent reductions on small transmitters.
3. By reducing the H.T.: For permanent changes where the number of valves cannot be reduced. Also used sometimes for small temporary changes where a very quick change is required.
4. By taking the output from an earlier stage: for permanent changes in all cases where it can be used.

Evidently certain of the above may be, and sometimes must be used in combination. Although some attempt has been made to indicate the occasions on which each method is used, the summary is not rigid and may be ignored where circumstances warrant.

8.1. Reducing the Number of Valves. In this case it is usual to maintain constant the operating conditions of the valves left in circuit, which is done by reducing the output coupling until, with the same value of grid drive, the anode feeds read the same as before the valves were removed. Under these conditions the anode peak volts, the efficiency and the power output per valve remain unchanged. The power reduction is therefore proportional to the number of valves taken out.

If the final stage is a modulated amplifier, though desirable for economy, it is not essential to remove valves from the modulator, but if valves are removed from the modulator care should be taken that the power-handling capacity of the modulator is not reduced more than the power reduction of the output stage. In general, as a safety precaution, when valves are removed from the modulator, the load line of the modulator should be drawn as follows in order to check that it is capable of supplying adequate voltage swing while maintaining linearity. The impedance presented by the modulated amplifier is given by

$$Z_m = \frac{\text{Modulated amplifier H.T.}}{\text{Total anode feed of modulated amplifier in the carrier condition}}$$

If Heising or series modulation is being used this gives the impedance facing the modulator. If the modulation is of the class B type (i.e. using a push-pull class B modulator) the impedance facing one-half of the push-pull is given by

$$Z_c = \frac{Z_m}{4} \times \frac{T_1^2}{T_2^2}$$

where T_1 is the total number of primary turns and T_2 is the total number of secondary turns on the modulating transformer. The load line Z_L is then equal to nZ_c, where n is the number of valves left in use on one side of the push-pull. This load line should be drawn on the field of anode characteristics for the type of valve in use in the modulator, through the point: anode volts = H.T. anode current = normal standing feed per valve in the absence of modulation.

The amount of the load line in use, and so the peak current, is determined by the intersection with the load line of the vertical through the point: anode volts = H.T. $-\hat{E}_a$, where $\hat{E}_a$ is the anode

peak volts on the modulator at 100% peak mod. The value of $\hat{E}_a$ is given by

$$\hat{E}_a = \frac{1}{2} \times \frac{T_1}{T_2} \times \text{H.T. on modulated amplifier.}$$

The field of valve characteristics must then be examined to see whether the characteristics drawn for equal increments of grid voltage are equally spaced over the part of the load line in use. Check should also be made that the peak (anode plus grid) current does not exceed about 0·9 of the total emission of the valve (this is unlikely) and that the mean current on 100% peak mod. with a sine wave input (which is equal to $\frac{1}{\pi}$ × the peak current) does not introduce too high an anode dissipation in the valves. For this purpose, the total power output of the modulator at 100% peak mod. may be subtracted from the total input anode power to give the anode dissipation. If it is found that the anode dissipation is too great, it does not necessarily prohibit the operating of the transmitter on programme, provided the dissipation on 50% mod. is not too great; it does, however, prevent the transmitter being tested with sine wave input up to 100% mod. See X.

Remember that after removing valves from R.F. stages the R.F. amplifiers concerned must be neuted before putting power on the circuit.

8.2. Reducing Coupling. If the output coupling of a class B or C amplifier is reduced and the circuit retuned, the impedance facing the anode is increased, and if the grid drive is adjusted to keep the anode peak volts constant the power output is reduced in inverse proportion to the increase of anode impedance. The standing feed (due to R.F. drive) is also reduced in approximately the same proportion, while the linearity of any R.F. power amplifier treated in this way is usually increased. Owing to the reduction in standing feed an economy in anode input power results, the percentage reduction being rather more than the percentage reduction of output power, since the impedance facing the anode is increased and so the efficiency is increased.

If the output amplifier treated in this way is a modulated amplifier the effect of reducing the output coupling and so increasing the anode impedance, is to increase the impedance facing the modulator in the same ratio that the standing feed of the modulated amplifier is reduced. In general, therefore, the modulator is made more linear by this procedure. The modulator also delivers less power for any given level of modulation and it would be a practical proposition to effect a further economy by reducing the power-handling capacity

of the modulator if the circuit arrangements permitted any of the means under discussion to be used.

Usually, however, reduction of output coupling is used only as a temporary expedient as far as large power reductions are concerned, and as a permanent expedient only for small power reductions or on small transmitters. In none of these cases is modification of the modulator worth while.

8.3. Reduction of H.T. This may be used for a permanent reduction of large amount when it is not possible to reduce the number of valves. It also provides a rapid method of providing a temporary power reduction when little time is available for making the change. The adjustments in the two cases are quite different.

In the case of a large permanent change, the stages involved are re-designed for the new value of H.T. and power output, in terms of anode impedance, grid drive and grid bias.

The possibility of using H.T. reduction for small temporary changes depends on the fact that if the H.T. is only reduced a small amount, e.g. 20%, linearity can be preserved sufficiently without any other change being made. In the case of low-power modulated transmitters, the H.T. should not be reduced by an amount greater than that which reduces the no-drive standing feed of the R.F. amplifier stages by about a third of the usual value. In the case of a class B high-power modulated transmitter, since the R.F. carrier is linearly proportional to the instantaneous value of the total H.T., as far as the modulated amplifier is concerned, a greater reduction of H.T. than 20% is permissible. Since, however, the H.T. is usually common to both modulator and mod. amp., the H.T. should not normally be reduced to a value below that at which the standing feed of the modulating amplifier is reduced by about a third. If it is required to be reduced further the bias on the modulating amplifier should be adjusted to maintain the standing feed constant at its normal value.

When H.T. is reduced the output of the transmitter should be monitored to check that undue distortion is not introduced.

9. Operation of Transmitters in Parallel on the Same Wavelength.

Provided the carrier-frequency voltages *and* audio-frequency envelopes of two transmitters are in phase and of equal amplitude, the outputs of the transmitters may be connected in parallel to supply to the load twice the power of either transmitter operating alone.

When changing over from the condition of one transmitter

operating alone to two transmitters in parallel, the load impedance at the point where paralleling takes place must be halved. In the same way, three transmitters may be operated into a third of the load impedance, or four transmitters into a quarter of the load impedance.

Transmitters which are to be operated in parallel in this way must be provided with means for adjusting the phase of the carrier drive at the input to the transmitter. A cathode-ray oscillograph should be connected to the output of the transmitter so that the phase of the output carrier voltages of each transmitter can be adjusted to equality *before* the transmitters are connected in parallel. It is very important that phasing should be adjusted correctly, otherwise each transmitter is faced with a reactive load and a falling off in efficiency results.

Parallel operation is usually confined to identical transmitters, in which case the change of envelope phase shift with frequency is the same in each transmitter. In this case, in order to ensure that the envelope phase is the same at the output of each transmitter, it is only necessary to ensure that the inputs of the transmitters are connected together in the correct sense.

Short-wave Transmitters Supplying the Same or Different Service Areas with the Same Programme on the Same Wavelength. Two short-wave transmitters supplied with the same programme are sometimes operated on the same wavelength and connected to arrays so that in certain locations their service areas overlap. This provides a means of economizing in short-wave channels.

While this is a straight case of common wave working, experiments indicate that the degradation of the quality of reception is negligible, probably because even when one transmitter is radiating alone the wave reaches the receiver by more than one path subject to random variation of phase shift. An increase of the audio-frequency output of the receiver is obtained by arranging that the transmitters are driven with audio frequency in the correct phase. Where common service areas occur, in such cases it is therefore customary to drive the transmitters with audio frequency in such sense that the envelopes of the radiated wave are in phase. It will be evident that the extent to which the envelopes can be brought into phase over the whole audio-frequency band depends on the extent to which the variation of audio-frequency phase shift with frequency (including envelope phase shift in the R.F. stages) is the same in each transmitter.

10. Prevention and Clearance of Faults.

The primary function of a maintenance engineer is to keep breakdown time to a minimum. To this end a proper maintenance routine must be established for every transmitter and its associated apparatus, while all possible steps must be taken to ensure the rapid location and clearance of all faults which occur. In the case of valves it is possible to some extent to anticipate failure by making periodic tests on all valves in service. Any valve which is outside prescribed limits is then thrown out of service.

10.1. Maintenance Routines on Valves. In the case of transmitter valves every valve in stock should be put into service for a short period once every six months.

A weekly gas test (see IX:18.4) should be carried out, on every transmitter valve in service.

A monthly test of mutual conductance should be made on all small valves in service.

Emission tests (see IX:18.6) are only carried out if the performance of a valve is suspected. Non-linear distortion is sometimes due to failure of emission, and any valve which shows distortion when operating on marked filament volts should be rejected.

10.2. Maintenance Routines on Transmitters. These should include all the equipment associated with the transmitter for power supply, valve cooling, etc. Every transmitter has its own idiosyncrasies and the maintenance routine on each transmitter should be designed to give special service to the parts which are known to need most attention. The following lists of points which require attention therefore constitute examples only: on some transmitters some of the items would not occur, on others extra items would appear: further points relating to transmitters proper have been included.

Daily Maintenance. Inspect all cooling hoses, both water and air. Look for any signs of overheating or flashover, and see that all connections are firm.

Weekly Maintenance. Overhaul and clean all transmitter units.

Remove all dust from all parts of transmitter, and see that plates of all high-voltage air condensers, spark gaps and high-voltage conductors are clean, free from abrasure and polished.

Check that all nuts and connections are tight, that all switches,

contactors and relays are in adjustment and that all relay contacts and switch contacts are clean and bright.

Check that all fuses and resistances are firmly held in their holders and make good contact with their terminals.

Check that no conductor or other element has become displaced.

In the case of transmitters with spare valves mounted in the units, interchange the working valve with the spare valve.

Points Requiring Attention Monthly. Check all that protection devices are sound both mechanically and electrically: interlock, water- and air-flow protection relays and alarms, etc.

Check oil levels in transformers, condensers and chokes.

Measure anode to earth resistance of water-cooled valves.

Check stock of fuses and of any component which is known to constitute a weak point.

All stocks of components should be checked every three months and replacements ordered to bring the stock up to complement. *In the case of large components replacements should be ordered immediately each component is used.*

10.3. Clearing Faults. In order that faults shall be dealt with rapidly it is necessary:

(*a*) that complete and up-to-date circuits of all units and all inter-unit connections, together with all necessary operating instructions, shall be available *at all times to all engineers who may at any time be concerned*;

(*b*) that all maintenance engineers concerned shall be intimately familiar both with the drawings of these circuits and with the physical layout of components and conductors, and shall understand the principle and mode of operation of all parts of the circuit, and all mechanical accessories;

(*c*) that tables of adjustments, dial settings and meter readings shall be permanently posted near the transmitter;

(*d*) that all staff concerned are advised immediately of any change of circuit setting, or adjustment, and are given all corresponding operating instructions;

(*e*) that all maintenance engineers shall be familiar with the general symptoms of normal faults and the symptoms of particular faults which are likely to occur on any particular transmitter for which they are responsible;

(*f*) that all necessary spares are always maintained in stock and that all staff know exactly where these are to be found;

(g) that all necessary tools and instruments are always available and in good repair.

10.31. Fault Analysis. Every maintenance engineer should prepare for himself a fault analysis of every transmitter he has to maintain. This consists of a list of: (1) fault symptoms which may be observed on a transmitter, (2) their immediate and (3) their ultimate cause, and (4) the steps taken in location.

11. Flashover.

11.1. Procedure when a Flashover Occurs. When an arc is formed due to voltage breakdown the procedure is as follows. If the arc immediately clears itself, or is suppressed by removal of the H.T. voltage from the transmitter, and no fault condition follows, no action is necessary unless the flashover occurs frequently, in which case an investigation must be made to find the cause.

Where automatic means are not provided for temporary removal of the H.T. this should be done manually if the arc persists.

If a fault condition follows the flashover this must be treated as a normal fault.

In all cases of flashover, at the end of the programme transmission, the locality of the flashover must be inspected and all damage caused by the flashover must be removed. This involves replacing faulty insulators, cleaning all insulating surfaces on which sputtered metal, carbon, or the products of burnt insulation, have been deposited, and removing all charred wood from the wooden formers of inductances. In the case of air condensers the small pock-marks due to the passing of the spark should be smoothed down with fine emery paper or metal polish. In the case of oil-filled condensers, the oil should be removed and scrapped, and all deposits due to the flashover should be thoroughly cleaned off.

11.2. Causes of Flashover. These are many. The unpardonable causes are inadequate clearances and sharp points on conductors. Other causes are voltage surges in the mains, direct strikes and induced surges in the aerial and feeder system by lightning, and fault conditions which give rise to high voltages. Among the last is impedance mismatch in the system comprised by the output coupling of the transmitter, the aerial feeder and aerial.

A cause of failure of wooden formers of R.F. inductances in high-power circuits is the presence of metal in close proximity to the wood. For this reason such formers are held together by wooden dowels. It is not clear whether this effect is due to the electric or the magnetic field. It appears that it is probably due to brushing from the metal caused by the electric field, in which case it is reasonable to classify it under voltage breakdown, although the symptoms are different from those of flashover: the metal becomes hot and the wood catches fire. For this reason ceramic formers are sometimes used instead of wood.

CHAPTER XVI

FEEDERS, AERIAL-COUPLING CIRCUITS AND AERIALS

THIS chapter is written to provide information for maintenance engineers at transmitter stations. It deals with aerials only so far as station engineers are concerned with their characteristics. The treatment therefore does not conform to a normally balanced treatment of the subject: for instance, fairly complete information is provided on the mechanical design and the behaviour of short-wave aerials, while no information is given on the theoretical design of these aerials in order to obtain a given required performance. The treatment of medium- and long-wave aerials is still more scanty. A treatment of aerials, even when limited to a minimum statement of the essential useful design information, is a proper subject for several textbooks.

As the discussion is rather mathematical, many readers will be grateful for some indication of an intelligent way of avoiding the analytical treatment. It should, however, be pointed out that the mathematical discussion has been simplified to an extreme degree: relevant transmission formulae are developed from first principles by new short methods which avoid the use of hyperbolic functions. Exceptions occur in the case of the formulae for propagation constant, attenuation constant, phase constant and characteristic impedance; these are stated without being proved.

The first section below which contains any mathematics is XVI:1.74 and the conclusions of this section are given in equation (35), which defines the sending end impedance of a line open circuited at the far end. Similarly, the conclusions of section XVI:1.75 are embodied in equation (38). These equations contain all the useful information that the practical engineer need extract from their respective sections.

The sections before XVI:1.74 appear to contain tiresome mathematical arguments, but if they are read carefully it will be seen that this is not the case, and they *should* be read carefully.

The whole purpose of section XVI:1.8 is to derive equations (46) and (47)/XVI:1, which are the most important equations in the whole chapter since they relate the terminating impedance and the sending-end impedance of a line.

The whole of XVI:2 which deals with Circle Diagrams must be understood thoroughly. The circle diagram embodies equations (46) and (47) in graphical form which saves an immeasurable amount of tedious calculation.

The intervening sections, up to XVI:11 inclusive, must be thoroughly digested except that in XVI:8.4 equations (6) and (6*a*) may be accepted as the conclusions without reading the proof.

XVI:12 deals with impedance matching in long-wave aerial circuits and presents a simple but complete description of the method of design of such circuits. The method of presentation is to derive the formulae which are obtained by simple arguments and to state the formulae which depend on rather more weighty analysis. This analysis is given in CIII for the benefit of those who want to refer to it.

1. The Behaviour of Feeders.

Conventions. Applicable to Section 1 only.

C = capacity per unit length.

D = spacing between conductors (of a line or feeder).

d = diameter of a conductor.

d_1 and d_2 = diameter of inner conductor and inner diameter of outer conductor of a concentric cable respectively.

E_0 = sending-end voltage = $\hat{e}_0 \sin \omega t$.

$\hat{e}_0$ = peak value of sending-end voltage.

e_0 = component of sending-end voltage giving rise to incident wave.

e_0' = component of sending-end voltage opposing reflected wave.

E_x = voltage between conductors of a feeder at distance x from the sending end.

e_x = component of voltage distance x along a line, due to the incident wave.

e_x = component of voltage distance x along a line, due to the reflected wave.

E_i = incident voltage.

E_r = reflected voltage.

f = frequency in c/s.

G = leakance per unit length.

h = height above ground.

I_0 = sending-end current = $\hat{i}_0 \sin \omega t$.

$\hat{i}_0$ = peak value of sending-end current.

I_x = current in a feeder at distance x from the sending end.

I_i = incident current.
I_r = reflected current.
I_t = transmitted current.
L = loop inductance per unit length.
l = length of transmission line.
m = an integer.
P = propagation constant per unit length of feeder = $\beta + j\alpha$.
R = loop resistance per unit length.
Z_0 = characteristic impedance (of a feeder or a line).
Z_s = sending-end impedance.
Z = impedance terminating a transmission line.
α = phase-shift constant per unit length of feeder.
β = attenuation constant per unit length of feeder.
λ = wavelength.
θ = angle of phase shift or angle of an impedance.
$\omega = 2\pi f$.

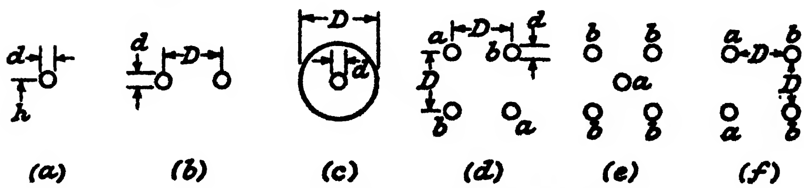


FIG. 1/XVI:1.—Types of Feeder.

The connecting link between a transmitter and a remote aerial is called a feeder. Six types of feeder are in general use. These are illustrated in Fig. 1.

The type at (*a*) is a single wire above ground and is normally used only for short lengths. It is not used for long lengths on account of earth losses and tendency to radiate and pick up energy from fields due to external sources.

The type at (*b*) is a balanced pair.

At (*c*) is shown a concentric cable, which is the best of all types from the point of view of radiation and pick up.

The type at (*d*) is known as a four-wire feeder. In this the two conductors *a* constitute one leg of the circuit and the two conductors *b* the other leg of the circuit. This type of circuit has an advantage over the types at (*a*) and (*b*) in that it is less liable to radiation and pick up.

The type at (*e*) is known as a five-wire feeder and is a cheap approximation to a concentric feeder; the centre conductor constitutes one leg of the circuit and the remaining four conductors in parallel constitute the other leg of the circuit. A closer approxi-

mation to a concentric feeder may be made by using a greater number of wires in the outer ring.

At (*f*) is shown a second type of four-wire feeder which is equivalent to two feeders of the type shown at (*b*), one above the other, connected in parallel. For the same conductor diameter and spacing as in the type at (*b*) the characteristic impedance of the feeder at (*f*) is 0·6 multiplied by the characteristic impedance of (*b*).

The types of feeder at (*b*) and (*f*) are in common use on short-wave circuits, while long- and medium-wave circuits use, in addition to these, the arrangements at (*c*), (*d*) and (*e*).

A feeder is a particular case of a transmission line and the formulae below define the performance of feeders considered as transmission lines. In the case of feeders, however, it is customary to use certain approximate formulae which are entirely satisfactory for the problems presented by feeders but are not always applicable to all other transmission-line problems.

A transmission line is characterized by:

1. *A loop resistance* R ohms per unit length. This is $(1/n) \times$ the resistance measured (in the absence of leakance, inductance and capacity) looking into the near-end terminals of n times unit length of the transmission line with the far end closed, where n tends to infinity.

2. *A leakance* G mhos per unit length. This is $(1/n) \times$ the conductance between legs which would be measured looking into the near-end terminals of n times unit length of the transmission line (in the absence of resistance, inductance and capacitance) with the far end open, where n tends to infinity.

3. *A loop inductance* L Henrys per unit length. In engineering practice this is $(1/n) \times$ the inductance which would be measured (in the absence of mutual capacity between legs and with $R = G = 0$) looking into the near-end terminals of n times unit length of the transmission line with the far end closed, where n tends to infinity.

4. *A capacity* C Farads per unit length. This is $(1/n) \times$ the capacity measured looking into the near-end terminals of n times unit length of the transmission line with $(L = R = G = 0)$ with the far end open, where n tends to infinity.

It is evident that these quantities are ideal quantities of which the use can only be justified by their usefulness : these quantities can be calculated and from them are determined the characteristic impedance and propagation constant of the transmission line.

In the case of a balanced line the loop resistance at any frequency is equal to twice the resistance of a single conductor of the same length as the line.

The values of loop inductance and capacity for three types of feeders are given in XVI:1.2 below.

1.1. Characteristic Impedance = Z_0. The impedance looking into one end of a transmission line of infinite length having uniformly distributed constants R, G, L and C is called the *characteristic impedance* and is indicated by the symbol Z_0. The value of Z_0 is given by :

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \quad . \quad . \quad . \quad . \quad (1)$$

In practice the magnitudes of R and G are respectively small compared with $j\omega L$ and $j\omega C$, so that for most practical purposes :

$$= \sqrt{\frac{L}{C}} \quad (2)$$

and is a pure resistance : i.e. an impedance of zero angle.

(It should be noted that while R and G may be neglected in computing characteristic impedance, to which they contribute a small component only, they constitute the only components of attenuation, apart from radiation, which is very small.)

The image impedances at the input and output of any length of a uniform transmission line are equal to one another and equal to Z_0.

1.2. Loop Inductance and Capacity per Unit Length and Characteristic Impedances of Three Types of Feeder.

Single Conductor : diameter d, height h above ground.

$$L = 7{\cdot}4 \times 10^{-4}\left[\log_{10} 4\frac{h}{d} + 0{\cdot}109\right] \text{ Henrys per mile} \quad . \quad (3)$$

$$C = \frac{0{\cdot}0388 \times 10^{-6}}{\log_{10} 4\frac{h}{d}} \text{ Farads per mile} \quad . \quad . \quad . \quad . \quad (4)$$

Since $\log_{10} 4\frac{h}{d}$ is usually large compared with 0·109

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{7{\cdot}4 \times 10^{-4}}{0{\cdot}0388 \times 10^{-6}}} \log_{10} 4\frac{h}{d} \text{ ohms}$$

$$= 138 \log_{10} 4\frac{h}{d} \text{ ohms} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Balanced Pair: conductor diameter d, spacing D.

$$L = 1{\cdot}48 \times 10^{-3} \left[\log_{10} 2\frac{D}{d} + 0{\cdot}109\right] \text{ Henrys per mile} \quad . \quad (6)$$

$$C = \frac{0{\cdot}0194 \times 10^{-6}}{\log_{10} 2\frac{D}{d}} \text{ Farads per mile} \quad . \quad . \quad . \quad (7)$$

$$Z_0 = \sqrt{\frac{1{\cdot}48 \times 10^{-3}}{0{\cdot}0194 \times 10^{-6}}} \log_{10} 2\frac{D}{d}$$

$$= 276 \log_{10} 2\frac{D}{d} \text{ ohms} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Concentric Cable; diameter of inner conductor d_1, inner diameter of outer conductor d_2.

$$L = 7{\cdot}4 \times 10^{-4} \left[\log_{10} \frac{d_2}{d_1} + 0{\cdot}109\right] \text{ Henrys per mile} \quad . \quad (9)$$

$$C = \frac{0{\cdot}0388 \times 10^{-6}}{\log_{10} \frac{d_2}{d_1}} \text{ Farads per mile} \quad . \quad . \quad . \quad (10)$$

$$Z_0 = \sqrt{\frac{7{\cdot}4 \times 10^{-4}}{0{\cdot}0388 \times 10^{-6}}} \log_{10} \frac{d_2}{d_1} \text{ ohms}$$

$$= 138 \log_{10} \frac{d_2}{d_1} \text{ ohms} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The above values of Z_0 are plotted in Fig. 2, together with the value of Z_0 for the special type of four-wire feeder in which each leg is constituted by one pair of diagonally opposed conductors. This arrangement has the advantage of minimizing interference. On Fig. 2 is also shown the effective characteristic impedance of a single vertical wire running from ground to a height h above ground, as it *appears* to a generator applied between the lower end of the wire and ground. This is rather an artificial concept, but a useful one on occasions, see XVI:13.1.

Fig. 1 (*e*) shows a five-wire feeder which is coming into general use as a cheap substitute for a concentric feeder which is comparatively expensive. The four outer conductors are joined together and take the place of the outer conductor in a concentric feeder. The centre conductor takes the place of the inner conductor in a concentric feeder. When the outer conductors are not grounded the impedance of this feeder is given by:

$$Z_0 = 69 \log_{10} \frac{D^2\sqrt{2D}}{d^2\sqrt{d}}$$

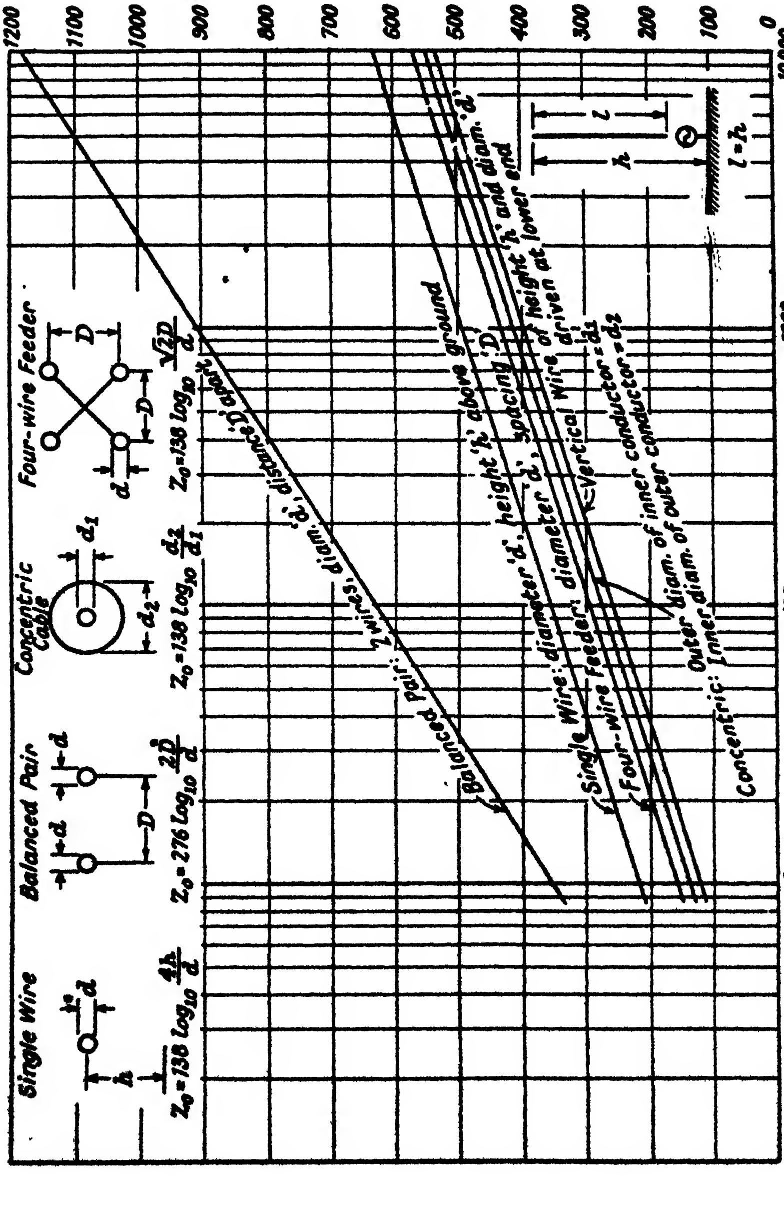


FIG. 2/XVI:1.—Characteristic Impedance of Feeders.

The feeder is always used with the outer conductors grounded, and under this condition the above formula is only approximate. The exact value of impedance with the outer conductors grounded is given by:

$$Z_0 = \frac{69\left(\log_{10}\frac{256h^5}{dD^4}\right)\left(\log_{10}\frac{D^2\sqrt{2D}}{d^2\sqrt{d}}\right)}{\log_{10}\frac{256h^5}{dD^4}+\frac{1}{10}\log_{10}\frac{D^2\sqrt{2D}}{d^2\sqrt{d}}}$$

I am indebted to H. Page for these two formulae, but we have been unable to discover the original author.

It will be found that in many cases the difference in impedance given by the two formulae is not large. Except where experience has shown this to be the case the last formulae should, however, be used.

1.3. Propagation Constant, Attenuation Constant and Phase Shift Constant. Relations between Voltages and Currents in Properly Terminated Feeder and Sending-End Voltages and Currents. For the practical radio engineer a full discussion of the propagation constant introduces unnecessary complication.

The explanation below, therefore, describes only the method of approximation which is employed in practice in connection with radio-frequency feeders. It will be understood further that all equations apply to the *steady state* condition which exists after all initial disturbances consequent on the application of the e.m.f. to the circuit have died away.

When a sinusoidal e.m.f. E_s of frequency f and angular frequency ω is applied to the input of a uniform transmission line of infinite length (or terminated in Z_0), a wave travels along the transmission line which gives rise to current I_x and voltage E_x at any point along the line distant x from the sending end defined by—

$$E_x = E_s\epsilon^{-Px} \qquad \text{(12)}$$

and

$$I_x = \frac{E_s\epsilon^{-Px}}{Z_0} \qquad \text{(13)}$$

where P is the propagation constant per unit length and ϵ is the base of Naperian logarithms = 2·71828 . . .

The propagation constant is a complex quantity of magnitude determined by the loop resistance R, the inductance L, the capacity C, and the leakance G, all per unit length. The fundamental formula is

$$P = \sqrt{(R+j\omega L)(G+j\omega C)}$$
$$= \sqrt[4]{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} \angle \tfrac{1}{2}\tan^{-1}\frac{\omega L}{R}+\tfrac{1}{2}\tan^{-1}\frac{\omega C}{G}$$
$$= \beta + j\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where

$$\beta = \sqrt[4]{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} \cos\left(\frac{1}{2}\tan^{-1}\frac{\omega L}{R}+\frac{1}{2}\tan^{-1}\frac{\omega C}{G}\right) \quad (14a)$$

and

$$\alpha = \sqrt[4]{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} \sin\left(\frac{1}{2}\tan^{-1}\frac{\omega L}{R}+\frac{1}{2}\tan^{-1}\frac{\omega C}{G}\right) \quad (14b)$$

The figure 4 before the root sign indicates that the fourth root is to be taken: 4 is *not* a multiplier.

If trigonometrical tables are available the above formulae are probably the most convenient to use for determining the values of β and α.

When such tables are not available the conventional formulae may be used. These are as follows, see Appendix VI, *Telephone Transmission*, by J. G. Hill.

$$\beta = \sqrt{\tfrac{1}{2}\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}+\tfrac{1}{2}(GR-\omega^2LC)} \quad . \quad (15)$$

When $\omega L >> R$ and $\omega C >> G$

$$\beta = \frac{R}{2}\sqrt{\frac{C}{L}}+\frac{G}{2}\sqrt{\frac{L}{C}} = \frac{R}{2Z_0}+\frac{GZ_0}{2} \quad . \quad . \quad . \quad . \quad . \quad (16)$$

and when G is negligible

$$\beta \doteq \frac{R}{2Z_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The quantity α is given by

$$\alpha = \sqrt{\tfrac{1}{2}\sqrt{(G^2+\omega^2C^2)(R^2+\omega^2L^2)} - \tfrac{1}{2}(GR-\omega^2LC)} \quad . \quad (18)$$

When $\omega L >> R$ and $\omega C >> G$ (see previous reference)

$$\alpha = \sqrt{\omega^2CL+\left(\frac{RC-GL}{2\sqrt{LC}}\right)^2} = \omega\sqrt{LC} \text{ when } RC = GL \quad (19)$$

and when G is negligible

$$\alpha = \sqrt{\omega^2CL+\frac{R^2C}{4L}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$
$$= \omega\sqrt{LC} \text{ when } R \text{ and } G \text{ are both negligible} \quad . \quad (20a)$$

β is called the *attenuation constant* and α is called the *phase-shift constant*. The chief use of the above equations in radio-frequency work is to show how the attenuation and phase shift vary with the

circuit constants. The calculation of attenuation and phase shift is seldom a practical proposition owing to the difficulty in determining the circuit leakance, except by a direct measurement of attenuation. The phase shift is usually assumed to correspond to a phase velocity equal to that of light: see XVI:1.5, 1.9 and 7.8.

Substituting the value of P from equation (14) in equation (12)

$$E_x = E_0 e^{-(\beta+j\alpha)x} = E_0 e^{-\beta x} \times e^{-j\alpha x} = E_0 e^{-\beta x}\angle\overline{\theta} \text{ where } \theta = \alpha x \quad (21)$$

Once the value of β and x are known the value of $e^{-\beta x}$ can be calculated and appears as a numerical fraction less than unity. *The voltage at any point along the line distance x from the sending end is, therefore, reduced in amplitude or attenuated by an amount corresponding to the fraction $e^{-\beta x}$ and retarded in phase (compared with E_0) by an angle $\theta = \alpha x$, i.e. E_x lags on E_0, by an angle $\theta = \alpha x$.*

Similarly, the current at any point in the line distant x from the source is given by—

$$I_x = \frac{E_0}{Z_0} e^{-\beta x}\angle\overline{\alpha x} \quad . \quad . \quad . \quad . \quad (22)$$

The transformation $e^{-j\theta} = 1\angle\overline{\theta}$ or $e^{j\theta} = 1\angle\theta$ is discussed in III:5. It follows from de Moivre's theorem $e^{j\theta} = \cos\theta + j \sin\theta$ $= \sqrt{\cos^2 \theta + \sin^2 \theta} \;\angle \tan^{-1}\dfrac{\sin\theta}{\cos\theta} = 1\angle\theta$, since $\cos^2\theta + \sin^2\theta = 1$.

Since equations (21) and (22) represent respectively voltages and currents lagging on the sending-end voltages and currents, these equations may be re-written, for the case where E_0 is written in full as $E_0 = \hat{e}_0 \sin(\omega t)$, as follows—

$$E_x = \hat{e}_0\, e^{-\beta x} \sin(\omega t - \alpha x) \quad . \quad . \quad . \quad (23)$$

$$I_x = \frac{\hat{e}_0}{Z_0} e^{-\beta x} \sin(\omega t - \alpha x) \quad . \quad . \quad . \quad (24)$$

Care should be taken not to confuse the conventions $\hat{e}_0$ = peak voltage and e = the Naperian base.

The instantaneous values of voltage and current along the line at any instant of time t may be found by inserting the value of t in equations (23) and (24). For instance, suppose $t = \dfrac{m}{f}$ where m is an integer and $f = \dfrac{\omega}{2\pi}$, then $\omega t = m \times 2\pi$ and

$$\sin(\omega t - \alpha x) = \sin(m \times 2\pi - \alpha x) = \sin - \alpha x = -\sin \alpha x.$$

Hence, at any time $\dfrac{m}{f}$ when the steady state has been established

$$E_x = \hat{e}_0\, e^{-\beta x} \sin(-\alpha x) \quad . \quad . \quad . \quad (25)$$

If this curve is plotted as a function of x there is seen to be at every instant a wave of voltage in the line such as is indicated in Fig. 3 by the full line.

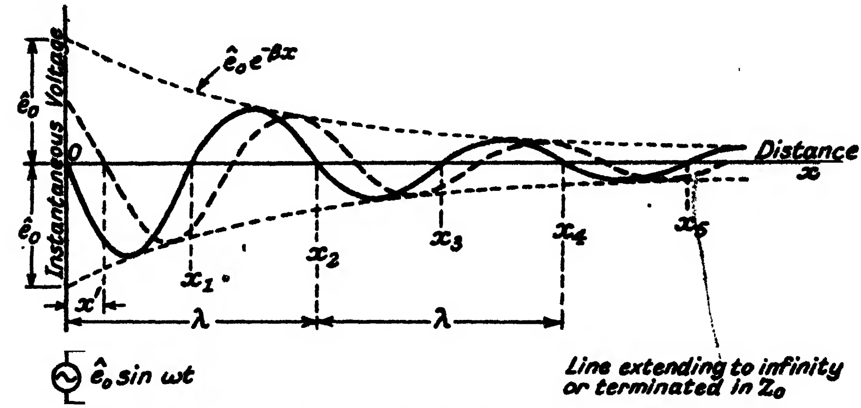


FIG. 3/XVI:1.—Instantaneous Values of Voltage along an Infinite Transmission Line.

It will be evident that such a rapid rate of falling away in amplitude represents a value of β much larger than would be tolerable in a feeder in practice.

1.4. Relation between Wavelength λ and Phase Constant α. Referring to Fig. 3, the distances between any neighbouring even-numbered points 0, x_2, x_4, etc., or between any neighbouring odd-numbered points x_1, x_3, x_5, etc., at which the voltage is zero, are all equal. This distance is called the wavelength λ.

Referring to equation (25), it is evident that when the distance x along the transmission line increases from, say, Ox_2 to Ox_4, the value of αx must change by 2π, i.e. at the same time that x changes by λ.

Hence $$\alpha\lambda = 2\pi$$

or $$\lambda = \frac{2\pi}{\alpha} \quad . \quad . \quad . \quad . \quad . \quad (26)$$

The same equation can evidently be derived by considering the current wave in the line.

1.5. Phase Velocity of a Progressive Wave. Relations between Phase Velocity, Wavelength and Frequency. The full-line curve in Fig. 3 is a picture of the instantaneous voltage distribution at some time $t = \frac{m}{f}$. Consider the voltage distribution at some subsequent time t' seconds later when ωt has increased by a small angle ϕ.

Evidently $\phi = \omega t'$

and $$E_x = \hat{e}_0 \, \varepsilon^{-\beta x} \sin(\omega t' - \alpha x) \quad . \quad . \quad . \quad (27)$$

The resultant curve of voltage is plotted as the dotted line in Fig. 3; it is evidently a curve in which the points of zero voltage x_1, x_2, x_3, etc., have been shifted a distance x' to the right. It is to be noticed that whereas previously the first point of zero voltage occurred when $\alpha x = 0$, see equation (25), the first point of zero voltage now occurs when $\omega t' = \alpha x'$, see equation (27). The first zero point has therefore advanced along the line a distance x' in time t', and since $\omega t' = \alpha x'$, the velocity of movement of the zero point is—

$$V = \frac{x'}{t'} = \frac{\omega}{\alpha} = \frac{2\pi f}{\alpha} = \lambda f \text{ (see (26))} \quad . \quad . \quad (28)$$

As the value of t' is increased, so the position of the wave of instantaneous voltages progressively moves along the line. The wave is, therefore, called a *progressive wave* and its velocity of travel along the line is given by equation (28). This velocity is called the *phase velocity*.

This velocity is not the velocity at which a transient disturbance travels along the line consequent on a suddenly applied voltage at the sending end; this velocity is called the *group velocity* (see VIII:3.2) and is not usually of importance in connection with feeder problems. In most problems on feeders it is usual to assume *as a first approximation* that the phase velocity, or velocity of propagation as it is generally called (with little justification), is equal to the velocity of light; 300 million metres a second. It is the phase velocity and not the group velocity which is involved in all calculations of wavelength, and equation (28) gives the necessary relation. For approximate calculations of wavelength in aerials a "velocity factor" of 0·95 is used: the velocity of propagation in the aerial is assumed to be 0·95 times the velocity of light.

In problems where an accurate determination of wavelength in feeders is important it is usual to make a determination by measurement, of the length of feeder occupied by a quarter-wave in the line at the frequency concerned. The method of doing this is described below in XVI:1.9.

1.6. Attenuation of Feeders. The attenuation of a feeder may, however, be calculated as follows, assuming it to be terminated in its characteristic impedance, provided the leakance is known.

If E_0 is the sending-end voltage and E_x is the voltage at any point P distance x along the feeder, the attenuation to point P in decibels is given by

$$L = 20 \log_{10} \frac{E_0}{E_x} = 20 \log_{10} e^{\beta x} \text{ (see equation (12))}$$

$$= 20\beta x \log_{10} e = 4{\cdot}343\left(\frac{Rx}{Z_0} + GxZ_0\right) \text{ decibels} \quad . \quad (29)$$

(see equation (16))

In open-wire lines of 500 ohms impedance GZ_0 is about 70% of $\frac{R}{Z_0}$ so that, if the leakance is unknown the attenuation may be taken as equal to 1·7 times the attenuation due to resistance.

As a rule of thumb the multiplying factor to convert resistance loss to total loss is $1 + 0{\cdot}7 \times \frac{Z_0}{500}$. See F. C. McLean and F. D. Bolt, *I.E.E. Jour.*, May 1946, p. 192.

Example. What is the attenuation per 1,000 ft. at 10 Mc/s of a balanced open-wire line consisting of 0·192-in. copper conductors with 12-in. spacing.

The ratio spacing/diameter $= \frac{D}{d} = 62{\cdot}5$, hence from Fig. 2

$$Z_0 = 580\ \Omega$$

At 10 Mc/s the resistance per 2,000 ft. of 0·192-in. conductor is 31·8 ohms. See numerical example No. 9 in II:99.

Hence $L = 4{\cdot}343 \times \frac{R}{Z_0} = 4{\cdot}343 \times \frac{31{\cdot}8}{580} = 0{\cdot}239$ db.

and multiplying by 1·8 to allow for leakance gives 0·43 db.

1.7. Standing Waves. In what follows, attenuation is neglected, i.e. assumed to be zero. If a generator is applied to the sending end of a transmission line of finite length which is either open circuited or short circuited at the far end, a progressive wave which will be called the incident wave, travels along the line away from the generator and is completely reflected so that there are two progressive waves of equal amplitude in the feeder travelling in opposite directions.

1.71. Instantaneous Voltages on an Open-Circuited Line. The case of an open-circuited line (i.e. a line open circuited at the far end) is illustrated in Figs. 4 and 5.

Fig. 4A gives a plot of the two voltage waves which are present on an open-circuited line at some instant. Figs. 4B to 4I show this

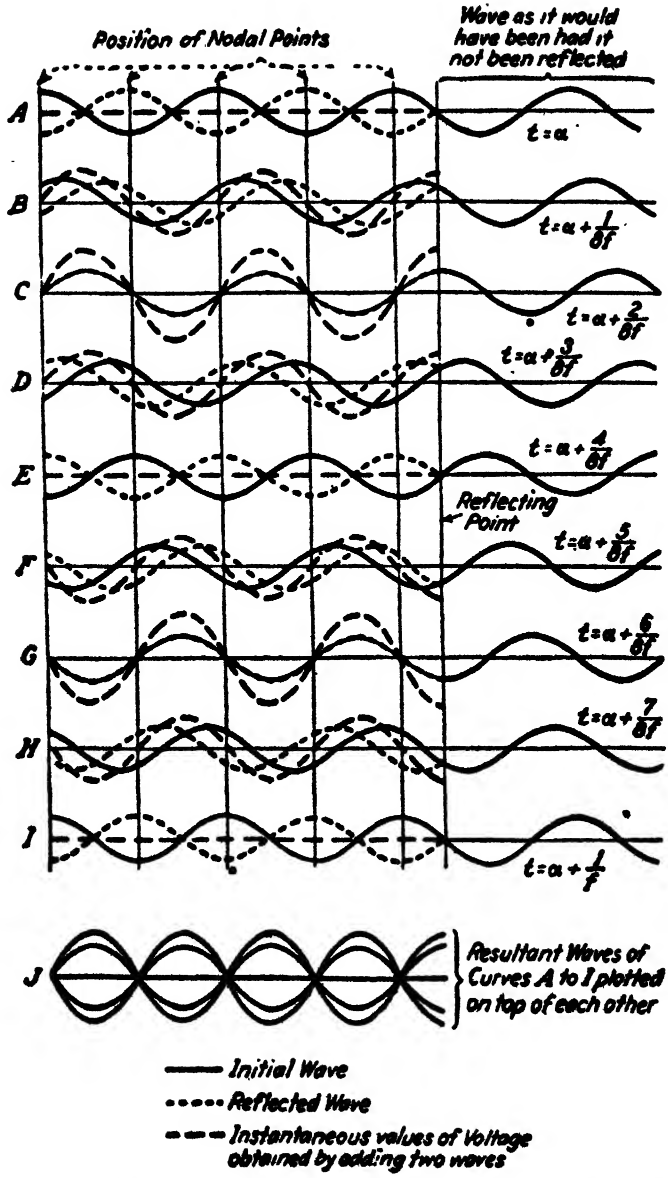


FIG. 4/XVI:1.—Instantaneous Distribution of Voltage on Open-Circuited Line or Current on Short-Circuited Line which has no Attenuation.

(By courtesy of The McGraw Hill Publishing Co. and Dr. W. L. Everitt.)

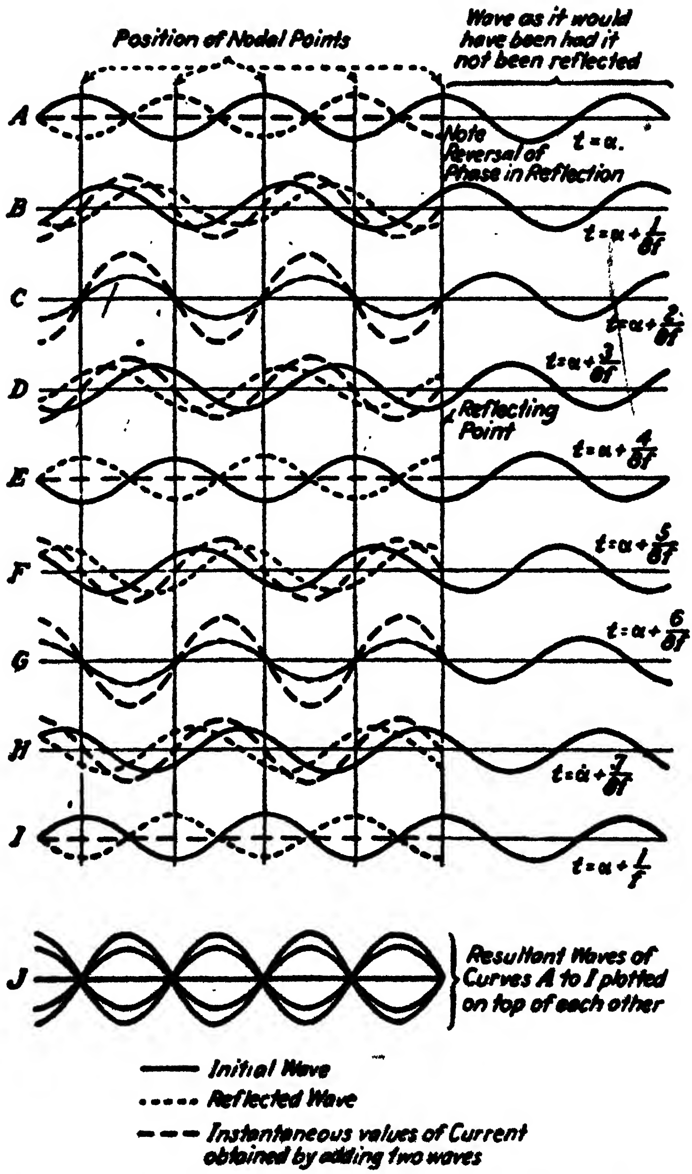


FIG. 5/XVI:1.—Instantaneous Distribution of Current on Open-Circuited Line or Voltage on Short-Circuited Line which has no Attenuation.

(By courtesy of The McGraw Hill Publishing Co. and Dr. W. L. Everitt.)

process at a number of progressively later instants indicated by, $t = a + \frac{1}{8f}$, etc., where a is an arbitrary value of time and f is the frequency of the applied e.m.f.

The total voltage at any point in the line at any instant is also shown in certain cases and is obtained by adding the ordinates of voltage due to incident and reflected progressive waves. In Fig. 4J the resultant instantaneous voltage waves at four instants are shown on one diagram. It will be seen that at some points the resultant voltage is always zero. These points occur at odd quarter-wavelengths from the open circuit, and are called voltage nodes. At even quarter-wavelengths from the open circuit the resultant voltage is always twice the value of either wave since the two voltages at these points are always equal and additive. These points are called voltage antinodes.

The nodes and antinodes do not move along the line and the resultant wave is therefore called a *standing wave*. At every point along the line the voltage reaches a maximum at the same instant, but the magnitude of this maximum varies along the line in the way indicated by Fig. 4J. In practice the voltage at the nodes is not quite zero since there is always some loss in the line so that a small progressive wave travels along the line to supply this loss.

These curves also represent the *current* on a short-circuited line.

1.72. Instantaneous Currents in an Open-Circuited Line. These are shown in Fig. 5. It will be noted that nodes of current occur at the same positions as antinodes of voltage, and vice versa.

Further, it should be noted that in the reflected wave the current undergoes a reversal of phase at reflection. It can be seen that this must be so since the total current at an open circuit must be zero ; a reversal of current phase compared with the phase in the incident wave is also consistent with a flow of energy in the opposite direction : i.e. reflection.

These curves also represent the voltages on a short-circuited line. The reflected wave of voltage is not reversed in phase with regard to the incident wave.

1.73. Instantaneous Voltages and Currents in a Short-Circuited Line. As already stated, these are also indicated in Figs. 4 and 5, the roles of current and voltage being interchanged as compared with the open-circuit case.

1.74. Derivation of Formulae for Voltages and Currents in Open-Circuited Line and for Sending-End Impedance.*

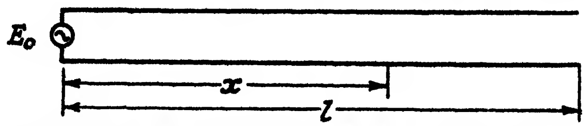


FIG. 6/XVI:1.—Open-Circuited Transmission Line of Length l.

Conventions referring to Fig. 6.

$E_s = e_s + e_0$ = the sending-end peak voltage.
e_s = the peak voltage giving rise to the incident wave.
e_0 = the peak voltage opposing the reflected wave.
$E_x = e_x + e_x$ = the instantaneous voltage at any point distant x from the sending end.
e_x = the instantaneous voltage due to the incident wave.
e_x = the instantaneous voltage due to the reflected wave.
l = the total length of the transmission line.
x = the distance from the sending end to any point along the line.
α = the phase-shift constant = the phase shift per unit length.

Then

$e_x = e_s \sin(\omega t - \alpha x)$
$e'_x = e_s \sin(\omega t - 2\alpha l + \alpha x)$, since to reach any point distant x from the source the reflected wave has travelled a distance $2l - x$.

Hence

$$E_x = e_s[\sin(\omega t - \alpha x) + \sin(\omega t - 2\alpha l + \alpha x)]$$
$$= e_s 2\sin(\omega t - \alpha l)\cos(\alpha l - \alpha x) \quad . \quad . \quad . \quad (30)$$

When $x = 0$, $E_x = E_s$ and hence

$$E_s = e_s 2\sin(\omega t - \alpha l)\cos\alpha l \quad . \quad . \quad . \quad (31)$$

Substituting in (30) the value of e_s derived from (31)

$$E_x = E_s \times \frac{2\sin(\omega t - \alpha l)\cos(\alpha l - \alpha x)}{2\sin(\omega t - \alpha l)\cos\alpha l}$$

$$\therefore E_x = E_s \times \frac{\cos(\alpha l - \alpha x)}{\cos\alpha l} \quad . \quad . \quad . \quad . \quad (32)$$

Similarly, if I_s is the sending-end current and I_x is the current at any point distant x from the sending end it may be shown that

$$I_s = \frac{e_s}{Z_s} \times 2\cos(\omega t - \alpha l)\sin\alpha l \quad . \quad . \quad (33)$$

and

$$I_x = I_0 \times \frac{\sin [\alpha(l - x)]}{\sin \alpha l} \quad . \quad . \quad . \quad (34)$$

From (31) and (33) the sending-end impedance

$$Z_s = \frac{E_0}{I_0} = Z_0 \times \frac{2 \sin (\omega t - \alpha l) \cos \alpha l}{2 \cos (\omega t - \alpha l) \sin \alpha l}$$

Now transform from scalar to mixed notation by introducing a transformation derived as follows:

$$\cos (\omega t - \alpha l) = \sin (\omega t - \alpha l + 90°) \equiv \underline{/90°} \sin (\omega t - \alpha l)$$

$$= j \sin (\omega t - \alpha l)$$

$$\therefore Z_s = -jZ_0 \cot \alpha l \quad . \quad . \quad . \quad . \quad . \quad (35)$$

1.75. Voltages and Currents in Short-Circuited Line and Sending-end Impedance. These may be derived in a similar way to that used for the open-circuit case and are as follows:

$$E_x = E_0 \times \frac{\sin [\alpha(l - x)]}{\sin \alpha l} \quad . \quad . \quad . \quad (36)$$

$$I_x = I_0 \times \frac{\cos [\alpha(l - x)]}{\cos \alpha l} \quad . \quad . \quad . \quad (37)$$

$$Z_s = jZ_0 \tan \alpha l \quad . \quad . \quad . \quad . \quad . \quad (38)$$

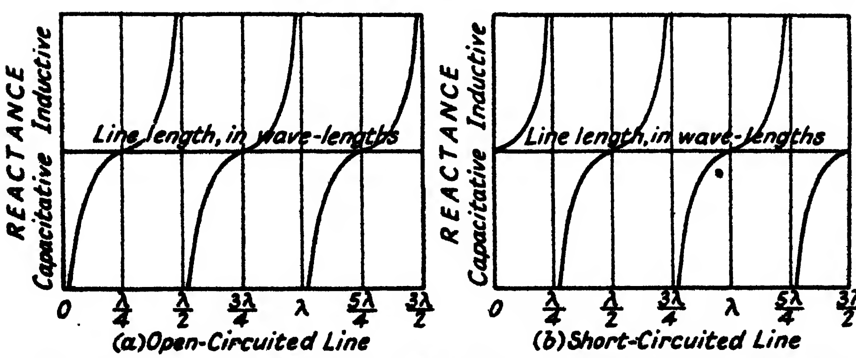


FIG. 7/XVI:1.—Variation with Line Length of Sending-End Impedance of Open-Circuited and Short-Circuited Lines with no Attenuation.

Fig. 7 shows the way in which the sending-end impedance of open- and short-circuited zero loss lines vary with the length measured in wavelengths and is obtained by plotting equations (35) and (38).

It is convenient sometimes to describe the length of a line in wavelengths; when the velocity of propagation V is known the wavelength $\lambda = \frac{V}{f}$.

A short-circuited or open-circuited line can evidently be used to provide any required value of reactance.

1.8. Line terminated in an Impedance not equal to its Characteristic Impedance. In this case a standing wave and a progressive wave exist in the line simultaneously. Fortunately, however, it is a comparatively simple matter to take account of the lumped voltages and currents due to both these waves, in one formula.

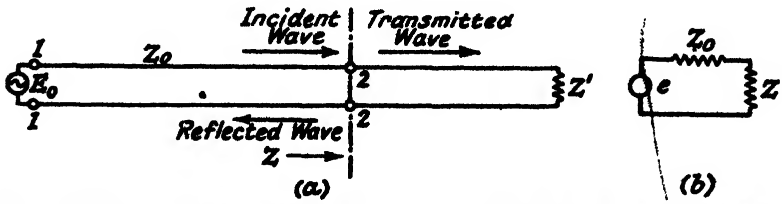


FIG. 8/XVI:1.—Transmission Line 1,1 to 2,2 terminated in an Impedance of Z, Unequal to its Characteristic Impedance Z_0.

In Fig. 8 at (*a*) is shown a transmission line of characteristic impedance Z_0 terminated at 2,2, in an impedance Z which may have any value. In the case shown this impedance is constituted by a second transmission line terminated in an impedance Z' which is only important in that it presents an impedance Z at 2,2.

When a generator is applied at terminals 1,1, a reflection of energy takes place at 2,2. A certain fraction of the energy and current supplied by the incident wave constitutes the so-called emergent or *transmitted wave* which supplies energy to the load Z provided by the second transmission line terminated in Z'. The remaining energy is returned to the sending end in the reflected wave.

1.81. Determination of Ratio between Incident and Reflected Currents and Voltages.

Let I_i = incident current.
I_t = transmitted current.
I_r = reflected current.

By Thévenin's theorem the magnitude of the transmitted current is determined by the equivalent circuit of 8(*b*) in which the transmission line on the left of 2,2, has been replaced by a generator of internal impedance Z_0 and internal e.m.f. e, equal to the open-circuit voltage at 2,2, i.e. due to E_0 when the circuit is broken at 2,2. The transmitted current is then given by

$$I_t = \frac{e}{Z_0 + Z} \qquad (39)$$

When $Z = Z_0$ the transmitted current is equal to the incident current, so putting $Z = Z_0$ in equation (39) we get

$$I_i = \frac{e}{2Z_0} \text{ so that } e = 2Z_0 I_i \quad . \quad . \quad . \quad (40)$$

The incident current is unchanged from this value when Z has any other value than Z_0.

Hence
$$I_t = \frac{2Z_0}{Z_0+Z} I_i$$

Also, numerically,

$$I_t = I_i - I_r$$

$$I_r = I_i - I_t = I_i\left(1 - \frac{2Z_0}{Z_0+Z}\right)$$

$$= \frac{Z - Z_0}{Z_0+Z} I_i$$

Since, however, the direction of I_r is reversed in sense from I_i and I_t, algebraically,

$$I_r = \frac{Z_0 - Z}{Z_0+Z} I_i \quad . \quad . \quad . \quad . \quad (41)$$

If E_i is the voltage at 2,2, due to the incident wave, then $E_i = I_i Z_0$. This must be the case because by definition the incident wave is the wave which would exist in the circuit if terminated in Z_0. Similarly, if E_r is the voltage due to the reflected wave, $E_r = I_r Z_0$.

Hence numerically,

$$\frac{E_r}{E_i} = \frac{I_r}{I_i} \text{ and } E_r = \frac{I_r}{I_i} E_i$$

Since, however, there is no reversal of phase in the voltage at reflection, algebraically,

$$E_r = -\frac{I_r}{I_i} E_i = \frac{Z - Z_0}{Z_0+Z} E_i \quad . \quad . \quad . \quad (42)$$

1.82. Sending-End Impedance of Line of Characteristic Impedance Z_0, terminated in an Impedance Z. This is determined in a similar manner to the sending-end impedance of an open-circuited line, and the same conventions are used. As before

$$e_x = e_0 \sin (\omega t - \alpha x)$$

The reflected voltage wave is, however, reduced in amplitude by the factor $\dfrac{Z - Z_0}{Z_0 + Z}$, so that

$$e'_x = e_0 \frac{Z - Z_0}{Z_0 + Z} \sin(\omega t - 2\alpha l + \alpha x)$$

Hence

$$E_x = e_0\left[\sin(\omega t - \alpha x) + \frac{Z - Z_0}{Z_0 + Z}\sin(\omega t - 2\alpha l + \alpha x)\right] \quad . \quad (43)$$

and when $x = 0$, $E_x = E_0$, hence

$$E_0 = e_0\left[\sin \omega t + \frac{Z - Z_0}{Z_0 + Z}\sin(\omega t - 2\alpha l)\right] \quad . \quad . \quad (44)$$

similarly

$$I_0 = \frac{e_0}{Z_0}\left[\sin \omega t + \frac{Z_0 - Z}{Z_0 + Z}\sin(\omega t - 2\alpha l)\right] \quad . \quad . \quad (45)$$

A plus sign exists before $\dfrac{Z_0 - Z}{Z_0 + Z}$ because the reversal of phase of the reflected current has already been covered in equation (41). Transforming from scalar to mixed notation, by means of the transformation $\sin(\omega t - 2\alpha l) \equiv \overline{\backslash 2\alpha l} \sin \omega t$; from (44) and (45) the sending-end impedance

$$\therefore\ Z_s = \frac{E_0}{I_0} = \frac{1 + \dfrac{Z - Z_0}{Z_0 + Z}\overline{\backslash 2\alpha l}}{1 + \dfrac{Z_0 - Z}{Z_0 + Z}\overline{\backslash 2\alpha l}} \times Z_0$$

Multiply through by $(Z_0 + Z)\underline{/\alpha l}$

$$\therefore\ Z_s = \frac{Z\underline{/\alpha l} + Z\overline{\backslash \alpha l} + Z_0\underline{/\alpha l} - Z_0\overline{\backslash \alpha l}}{Z_0\underline{/\alpha l} + Z_0\overline{\backslash \alpha l} + Z\underline{/\alpha l} - Z\overline{\backslash \alpha l}} \times Z_0$$

But

$$Z\underline{/\alpha l} = Z(\cos \alpha l + j \sin \alpha l),\quad Z\overline{\backslash \alpha l} = Z(\cos \alpha l - j \sin \alpha l),\ \text{etc.}$$

$$\therefore\ Z_s = \frac{Z \cos \alpha l + jZ_0 \sin \alpha l}{Z_0 \cos \alpha l + jZ \sin \alpha l} \times Z_0$$

$$= \frac{(Z + jZ_0 \tan \alpha l)Z_0}{Z_0 + jZ \tan \alpha l} = \frac{\left(1 + j\dfrac{Z_0}{Z}\tan \alpha l\right)Z_0}{\dfrac{Z_0}{Z} + j \tan \alpha l} \quad . \quad . \quad (46)$$

It follows that if the impedance looking towards the load at any point in a uniform transmission line is known, the impedance at any other point nearer to the generator can be determined from (46).

Solving equation (46) for Z

$$Z = \frac{(Z_s - jZ_0 \tan \alpha l)Z_0}{Z_0 - jZ_s \tan \alpha l} \quad . \quad . \quad . \quad (47)$$

If the sending-end impedance Z_s of a transmission line is known, it is therefore possible to determine from (47) the impedance looking towards the load at any other point nearer to the load.

Equations (46) *and* (47) *are of great importance in work on the impedance matching of short-wave feeders, since by their use the circuit impedance at points which are electrically inaccessible can be determined with comparative ease. Further, one measurement of impedance determines the impedance at every point along a feeder.*

In Figs. 1/XVI:2 and 2/XVI:2 these equations have been incorporated in charts which are known colloquially as *circle diagrams*.

1.9. Determination of Wavelength in a Feeder or Transmission Line. This is done with an R.F. bridge by finding the length of line which, when shorted at the far end, presents an open circuit at the near end. Such a length of line is a quarter of a wavelength long. This evidently enables the phase velocity in the feeder to be determined, since phase velocity = wavelength × frequency at which wavelength is measured.

1.10. Nodes and Antinodes in a Line terminated in a Mismatched Load Taking Power. Such a line contains a standing wave and a progressive wave.

At the voltage antinodes (current nodes) the incident and reflected voltages are in phase and therefore add, while the incident and reflected currents are 180° out of phase and therefore subtract. The impedance at such a point is therefore a maximum and of zero angle: a pure resistance.

At the voltage nodes (current antinodes) the incident and reflected voltages are 180° out of phase and therefore subtract, while the incident and reflected currents are in phase and therefore add. The impedance at such a point is therefore a minimum and also of zero angle: a pure resistance.

Hence, when using a circle diagram, the voltage antinodes can be identified as the points where the circuit impedance is a pure resistance and is a maximum, while the voltage nodes are the points where the circuit impedance is a pure resistance and is a minimum.

2. Circle Diagrams.

Conventions Applicable to Section 2 only.

R = series or parallel resistive component of impedance, as stated.

Z_0 = characteristic impedance of line.

X = series or parallel reactive component of impedance, as stated.

λ = wavelength.

$G_r = \frac{Z_0}{R}$ where R is the parallel resistive component of an impedance = resistive admittance ratio.

$G_x = -\frac{Z_0}{X}$ where X is the parallel reactive component of an impedance = reactive admittance ratio.

The object of these diagrams is to relate impedances at different points of a zero loss transmission line. For instance, if the impedance terminating the transmission line is known, the input impedance can be determined, and vice versa.

Circle Diagram in Fig. 1. The circle diagram should be viewed with the axis labelled " Resistance Component " placed vertically, in such a way that the figures written along this axis are the right way up. In this case, positive reactance components will appear on the right-hand side of the diagram, and negative reactance components to the left.

Distance Scale in Fractions of a Wavelength. Round the periphery of the diagram is a single scale, marked in fractional values of a wavelength, such that the complete circle represents half a wave. For convenience in reading, this graduated scale has been marked with four sets of figures, any of which may be used at any time, but one of which may generally be found to be more convenient than the others from an arithmetical point of view. The purpose of this scale is to mark out on the chart angular distances which correspond to linear distances measured along a transmission line, expressed in fractions of a wavelength.

Concentric Circles. The dotted concentric circles are drawn parallel to the paths that the eye should follow in proceeding from one known impedance point to the unknown impedance point which it is required to find.

Resistance Circles. The eccentric circles with centres on the

vertical axis are circles of constant resistance, and will be referred to as resistance circles. The values of resistance, corresponding to each of these circles, are marked along the vertical axis.

Reactance Curves. The black curves which form a series of orthogonal trajectories with the resistance circles, i.e. which cut the resistance circles at right angles, are curves of constant reactance. The value of reactance on each curve is marked round the periphery of the circle. Positive reactances are on the right of the vertical axis, and negative reactances are on the left of the vertical axis.

Method of Entering Resistance and Reactance. If Z_0 is the characteristic impedance of the line, and R is the resistance component of the impedance at any point of the circuit, this is entered in the diagram as $\frac{R}{Z_0}$, e.g. if $Z_0 = 600$ and $R = 300$, the resistance is entered as 0·5. Similarly, a reactance of $\pm X$ is entered as $\pm\frac{X}{Z_0}$. The quantities $\frac{R}{Z_0}$ and $\pm\frac{X}{Z_0}$ may be called the resistance and reactance ratios respectively.

Method of Entering an Impedance. If the impedance is $R+jX$, this is entered by finding the point at which the resistance circle $\frac{R}{Z_0}$ intersects the reactance curve $\pm\frac{X}{Z_0}$.

Method of Use of Chart. *Example.* A transmission line of characteristic impedance 600 Ω is terminated with an impedance of $1{,}200+j1{,}800\ \Omega$. The length of the line is 2 m. and the wavelength is 16 m. What is the sending-end impedance of the line?

In all these problems the terminating impedance is regarded as the load, and any other point along the line is regarded as being "towards the generator from the load".

In this case, therefore, after having entered the impedance $\frac{1{,}200}{600}+j\frac{1{,}800}{600} = 2+j3$, it is necessary to proceed round the circle towards the generator, i.e. in a clockwise direction, in order to move towards the sending-end impedance. The length of line involved is evidently 0·125 of a wavelength.

Enter the point $2+j3$ on the diagram (since the reactance is positive the point is on the right-hand side of the diagram) and place a scale running through the centre of the diagram and the impedance point $2+j3$ and cutting the periphery of the diagram. It cuts the periphery of the diagram at $0{\cdot}287\lambda$ (λ = wavelength) on one of the scales.

Proceed towards the generator (in a clockwise direction) 0·125 of a

$\frac{1}{8}$ wavelength round this scale, i.e. to the point 0·287 − 0·125 = 0·162λ. Place a rule through 0·162λ and the centre of the diagram, and note the point where this cuts the concentric circle passing through the point $2 + j3$. This is actually a circle drawn just outside the dotted circle passing through the point on the vertical axis marked 0·15 on the resistance scale. At this point, read off the impedance which is approximately $0·5 - j1·49$. The sending-end impedance is, therefore, 300 Ω − j894 Ω.

This example makes it quite clear how the converse proposition would have been handled, e.g. the sending-end impedance of the same line is 300 − j894 Ω, what is the terminating impedance? It is evidently only necessary to retrace the steps of the example above. It may be noticed that movement round the chart is now in the reverse direction, i.e. towards the load or anti-clockwise.

2.1. Method of Entering Parallel Represented Impedances. Although the normal convention for expressing impedances is in the form $A + jB$, a form which is sometimes useful is $R//jX$, i.e. R in parallel with jX. See V.16 for conversion chart. This is particularly the case in bridge measurements where the known impedance consists of a resistance in parallel with a reactance.

The transmission line calculator (circle diagram) can be used to handle directly impedances in the form $R//jX$ and to give answers in the same form.

Method of Entering Impedances given in the form $R//jX$. Required to know what impedance appears at any specified point in a transmission line of characteristic impedance Z_0, when an impedance $R//jX$ is observed at some other point.

Derive the quantities $G_r = \dfrac{Z_0}{R}$ and $G_x = -\dfrac{Z_0}{X}$ . . . (1)

These may be called the resistive and reactive admittance ratios, respectively.

Enter the quantities G_r and G_x in the circle diagram as if in the form $G_r + jG_x$. (If X is positive then, of course jG_x will be prefixed with a minus sign and entered on the chart in the negative reactance region.)

Read off the apparent answer in the form $G_{r2} + jG_{x2}$.

Then if the answer is required in the form $R_2//X_2$

$$G_{r2} = \frac{Z_0}{R_2} \ . \ . \ . \ (a) \quad \text{and} \quad G_{x2} = -\frac{Z_0}{X_2} \ . \ . \ . \ (b) \qquad . \quad (2)$$

whence

$$R_2 = \frac{Z_0}{G_{r2}} \ . \ . \ . \ (a) \quad \text{and} \quad X_2 = -\frac{Z_0}{G_{x2}} \ . \ . \ . \ (b) \qquad . \quad (3)$$

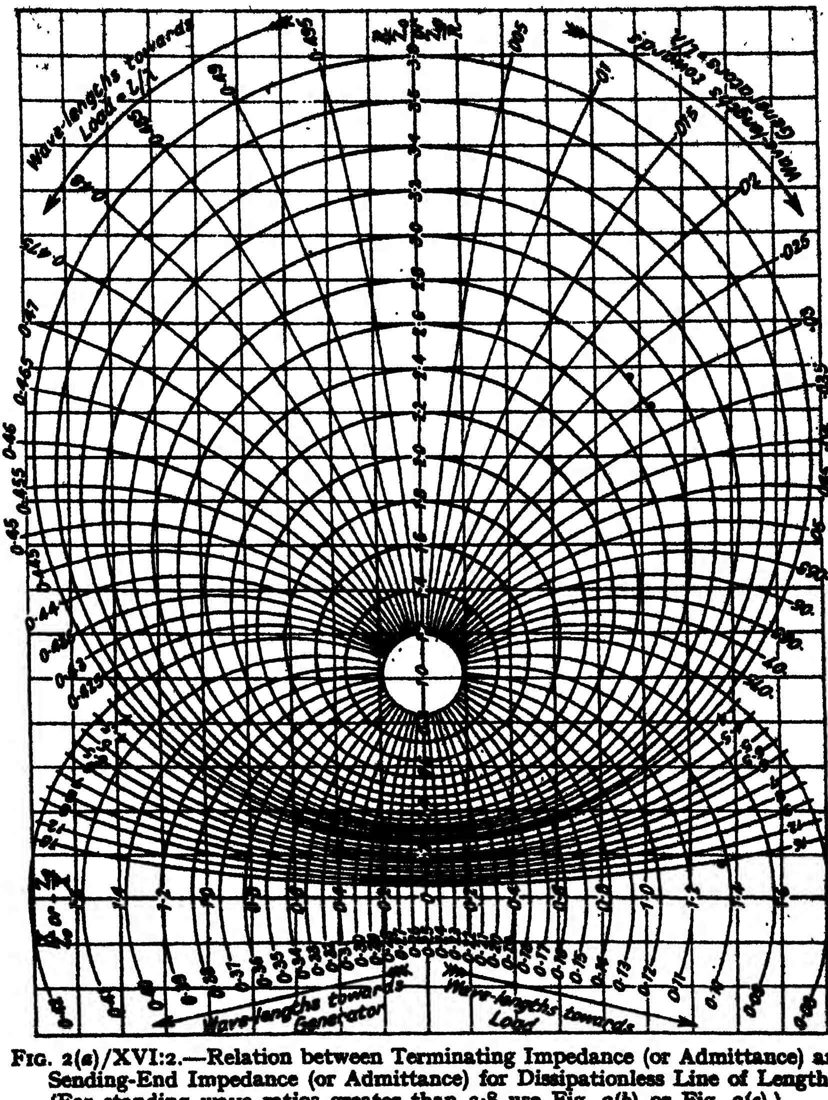
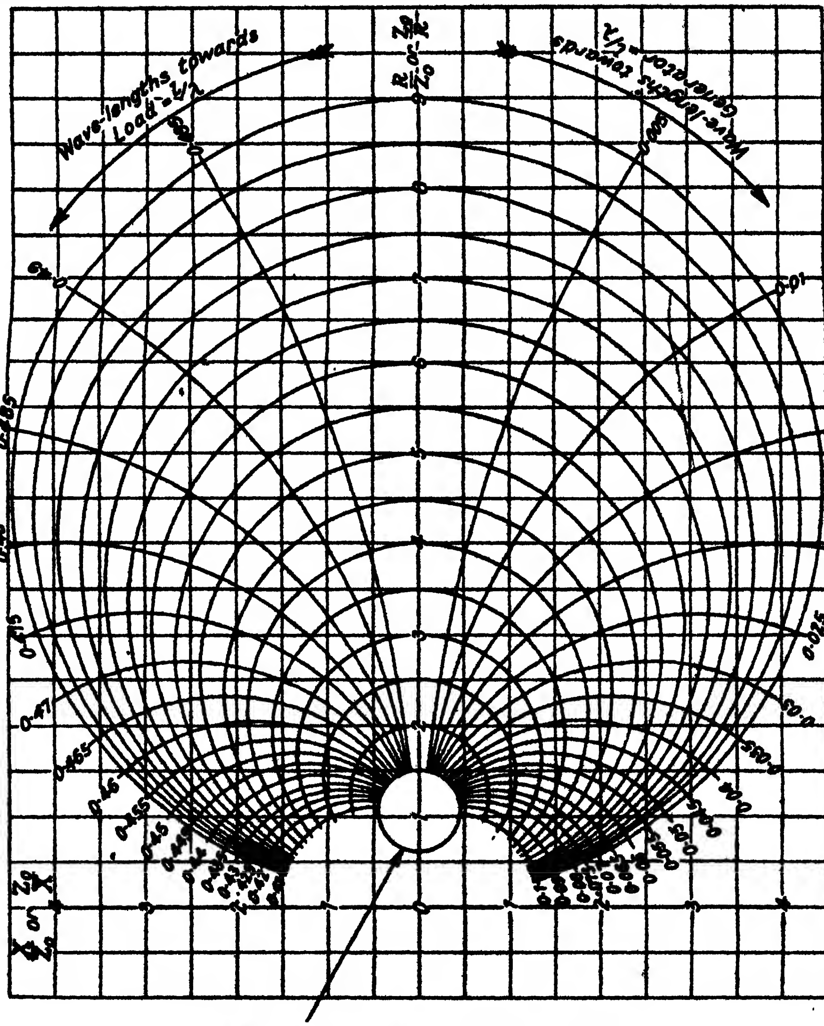


FIG. 2(*a*)/XVI:2.—Relation between Terminating Impedance (or Admittance) and Sending-End Impedance (or Admittance) for Dissipationless Line of Length *l*. (For standing wave ratios greater than 3·8 use Fig. 2(*b*) or Fig. 2(*c*).)

If the answer is required in the form $A+jB$, then conversion must be made by means of equation (1) V:16 or by means of the conversion chart in Fig. 3/V:16.

The reader is now in a position to evolve for himself the way to enter admittances in the chart.

Circle Diagram in Fig. 2. This diagram uses a form of representation which is different from the previous one. Impedances are entered on and read off from the rectilinear co-ordinates.

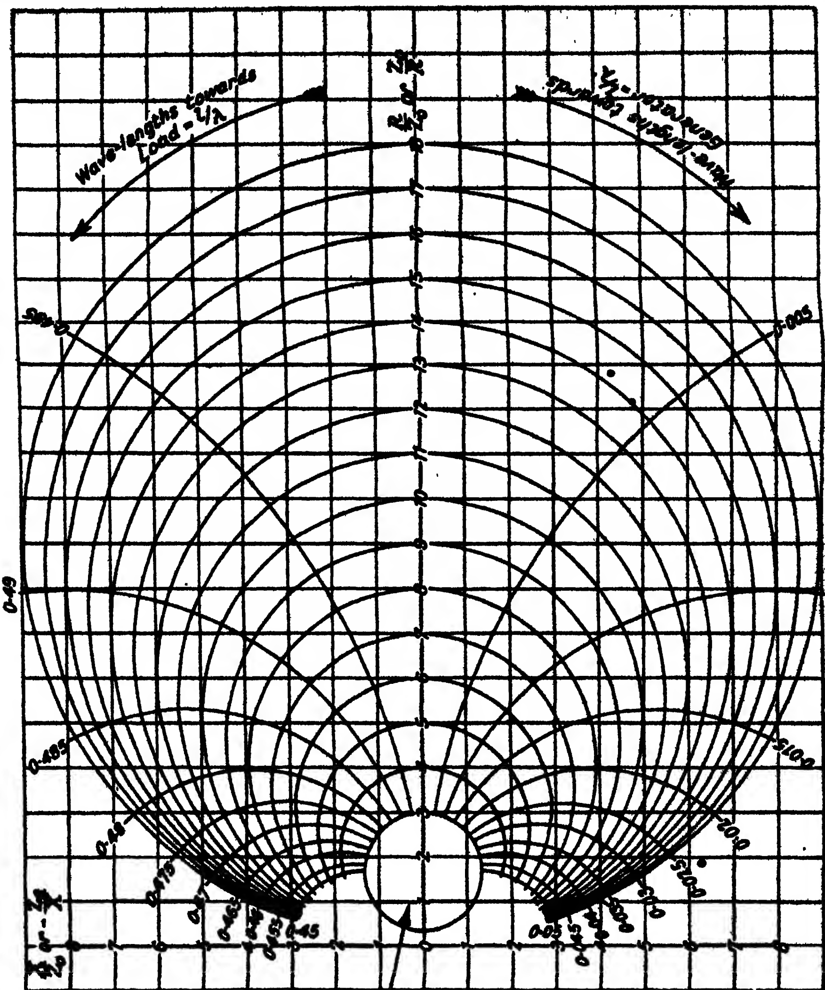


Inside this region transfer to Fig. 2(a)/XVI:2.

Fig. 2(b)/XVI:2.—Relation between Terminating Impedance (or Admittance) and Sending-End Impedance (or Admittance) for Dissipationless Line of Length l.

The eccentric circles replace the dotted circles in Fig. 1 in that in travelling from one point of the diagram to another the path should traverse one of these circles or an intermediate circle. The system of arcs forming a system of orthogonal trajectories with the eccentric circles, constitute lines of constant distance along the transmission line.

As before, the sense of direction is indicated according to whether it is towards the generator or towards the load.



Inside this region transfer to Fig. 2(*a*)/XVI:2.

FIG. 2(*c*)/XVI:2.—Relation between Terminating Impedance (or Admittance) and Sending-End Impedance (or Admittance) for Dissipationless Line of Length *l*.

Example. A line of 600 ohms impedance $0{\cdot}35\lambda$ in length is terminated in an impedance $R+jX = 300+j180$ ohms, what is the sending-end impedance?

$$\frac{R}{Z_0} = 0{\cdot}5, \quad \frac{X}{Z_0} = +0{\cdot}3$$

Entering these in Fig. 2 (*a*)/XVI:2, the point is reached where the line corresponding to $\frac{l}{\lambda} = 0{\cdot}31$ intersects the 6th circle from the

inside. Since the sending-end impedance is required the diagram must be traversed "towards the generator", i.e. in the direction of increasing $\frac{l}{\lambda}$ to the point where $\frac{l}{\lambda} = 0{\cdot}31 + 0{\cdot}35 = 0{\cdot}66$. This point is marked 0·16. Traversing the diagram 0·35 of a wavelength towards the generator, the point is reached where the 6th circle intersects the line $\frac{l}{\lambda} = 0{\cdot}16$. At this point the resistance ratio is 0·59 and the reactance ratio is $-0{\cdot}46$. Hence the sending-end impedance is $600(0{\cdot}59 - j0{\cdot}46) = 354 - j276$.

Parallel Represented Impedances. These are dealt with as in the case of Fig. 1.

3. Impedance Matching in Feeders.

If a feeder is terminated in an impedance unequal to its characteristic impedance, standing waves are set up in the feeder in *addition* to the progressive wave supplying power to the load. These standing waves cause increased copper loss owing to the increased currents; they introduce an increased leakance loss owing to the increased voltages introduced. In addition, in lines carrying high power, e.g. 50 to 100 kW, there is an increased risk of voltage breakdown, and, if the standing waves are sufficiently large, transmission of the required power over the feeders is impossible. For this reason it is necessary to avoid standing waves by matching the load impedance to the feeder at a point as near to the load as possible. This load is, of course, usually constituted by an aerial or aerial array. In the case of medium-wave aerials, matching of aerials to feeders is effected by means of aerial coupling circuits located in aerial tuning huts immediately below the aerials, while in the case of short-wave aerials, matching is by feeders as described later.

3.1. Magnitude of Standing Waves. This is normally specified by the *Standing Wave Ratio*, which is the ratio of the current at a current antinode to the current at a current node. It will be shown that the standing wave ratio also defines the ratio between the voltage at a voltage antinode and the voltage at a voltage node. The ratio of the voltage at a voltage antinode to the voltage across a circuit of the same impedance carrying the same power and terminated in its characteristic impedance is equal to the square root of the standing wave ratio.

Assume a line or feeder of zero attenuation, of characteristic impedance Z_0, terminated in an impedance Z. The voltage at any

point along the line is equal to the algebraic sum of the incident and reflected voltages: these add arithmetically at voltage nodes and subtract arithmetically at voltage antinodes. Similarly, the incident current adds to the incident current at current antinodes and subtracts arithmetically at current nodes.

Conventions Applicable to Section 3 only.

E_i = the voltage due to the incident wave.
E_r = the voltage due to the reflected wave.
I_i = the incident current.
I_r = the reflected current.
$k = E_r/E_i$ = (arithmetically) $I_r/I_i = \dfrac{Z_0 - Z}{Z_0+Z}$; see XVI:1.81.
E_n = the voltage at voltage nodes (i.e. at current antinodes).
I_n = the current at current nodes (i.e. at voltage antinodes).
E_a = the voltage at voltage antinodes (i.e. at current nodes).
I_a = the current at current antinodes (i.e. at voltage nodes).
Z_0 = the characteristic impedance of the feeder.
Z_{va} = feeder impedance at voltage antinodes (i.e. at current nodes).
Z_{vn} = feeder impedance at voltage nodes (i.e. at current antinodes).
r = the standing wave ratio.

Then

$$I_i = E_i/Z_0 \qquad (1)$$
$$E_a = E_i+E_r = (1 + k)E_i \qquad (2)$$
$$E_n = (1 - k)E_i \qquad (3)$$
$$I_a = (1 + k)I_i \qquad (4)$$
$$I_n = (1 - k)I_i \qquad (5)$$

When the circuit is terminated in its characteristic impedance, that is, when $Z = Z_0$, there is no reflection and $k = 0$, so that

$$E_a = E_n = E_i = E_0, \text{ say. Also } I_a = I_n = I_i = I_0, \text{ say} \qquad (6)$$

The suffixes zero have here been introduced to indicate the condition of no reflection. Incidentally, it will be clear that in a line with no attenuation the values of E_0 and I_0 will be equal to the sending-end voltage and current respectively.

3.11. The Standing Wave Ratio is given by—

$$r = \frac{E_a}{E_n} = \frac{I_a}{I_n} = \frac{1+k}{1-k}; \text{ see (2) to (5)} \qquad (7)$$

From (1), (2) and (5):

$$Z_{va} = \frac{E_a}{I_n} = \frac{1+k}{1-k}Z_0 = rZ_0 \qquad (8)$$

Similarly

$$Z_{vn} = Z_0/r \quad . \quad . \quad . \quad . \quad (9)$$

Hence

$$\frac{Z_{va}}{Z_{vn}} = r^2 \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Now equate the power transmitted in a line without standing waves to the power transmitted in a line with standing waves at the points of voltage nodes and antinodes:

$$\frac{E_0^2}{Z_0} = I_0^2 Z_0 = \frac{E_n^2}{Z_{vn}} = I_a^2 Z_{vn} = \frac{E_a^2}{Z_{va}} = I_n^2 Z_{va} \quad . \quad (11)$$

Substituting from (9) into (11):

$$\sqrt{r} = \frac{E_a}{E_0} = \frac{I_a}{I_0} = \frac{E_0}{E_n} = \frac{I_0}{I_n} \quad . \quad . \quad . \quad (12)$$

The relation $E_a = E_0\sqrt{r}$, derived from (12), is particularly important in determining how much the presence of standing waves in a feeder cause the voltage to rise above the voltage in a properly terminated line.

Equations (7) to (8) may be summarized

$$r = \frac{E_a}{E_n} = \frac{I_a}{I_n} = \sqrt{\frac{Z_{va}}{Z_{vn}}} = \frac{Z_{va}}{Z_0} = \frac{Z_0}{Z_{vn}} \quad . \quad . \quad (13)$$

A very important point which should be noticed is that *since like nodes are repeated at half-wave intervals along a uniform dissipationless transmission line, impedances half a wavelength apart are equal.*

3.12. Use of Circle Diagram to determine the Magnitude of the Standing Wave Ratio in a Feeder. The magnitude of the standing waves in any particular case can be determined rapidly from Figs. 1/XVI:2 and 2/XVI:2.

Example. A line of characteristic impedance $Z_0 = 600$ ohms is terminated in an impedance $390 + j570$. What are the magnitudes of the standing wave voltages and currents expressed in terms of the voltages and currents in a properly terminated line transmitting the same power?

The resistance ratio = 0·65 and the reactance ratio 0·95. Plotting these values on Fig. 1/XVI:2, the dotted circle passing through these points is found to pass through two points on the axis of resistance (which are therefore points of zero reactance): 0·3 and 3·333, corresponding to circuit impedances at the corresponding points equal to 180 and 2,000 ohms.

Reference to XVI:1.10 shows that these impedances are respectively those at the voltage nodes and antinodes. The standing

wave ratio is then given by $r = \sqrt{\frac{2{,}000}{180}} = 3{\cdot}333$.

3.2. Determination of Terminal Impedance from Measurements of I_n and I_a, the Currents respectively at Current Nodes and Antinodes. If Z_0 is the characteristic impedance of the line, from (13) the impedance at any voltage antinode (or any current node)

$$= Z_{va} = \frac{I_a}{I_n} Z_0$$

similarly, the impedance at any voltage node (or any current antinode)

$$= Z_{vn} = \frac{I_n}{I_a} Z_0$$

Hence the value of Z_{va} and Z_{vn} can be determined by measuring I_a and I_n, or, what is more important, by measuring the ratio of I_a to I_n.

Once this is done the value of the terminating impedance can be determined by entering the value of Z_{va} or Z_{vn} in Fig. 1/XVI:2 or 2/XVI:2 and proceeding round the diagram "towards the load" an angular distance respectively corresponding to the distance in wavelengths of the voltage antinode or node from the terminal impedance.

It is to be noted that it is not necessary to measure the absolute magnitude of I_a and I_n but only the ratio between them. For this purpose any form of measuring instrument which will give a reading proportional to the current in the line is suitable: it is not necessary that the instrument should read line current in amperes. A radio-frequency ammeter is usually mounted on a short pole and connected to a tuned loop of wire which links the flux due to the current in the line. Hooks of insulating material are provided by which the assembly may be hung from the line so that the coupling to the line is always constant.

The circuit arrangement is shown in Fig. 1 (*a*) for the case of an unbalanced line and at (*b*) for a balanced line. In the case of the balanced line care must be taken to make the wiring symmetrical.

While this method of measurement of impedance is often very useful, particularly to check up conditions existing on power, in general, feeder impedance measurements are made with radio-frequency bridges (see XX:9).

3.3. Impedance Matching in Short-Wave Feeders. Use of Tails for Impedance Matching. This depends on the repre-

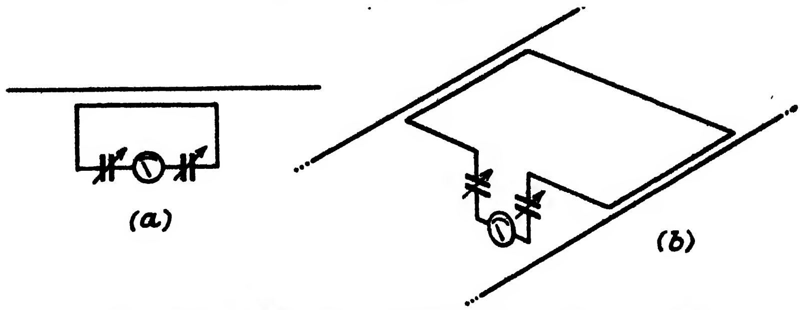


FIG. 1/XVI:3.—Method of Coupling R.F. Ammeter to Line. (a) In case of Unbalanced Line. (b) In case of Balanced Line.

sentation of impedance as parallel resistance and reactance. An impedance represented by $R//jX$, when shunted with a reactance $-jX$, presents an impedance of zero angle equal to R. Hence if any point can be found along a transmission line or feeder where the impedance looking towards the load, $R//\pm jX$, is such that $R = Z_0$, by shunting a reactance $\mp jX$ across the line at that point the impedance is converted to Z_0. Whatever the value of the terminating impedance such a point can always be found. This will be made clear by reference either to Fig. 1/XVI:2 or 2/XVI:2.

If the terminating impedance or sending impedance is $R//jX$, the quantities to be entered in the charts are $G_r = \frac{Z_0}{R}$ and $G_x = -\frac{Z_0}{X}$. Referring to Fig. 1/XVI:2, it will be seen that the dotted circles all cut the unity resistance circle. Hence, whatever point in the chart is determined by the values of G_r and G_x, by proceeding an appropriate distance round the chart in *either* direction, two points can be found at which the referred resistive admittance ratio G_{r2} is unity, i.e. where the dotted circle through the point G_r, G_x cuts the unity resistance circle. At one of these points the reactive admittance ratio is positive and at the other it is negative. At these points the parallel resistance $R_2 = \frac{Z_0}{G_{r2}} = Z_0$, and if the reactive admittance ratio at these points is $\pm G_{x2}$ the required value of added parallel reactance is $X_2 = -\frac{Z_0}{\pm G_{x2}} = \mp\frac{Z_0}{G_{x2}}$. Hence by shunting the circuit at either of the corresponding distances along the transmission line with $\pm X_2$ respectively, the circuit impedance may be reduced to Z_0. Which of the two possible points is chosen depends chiefly on which of the two required reactances is most easily realized, but may also be influenced by which location in space is most convenient for applying the shunt reactance.

The required shunt reactance, if a capacity, may be realized by a condenser, constituted in the case of open wire feeders by metal plates suspended from the feeder conductors. Alternatively, a capacity reactance may be realized by a length of short-circuited or open-circuited feeder shunted across the circuit at the points in question. This is called a *tail.* From mechanical considerations a short-circuited tail is most simple to construct and is generally used; examination of Figs. 7 (*a*) and (*b*) XVI:1 shows that the shortest length of tail is realized by the use of an open circuit when a capacitative reactance is required and by a short circuit when an inductive reactance is required. A short-circuited tail is, however, most simple to construct mechanically, and hence, where possible, the tail is inserted at a point where an inductive reactance is suitable.

The length of tail required is determined from equations (35) and (38)/XVI:1 by putting $\alpha = \frac{2\pi}{\lambda}$.

Open-circuited tail: $Z_s = jX = -jZ_0 \cot \frac{2\pi l}{\lambda}$

where X is the required reactance ***with appropriate sign inserted.***

$$\therefore\ l = \frac{\lambda}{2\pi} \cot^{-1}\left(\frac{-X}{Z_0}\right) \text{ if } \cot^{-1}\left(\frac{-X}{Z_0}\right) \text{ is expressed in radians} \quad (14)$$

$$= \frac{\lambda}{360} \cot^{-1}\left(\frac{-X}{Z_0}\right) \text{ if } \cot^{-1}\left(\frac{-X}{Z_0}\right) \text{ is expressed in degrees} \quad (15)$$

Short-circuited tail: $Z_s = jX = jZ_0 \tan \frac{2\pi l}{\lambda}$

where X is the required reactance ***with appropriate sign inserted.***

$$\therefore\ l = \frac{\lambda}{2\pi} \tan^{-1}\frac{X}{Z_0} \text{ if } \tan^{-1}\frac{X}{Z_0} \text{ is expressed in radians} \quad (16)$$

$$= \frac{\lambda}{360} \tan^{-1}\frac{X}{Z_0} \text{ if } \tan^{-1}\frac{X}{Z_0} \text{ is expressed in degrees} \quad (17)$$

Example. A 600-ohm transmission line connected to an aerial array is cut one wavelength from the array and the impedance at that point looking towards the array is found to be 1,200// +j4,200 ohms, find location and length of required tail.

$$G_r = \frac{600}{1{,}200} = 0{\cdot}5 \qquad G_x = -\frac{600}{4{,}200} = -0{\cdot}143$$

Enter these in Fig. 1/XVI:2 and draw a radius through the point plotted: it cuts the extreme outside scale at $0{\cdot}2185\lambda$. Proceed round the diagram towards the load, i.e. in an anti-clockwise

direction, until the equivalent concentric circle through the entered point cuts the unity ratio resistance circle in the "positive reactance" side of the diagram, and at this referred point draw another radius; it cuts the extreme outer scale at $0{\cdot}4030\lambda$. Further, at the referred point where $G_r = 1$, $G_x = 0{\cdot}73$, so that the parallel reactance is $-\dfrac{600}{0{\cdot}73} = -823\ \Omega$. The distance of the referred point from the measuring point is $(0{\cdot}5 - 0{\cdot}4030)\lambda + 0{\cdot}2185\lambda = 0{\cdot}3155\lambda$. A second point of equal impedance evidently exists half a wavelength nearer the array, and if physically convenient the tail should be located at this second point, otherwise it should be located $0{\cdot}3155\lambda$ from the measuring-point.

Since the parallel reactance at the referred point is $-823\ \Omega$ the tail should present a reactance equal to $+823\ \Omega$ and using a short-circuited tail its length is given by equation (17) as follows:

$$l = \frac{\lambda}{360}\tan^{-1}\frac{823}{600} = \frac{54}{360}\lambda = 0{\cdot}15\lambda$$

It is evident that by choosing the alternative point on the "negative reactance" side of the diagram, where G_r is also unity, impedance matching could have been carried out with a tail presenting a capacitative reactance. The shortest possible tail would then be an open-circuited tail.

In practice, when very exact matching is required, owing to approximations in measurement and in reading the chart, it is generally necessary to adjust slightly the position and length of the tail until the impedance at the measuring-point reaches 600 ohms

Since the object of matching is the reduction of standing waves, the degree of impedance match is very conveniently expressed by the value of the standing wave ratios; see equation (13).

3.4. Impedance Matching by Use of Inserted Length of Feeder of Different Impedance from the Main Feeder. This method is due to E. W. Hayes. Fig. 2 shows a feeder of characteristic impedance Z_0 terminated in a load L and divided

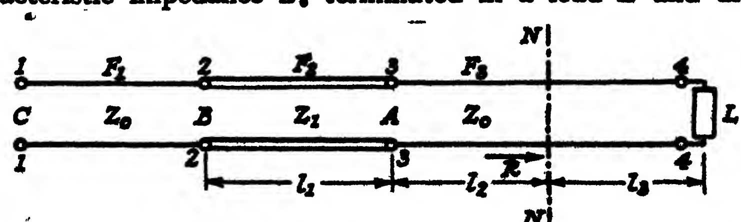


FIG. 2/XVI:3.—Impedance Matching by Inserted Length of Feeder of Different Characteristic Impedance from the Main Feeder.

into two parts F_1 and F_2 by an inserted length of feeder F_3 of characteristic impedance Z_1. NN is a point at which the referred impedance of L constitutes a pure resistance of magnitude R; it is evidently either a node or an antinode: l_1 is the length of inserted feeder, and l_2 is the distance from the end of the inserted feeder to the chosen node or antinode in F_2. If a system as in Fig. 2 has been set up with a driving-point impedance looking right from 2,2 equal to Z_0, it follows that the conditions of maximum energy transfer must be satisfied at every point in the system when the driving-point (2,2) is terminated in a resistance Z_0, the feeder F_1 being removed. The condition for maximum energy transfer is that, if the system is broken at any point, the impedances looking each way are conjugate. With F_1 removed and 2,2 shunted by Z_0, break the circuit at 3,3 and equate the impedance looking left from 3,3 to the conjugate of the impedance looking right from 3,3 in order to express the condition for maximum energy transfer:

$$\frac{Z_1(Z_0+jZ_1A)}{Z_1+jZ_0A} = \frac{Z_0(R - Z_0B)}{Z_0 - jRB} \qquad (18)$$

where R is the resistance at NN.

Let $$A = \tan\frac{2\pi l_1}{\lambda} \text{ and } B = \tan\frac{2\pi l_2}{\lambda}$$

$$\lambda = \text{wavelength.}$$

Z_1 and Z_0 are assumed to be zero angle impedances: i.e. pure resistances.

Put $$\frac{Z_1}{Z_0} = r_1 \quad \therefore\ Z_1 = Z_0r_1 \qquad (19)$$

and $$\frac{R}{Z_0} = r_2 \quad \therefore\ R = Z_0r_2 \qquad (20)$$

Substitute in (18) the values of Z_1 and R from (19) and (20) and cancel Z_0

$$\therefore \frac{r_1(1+jr_1A)}{r_1+jA} = \frac{r_2 - jB}{1 - jr_2B}$$

Cross-multiplying and separating real and imaginary parts

$$r_1(1+r_1r_2AB) = r_1r_2+AB \qquad (21)$$
$$r_1(r_1A - r_2B) = r_2A - r_1B \qquad (22)$$

Solving (22) for B

$$\therefore B = \frac{A(r_1^2 - r_2)}{(r_1r_2 - r_1)} \qquad (23)$$

Substituting (23) in (21) and solving for A and B in terms of r_1 and r_2

$$A = \pm \frac{r_1(r_2 - 1)}{\sqrt{(r_1^2 - r_2)(r_1^2 r_2 - 1)}} \quad B = \pm\sqrt{\frac{r_1^2 - r_2}{r_1^2 r_2 - 1}} \quad . \quad (24)$$

A and B are only real for values of r_2 lying between $\frac{1}{r_1^2}$ and r_1^2. This reflects the limitation imposed by the value of $\frac{Z_1}{Z_0}$ on the degree of mismatch permissible between R and Z_0 without making matching impossible.

There are evidently two solutions of equation (24), one taking the positive values of A and B together, and the other taking the negative values of A and B. It must be remembered that A and B are each the tangent of an angle, and that corresponding to any one value of A or B there are an infinite number of angles separated by 180° in the range from minus infinity to plus infinity. It is convenient to regard only the lengths corresponding to the angles in the first two quadrants : the positive arithmetical values of A and B will give rise to angles lying in the first quadrant, while the negative values will give rise to angles lying in the second quadrant, i.e. the angles will be positive in both cases. An infinite series of other solutions can then be obtained by adding to or subtracting from the resulting values of l_1 and l_2 any number of half-wave-lengths ; but normally none of these solutions is of interest and may be neglected.

The convention for positive lengths is of importance ; the lengths l_1 and l_2 shown in Fig. 2 will be considered to be positive lengths. Evidently a negative value of l_1 has no meaning. Negative values of l_2 will be taken to refer to lengths measured to the right of NN in Fig. 2.

In practice the feeder of characteristic impedance Z_1 is sometimes constituted by suspending from each leg of the main feeder a series of loops of wire as shown in Fig. 3. The sag of each loop is usually made equal to the spacing between the conductors, in which case the modified characteristic impedance is 0·6 Z_0, where Z_0 is the impedance of the unmodified feeder. The span of each loop is not critical, a span of 5 or 6 ft. is convenient.

Some surprise may be expressed because the impedance of a feeder, built as in Fig. 3, is the same as that of the four-wire feeder of Fig. 1 (*f*) ; see XVI:1. This is substantially true because, with small separations between the feeder and each element of the added

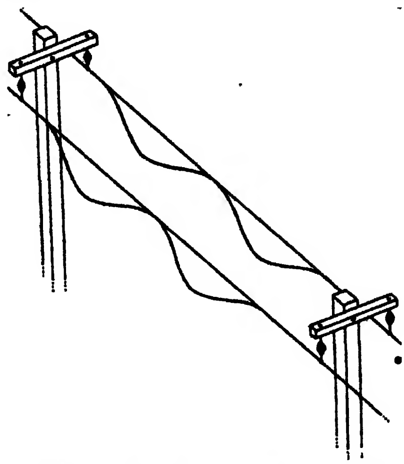


FIG. 3/XVI:3.—Method of Hanging Loops on Conductors to Adjust the Characteristic Impedance.

loops, the drop in characteristic impedance is nearly equal to the drop in impedance consequent on the full separation.

From the equations above the relation between r_2 and l_1, and between r_2 and l_2 are plotted in Fig. 4 for the case where $Z_1 = 0{\cdot}6Z_0$, i.e. $r_1 = 0{\cdot}6$. l_1 and l_2 are expressed as fractions of the wavelength λ. The curve shown is only plotted for values of r_2 less than unity, i.e. $R < Z_0$ because the distance l_2 from 4,4, the load terminals is usually chosen so that at NN a low-impedance point occurs, constituted by a point of voltage node, when energy is supplied from an R.F. source to 1,1, the input of the system.

Using the curve of Fig. 4, the procedure for impedance matching in the case of a feeder connected to an aerial or other load is as follows. A convenient point is chosen distant from the aerial or other load by such a length of feeder that a node of voltage exists; this is then the location NN. The impedance of this point is measured either directly or as a referred impedance, and this determines the value of R and so of $r_2 = \dfrac{R}{Z_0}$. The values of l_1 and l_2 are then read off from Fig. 4, or from equivalent curves when Z_1 has any other value than $0{\cdot}6Z_0$.

In practice, where very exact impedance matching is required, slight adjustments in the length and position of the inserted section may be necessary. These are carried out, observing the impedance at 2,2, or any convenient point to the left of 2,2, until the impedance is made equal to Z_0.

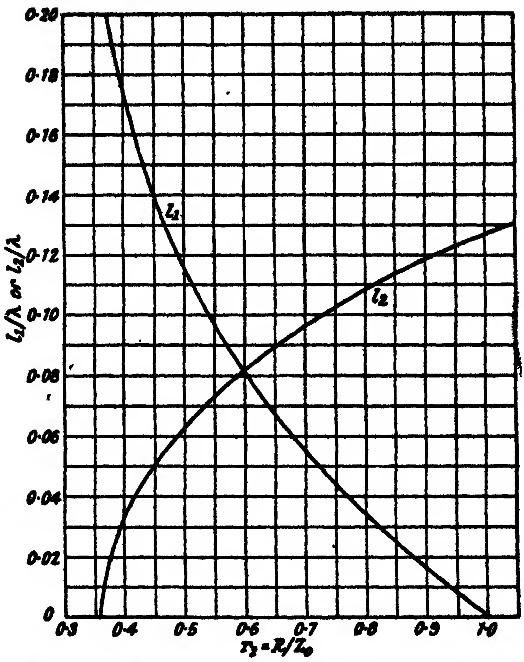


FIG. 4/XVI:3.—Design Curves for Inserted Matching Sections using Inserted Feeder of Impedance 0·6 × Main Feeder Impedance.
(By courtesy of Mr. E. W. Hayes and the B.B.C.)

4. The Quarter-Wave Line.

When the phase constant α and the length of a line have such dimensions that $\alpha l = \frac{2\pi}{\lambda}l = \frac{360^\circ}{\lambda}l = 90^\circ$, so that $l = \frac{90^\circ}{360^\circ}\lambda = \frac{1}{4}\lambda$: the line is a quarter of a wavelength long and is called a quarter-wave line. Such a line has peculiar properties.

If terminated in an impedance Z (see Fig. 1), from equation (46) the input impedance is given by

$$Z_s = \frac{(Z + jZ_0 \tan 90^\circ)Z_0}{Z_0 + jZ \tan 90^\circ} \quad \frac{Z_0^2}{Z} \qquad . \quad . \quad (1)$$

A quarter-wave line of characteristic impedance $Z_0 = \sqrt{Z_s Z}$ can therefore be used as an impedance transformer between an impedance Z_s and an impedance Z.

The properties of quarter-wave lines are not confined to lines with distributed constants: networks with lumped constants exist which, as far as their terminal currents and voltages are concerned, behave exactly as a quarter-wave line with distributed constants. For all practical purposes, therefore, when required, such networks may be used to provide the qualities of a quarter-wave line. Simple

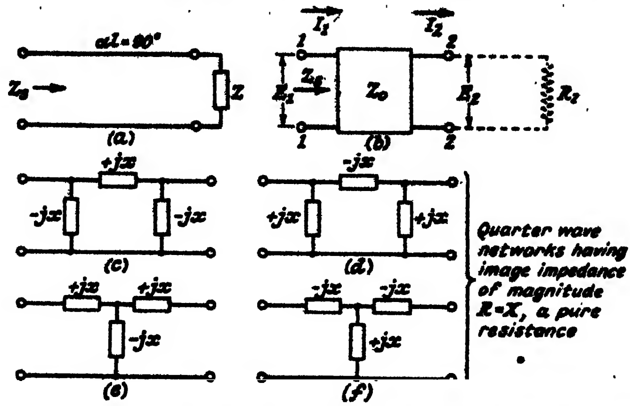


FIG. 1/XVI:4.—Quarter-Wave Line and Quarter-Wave Networks.

types of such networks are illustrated in Fig. 1 at (*c*), (*d*), (*e*) and (*f*). These are made up from positive and negative reactances of magnitude X and have resistive (zero angle) image impedances of magnitude equal to X. The phase-shift constant of a network, analogous to the value of αl for a line, is the phase shift through the network when terminated in its image impedances. Any network having image impedances which are pure resistances over a part of the frequency spectrum and which has a phase-shift constant equal to 90° at a frequency where the image impedances are resistive, can be used to constitute a quarter-wave network at that frequency. Another type of quarter-wave line is described in CIII:1.

Fig. 1 (*b*) illustrates the general case of a quarter-wave line or network with input and output voltages as shown. The relation between E_1 and I_2, for instance, may be derived as follows:

$$Z_s = \frac{Z_0^2}{R_1}$$

The input power = the output power.

$$\therefore \frac{E_1^2}{Z_s} = \frac{E_1^2 R_2}{Z_0^2} = I_2^2 R_2$$

$$I_2 = \frac{E_1}{Z_0} \qquad (2)$$

a rather startling conclusion since I_2 is independent of R_2. If a constant-voltage generator is applied at 1,1 the output of 2,2 behaves as a constant-current generator.

Similarly $$E_2 = Z_0 I_1 \quad . \quad . \quad . \quad . \quad (3)$$

so that if a constant-current generator is applied at terminals 1,1 the output 2,2 behaves as a constant-voltage generator.

at the wanted frequency and the s.c. tail must be made more than a quarter of a wavelength long. In this case the minimum length of short-circuited tail is

$$\theta_r = 180° + 90°\left(\frac{r}{w} - 1\right) \quad . \quad . \quad . \quad (5)$$

It is evidently permissible to change either tail from o.c. to s.c., or vice versa, if the length of the tail is simultaneously changed by $\frac{1}{4}\lambda$.

In practice, for ease of adjustment, both tails would probably be short-circuited tails. The rejecting tail would be adjusted on the bridge to present a short circuit at the unwanted frequency, and the compensating tail would also be adjusted on the bridge until the reactance of the rejecting tail disappeared at the wanted frequency.

5.1. The Tapped Quarter-Wave Line. *Use of a single length of quarter-wave feeder open circuited one end and short circuited the other.* Consider the impedance looking into a tapping T taken any distance along a length of quarter-wave feeder open circuited

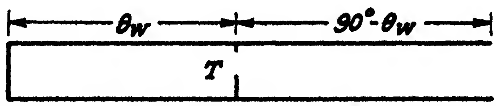


FIG. 1/XVI:5.—Tail Quarter-Wave Long at Wanted Frequency.

one end and short circuited the other end; see Fig. 1. Assume that it is a quarter-wavelength long at the wanted frequency.

Let the angular length from the tapping to the short circuit be θ_w, then the angular length from the tapping to the open-circuited end is $90° - \theta_w$.

The impedance presented at T by the short-circuited length is

$$jZ_0 \tan \theta_w$$

and the impedance presented at T by the open-circuited length is

$$-jZ_0 \cot (90° - \theta_w) = -jZ_0 \tan \theta_w$$

Hence, *at the frequency at which the line in Fig.* 1 *is a quarter wave long*, the impedance looking into T is infinity, regardless of the position of T. In practice the line has resistance which limits the impedance to a finite value, but the impedance is always high.

Hence, if the tapping T on the subsidiary length of feeder of Fig. 1 is connected across a main feeder, attenuation is introduced at all frequencies except the wanted frequency. This attenuation is not so high as in the case just discussed, where an open-circuited

line a quarter-wave long at the unwanted frequency is connected across the feeder, but if there is more than one unwanted frequency to be dealt with, the tapped quarter-wave line has the advantage of simplicity of adjustment. Further, if there is more than one unwanted frequency the previous technique may become cumbersome. The position of the tapping on the subsidiary feeder in Fig. 1 determines the sharpness of the resonance in the neighbourhood of the wanted frequency, and the magnitude of the impedance presented at T at the wanted frequency, as determined by losses. As the s.c. end is approached the sharpness of resonance increases and the magnitude of the impedance at the wanted frequency drops.

6. Conductor Sizes and Power-Carrying Capacity of Feeders.

The power-carrying capacity of two-wire open-wire balanced feeders is determined by conductor diameter and spacing. The normal spacing in the B.B.C. is 12 ins. and departures from this are rare. The limitation on power-handling capacity is the occurrence of *corona*. Corona appears at night as a luminous halo surrounding the conductors due to ionization of the air and passage of current from one conductor to another. Its effect is to increase the attenuation of the feeder and to introduce predisposition to breakdown by flashover. At a somewhat lower voltage than that at which corona occurs a phenomenon called *dark* corona is observed. This can be observed in daylight due to the refraction of light passing near the conductors and appears as a distortion of objects seen behind the conductors. If feeders are operated so that dark corona is absent an adequate factor of safety is provided.

In the B.B.C., No. 6 s.w.g. (0·192 in. dia.), conductors are used for feeders, which give an impedance, on a two-wire feeder, of 580 Ω. Experience at the B.B.C. station at Daventry has shown that with a standing wave ratio equal to 1·7, such a feeder will carry a modulated power corresponding to 70 kW unmodulated carrier without risk of breakdown. Under these conditions the voltage between conductors is $\sqrt{1{\cdot}7} = 1{\cdot}305$ times the voltage which would occur in the absence of standing waves.

With a power of 70 kW traversing a properly matched feeder of 580 ohms impedance, the peak voltage on unmodulated carrier is given by

$$\frac{\hat{v}^2}{2 \times 580} = 70{,}000$$

Hence $\hat{v} = 9{,}000$ volts.

With a carrier modulated 100% this value is doubled and the peak voltage is 18,000 volts.

With a 1·7 : 1 standing wave the peak voltage is increased to 18,000 × 1·3 = 23,400.

On a 6 S.W.G. two-wire feeder it is generally found that corona and flashover begin to appear at about 33 kV peak volts.

Recently, four-wire feeders of the type shown in Fig. 1(*f*)/XVI:1, which are equivalent to two two-wire feeders in parallel one above the other, have been introduced, using 6-S.W.G. conductors. Such feeders have a characteristic impedance equal to 0·6 × 580 = 348 Ω and have been found to carry 130 kW with complete freedom from breakdown.

7. Short-Wave Transmitting Aerials.

While a variety of types of short-wave aerials are in use, arrays of horizontal half-wave dipoles have the advantages of radiating horizontally polarized waves and of simplifying problems of circuit balance.

A half-wave dipole consists of a conductor half a wavelength long (see Fig. 1).

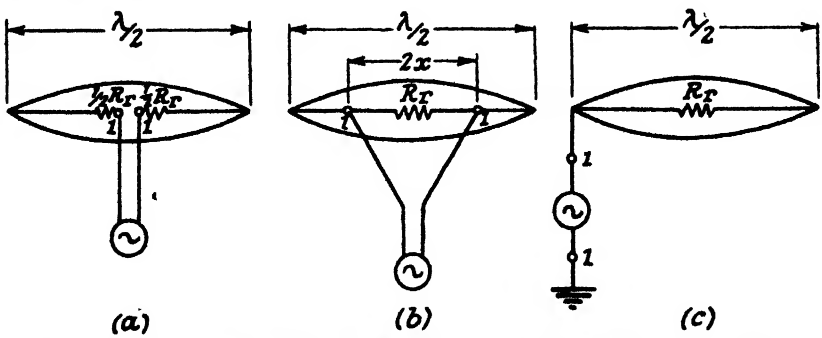


FIG. 1/XVI:7.—Half-wave Dipoles showing Standing Wave of Current. (*a*) Centre-Fed Dipole. (*b*) Shunt-Fed Dipole. (*c*) End-Fed Dipole.

It is important to note that, since the velocity of propagation in the aerial conductor is less than the velocity of propagation in free space, the length of a half-wave dipole is less than a half of the wavelength as determined from the standard formula $\lambda = 300{,}000{,}000/f$, where f is the frequency. It is usually about 0·95 times the free space half-wavelength.

In practice, however, a dipole is usually adjusted to such a length that it presents no reactance at its driving-point (see Fig. 1/XVI:7). This is done by observing the impedance at the driving-point on

an R.F. bridge and adjusting the length until the reactance is zero. In this case the effect of the radiation reactance (see XVI:7.41) is also cancelled. The ratio of the length of the dipole to a half-wavelength in free space is therefore not exactly equal to the ratio between the propagation velocities in the aerial and in free space (see XVI:7.8).

For approximate calculations the figure 0·95 is, however, sometimes used to determine the length of aerial.

A dipole may be vertical, in which case vertically polarized waves are radiated, or horizontal, in which case horizontally polarized waves are radiated.

In vertically polarized waves, for a horizontal direction of propagation, the electric field is represented by a potential gradient in the vertical plane, the magnetic field (i.e. lines of force) being horizontal: it is current usage to describe this condition by saying that the electric vector is vertical and the magnetic vector is horizontal. Both vectors are perpendicular to the direction of propagation; this condition always holds, whatever the direction of propagation.

In horizontally polarized waves, for a horizontal direction of propagation, the electric vector is horizontal and the magnetic vector is vertical.

While a dipole in free space radiates maximum energy in a direction perpendicular to itself it also radiates energy in all other directions, reaching zero radiation of energy in the direction parallel to its length.

The radiation from a dipole above a perfectly conducting ground is represented by the free space radiation of the dipole, plus the radiation from a virtual dipole carrying the same current as the real dipole, located at the position of the image of the dipole in the ground regarded as a mirror. The position of the image is evidently the same distance below ground as the height of the dipole above ground. Since vertically polarized waves are reflected without reversal of phase while horizontally polarized waves are reflected with 180° reversal of phase, the virtual currents in the vertical dipole are in phase with those in the real aerial in the case of a vertical dipole, and 180° out of phase with the currents in the real dipole in the case of a horizontal dipole.

In practice a dipole radiates energy into a hemisphere described on the ground with the dipole as centre, the energy density varying with direction.

In the case of radiation from a vertical aerial the magnetic vector

is always horizontal, while for directions of propagation making an angle with the horizontal the electric vector is tilted so as to remain normal to the direction of propagation and the magnetic vector.

In the case of radiation from a horizontal dipole in a plane normal to the direction of the dipole, the electric vector is always horizontal while the magnetic vector tilts so as to remain normal to the direction of propagation and the electric vector. In the case of radiation in the vertical plane through the dipole the magnetic vector is always horizontal, while the electric vector tilts so as to remain normal to the direction of propagation. For directions lying outside these planes both electric and magnetic vectors make an angle with the vertical and do not lie in the vertical plane through the direction of propagation.

A certain amount of vagueness exists in current opinions as to whether the definitions of horizontally and vertically polarized waves are confined to the case of horizontal propagation, or whether those definitions refer to the whole radiation respectively for horizontal and vertical aerials.

The Standards Committee of the American Institute of Radio Engineers has no such doubts and gives the following definitions:

Vertically Polarized Wave. A linearly polarized wave whose direction of polarization is vertical.

Horizontally Polarized Wave. A linearly polarized wave whose direction of polarization is horizontal.

The direction of polarization is defined in effect as the direction of the displacement vector, or electric vector: the direction in which the gradient of electric potential is a maximum.

Probably the best convention to use is neither of these but to say that a horizontally polarized wave is any wave in which the electric vector is horizontal and a vertically polarized wave is one in which the magnetic vector is horizontal, regardless of the direction of propagation in each case. Waves with skew planes of polarization which do not fit into either of these categories can then be said to be composed of the two kinds of polarization in appropriate proportions.

This convention applies in what follows.

The object of using an array of simple aerials is to secure an increase of radiation in one direction at the expense of radiation in other directions. When the concentration of energy in one direction is intense the array is called a beam array.

For short-wave arrays on long-distance service, horizontal dipoles are often used because they give rise to structures balanced to

ground, and because the long-distance propagation of substantially horizontally polarized waves is found to be more efficient than the propagation of vertically polarized waves. This remark applies in the range of frequencies where the physical dimensions of half-wave dipoles make the erection of beam arrays a practical proposition and which give efficient long-distance transmission. The superiority of the horizontally polarized waves is related to their efficiency of reflection both in the ionosphere and from the ground.

7.1. Arrangement and Method of Designation of Short-Wave Arrays. Many types of short-wave array are of course used. The type most common in the B.B.C. is normally built from assemblies of half-wave dipoles, usually arranged in pairs on each side of a balanced feeder as shown in Fig. 13. It is important to note that, while this assembly looks like an assembly of centre-fed dipoles as in Fig. 1 (*a*), it is actually an assembly of end-fed dipoles and its overall width is a full wavelength and not a half-wave-length.

The vertical spacing between pairs of dipoles is half a wave-length, and, as the feeder is crossed between each pair of dipoles, all the dipoles are driven in phase so that their fields all add in a direction normal to the plane of the aerial. Owing to reflection from the ground the direction of maximum radiation is not directly normal to the aerial, but in a direction, making an angle with the horizontal, above the horizon. The array of Fig. 13 is called an H2/4/0·5 array; this means that it is composed of *Horizontal* half-wave dipoles, that *it is two dipoles wide* / that *it is four dipoles high* spaced half a wave apart / and that the bottom pair of dipoles *is half a wave above ground*. A single dipole half-wave above ground would be an H1/1/0·5. Two dipoles arranged like the bottom pair of dipoles in Fig. 13 would constitute an H2/1/0·5 array. Fig. 11 shows an H4/4/2 array. Various other information is sometimes incorporated in the description of an array such as the wavelength or frequency, the bearing, whether the array has a reflector and if it is reversible. Since the reflectors consist of arrays identical with the transmitting array, situated usually a quarter of a wavelength behind the array and undriven, an array can be reversed by changing the drive from the array proper to the reflector. When means for doing this are provided the array is said to be reversible. Finally, an indication of whether the array is slewed is given (see XVI:7.2 below). The various conventions for describing arrays are summarized below.

Summary of Methods of Designating Arrays. It is con-

venient to have a code by which the type of aerial can easily be designated, and the one used in the B.B.C. is as follows:

The first letter H or V stands for "Horizontal" or "Vertical".

The second letter R stands for "Reflector".

The third letter R stands for "Reversible".

The fourth letter S stands for "Slewed".

The first figure gives the number of dipoles side by side.

The second figure gives the number of dipoles one above the other.

The third figure gives the height of the bottom dipole in wavelengths above ground.

The fourth figure gives the waveband in metres.

The fifth and successive figures gives the azimuthal bearing(s) of the centre of the main lobe of radiation in degrees east of true north.

Letters which do not apply are left out.

Thus HRRS4/4/0·5/49·6/114°, 126°, 294°, 306° indicates a horizontal array, having a reversible reflector, and capable of being slewed. There are 4 tiers of dipoles side by side and 4 rows of dipoles one above the other with the bottom row 0·5 wavelength above the ground. The aerial is cut for 49·6 metres and, with reversible reflector and slew, can transmit on each of the bearings 114°, 126°, 294°, 306° east of true north.

HR4/2/0·6/16·8/80° indicates a horizontal array, having a reflector. There are 4 tiers of dipoles side by side, and 2 rows of dipoles one above the other, with the bottom row 0·6 wavelength above the ground. The aerial is cut for 16·8 metres and can transmit only on a bearing of 80° east of true north.

7.11. Performance of Arrays. The performance of arrays is determined by their directional characteristics, that is by the relative amounts of power transmitted in each direction.

7.111. Power-Density Diagrams. The radiation characteristics of an aerial are portrayed by drawing a vector * in each direction, of length proportional to the strength of radiation in that direction. The strength of radiation may be expressed absolutely in terms of field strength at a given distance, or in terms of the power density at any given distance, e.g. 1 kilometre. Alternatively, as in the diagrams which follow, the solid polar diagram, as the resulting figure is called, may be represented in terms of *relative* power density, the maximum power density being arbitrarily represented by 100.

* Remember that the fundamental definition of a vector is any quantity having magnitude and direction.

Figs. 2 to 5 inclusive show a projection of the contours of the solid polar diagram of power density of a single dipole at different heights above ground : $0{\cdot}25\lambda$, $0{\cdot}35\lambda$, $0{\cdot}5\lambda$ and $0{\cdot}75\lambda$ above ground. This diagram represents a half-hemisphere above ground described with the dipole as centre, so that the plane in which the dipole lies constitutes one boundary of the half-hemisphere. *The radiation into the other half-hemisphere is identical.* On the diagram the horizontal lines represent angles of elevation and the curved lines represent angles of deviation to right and left of the centre meridian, i.e. azimuth. The two sets of lines therefore correspond to latitude (Elevation) α and longitude (Azimuth) β on the sphere referred to the centre meridian and the horizon.

If the observer imagines himself standing at the dipole looking at the horizon in a direction at right angles to the dipole, the point $\alpha = 0$, $\beta = 0$ on the diagram would be straight in front of him.

The reason for the peculiar shape of the diagram is the use of a form of projection such that everywhere on the diagram equal solid angles are represented by equal areas on the diagram. This has practical uses which are not relevant to the present discussion : see CVI.

It will be seen that a dipole $\frac{1}{4}\lambda$ above ground (Fig. 2) radiates the maximum power density in a vertical direction. As the height above ground is progressively increased the angle of elevation of the angle of maximum radiation progressively falls until the height reaches $0{\cdot}5\lambda$, see Fig. 4, at which the angle of maximum radiation is 30°. When the height is increased to $0{\cdot}75\lambda$, see Fig. 5, the vertical radiation has reappeared and there are two directions of maximum radiation, one vertical and one at a vertical angle of 18°.

Fig. 6 shows the polar diagram of two half-wave dipoles driven in phase one above the other half a wavelength apart with the lowest dipole half a wavelength above ground. Comparison with Fig. 4, which gives the polar diagram of a single dipole half a wave above ground, shows that the effect of adding the second dipole is to reduce the angle of maximum radiation from 30° to 17° and to compress the radiation so that the vertical dimensions of the beam are reduced.

Generally speaking, the effect of adding more radiating elements in a vertical direction is to reduce the depth of the radiated beam and reduce the angle of maximum radiation, while the effect of increasing the height of an array is to reduce the angle of maximum radiation and introduce *lobes* of radiation at higher angles of elevation. Similarly, the effect of increasing the width of an array by

Chart Calibration

$$ED/\sqrt{kW} = 4{\cdot}12\sqrt{\frac{P_e}{100}}\ \text{mV/M} \times \text{kM.}$$

Where P_e = contour marking read from this chart and 'D' is distance along radius vector. Power density in watts per radian/$\sqrt{kW}$ = kP_e where k = 4·49.

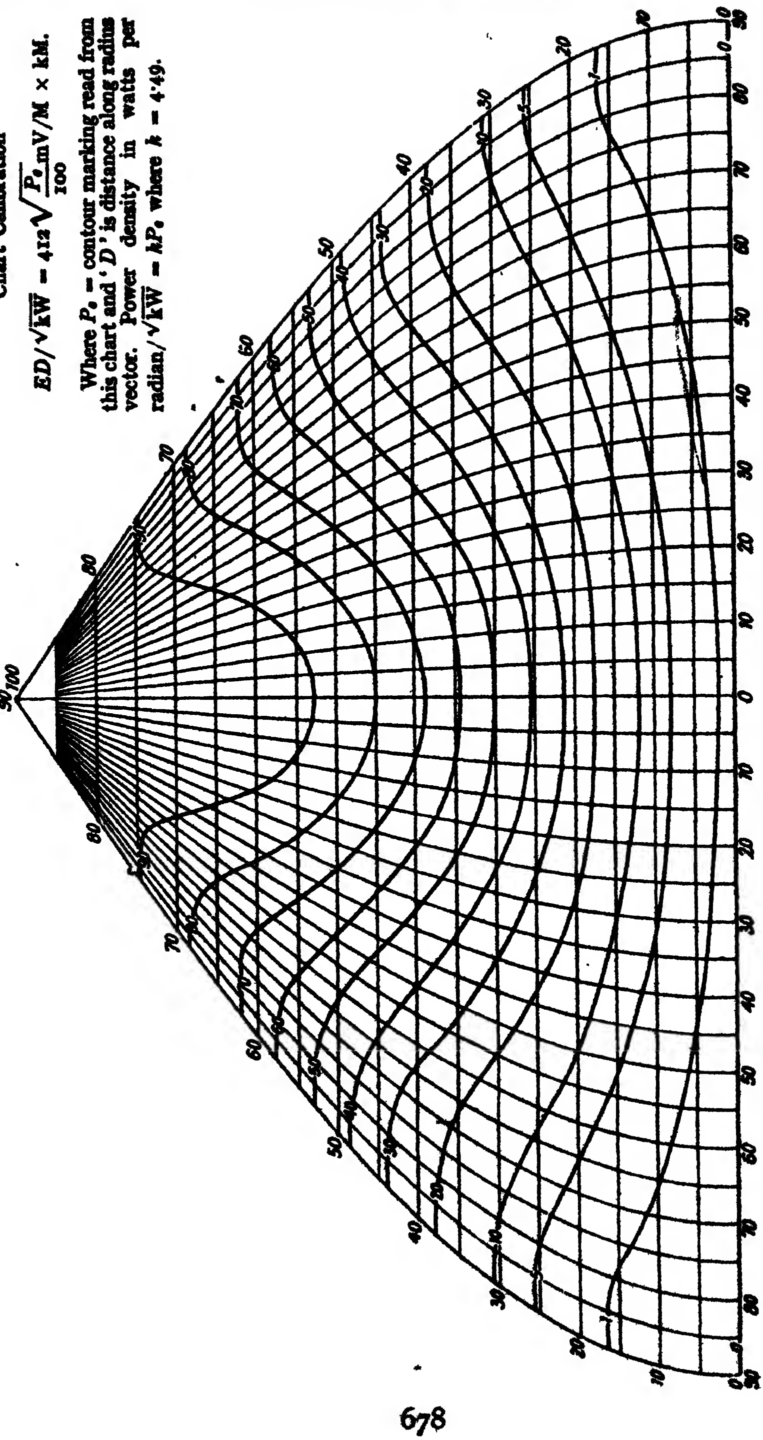


FIG. 2/XVI:7.—Power Distribution Diagram—Antenna Array H1/1/0·25.
Ranges of Azimuth and Elevation for power of 50:—Elevation = 30° – 90°; Azimuth = ±90°; Directivity = 5·64.

Chart Calibration

$$ED/\sqrt{\text{kW}} = 382\sqrt{\frac{P_e}{100}}\ \text{mV/M} \times \text{kM}.$$

Where P_e = contour marking read from this chart and 'D' is distance along radius vector. Power density in watts per radian/$\sqrt{\text{kW}}$ = kP_e where k = 3·89.

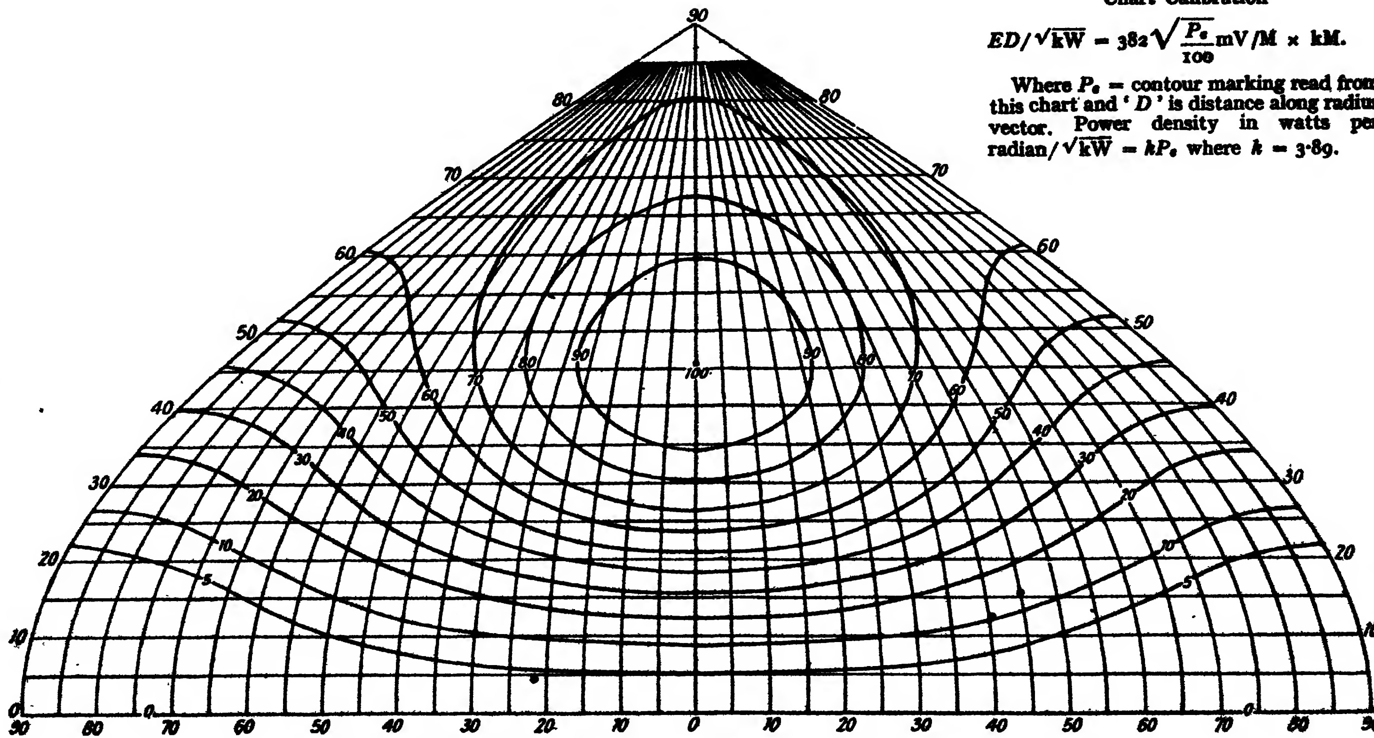


FIG. 3/XVI:7.—Power Distribution Diagram—Antenna Array H1/1/0·35.

Ranges of Azimuth and Elevation for power of 50:—Elevation = 22° – 90°; Azimuth = 0° ±90°; Directivity = 4·9.

Chart Calibration

$$ED/\sqrt{\text{kW}} = 458\sqrt{\frac{P_e}{100}}\,\text{mV/M} \times \text{kM}$$

Where P_e = contour marking read from this chart and 'D' is distance along radius vector. Power density in watts per radian/$\sqrt{\text{kW}} = kP_e$ where k = 5·54.

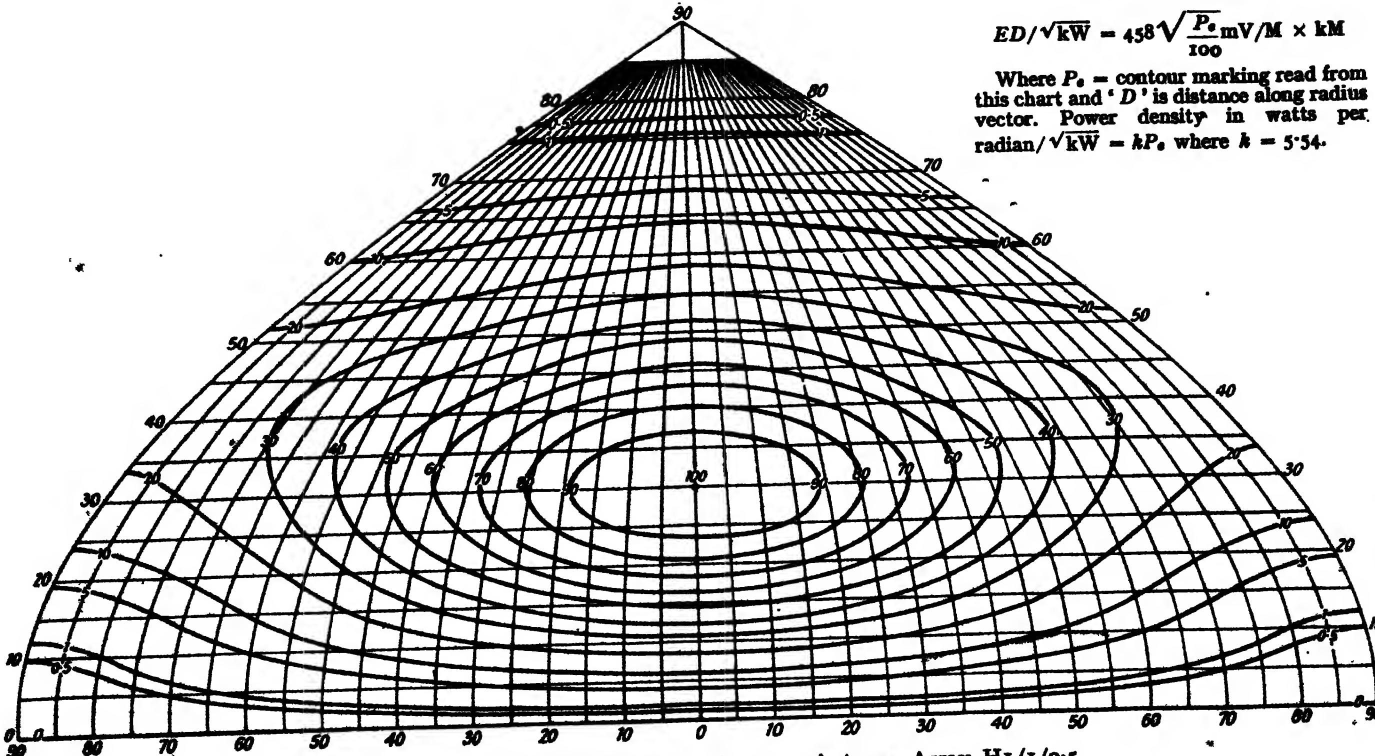


FIG. 4/XVI:7.—Power Distribution Diagram—Antenna Array H1/1/0·5.
Ranges of Azimuth and Elevation for power of 50:—Elevation = 30° ±17°; Azimuth = ±48°; Directivity = 6·95.

Chart Calibration

$ED/\sqrt{kW} = 436\sqrt{\frac{P_e}{100}}$ mV/M × kM

Where P_e = contour marking read from this chart and 'D' is distance along radius vector. Power density in watts per radian/$\sqrt{kW}$ = kP_e, where k = 5·04.

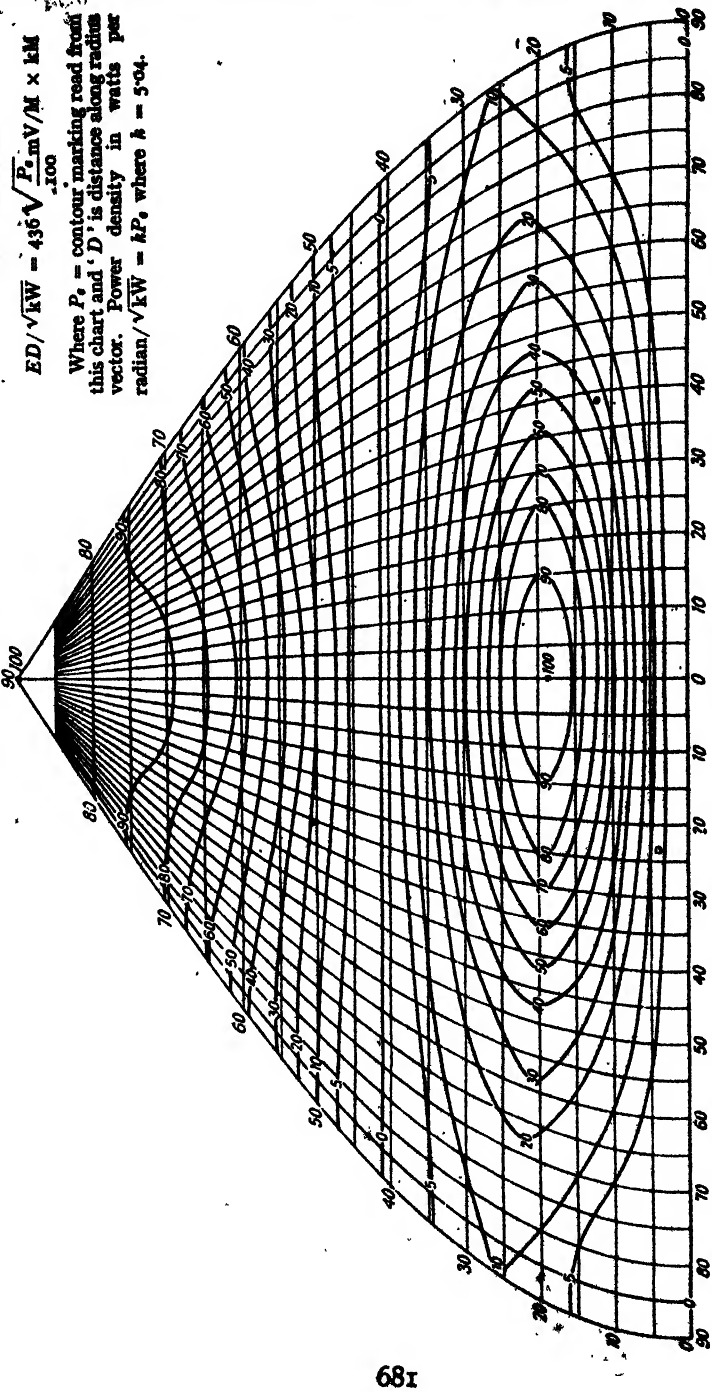


FIG. 5/XVI:7.—Power Distribution Diagram—Antenna Array H1/1/0·75.

Ranges of Azimuth and Elevation for power of 50:—Elevation = 90° – 56·5°; Azimuth = ±90°. Elevation = 20° ±10°; Azimuth = ±42°. Directivity = 6·3.

Chart Calibration

$ED/\sqrt{kW} = 600\sqrt{\frac{P_e}{100}}$ mV/M × kM.

Where P_e = contour marking read from this chart and 'D' is distance along radius vector. Power density in watts per radian/$\sqrt{kW}$ = kP_e where k = 9·55.

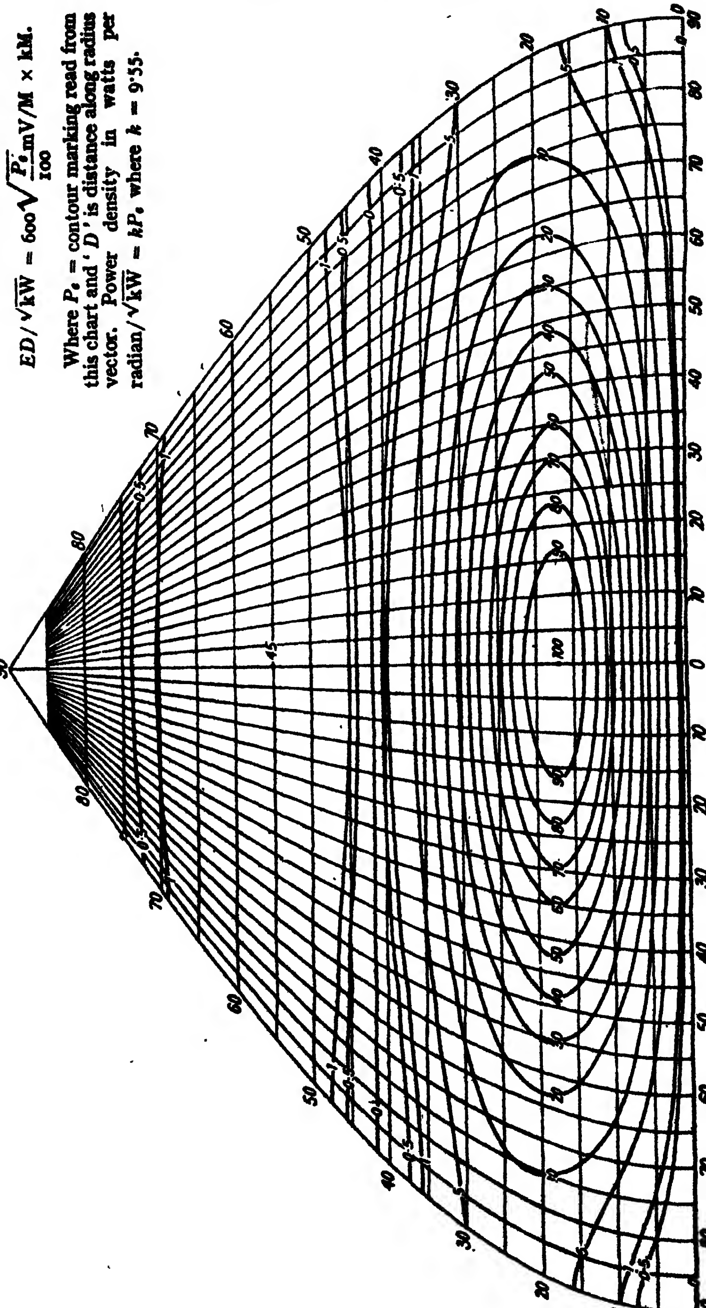


FIG. 6/XVI:7.—Power Distribution Diagram—Antenna Array H1/2/0·5.

Ranges of Azimuth and Elevation for power of 50:—Elevation = 17° ±10°; Azimuth = ±10°; Directivity = 12.

Chart Calibration

$ED/\sqrt{kW} = 1123\sqrt{\frac{P_e}{100}}$ mV/M × kM.

Where P_e = contour marking read from this chart and 'D' is distance along radius vector. Power density in watts per radian/$\sqrt{kW}$ = kP_e where k = 37·5.

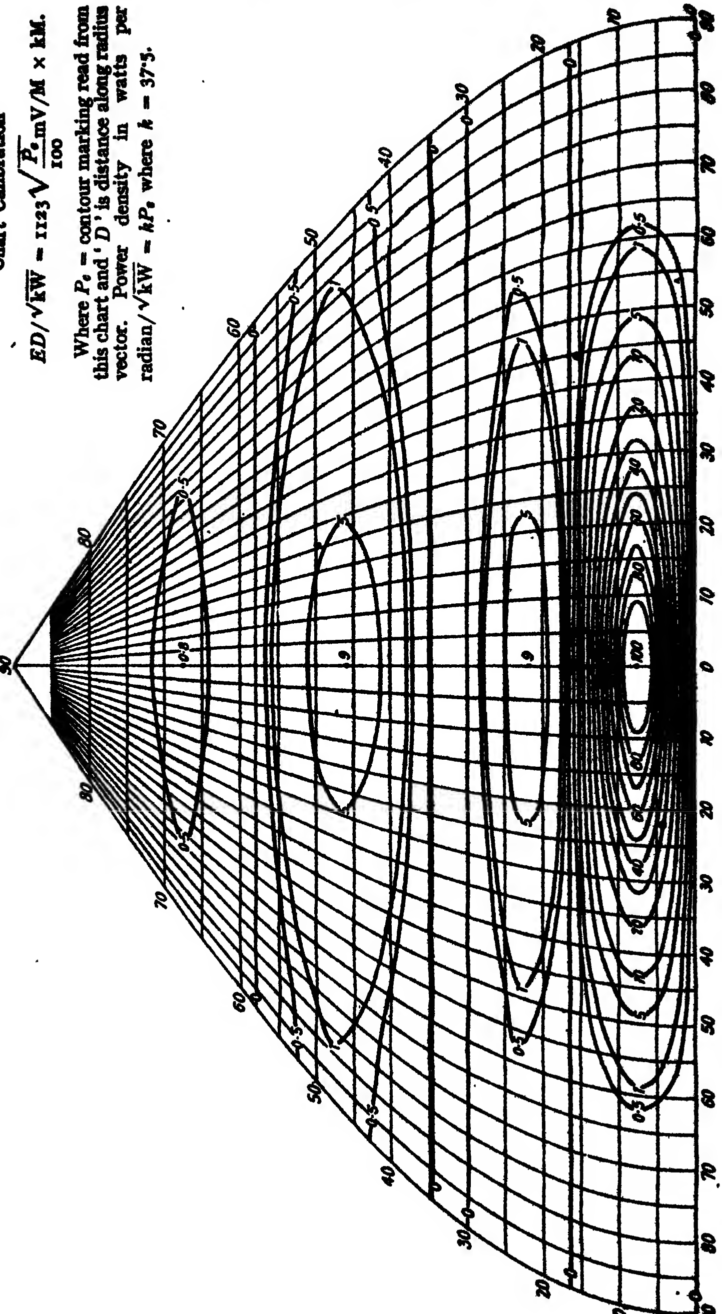


FIG. 7/XVI:7.—Power Distribution Diagram—Antenna Array H2/4/1.

Ranges of Azimuth and Elevation for power of 50:—Elevation = 7·4° ±4°; Azimuth = ±24°; Directivity = 48.

Chart Calibration

$ED/\sqrt{\text{kW}} = 1550\sqrt{\frac{P_e}{100}}$ mV/M × kM

Where P_e = contour marking read from this chart and 'D' is distance along radius vector. Power density in watts per radian/$\sqrt{\text{kW}}$ = kP_e where k = 63·6.

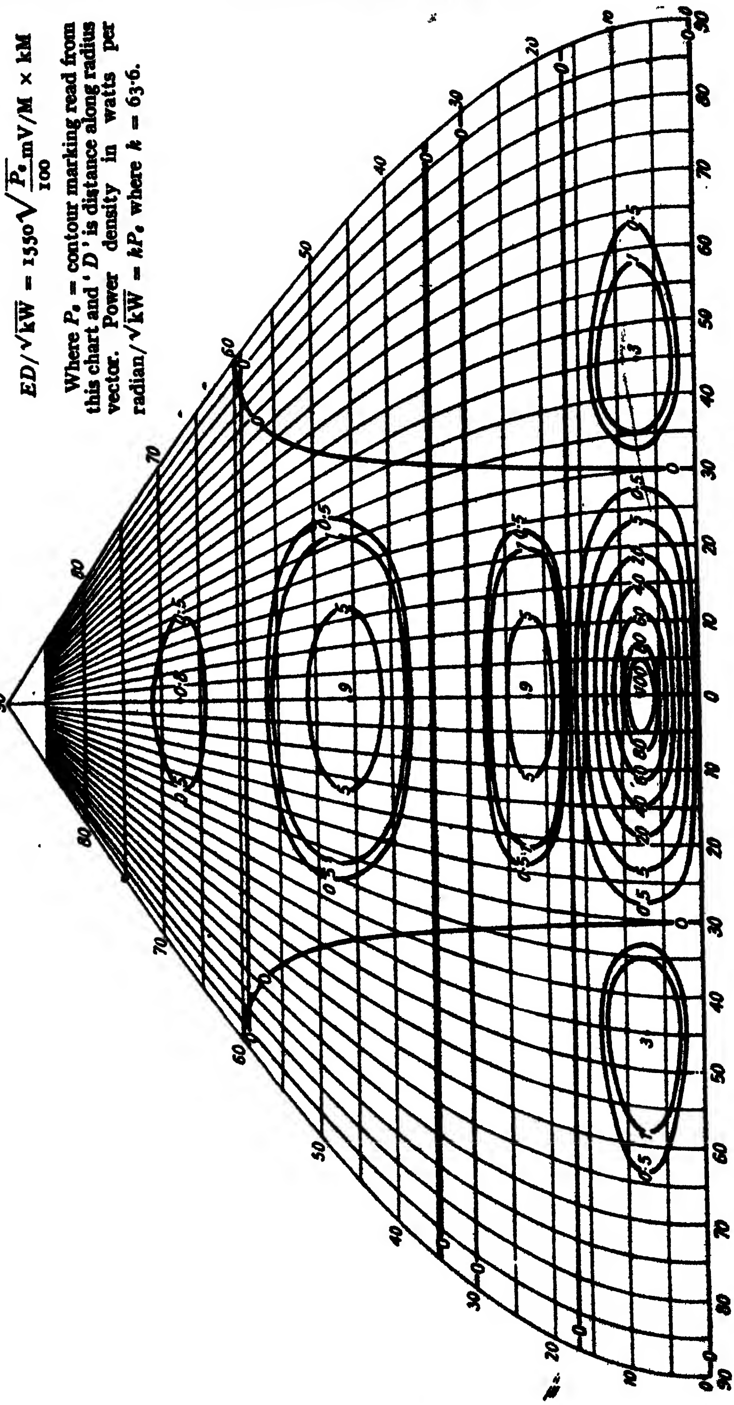


Fig. 8/XVI:7.—Power Distribution Diagram—Antenna Array H4/4/1.

Ranges of Asimuth and Elevation for power of 50:—Elevation = 7·4° ±4°; Azimuth = ±13°; Directivity = 80.

Chart Calibration

$$ED/\sqrt{\mathrm{kW}} = 1650\sqrt{\frac{P_e}{100}}\,\mathrm{mV/M} \times \mathrm{kM}$$

Where P_e = contour marking read from this chart and 'D' is distance along radius vector. Power density in watts per radian/$\sqrt{\mathrm{kW}} = kP_e$ where $k = 72$.

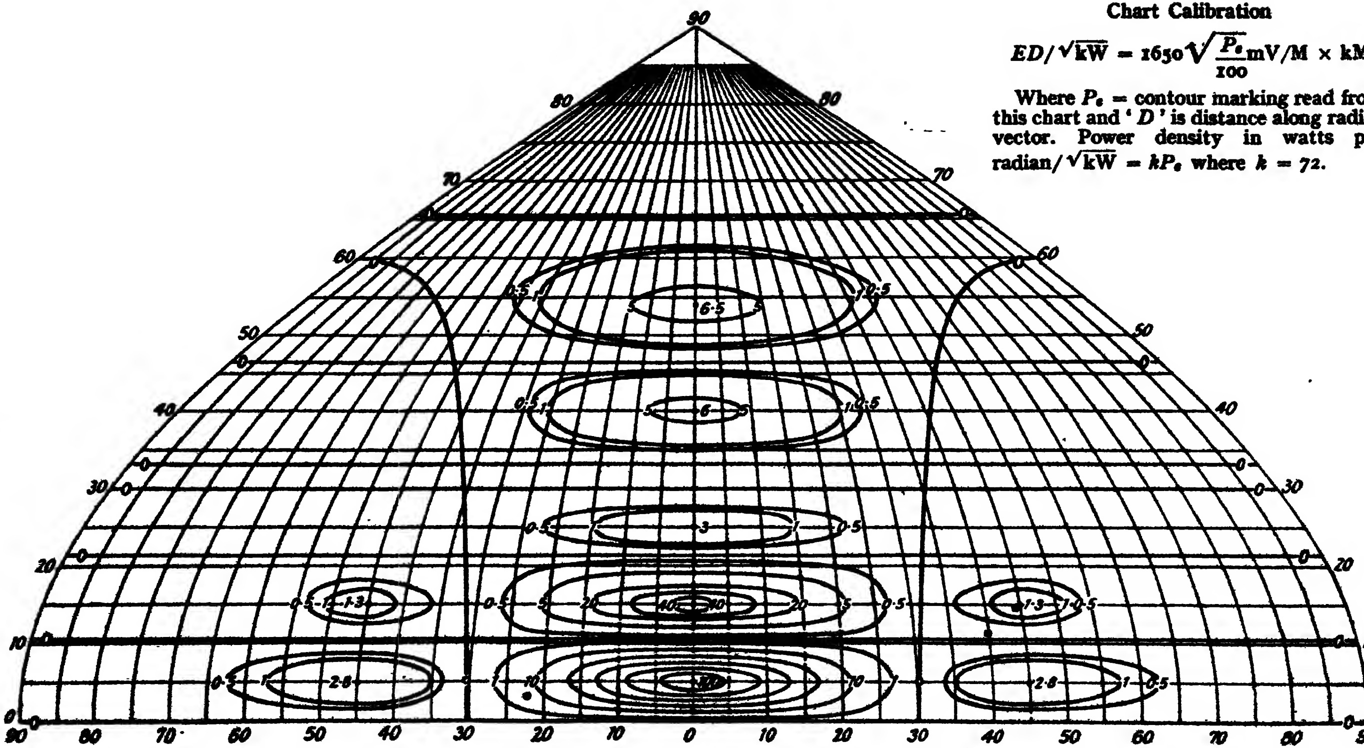


FIG. 9/XVI:7.—Power Distribution Diagram—Antenna Array H4/4/2.
Ranges of Azimuth and Elevation for power of 50:—Elevation = 5° ±2½°; Azimuth = ±13°; Directivity = 90·4.

Chart Calibration

$$ED/\sqrt{\text{kW}} = 1400\sqrt{\frac{P_e}{100}}\ \text{mV/M} \times \text{kM.}$$

Where P_e = contour marking read from this chart and 'D' is distance along radius vector. Power density in watts per radian/$\sqrt{\text{kW}}$ = kP_e where k = 51·7.

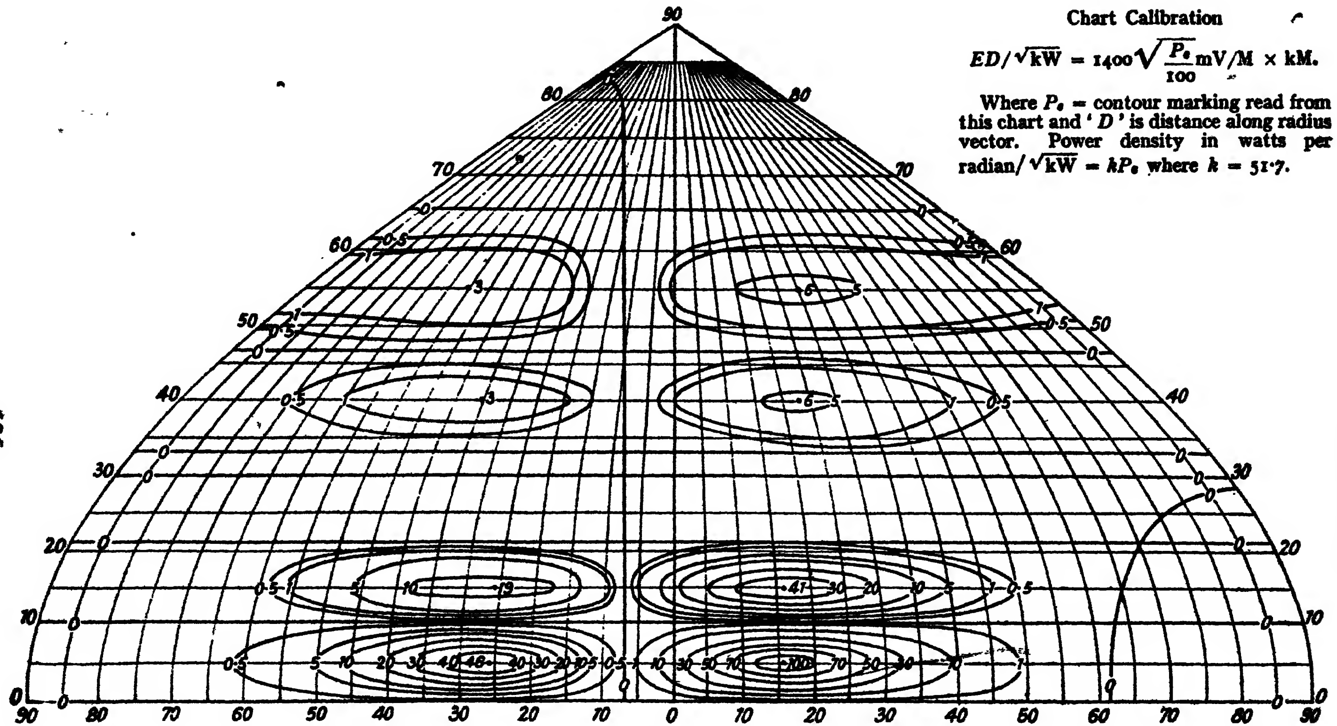


FIG. 10/XVI:7.—Power Distribution Diagram—Antenna Array HS4/4/2. (Slew 16°.)
Ranges of Azimuth and Elevation for power of 50:—Elevation = 5° ± 2½°; Azimuth = 16° ± 12°; Directivity = 66; Phase difference between antenna halves = 136°

(Figs. 2-10/XVI:7 by courtesy of the B.B.C.)

adding elements side by side with existing elements is to reduce the width of the beam.

Figs. 7, 8, 9 and 10 show respectively the polar diagrams of an H2/4/1, and H4/4/1, an H4/4/2 and an HS4/4/2 array.

7.112. Calibration of Power-Density Diagrams. On each power density diagram is given a formula for the *ED* per root kilowatt.

The ED per root kilowatt of an aerial is the field strength (in these diagrams in millivolts per metre) for a total radiated power of 1 kW, at a distance of one kilometre from the aerial, measured along the straight line joining the aerial to the point at which the field strength is specified, i.e. measured along the radius vector.* All this merely means that, if the field strength is specified for an angle of radiation of 45° above the horizontal, it is specified at a point in space distant from the aerial by one kilometre along a line drawn at 45° to the horizontal.

Referring, for instance, to Fig. 7, the formula given in the top right-hand corner reads

$$ED/\sqrt{\text{kW}} = 1{,}123\sqrt{\frac{P_c}{100}}\,\text{mV/M} \times \text{kM}$$

This means that the field at one kilometre distance from the aerial, for one kilowatt radiated, is

$$1{,}123\sqrt{\frac{P_c}{100}} \text{ millivolts per metre}$$

where P_c = the relative power density indicated by the contour lines at each point on the diagram.

The above relations are evolved in CVI.

Example. Find the field strength at one kilometre from an H2/4/1 array along the radius vector to the point 130 metres above ground, situated 20° off the centre line of radiation of the array—i.e. on line of longitude (on diagram) 20°, for 100 kW radiated.

Since one kilometre equals 1,000 metres, the sine of the angle of elevation is 0·130 and the angle of elevation is therefore 7° 30′. From the diagram on Fig. 7, the value P_c at the intersection of longitude (azimuth) 20° and latitude (elevation) 7° 30′ is 59.

* This is not the proper definition, but the most useful. The full definition is of course that the *ED* per root kilowatt is the product of the field strength *E* at any point distance *D* along the radius vector from the aerial and the distance *D* divided by the square root of the number of kilowatts radiated. Both definitions assume non-dissipative propagation.

The field strength for one kilowatt radiated is therefore

$$1{,}123\sqrt{\frac{59}{100}} = 863 \text{ millivolts per metre}$$

The field strength for 100 kW radiated is therefore

$$8{,}630 \text{ mV/metre},$$

since the radiated field strength is proportional to the square root of the radiated power.

The field strength is inversely proportional to distance (assuming no dissipation), so that, neglecting dissipation, the field strength at 100 kilometres, for instance, is 86·3 mV/metre.

In practice it is more usual to want to know the radiation at given angles of azimuth and elevation so that the calculation is normally slightly simpler.

7.2. Slewing of Arrays. Slewing is adopted when an existing array has an angle of maximum radiation which is not quite in the right direction. By driving adjacent sections of the array in different relative phase it is possible to translate the angle of maximum radiation away from the centre meridian. The polar diagram of Fig. 10 is obtained by considering the H4/4/2 array corresponding to Fig. 9 to consist of two H2/4/2 arrays side by side, and driving them with a mutual phase difference of 136°. The method of doing this is illustrated in Fig. 11, which shows an HS4/4/2 array. If driven at B the two halves of the array are normally in parallel and the polar diagram of Fig. 9 results. If driven at A, then the phase of the left-hand stack of dipoles leads on the phase of the right-hand stack by twice the phase shift in the length AB, and the diagram of Fig. 10 results. For a slew of 16° (i.e. for a horizontal deviation of the angle of maximum radiation of 16°) the required phase difference between the two stacks is 136°. Hence $AB = \frac{1}{2}(136°/360°)\lambda$.

Fig. 11 also serves to illustrate the method of crossing the vertical lengths of feeder between dipoles, so that the 180° phase shift occurring in half a wavelength of feeder is cancelled and all dipoles are driven in phase. This method is standard in all the arrays here being discussed.

7.3. Reflectors. It will have been noticed that since all the arrays previously discussed radiate equally into each half-hemisphere, if an array is set up to serve a given area, half the radiated power is lost because it is radiated in the opposite direction. Further, on the 16- and 19-metre bands particularly, trouble with "echo" may be experienced since the back radiation goes round the world in the opposite direction from the forward radiation and

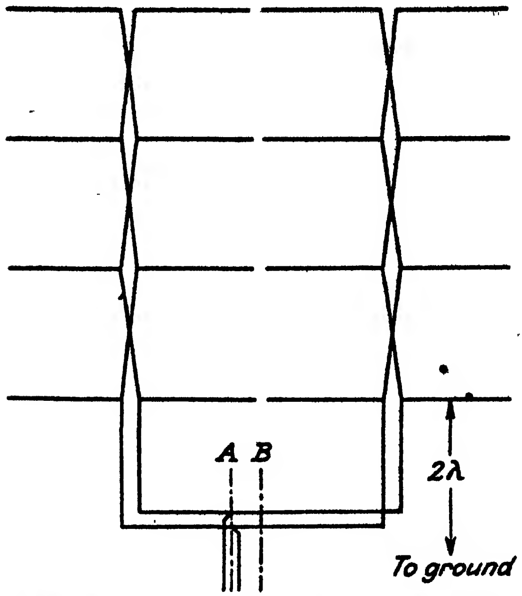


FIG. 11/XVI:7.—Method of Slewing illustrated by Case of an H4/4/2 Array. Distance $AB = \frac{1}{2}\frac{136°}{360°}\lambda$ for 16° Slew (136° Phase Difference).

arrives at the service area a fraction of a second later, of reduced strength, but strong enough to cause annoyance.

Echo is eliminated or largely reduced, and greater efficiency in the forward radiation is obtained, by the use of a reflector behind the array. The reflector consists of an array identical with the main array, located a quarter of a wavelength behind the main array with a short feeder running up to it exactly as does the feeder to the main array.

Where it is not possible to make measurements of the *front-to-back ratio*: the relative efficiency of forward and back radiation: the feeder to the reflector is terminated in a short circuit an odd number of quarter-wavelengths away from the bottom member of the reflector. This has the effect of effectively "removing the feeder"; in other words, the feeder presents a very high impedance towards the reflector array. With such a procedure a front-to-back ratio of about 15 db. is obtained.

Where it is possible to make measurements of front-to-back ratio this ratio may in practice be raised to nearly 25 db. by adjustment of the position of the short circuit on the feeder coming away from the array, until an optimum front-to-back ratio is obtained.

In certain cases a single array is operated over a small band of

frequencies; in such a case the short on the reflector feeder is located an odd number of quarter-wavelengths from the array at the midband frequency.

Since the reflector is identical with the aerial array, it is possible to reverse the direction of transmission by interchanging the roles of aerial and reflector. An array fitted with means for doing this is said to be "Reversible".

7.4. The Impedance Presented by a Half-Wave Dipole. The radiation resistance of a dipole varies with height above ground and also due to the presence of neighbouring dipoles and supporting stays. The radiation resistance of a dipole is the value of resistance which, placed at the centre of the current loop (position of maximum current), would dissipate the same power as is radiated by the dipole (assuming the current at the current loop to remain unchanged). The dipoles used in short-wave work are half-wave dipoles, and, when driven by any of the normal methods, a standing wave is set up as in Fig. 1 at (*a*), (*b*) and (*c*), which shows respectively a centre-fed dipole, a shunt-fed dipole and an end-fed dipole. The impedance presented by a dipole depends on the way it is fed.

7.41. Centre-Fed Dipole. In the case of a centre-fed dipole the impedance at 1,1 is approximately equal to the radiation resistance because, since each half of the dipole is a quarter-wave long and open circuited at the far end, the reactance at 1,1 is zero. The impedance is not exactly equal to the radiation resistance because there is also a radiation (inductive) reactance: it is usual to neglect this in practical calculations, although of course it is automatically taken care of by measurement.

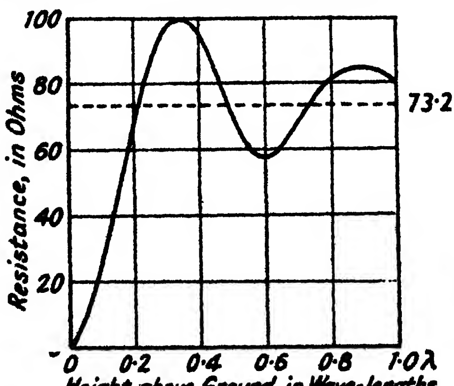


FIG. 12/XVI:7.—Variation of Radiation Resistance of Horizontal Dipole with Height above Ground.

The radiation resistance R_r of a dipole in free space is 73·2 ohms, and this is approximately the impedance appearing at 1,1. The feeder for a single centre-fed dipole is therefore preferably one with a characteristic impedance in the neighbourhood of 75 ohms. The way in which the radiation resistance varies with height above ground is shown in Fig. 12.

7.42. Shunt-Fed Dipole. The impedance looking into terminals 1,1 is given by

$$Z_{11} = Z_0\left(\frac{Z_0}{R_r}\sin^2\frac{2\pi x}{\lambda} + j\frac{1}{2}\sin\frac{4\pi x}{\lambda}\right)$$

Here x is the distance of each tapping-point from the centre of the dipole (see Fig. 1 (*b*)), R_r is the radiation resistance given by Fig. 12, and Z_0 is the characteristic impedance of each half of the dipole given by Fig. 2/XVI:1, taking the curve for a "vertical wire" of height h and diameter d. h is made equal to half the length of the dipole and d is equal to the diameter of the conductor from which the dipole is made. The reason for treating each half of the dipole as a vertical wire above ground is that a virtual earth is created at the neutral plane wiring, normally through the centre of the dipole. The above formula is due to Group-Captain W. P. Wilson.

7.43. End-Fed Dipole. In this case the radiation resistance is seen from 1,1 through a quarter-wavelength of feeder of characteristic impedance Z_0, as defined by Fig. 2/XVI:1 (see immediately above).

The impedance at 1,1 is, therefore, given approximately by

$$Z_{11} = \frac{Z_0^2}{R_r}$$

Example. Determine the impedance of an end-fed half-wave dipole 0·5λ above ground, built for 60 metres, from No. 12 gauge conductor, 0·104 in. diameter. (Neglect velocity factor.)

The radiation resistance for a horizontal dipole half-wave above ground is nearly equal to the free space radiation resistance.

Half the dipole-length = 15m = 15 × 3·28 × 12 ins. = 590 ins.

$\frac{h}{d} = \frac{590}{0\cdot104} = 5{,}670$, and from Fig. 2/XVI:1 $Z_0 = 500\ \Omega$,

$Z_{11} = \frac{500^2}{73\cdot2} = 3{,}420\ \Omega$

In practice, with 12-wire gauge and the lengths of radiating element used, the observed impedance is about 3,600 (see 7·8 below). This value may be assumed as an approximation independent of the dipole height.

In determining the value of Z_0 from h/d each half of the dipole must

be considered as being normal to a zero potential plane bisecting the dipole at right angles. This plane takes the place of ground so that the curve to use on Fig. 2/XVI:1 is that for a vertical wire of height h.

7.5. Impedance of an Array. The general method of calculating the impedance of an array will be illustrated by an example.

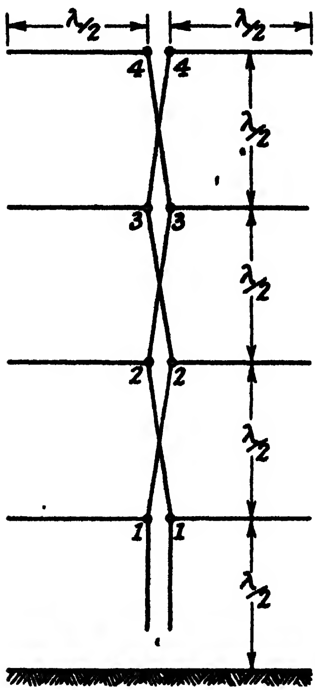


FIG. 13/XVI:7.—H4/2/0·5 Array.

An H2/4/0·5 array is shown in Fig. 13.

It is important to note that although the arrangement looks like an array of centre-fed dipoles, the members each side of the feeder are full half-wave dipoles and are therefore end fed.

On account of the half-wavelength spacing, the impedance at point 4,4 is referred to point 3,3 unchanged in magnitude; similarly, the impedance 3,3 is referred to 2,2 unchanged in magnitude. The impedance at 1,1 is therefore a quarter of the impedance due to any one pair of dipoles and is therefore given by

$$Z_{11} = \tfrac{1}{4} \times 2 \times 3{,}600 = 1{,}800$$

See XVI:7.43.

In practice, the input impedance of an array is influenced by many factors which may cause its value to depart from the value determined as above. The height above ground, the ground conductivity, and any mutual impedance between the elements of the array and any other conductors in the neighbourhood, are all factors which in practice are automatically taken into account by measuring the input impedance to an array on an R.F. bridge, instead of calculating it. Where a reflector is used, the mutual impedance between the reflector and the driven array, as well as the termination of the reflector (see XVI:7.3) have a large influence on the impedance of the array.

The method of calculating any other type of array will now be clear.

7.6. Size of Conductor used in Dipole Arrays. This depends on the power radiated by each dipole. In an H4/4 array, for

instance, there are 16 dipoles, and it is assumed that the power is divided equally between them, an assumption very close to fact, so that each dipole radiates only $\frac{1}{16}$th the total power.

The normal gauge used is No. 12 S.W.G. (0·104 in. dia.), since this is the smallest gauge which has sufficient mechanical strength. A dipole constructed of No. 12 gauge is capable of radiating about 15 kW without excessive corona or high voltages leading to flash-over.

A dipole of No. 6 gauge (0·192 in. dia.) is capable of radiating 50 kW, while a dipole constructed of two No. 6 gauge conductors with 9 ins. spacing is capable of radiating 70 kW. For higher powers it is necessary to go to a cage form of construction.

The effect of using two wires in parallel is to reduce the characteristic impedance to about 0·7 times the impedance when a single wire is used.

7.7. Kraus Aerial. Fig. 14 shows a Kraus aerial which may be regarded as a special type of dipole and can be built up into arrays just as can ordinary dipoles.

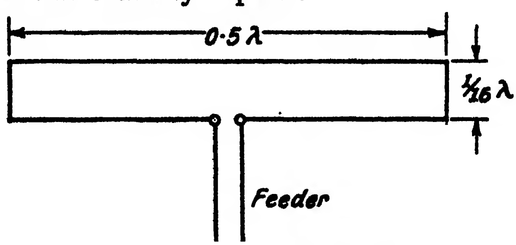


FIG. 14/XVI:7.—Kraus Aerial.

The particular advantage of this type of aerial resides in the fact that the impedance presented towards the feeder at terminals 1,1, is substantially 300 Ω, and a single aerial of this type is therefore much more closely matched to its feeder than the normal type of dipole.

7.8. Lengths of Half-Wave Dipoles. To satisfy the requirement that the input impedance of a dipole shall have zero angle it is necessary and customary to adjust the length of a "half-wave" dipole to a value which is not exactly equal to half the wavelength.

This is for two reasons: because the propagation velocity in the aerial differs from that in free space, and because the aerial, in addition to presenting a resistance at its driving point, corresponding to its radiation resistance, presents also a reactance corresponding to its radiation reactance (see XVI:7.4).

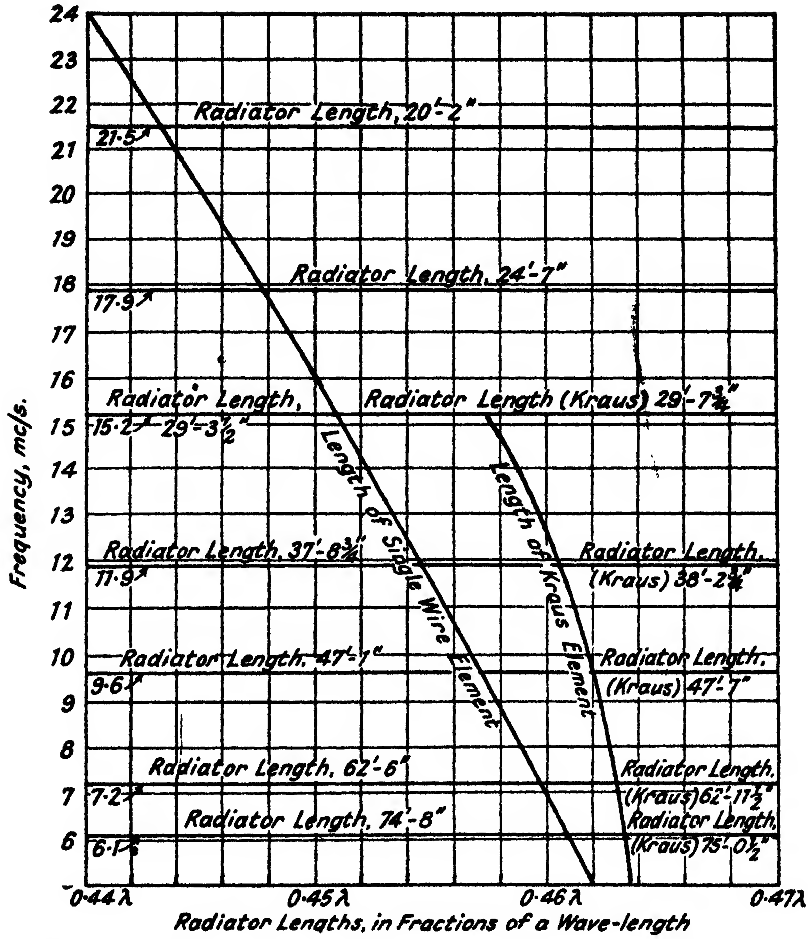


Fig. 15/XVI:7.—Lengths of Radiating Elements, Constructed of No. 12 S.W.G. Wire.

(By courtesy of the B.B.C.)

Fig. 15 shows curves of the relation between "half-wave" radiator lengths in fractions of a wavelength, and frequency. The left-hand curve applies to normal single-wire dipoles and the right-hand curve applies to Kraus aerials.

These curves were obtained experimentally by adjusting the length of an end-fed half-wave radiating element at each frequency until the input reactance disappeared. To balance the circuit, two such end-fed dipoles were used, fed by a balanced feeder.

With elements of these lengths the impedance looking into an end-fed dipole is normally 3,600 Ω, so that the impedance looking

into a pair of end-fed dipoles arranged end to end and driven respectively one from each leg of a balanced feeder, is 7,200 Ω. The impedance looking into an H2/4/– array is therefore in the neighbourhood of 1,800 Ω as shown above.

Experiment has shown also that radiating elements constructed of two stranded wires of 7/18 spaced 6 ins. apart offer a resistive input impedance when given the lengths indicated in Fig. 15 for single-wire elements.

8. Impedance Matching in Medium and Long-Wave Feeders.

In the case of medium- and long-wave transmitters aerial coupling networks of inductance and capacity are provided to match the aerial impedance to the feeder so that the latter is terminated in its proper characteristic impedance. This prevents standing waves in the feeder with consequent risk of flashover and increased power losses. In the case where balanced feeders are used a second object is also achieved; the transformation from the balanced-feeder circuit to the unbalanced aerial circuit. These aerial coupling circuits are installed in *aerial tuning huts* located immediately at the foot of the aerial.

8.1. Essential Points to be observed in Adjustment of Aerial Circuits. *Before considering in detail the technical procedure of adjustment, it is necessary to stress a number of essential points.*

1. Never touch or work on a circuit which has not been connected to a reliable earth by means of an earth-rod and clip.

2. Before touching any part of a circuit, even when earthed, check that it is dead by means of an insulated test-rod.

3. Never remove an earth-rod from a circuit or apply power to it until you have checked that everybody is standing clear.

4. Before making bridge measurements on an aerial, check that masts and any other aerials on the site are grounded through impedance equal to their working impedances. A half-wave mast is *usually* earthed, and a quarter-wave mast is *usually* left open at the base, while a $\frac{5}{8}\lambda$ wave mast is left open or earthed according to the results obtained on the polar diagram. Other aerials are left connected to their coupling circuits, unless these have not been designed, in which case they are treated as masts, and some re-adjustment of the circuit under measurement may be necessary when subsequently these aerials are connected to their coupling circuits. Conditions of other aerials and masts should be noted on records of all aerial impedance measurements.

5. Before passing any part of a circuit as satisfactory, calculate the currents and voltages which it will have to stand, and check that they do not exceed its rating. Remember to add in pick-up currents and voltages induced by any other aerial systems nearby.

6. Before making bridge measurements, check oscillator frequency against station crystals where these exist on the required frequency.

7. Before coming up on power on an aerial tuning circuit, insert meters of adequate rating to measure:

Aerial current.
Primary closed circuit current.
Feeder currents.

8. Never come up on power for the first time on a circuit without having someone watching it for flashover. In the case of circuits newly set up and put into service, an engineer should be left to watch the circuit for at least the first two hours of the first run on power, after which the circuit should be visited at intervals of two hours for the first day.

9. Before coming up on power on a circuit, make sure that all temporary connections have been replaced with conductors of adequate gauge; that all clearances are adequate and that all sharp corners have been rounded off.

10. Before coming up on power in an A.T.H. detune all receivers, oscillators, etc., away from the working frequency.

11. When circuit is complete, investigate amount of pick-up from other aerials radiating in the neighbourhood. It is necessary to check that this does not

(*a*) Cause component ratings to be exceeded.
(*b*) Give rise to intermodulation at the anodes of the last stage of the transmitter.

12. Circuits set up in aerial tuning huts must be enclosed as soon as possible with guard gates reaching from the floor to the ceiling.

8.2. Aerial-Coupling Unit. Where a balanced feeder system is used to feed an unbalanced aerial, the transition from balanced to unbalanced circuit is carried out by means of an aerial coupling unit. This consists of two coaxial coupled coils each wound in two halves.

As an example, the aerial coupling unit used with a B.B.C. medium power (e.g. 50 kW) "Regional" type transmitter will be considered. The circuit of this is shown in Fig. 3.

The outer or *coupling* coil consists of two balanced coils each

consisting of five turns of ¾-in. copper tubing 36 ins. diameter wound in the same sense. The axial separation between these coils is 4 ins. and the axial length of each coil is 10 ins., making an overall length of 24 ins. Its inductance is approximately 80 microhenrys and its maximum current-carrying capacity is 60 amperes. This coil is used as the balanced primary coil on the feeder side.

The inner or coupled coil consists of two coils end to end on the same axis 18 ins. in diameter *wound respectively in opposite sense*, each consisting of 18 turns of $\frac{7}{32}$-in. copper rod. The axial length of each coil is 9 ins. To vary their couplings with the outer the two coils are arranged to slide axially so that they can be adjusted from 6 ins. mutual separation (between inners of conductor) to 22 ins. separation. This coil is used with its two halves in parallel aiding, on the secondary or aerial side. This arrangement is used to balance the capacities to ground of the outer coil. The current-carrying capacity of the conductor is 15 amps., so that the two coils together can carry 30 amps.

The voltages across either primary or secondary should not exceed 6,000 volts R.M.S. in the (unmodulated) carrier condition, on account of the limitations imposed by the type of condenser in use.

Before considering the method of setting up aerial coupling circuits it is convenient to summarize the relation between series and parallel impedances and to describe the method of design of L matching networks, the last of which is not dealt with elsewhere.

8.3. Equivalence of Series and Parallel Impedances (see V:16 for conversion chart). It was shown in V:16 that the two

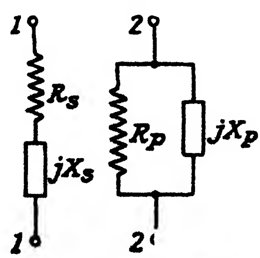


FIG. 1/XVI:8.—Series and Parallel Representation of Impedances.

circuits in Fig. 1 present the same impedance respectively between terminals 1,1, and 2,2, when at the frequency of measurement:

$$R_p = R_s\left(1 + \frac{X_s^2}{R_s^2}\right) \text{ and } X_p = X_s\left(1 + \right. \qquad (1)$$

An alternate form of this relation is

$$R_s = \frac{R_p}{1+\frac{R_p^2}{X_p^2}} \text{ and } X_s = \frac{X_p}{1+\frac{X_p^2}{R_p^2}} \quad . \quad . \quad . \quad (2)$$

It follows, therefore, that any impedance can be balanced on the known side of a bridge, either by using two elements in parallel or by two other elements in series. Whichever arrangement is used, when balance is obtained, the values of the other arrangement which would give balance can be found from equations (1) or (2).

It was also pointed out in V:16 that any impedance can be represented either in the form R_s+jX_s or in the form $R_p//+jX_p$ which is the abbreviation for R_p in parallel with $+jX_p$.

Example 1. An aerial driving-point impedance is measured as $500+j700$ and the volts to ground from the driving-point must not exceed 6,000v R.M.S. in the carrier condition. What carrier power can be supplied to the aerial considering this limitation only?

$$R_p = R_s\left(1+\frac{X_s^2}{R_s^2}\right) = 500\left(1+\frac{700^2}{500^2}\right) = 1{,}480\ \Omega$$

$$\text{Carrier power} = \frac{6{,}000^2}{1{,}480} = 24{,}300 \text{ watts} = 24{\cdot}3 \text{ kW.}$$

Example 2. An aerial driving-point impedance is measured as $1{,}000//+j500$. What R.M.S. carrier current must be supplied to the aerial to give 50 kW of carrier power, and what will be the voltage to ground under this condition?

$$R_s = \frac{R_p}{1+\frac{R_p^2}{X_p^2}} = \frac{1{,}000}{1+\frac{1{,}000^2}{500^2}} = 200$$

R.M.S. *Current into Aerial* $\therefore\ I^2 200 = 50{,}000$

$\therefore\ I = 15{\cdot}8$ amps.

Voltage to Ground. $\dfrac{E^2}{R_p} = \dfrac{E^2}{1{,}000} = 50{,}000$

$\therefore\ E = 7{,}070$ volts R.M.S. carrier (i.e. volts R.M.S. with no modulation).

The type of R.F. condenser in most common use in the B.B.C. for aerial tuning and feeder matching is designed for a maximum working R.F. voltage of 12,000 volts at 100% peak modulation; this corresponds to an unmodulated carrier level voltage of 6,000 volts R.M.S. Hence in the example above a special condenser would be necessary to place in parallel with the aerial, i.e. between the aerial driving point and ground, see, for instance, Fig. 3/XVI:8.

8.4. *L* Networks for Use as Impedance Transforming Sections. *L*-type networks constitute a type of impedance-transforming network which is not capable of being designed as a

bandpass filter, but provided the step-up ratio is not so high as to make the kVA/kW ratio exceed 5, the increased attenuation at sideband frequencies can be tolerated. *L* networks are normally designed and applied to work between zero angle impedances.

L networks are of two types as shown in Fig. 2 (*a*) and (*b*); the circuit at (*b*) is not generally used, however, because it gives no suppression of harmonics of the carrier frequency.

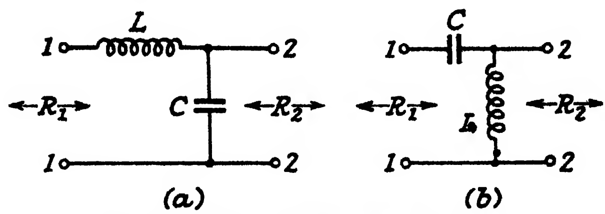


FIG. 2/XVI:8.—*L* Matching Networks.

In each case the shunt element is regarded as being adjusted so that the parallel reactance at 2,2, is neutralized: the shunt element possesses a reactance equal and opposite in sign to the parallel equivalent reactance of R_1 in series with the series reactance, while R_2 is equal to the equivalent parallel resistance.

Hence: In case (*a*)

$$-\frac{1}{Cj\omega} = Lj\omega\left(1+\frac{R^2}{L^2\omega^2}\right) \quad . \quad (3)$$

and

$$R_2 = R_1\left(1+\frac{L^2\omega^2}{R^2}\right) \quad . \quad (4)$$

$$\therefore \frac{R_2}{R_1} - 1 = \frac{L^2\omega^2}{R_1^2}$$

$$\therefore L = \frac{R_1}{\omega}\sqrt{\frac{R_2}{R_1} - 1} \quad . \quad . \quad (5)$$

From (3)

$$\frac{1}{C\omega} = \frac{R_1^2}{L\omega}\left(1+\frac{L^2\omega^2}{R_1^2}\right)$$

$$\therefore \frac{1}{C} = \frac{R_1R_2}{L}$$

$$\therefore C = \frac{1}{R_2\omega}\sqrt{\frac{R_2}{R_1} - 1} \quad . \quad (6)$$

In case (*b*)

$$Lj\omega = -\frac{1}{Cj\omega}(1+R^2C^2\omega^2) \quad . \quad (3a)$$

and

$$R_2 = R_1\left(1+\frac{1}{R_1^2C^2\omega^2}\right) \quad . \quad . \quad (4a)$$

$$\therefore \frac{R_2}{R_1} - 1 = \frac{1}{R_1^2C^2\omega^2}$$

$$\therefore C = \frac{1}{R_1\omega\sqrt{\frac{R_2}{R_1} - 1}} \quad . \quad . \quad (5a)$$

From (3*a*)

$$L\omega = R_1^2C\omega\left(1+\frac{1}{R_1^2C^2\omega^2}\right)$$

$$\therefore L = R_1R_2C$$

$$\therefore L = \frac{R_2}{\omega\sqrt{\frac{R_2}{R_1} - 1}} \quad . \quad . \quad . \quad (6a)$$

Equations (5) and (6) and equations (5*a*) and (6*a*) give the necessary design formulae respective to these two networks. As already indicated, *the usual type is the one using series inductances, since this provides suppression of harmonics of the carrier.* The type in Fig. 2 (*b*) could, however, be used in the case of an inductive aerial with over-voltage on it, the inductance being constituted by the equivalent parallel inductance of the aerial.

A very simple way of regarding the L-type network is to consider the shunt reactance as reducing the equivalent series resistance (e.g. due to R_2 and the shunt reactance in parallel) to the value determined from equation (2) (e.g. to R_1), while the series element neutralizes the resultant series reactance. A practical way of building an L network of the usual (series inductance) type is to insert a shunt capacity across the circuit and adjust it (by calculation, or by measurement) until the series resistance reaches the required value, and then adjust the series inductance until the capacitative series reactance is neutralized.

Where an aerial parallel impedance * presents a capacitative reactance, this must be made to constitute part of the shunt capacity of the L network, so that the value of physical capacity used is less than the value given by equation (6). When the aerial parallel reactance is inductive, more parallel capacity has to be added.

In most cases the inductance of the coupled (secondary) coil of the aerial coupling unit can be made to constitute the series inductance of the L network matching to the aerial.

The primary side of the coupling is also effectively a balanced equivalent of the L network in which the series inductance is partially cancelled by series capacity, e.g. C_1 in Fig. 5, shunt capacity being connected across the feeder. The object of condenser C_1 in Fig. 5 is to enable L_1 to be given sufficient turns to provide adequate mutual coupling without presenting too high an inductance to provide proper matching.

8.5. Method of Setting Up Aerial-Coupling Circuits. The examples given are for the case of a balanced feeder of 550 Ω impedance, but similar methods apply in the case of balanced feeders of other values of impedance. Where an unbalanced feeder is used the coupling coil is unnecessary since matching can be achieved by the use of L networks, see below.

A typical aerial-coupling circuit is shown in Fig. 3.

Terminals 1,1 and 2,2 are joined when the circuit is in service, being broken only for purpose of measurement while the circuit is

* Measured between the lower end of the aerial and ground.

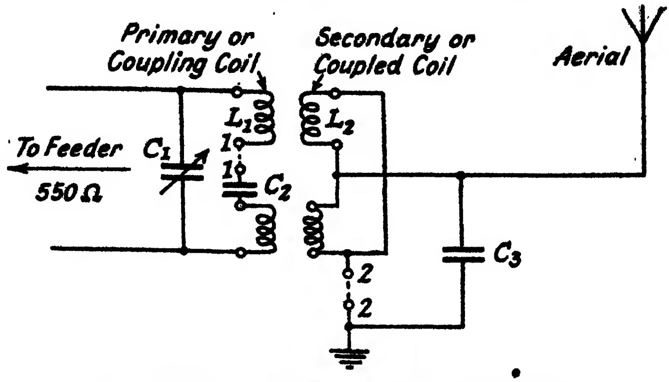


FIG. 3/XVI:8.—Typical Aerial Coupling Circuit.

being set up. Elements L_2 and C_3 are treated as constituting an L network stepping down from the aerial parallel resistance to the series resistance seen at 2,2. Allowance is made for the fact that C_3 is in parallel with the aerial parallel reactance. Similarly C_1 and L_1, reduced in effective inductance by the presence of C_2, constitute an L network stepping up from the series resistance seen at 1,1, with the feeder disconnected, to the feeder impedance of 550 Ω.

The steps involved in setting up the aerial coupling circuits are described *in terms of equipment at a typical* 50 *kW B.B.C. Regional Transmitter* and may be divided up as follows:

1. Measure aerial impedance.
2. Calculate volts between aerial driving-point and ground necessary to supply required power to aerial, and check that these do not exceed 6,000 volts R.M.S. carrier. See last paragraph of XVI:8.3.
3. Decide on form and values of circuit on secondary side of aerial coupling unit. See XVI:8·51 and 8.6. Set up this circuit
4. Set up secondary side so that impedance level about 80 ohms. This refers to the series resistance observed when measuring, with the primary open circuited, the impedance between earth and the earthy side of the secondary coils (i.e. looking into terminals 2,2, Fig. 3), the top end of the secondary coils being connected to the aerial through the circuit set up under (3), and the secondary inductance being adjusted until the impedance is substantially a pure resistance. A series resistance of 80 ohms observed in this way corresponds to a current of 25 amps. for 50 kW. Since the secondary is only rated at 30 amps. total, it is evident that the resistance should not be allowed to fall below this value. It should not be allowed to exceed 100 ohms

or else it will not be possible to obtain a sufficiently high referred resistance in the primary circuit.

If the referred resistance in the primary circuit is too low, the impedance looking into the primary winding, when tuned as in para. 5 immediately below, will be greater than 550 ohms.

(It is evidently a simple matter to derive equivalent figures of resistance and currents for other values of power. If higher currents are required, the coupling coil conductors have to be designed accordingly.)

5. Set up primary side. This consists in adjusting the primary turns and the mutual coupling (by sliding the coupled coils in or out) until the series resistance in the primary circuit is about 30 ohms. This refers to the series resistance observed when measuring (with the secondary circuits as derived above connected to the aerial) the series impedance at the mid-point of the primary coils (i.e. looking into terminals 1,1, Fig. 3), the feeder being disconnected and the primary tuned to series resonance with appropriately located capacity. This capacity may be conveniently constituted by the two condensers C_1 and C_2. C_1 is usually a variable oil condenser and C_2 is a fixed condenser. The function of C_2 is to neutralize some of the reactance of the primary coil so that the resistance facing the feeder will not be stepped up to too high a value when C_1 is tuned to give an impedance of zero angle facing the feeder.

After adjustment of the series resistance to 30 ohms, C_2 and C_1 are adjusted until the impedance facing the feeder is as nearly as possible $550\underline{/0}$ ohms. Since C_2 is adjustable in steps, final fine adjustment to $550\underline{/0}$ ohms is made by sliding the coupled coil to change the mutual, tuning on C_1. In cases where it is required to adjust a circuit to a given impedance a very useful method is to set up this impedance on the known side of the bridge and vary the circuit until balance is obtained. The above is a case where this method applies.

8.51. Choice of Form of Circuit on Secondary Side of the Coupling Unit. When R_{ae} the series resistance component of the aerial is between 80 and 100 ohms, no impedance transformation is necessary (other than that provided by the coupling unit), and all that is required is a series reactance to neutralize the total series reactance of the secondary circuit as indicated in Figs. 4 (*a*) and (*b*), in which the aerial impedance in its series form is shown on the right of the chain-dotted line.

In certain cases it may be possible to adjust X_e to be equal to $-X_{ae}$, in which case no further reactance is required.

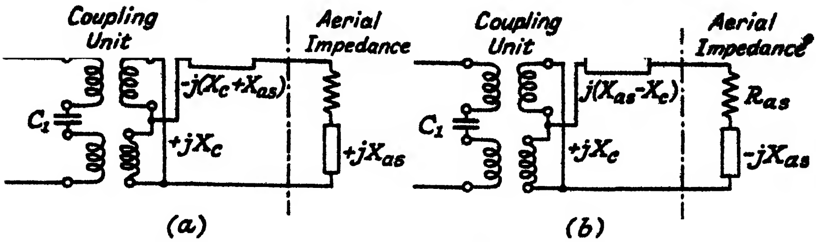


FIG. 4/XVI:8.—Aerial-Feeder Matching Circuits.

The circuits of Figs. 5 (*a*) and (*b*) can be made to cover nearly all other conditions except where over-voltages are experienced. In both these figures the aerial impedance is shown on the right-hand side of the dotted line in its parallel form.

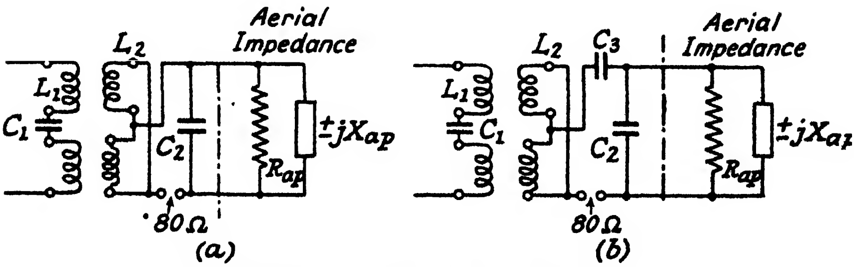


FIG. 5/XVI:8.—Aerial-Feeder Matching Circuits.

The circuit of Fig. 5 (*a*) uses the conventional *L* impedance matching network in which the series inductance is constituted by L_2 the coupled coil, and the shunt capacity by C_2 in conjunction with the parallel reactance of the aerial. The value of C_2 is evidently given by the value required to step up from $R_1 = 80\ \Omega$ to $R_2 = R_{ap}$, minus or plus the equivalent parallel capacity of the aerial according to whether this is respectively positive or negative.

The circuit in Fig. 5 (*b*) is the same circuit in which the reactance of the series inductance (supplied by the coupled coils) is partially cancelled by C_3 so as to obtain a lower step-up ratio when required.

The converse circuit in which extra inductance is added in series between the coupled coils and the aerial is hardly ever required, since when such a high step-up ratio occurs the impedance is so high that over-voltages occur. Such a case is shown in Fig. 6 and is considered below.

If, for instance, when setting up circuits to radiate 50 kW carrier, the parallel resistance of the aerial is greater than 720 Ω, the volts to ground from the aerial driving point will exceed 6,000, and if

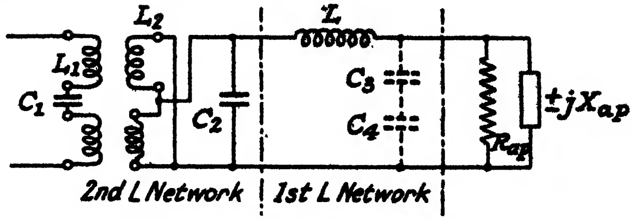


FIG. 6/XVI:8.—Aerial-Feeder Matching Circuit.

it is necessary to insert shunt capacity, two condensers must be used in series to break down the voltage.

Alternatively, a series inductance must be used in conjunction with the aerial shunt reactance to transform the circuit down to a pure resistive circuit of lower impedance (but still greater than 80 Ω) and lower voltage. It is necessary to watch that the volts across the inductance which are equal to the difference between the voltages, respectively at the points of different impedance level joined by the inductance, do not become excessive. A second L network may then be used to transform the impedance down to 80 Ω. The resulting circuit is as shown in Fig. 6 with C_3 and C_4 omitted. Variations of this circuit may be introduced: C_3 and C_4 may be introduced and/or C_2 may be omitted.

If X_{ap} is capacitive the condensers C_3 and C_4 can be omitted provided a low enough impedance is obtained at C_2 to keep the volts across it down to a value not exceeding its rating. If X_{ap} is inductive the series capacity type of L network may be constituted by a series condenser in conjunction with X_{ap}.

8.6. Matching of Low Impedances. If the impedances to be matched are low it may be found that the value of shunt condenser required in an L network of normal design is too large to be realized economically; or with the condensers which are available.

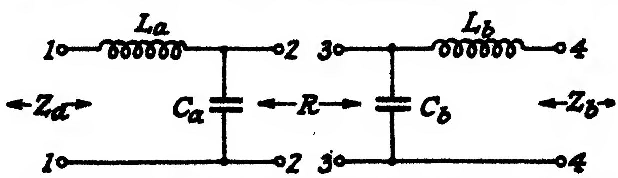


FIG. 7/XVI:8.—Method of Matching Low Impedances.

In such case the circuit of Fig. 7 may be used which merely consists of two normal L networks back to back. Fig. 7 shows the arrangement for matching an impedance Z_a to an impedance Z_b. One L network steps up from Z_a to a resistive impedance R, of magnitude determined as below; the other steps down from R to Z_b.

The value of R is determined by the permissible voltage across the condenser or condensers which are to be used for realizing condensers C_a and C_b, and by the power to be transmitted. (For optimum design see XXV:11.)

If P = the unmodulated carrier power to be transmitted.
V = the R.M.S. value of the unmodulated carrier voltage which is permissible across C_a and C_b,

then R must not be greater than the value satisfied by $P = \frac{V^2}{R}$.

In other words, R must not be greater than $\frac{V^2}{P}$. It may be an advantage to make R as close as possible to this value since this reduces the value of C_a+C_b to the smallest permissible value.

If $Z_a = R_a+jX_a$ and $Z_b = R_b+jX_b$, then one L network is designed to match from R_a to R, and the other to match from R to R_b. The values of L_a and L_b are then equal to the values of L given by equation (5) respectively, plus or minus the values of X_a and X_b, according to whether X_a and X_b are negative or positive.

The method of setting up this circuit is as follows:

(1) Find the maximum permissible value of R.
(2) Calculate the values of C_a and C_b to match to R.
(3) Find the nearest value of condenser larger than C_a plus C_b. Insert this condenser in circuit in shunt at the junction of the two networks with 2,2 joined to 3,3 and connect the load Z_b at 4,4.
(4) Observe the impedance looking in at 1,1 and adjust L_b until the series resistance component of this impedance equals R_a.
(5) Adjust L_a until the series reactance component of the impedance looking in at 1,1 is equal to minus X_a (e.g. if X_a is a negative reactance, the reactance looking into terminals 1,1 will, of course, be positive, and vice versa).

9. Insertion of Rejector Circuits for the Prevention of "Throw-in".

When two transmitters are operated on the same site, owing to the proximity of the aerials, each aerial picks up some of the radiation from the other transmitter and feeds power via the aerial coupling circuits and feeder, to the anodes of the valves in the output of the other transmitter. This effect is usually called "Throw-in". When the wavelengths are not widely separated and the aerials are not very far apart in space (and from this point of view the average

so that the voltage across the rejector circuit

$$= 18{\cdot}55 \times 246 = 4{,}560 \text{ volts R.M.S.}$$

The current through $L_R = \dfrac{4{,}560}{60} = 76$ amps.

The current through $C_R = \dfrac{4{,}560}{79{\cdot}3} = 58$ amps.

The difference between these currents $= 76 - 58 = 18$ amps. This constitutes a check on the working (which in the case above is by slide rule), since this difference should be equal to the aerial current: i.e. to 18·55 amps. in the present case.

The current through $C_s = 18{\cdot}55$ amps. and the voltage across $C_s = 18{\cdot}55 \times 239 = 4{,}430$ volts R.M.S.

At 767 kc/s it should be assumed that the whole open-circuit voltage of the aerial is effective across the rejector. The reactance of C_R is $-69\ \Omega$ and the reactance of L_R is also $69\ \Omega$. The current through each is therefore $\dfrac{1{,}350}{69} = 19{\cdot}6$ amps.

The R.M.S. current through C_R is therefore

$$\sqrt{58^2 + 19{\cdot}6^2} = 61{\cdot}3 \text{ amps.}$$

and the R.M.S. current through L_R is

$$\sqrt{76^2 + 19{\cdot}6^2} = 78{\cdot}5 \text{ amps.}$$

Since the diameter of the conductor in the inductance is 1 in., its current-carrying capacity is nominally $3 \times 25{\cdot}4 = 76{\cdot}2$ amps. It will, however, carry 78·5 amps. without undue overheating.

The kVA in $C_R = 4{\cdot}56 \times 58 + 1{\cdot}350 \times 19{\cdot}6 = 290{\cdot}4$.

The kVA will be divided between the two condensers constituting C_R in proportion to their capacities.

The voltage effective across C_R must be taken as the sum of the voltages due to the 668 and 767 kc/s currents since these voltages are periodically in phase. The voltage effective across C_R is therefore $4{,}560 + 1{,}350 = 5{,}910$ volts.

An important point occurs in connection with the practical setting up of circuits such as are shown in Fig. 1. This is that owing to stray capacity between the elements C_s, L_R and C_R and ground, the impedance looking towards the aerial at point B is not equal to the impedance obtained by adding together the aerial driving-point impedance, the rejector impedance and the impedance of C_s. This introduces no practical difficulty since the impedance at B is always measured on a bridge and the coupling circuit is then designed to match the feeder to this impedance.

10. Operation of Two Transmitters on Different Wavelengths into a Single Aerial.

By the use of rejector circuits two transmitters operating on different wavelengths may be connected directly to the same aerial.

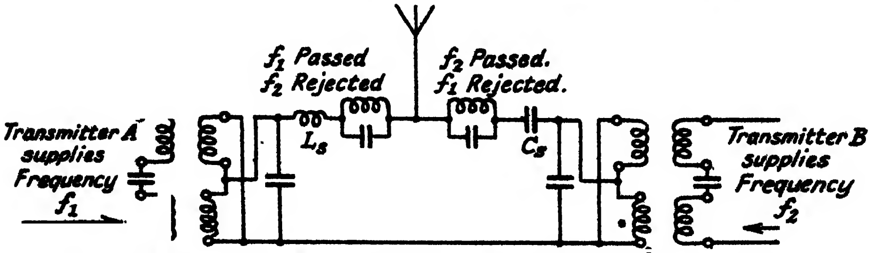


FIG. 2/XVI:9.—Operation of Two Transmitters into a Single Aerial. Frequency f_1 greater than f_2.

The general circuit arrangement is shown in Fig. 2 and the method of design of the rejector circuits and the elements C_s and L_s is exactly the same as in the case when such elements are used to combat throw-in. Each rejector circuit must be designed to withstand the total voltages and currents consequent on simultaneous transmission by both transmitters.

11. Losses in Aerial Tuning Huts.

It sometimes happens that when a circuit, which has been matched to an aerial by measurement, is powered, the power going into the aerial (as determined by the aerial ammeter and the series resistance component of the aerial impedance) is very much less than the power delivered by the transmitter, as determined either by the anode input power and the anode efficiency or by the feeder current and feeder impedance. When the magnitude of the discrepancy is too large to be explained by inaccuracies in R.F. meters, it is due to power supplied either to stray capacities with loss in them, or else to lossy circuits coupled to one or more of the coils in the A.T.H.

The stray capacities may occur between any element of appreciable size having a high voltage to ground, which may be a coil or a condenser, and the floor or walls of the A.T.H. In this case the loss can sometimes be located by the fact that the wall or floor in the part where the loss is occurring is rather warmer than in other parts. The method of reducing this loss is, first, to locate any high-voltage points of the circuit as far from the floor and walls of the A.T.H. as is conveniently possible, and secondly, if the loss is still serious, to cover the floor and walls where the loss is

taking place with sheets of $\frac{1}{16}$ in. copper sheeting well bonded together and earthed through a copper strip of at least $\frac{1}{8}$ sq. in. in section, e.g. 1″ × $\frac{1}{8}$″ in section.

Loss due to currents induced electro-magnetically into the walls and floor is usually not so serious as that due to stray capacity, and can generally be avoided by orientating coils so that no coupling occurs with any part of the walls or floor near the coils.

If the wooden former of a coil is at all damp the power absorbed by the wood may heat it sufficiently to set it on fire. This may appear rather a paradox, but it constitutes a very real danger, and for this reason when new wooden formers are first brought into use the circuit is run on reduced power for a sufficient time to dry the wood.

12. Impedance Matching in Long-Wave Aerial Circuits.

Long-wave aerials present a special problem because, on account of the lower carrier frequency, the percentage band width which the circuits have to pass for any given band of audio frequencies, is considerably increased as compared with shorter waves. For instance, to pass audio frequencies up to 5 kc/s with a carrier of a megacycle it is necessary to pass sidebands at 995 kc/s and 1,005 kc/s and the percentage band width is $\frac{10{,}000}{1{,}000{,}000} = 1\%$. To pass the same band width with a carrier frequency of 200 kc. corresponding to 1,500 metres, a band width of 5% is required. This may not appear to be a very large percentage band width, but in practice considerable difficulties are experienced owing to the variation of the reactance of the aerial over the band of frequencies.

An identical problem occurs of course in television aerials which may have to transmit a band of video frequencies 2 megacycles wide on a carrier frequency of, say, 40 megacycles. The sideband width is then 10% of the carrier frequency and conditions are even more stringent. *While the discussion below is specifically in terms of long-wave aerials, it applies with equal force to television aerials.*

In all transmitters the impedance presented towards the anodes of the final amplifier by the aerial impedance, seen through the feeder and transmitter and aerial coupling circuits, varies with frequency. At medium and short waves, since the audio-frequency band width is a small percentage of the carrier frequency, this effect is comparatively unimportant, but at long waves, as stated above, it is appreciable.

The aerial impedance can usually be represented by an inductance

and a capacity in series with a resistance. By adding a reactance (usually an inductance) in series with the aerial impedance, the reactance at the carrier frequency can be reduced to zero, and the resultant impedance is then substantially symmetrical about the carrier frequency, rising at sideband frequencies as the distance of the sideband frequency from the carrier increases. This type of impedance is called a *symmetrical rising characteristic.*

At a point along the transmission line from transmitter to aerial, where the phase shift to the aerial is about 90°, the impedance looking towards the aerial is inverted and appears as a *symmetrical falling characteristic*, that is to say, the impedance at the sideband frequencies decreases as the distance of the sideband frequencies from the carrier frequency increases.

It is important to note that in the case of both symmetrical rising and symmetrical falling characteristics, the impedance at each of any two sideband frequencies (i.e. corresponding to any one modulating frequency) is the same, although the impedance at the sideband frequencies differs from that at the carrier frequency. The impedance at the sideband frequencies in both these symmetrical cases is not a pure resistance but possesses an angle.

In the case where power factor correction of the aerial circuits is used, as for instance in the B.B.C. long-wave aerial circuits at Droitwich, a type of symmetrical case occurs in which the impedance at sideband frequencies is completely resistive at two symmetrically disposed sideband frequencies, and at the carrier frequency, and is substantially resistive over the whole band.

It will be evident that at intervals separated by 90° phase shift (at the carrier frequency), along the transmitter aerial chain, points of symmetrical rising characteristic and symmetrical falling characteristic alternate. In between the points of symmetric impedance characteristic the impedance looking towards the aerial is *asymmetric.* That is to say, the impedance at each sideband frequency is not the same.

It is evident therefore that, by building out the network constituted by the transmitter output circuits plus feeder and aerial coupling circuits, to have a required phase shift at the carrier frequency, any form of impedance characteristic (i.e. symmetric rising or falling, or asymmetric) may be presented at the anodes of the output valves of the transmitter. It is even possible, in the case of an aerial with an asymmetric impedance characteristic, to introduce a measurable approach to symmetry by building up the phase shift to an appropriate non-integral multiple of 90°.

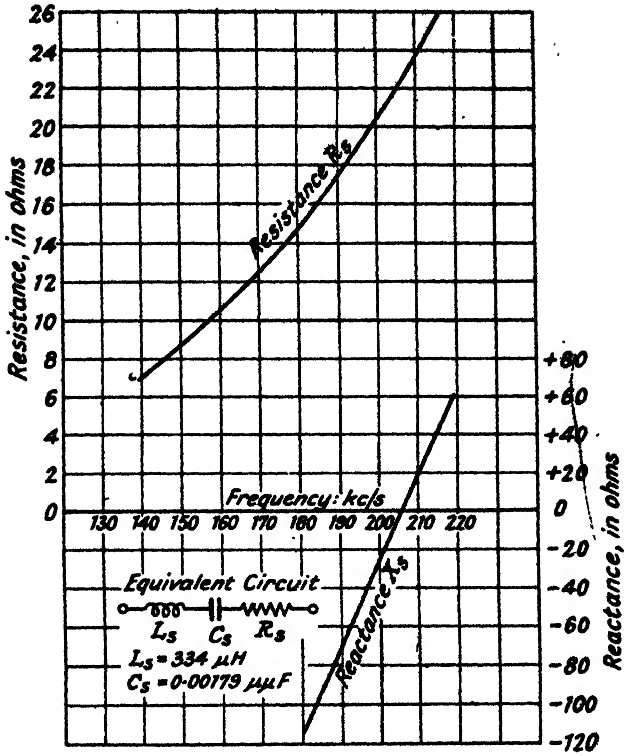


FIG. 1/XVI:12.—Resistance and Reactance Components of Impedance observed between Lower End of Downlead of Droitwich 1,500-m. Aerial and Ground. Equivalent Series Resonant Circuit simulating Reactance.

(By courtesy of the B.B.C.)

12.11. Simulation of Reactance Curve by Series Resonant Circuit.

Conventions applicable to 12.11 *only.*

Let X_1 = the reactance of the aerial at frequency f_1.
X_2 = the reactance of the aerial at frequency f_2.
L = inductance in series resonant circuit.
C = capacity in series resonant circuit.
$\omega_1 = 2\pi f_1,\ \omega_2 = 2\pi f_2.$

Then if the resonant circuit presents the same reactance as the aerial

$$L\omega_1 - \frac{}{C\omega_1} = X_1 \tag{1}$$

and

$$L\omega_2 - \frac{1}{C\omega_2} = X_2 \tag{2}$$

From (1) $$L = \frac{X_1}{\omega_1} + \frac{1}{C\omega_1^2} \quad . \quad . \quad . \quad . \quad (3)$$

and substituting in (2):

$$\therefore \frac{\omega_2}{\omega_1} X_1 + \frac{\omega_2}{C\omega_1^2} - \frac{1}{C\omega_2} = X_2$$

$$\therefore C = \frac{\dfrac{\omega_2}{\omega_1^2} - \dfrac{1}{\omega_2}}{X_2 - \dfrac{\omega_2}{\omega_1} X_1} = \frac{\omega_2^2 - \omega_1^2}{X_2\omega_1^2\omega_2 - X_1\omega_1\omega_2^2} = \frac{f_2^2 - f_1^2}{2\pi f_1 f_2(X_2 f_1 - X_1 f_2)} \quad (4)$$

Substituting (4) in (2)

$$L = \frac{X_1}{\omega_1} + \frac{1}{\omega_1^2} \times \frac{2\pi f_1 f_2(X_2 f_1 - X_1 f_2)}{f_2^2 - f_1^2}.$$

$$= \frac{1}{\omega_1}\left(X_1 + \frac{f_2(X_2 f_1 - X_1 f_2)}{f_2^2 - f_1^2}\right)$$

$$= \frac{1}{2\pi} \times \frac{X_2 f_1 f_2 - X_1 f_2^2 + X_1 f_2^2 - X_1 f_1^2}{f_1(f_2^2 - f_1^2)}$$

$$= \frac{1}{2\pi} \times \frac{X_2 f_2 - X_1 f_1}{f_2^2 - f_1^2} \quad . \quad . \quad . \quad . \quad (5)$$

Referring to Fig. 1, if $f_1 = 194$ kc/s. $X_1 = -52$
and if $f_2 = 206$ kc/s. $X_2 = 0$

$\therefore$ from (4)

$$C = \frac{206^2 - 194^2}{2\pi . 194 . 206(0 + 52 . 206000)} 10^6 \ \mu F = 0{\cdot}00179 \ \mu F \quad . \quad (6)$$

From (5)

$$L = \frac{1}{2\pi} \times \frac{0 + 52 . 194000 . 10^6}{(206^2 - 194^2) . 10^6} \ \mu H = 334 \ \mu H \quad . \quad (7)$$

Check. Using the above values of L and C

$$L\omega - \frac{1}{C\omega} = 0 \text{ at } 206 \text{ kc.}$$

$$= 52 \text{ at } 194 \text{ kc.}$$
$$= -26\ \Omega \text{ at } 200 \text{ kc.} \quad \text{(Slide rule)}$$

and all those points lie exactly on the reactance curve of Fig. 1.

12.12. Neutralization of Aerial Reactance. To obtain symmetrical transmission of the sidebands, it is necessary to make the circuit resonate at 200 kc/s. This can be done by inserting a series inductance having an inductive reactance of 26 Ω at 200 kc/s.

This corresponds to an inductance L_e of 20·7 μH, so that if

this is added the total effective series inductance is $334+20{\cdot}7 \doteq 354{\cdot}7\,\mu$H.

It will be appreciated that the possibility of adding an inductance (or a capacity) in series in this way only exists when the aerial is operated near its quarter-wave condition, that is to say, the aerial is of such a length that a quarter-wavelength standing wave exists in the aerial. Alternatively, if the aerial is less than a quarter of a wavelength high, it may be effectively extended to quarter-wavelength, from the point of view of the reactance at the base of the aerial by addition of a capacity top. The capacity top must evidently present the same reactance towards the top of the aerial as would the necessary extension in height of the aerial to make it a quarter-wave long.

When the extra inductance has been added the reactance of the combination of aerial and inductance is zero at 200 kc/s, $+26{\cdot}0\ \Omega$ at 206 kc/s and $-26{\cdot}0\ \Omega$ at 194 kc/s. The resistance at 206 kc/s is $22{\cdot}4\ \Omega$ and at 194 kc/s is $18{\cdot}6\ \Omega$, so that the power factor at 206 kc/s is 0·65 and at 194 kc/s is 0·58. Hence, apart from the unavoidable transition loss due to the variation of the aerial resistance over the band of frequencies transmitted, there is a loss due to reactance which reaches nearly 4 db. at 206 kc/s and nearly 5 db. at 194 kc/s, increasing to higher values at frequencies further removed from the carrier frequency 200 kc/s.

This loss is in itself undesirable, but in addition, if normal methods of impedance matching are used, the impedance presented at the anodes of the output amplifier of the transmitter will have a large reactance component at the sideband frequencies.

12.13. Power Factor Correction at Two Sideband Frequencies. Use of Quarter-Wave Network.

Conventions.

R_a = aerial resistance at any frequency f.

X_a = aerial reactance at any frequency f.

$\omega = 2\pi f$.

L_e = inductance neutralizing aerial reactance at carrier frequency.

X_e = reactance of L_e at any frequency f.

$Z_1 = R_a+j(X_e+X_a) = X_e$+aerial impedance.

Z_n = characteristic impedance of quarter-wave network N at any frequency f.

L_n = inductance of each winding of unity ratio coupled circuit as shown in Fig. 3 (*a*).

M = mutual inductance of coupled circuit in Fig. 3 (*a*).
L_p = primary inductance of unequal ratio coupled circuit constituting quarter-wave network.
Z_2 = impedance looking into input of quarter-wave network as indicated in Fig. 2.
L_e and C_e = inductance and capacity respectively, of power factor correcting circuit shown in Fig. 2.
$X_e = L_e\omega - 1/C_e\omega$ = reactance of power factor correcting circuit.
Z_3 = impedance looking into terminals 3,3, in Fig. 2.
R_2, R_3, X_2, and X_3 = respectively the resistance and reactance components of Z_2 and Z_3.
Z_4 = impedance of Z_3 seen through a quarter-wave network.
R_4 and X_4 = resistance and reactance components of Z_4.

The method used for partial neutralization of reactance, which gives complete neutralization at two sideband frequencies, is ingenious and simple. A quarter-wave network N is inserted between the matching circuit connected to the feeder and the inductance L_e in series with the aerial as shown in Fig. 2.

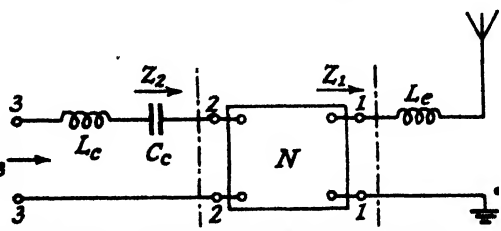


FIG. 2/XVI:12.—Principle of Reactance Neutralization.

The impedance looking into the input of the network N is

$$Z_2 = \frac{Z_n^2}{Z_1} = \frac{Z_n^2}{R_s + j(X_e + X_s)}$$

$$= \frac{Z_n^2[R_s - j(X_e + X_s)]}{R_s^2 + (X_e + X_s)^2} \quad . \quad . \quad . \quad . \quad (8)$$

$$= \frac{Z_n^2 R_s}{R_s^2 + (X_e + X_s)^2} - j\frac{Z_n^2(X_e + X_s)}{R_s^2 + (X_e + X_s)^2} \quad . \quad . \quad (9)$$

$$= R_2 + jX_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (9a)$$

In general, more than one type of quarter-wave network is suitable for use in the position occupied by N, but the type of quarter-wave network which was used in practice at Droitwich was provided by two coupled inductances as shown in Fig. 3 (*a*), where

M is the mutual inductance between the two circuits. (In the Droitwich circuit an unequal ratio coupling was used to provide a higher impedance on the primary side of the network but for simplicity a unity ratio network is considered here.) One equivalent circuit is shown at (*b*) and a second at (*c*). In Fig. 3 (*c*) the circuit between the chain-dotted lines constitutes a quarter-wave network *at all frequencies*; its characteristic impedance is evidently $M\omega$, proportional to frequency. If L_n is made equal to L_e, the inductance L_n on the right-hand side of Fig. 3 (*c*) can be made to perform the function of L_e.

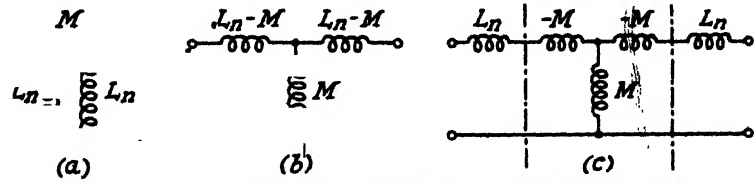


FIG. 3/XVI:12.—Coupled Inductances and Equivalent Circuits.

Assume that $L_n = L_e = 20{\cdot}7\ \mu$H and that the coupling factor of the mutual inductances is 0·6, then at 200 kc/s the value of Z_n the characteristic impedance of the quarter-wave network is $M\omega = 0{\cdot}6L_n\omega = 0{\cdot}6 \times 26 = 15{\cdot}6\ \Omega$, and at any frequency f kc/s

$$Z_n = 15{\cdot}6 \times \frac{f}{200} \qquad . \quad . \quad . \quad (10)$$

Now assume that network N in Fig. 2 consists of two coupled inductances of 20·7 μH with a coupling factor of 0·6. Assume further that the impedance Z_s is the impedance looking into this coupling circuit minus the reactance $L_n\omega$ implicit in the coupling circuit.

12.14. Calculation of Circuit Impedances. *Conventions as in* 12.13. The value of Z_s is then given by equation (9) with Z_n given the value from (10).

X_s the reactance component of Z_s is calculated in Table I for the case where $Z_n = 15{\cdot}6f/200$ and $L_e = 20{\cdot}7\ \mu$H, the values of R_s and X_s at each frequency being taken from Fig. 1; the value of X_s is given in column 9 and plotted in Fig. 4.

It is to be noticed that the values of reactances are positive below 200 kc/s and negative above 200 kc/s. They are, therefore, opposite in sign to the values of reactance of a series circuit* resonant at 200 kc/s such as L_e, C_e (Fig. 2), which can, therefore, be used

* A technique using a parallel resonant circuit in shunt across the circuit may equally well be developed.

TABLE

	1	2	3	4	5	6	7	8	9	10	11
	f kc/s	R_s	R_s^2	X_s	X_e	X_s+X_e	$(X_s+X_e)^2$	$R_s^2+(X_s+X_e)^2$	X_3	$X_c = L_c\omega - \frac{1}{C_c\omega}$	$X_2 = X_c+X_3$
0·9	180	14·8	218·0	− 116	23·4	− 92·6	8,550	8,768	+2·08	− 20·4	− 18·3
0·95	190	17·4	302	− 70	24·7	− 45·3	2,050	2,352	+4·87	− 9·9	− 5·03
0·97	194	18·6	346	− 52	25·2	− 26·8	730	1,076	+5·7	− 5·7	0
0·98	196	19·2	368	− 43	25·5	− 17·5	305	673	+6·1	− 4·0	+2·1
0·99	198	19·8	392	− 34	25·7	− 8·3	69	461	+4·3	− 2·0	+2·3
1·0	200	20·4	417	− 26	26·0	0	0	417	0	0	0
1·01	202	20·7	435	− 16	26·25	+ 8·25	68	510	− 4·0	+ 2·0	− 2·0
1·02	204	21·6	467	− 8	26·55	+18·25	332·5	799·5	− 5·78	+ 4·0	− 1·78
1·03	206	22·4	501	0	26·8	+26·8	718	1,219	− 5·7	+ 5·7	0
1·05	210	23·8	566	+19	27·3	+46·3	2,140	2,706	− 4·6	+ 9·4	+ 4·8
1·1	220	27·8	773	+64	28·6	+72·6	5,250	6,023	− 3·57	+18·4	+14·83

to secure reactance cancellation at two frequencies each side of the carrier frequency. The choice of these frequencies is evidently a question of compromise and depends on the band width over which effective transmission is to be obtained, the variation of X_2, and the behaviour of the transmitter when faced with an impedance varying with frequency and having reactance.

The best compromise is probably obtained by neutralizing the reactance at the sideband frequencies corresponding to the highest frequency in the audio-frequency range to be transmitted. Assuming the audio-frequency range to extend up to 6 kc/s, the case will be considered where neutralization is effected at 194 and 206 kc/s.

The values of L_c and C_c are then obtained as follows. The

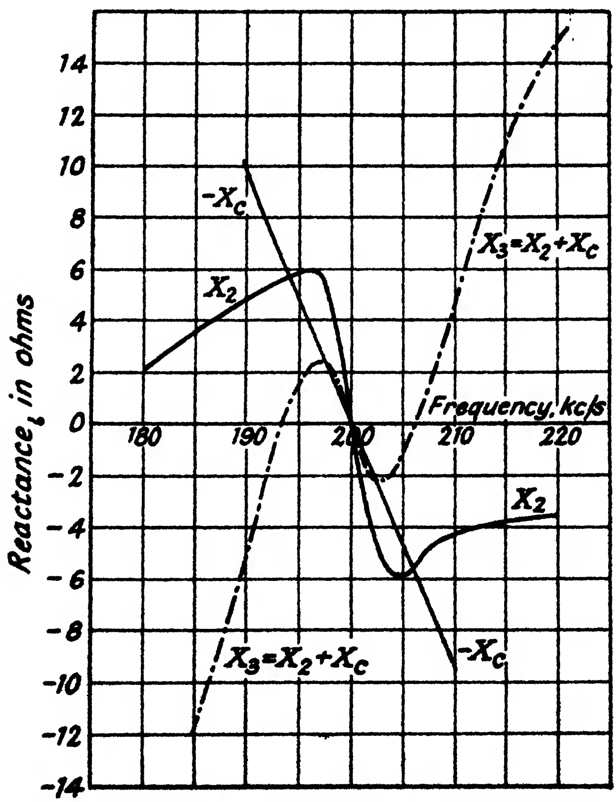


FIG. 4/XVI:12.—Plot of X_2, $-X_c$ and $X_3 = X_2 + X_c$ where $X_c = L_c\omega - \frac{1}{C_c\omega}$.

magnitude of X_2 at 194 kc/s and at 206 kc/s is 5·7 Ω. The mean value is, therefore, 5·7 Ω.

Now assume that $L_e\omega - \frac{1}{C_e\omega} = 5{\cdot}7\ \Omega$ at 206 kc/s. If X_0 is the reactance of L_e at 200 kc/s, since L_e and C_e resonate at 200 kc/s.

$$\frac{206}{200}X_0 - \frac{200}{206}X_0 = 5{\cdot}7 \quad \therefore X_0 = 96\ \Omega$$

$$\therefore L_e = \frac{96{\cdot}3}{2\pi \times 200{,}000}\text{H} = 76{\cdot}7\ \mu\text{H and } C_e = 0{\cdot}00827\ \mu\text{F}$$

The value of $L_e\omega - \frac{1}{C_e\omega}$ will then be very close to $5{\cdot}7\ \Omega$ at 194 kc.

The value of $X_e = L_e\omega - \frac{1}{C_e\omega}$ is given by $X_e \doteq X_0\frac{f}{f_0} - X_0\frac{f_0}{f}$ where $f_0 = 200$ kc/s, and X_0 is the reactance of L_e at 200 kc/s as above.

The value of X_e is tabulated in column 10 of Table I, and the value of $-X_e$ is plotted in Fig. 4 for comparison with X_2. The value of $X_s = X_e + X_2$, which is the reactance component of Z_s in Fig. 2 is tabulated in column 11 of Table I and also plotted in Fig. 4.

It will now be evident that since $L_n = 20{\cdot}7\ \mu$H and $L_e = 76{\cdot}7\ \mu$H, L_e can be constituted by the inductance L_n inherent in the mutual coupling plus an external real inductance $L_e' = L_e - L_n = 56{\cdot}0\ \mu$H.

The magnitude of the resistance component of Z_s (Fig. 2) at the carrier frequency is given by $\frac{Z_n^2}{R_s} = \frac{15{\cdot}6^2}{20{\cdot}4} = 11{\cdot}9\ \Omega$, which is a very low impedance and must be stepped up before connection to the feeder.

A modification of considerable importance which should be introduced in practice is the use of an unequal ratio mutual coupling for the quarter-wave network, in order to obtain a higher impedance on the primary side and so a higher value of Z_s. Provided the secondary inductance is made equal to L_e, the primary inductance may be given any higher value, and should be given as high a value as possible. When this is done the only effect is to multiply all impedances on the primary side by the ratio $r = \frac{L_p}{L_s}$, where L_p is the primary inductance replacing L_n in Fig. 3 (a) and L_s is the value of secondary inductance equal to L_e.

The reactance of L_e and C_e must, therefore, be increased in the ratio r and evidently the values of all reactances plotted in Fig. 4 will be increased in this ratio, as well as the value of the resistive component of Z_s given by equation (9).

In practice, by increasing the primary inductance, it will probably not be possible to increase the value of Z_s, above the unity ratio value, by much more than between two and four times. This is because there is a limit to the value of primary inductance which can conveniently be realized in such a coupling and because of the limit to the voltage which may be generated across condenser C_c. It is, therefore, necessary to insert an impedance transforming network between the input to the power factor correcting circuit L_c,C_c and the feeder. Although at Droitwich an L network was used for this purpose, in a circuit where the sideband pass range is a considerable fraction of the carrier frequency, such a circuit does not appear to constitute the best arrangement. The circuit of Fig. 1/VII:14 appears to be preferable since it has bandpass properties while an L network has not. By designing this circuit for a high-percentage band width the variation of its characteristics over the pass range can be reduced. Further, it is a very simple matter to design this network so that the carrier frequency is located at the point where 90° phase shift occurs (see CIII:1). In certain cases this may have some advantage since it is essential that the phase shift of the complete circuit between the anodes of the transmitting amplifier and the input to the power factor correcting circuit L_c,C_c should constitute an odd integral multiple of 90°. Sections of the circuit which in their own right contribute 90° phase shift therefore constitute useful elements in building up the total phase shift of the circuit. An exception occurs when the impedance Z_s is asymmetric, in which case an improvement in symmetry may be obtained with a phase shift of the complete circuit which is a non-integral multiple of 90°

The requirement on the total phase shift is imposed by the following considerations. The impedance Z_s if stepped up to present, at the carrier frequency. the correct load impedance at the anodes of the output amplifier, would present at the sideband frequencies an impedance which, after a small fall in value, increases on each side of the carrier frequency as departure is made from the carrier frequency. This would give rise to excessive voltages at sideband frequencies remote from the carrier. For this reason an impedance characteristic Z_4 is obtained which is the impedance Z_s seen through a quarter-wave network. This impedance is then transferred to the anodes of the output amplifier through a system of networks having a phase shift equal to an *even* integral multiple of 90°, and providing any necessary step-up in impedance.

To give a rough indication of the type of impedance presented

TABLE II

1	2	3	4	5	6	7	8	9	10	11	12
f kc/s	R_e	$R_e^2+(X_e+X_s)^2$	$R_3 = R_2$	R_3^2	X_3	X_3^2	$R_3^2+X_3^2$	R_4	X_4	Response db.	Relative Anode Peak Volts
180	14·8	8,768	0·33	0·11	− 18·3	335	335·1	9·12	+ 396	—	1·18
190	17·4	2,352	1·63	2·65	− 5·03	25·3	27·95	422	+1,300	5·86	1·315
194	18·6	1,076	3·96	15·7	0	0	15·7	1,820	0	2·6	1·28
196	19·2	673	6·67	44·5	+2·1	4·42	48·92	985	− 310	1·13	1·17
198	19·8	461	10·3	106·0	+2·3	5·28	111·28	670	− 150	0·25	1·045
200	20·4	417	11·9	141·0	0	0	141·0	607	0	0	1·0
202	21·0	510	10·2	104·0	− 2·0	4·0	108·0	682	+133	0·25	1·045
204	21·6	799·5	6·82	46·4	− 1·78	3·17	49·57	992	+259	1·13	1·16
206	22·4	1,219	4·76	22·7	0	0	22·71	1,515	0	2·08	1·25
210	23·8	2,706	2·35	5·5	+ 4·8	23·0	28·5	595	− 1,220	4·52	1·34
220	27·8	6,023	1·36	185	+14·83	218	219·85	44·8	− 488	—	1·23

at the anodes of the output amplifier, consider the simple case where Z_4 is the impedance of Z_3, determined by the values used in Table I, seen through an ideal quarter-wave line (having 90° phase shift at all frequencies and constant characteristic impedance) of characteristic impedance equal to 85 ohms.

$$Z_4 = \frac{85^2}{Z_3} = \frac{85^2}{R_3+jX_3} = \frac{85^2(R_3 - jX_3)}{R_3^2 + X_3^2}$$

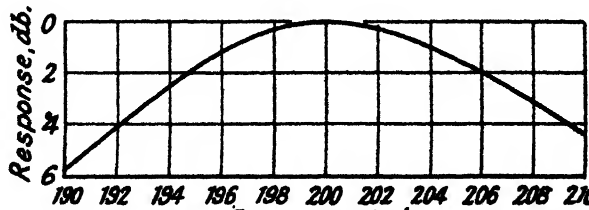
where $$R_3 = R_2 \text{ and } X_3 = X_c+X_2$$

R_2 and X_2 being defined by equations (9) and (9*a*).

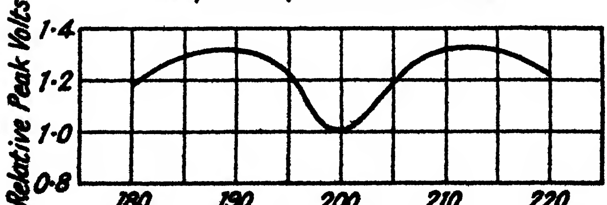
The value of X_3 is given in column 11 of Table I, while the value of R_3 and $Z_4 = R_4+jX_4$ is calculated in Table II on page 721

12.15. Frequency Response. An approximate idea of the frequency distortion of such a circuit can be obtained by assuming that Z_4 faces a valve or bank of valves with a total effective internal anode impedance equal to half the value of R_4 at the carrier frequency. If this internal anode impedance is R_0 and the effective internal e.m.f. in the valve is e, the power delivered at each frequency is

$$P = \frac{e^2}{R_4}\left|\frac{R_4}{R_0+R_4+jX_4}\right|^2 = \frac{R_4e^2}{(R_0+R_4)^2+X_4^2}$$



(a) Response of ideal circuit from anodes of output amplifier to aerial.



(b) Relative values of anode peak volts in ideal circuit.

FIG. 5/XVI:12.

and the response curve in decibels is given by

$$\text{Response} = 10 \log_{10} \frac{P}{P_0} \text{ db.}$$

where P_0 is the power delivered at the carrier frequency.

The response is tabulated in column 11 of the table above and plotted on Fig. 5 (*a*).

Similarly an *approximate* indication of the *variation* of the peak anode volts on the transmitter can be obtained from the formula

$$E = \frac{R_4+jX_4}{|R_0+R_4+jX_4|} = e\sqrt{\frac{R_4^2+X_4^2}{(R_0+R_4)^2+X_4^2}}.$$

The ratio of the peak anode volts to the peak volts at the carrier frequency is tabulated in column 12 of the table above and plotted on Fig. 5 (*b*).

It will be realized that, apart from the assumed value of valve impedance, these curves, while of similar form to those obtained in practice, do not represent practical conditions, because the system of quarter-wave networks used in practice is not made up of ideal networks but of networks in all of which the phase shift varies with frequency, and in some of which the characteristic impedance varies with frequency. In practice this does not introduce any extra complication because the impedances at each point of the circuit are not derived by calculation but are observed on a radio-frequency bridge.

In this connection it is important to note that the practical setting up of a circuit such as the above involves much less calculation than has been necessary above in order to demonstrate the principles involved.

12.16. Analysis of Circuit used at Droitwich and Method of Design of Component Networks to Build up Overall Required Phase Shift. The practical problem is concerned with transferring the impedance Z_3 to the anodes of the valves of the final stage through a series of networks having a combined phase shift equal to an odd integral multiple of 90° at the carrier frequency. For this reason, for purposes of adjustment, the circuit running back to the transmitter is divided into a series of quarter-wave networks, each having image impedances which are matched to adjacent networks at the carrier frequency.

(An exception to the rule occurs where the impedance Z_3 when plotted against frequency is unsymmetrical about the carrier frequency. In this case symmetry may be partially restored by a suitable choice of the phase shift between Z_3 and the valve anodes.)

Fig. 6 shows an analysis of the Droitwich circuit, the actual circuit being shown at (*a*) and the dissected circuit at (*b*). Starting from the aerial the design of circuit has been explained up to and including the power factor correcting circuit, between which and the anodes of the output stage the total phase shift must be equal to an odd integral multiple of 90°.

Network I, impedance ratio 560 : 100 ohms, has 90° phase shift and is constituted partly by an L network in the aerial tuning hut and partly by a length of the feeder equal to $0{\cdot}07\lambda$. The method of design of Network I is derived in CIII:3.2 and is as follows:

Z_0 = feeder impedance = input impedance of Network I.

α = phase shift constant of feeder ($= 0{\cdot}07 \times 360°$ in the example in Fig. 6).

Z_2 = output impedance of Network I ($= 100\ \Omega$)

L_a and C_β are components of Network I as indicated in Fig. 6.

$\omega = 2\pi f$ where f is the carrier frequency.

Then
$$\alpha = \sin^{-1}\sqrt{\frac{Z_2}{Z_0}} \qquad . \quad . \quad . \quad . \quad . \quad (11)$$

$$L_a\omega = \sqrt{Z_0 Z_2 - Z_2^2} \qquad . \quad . \quad . \quad . \quad . \quad (12)$$

$$\frac{1}{C_\beta\omega} = \frac{Z_0\sqrt{Z_0 Z_2 - Z_2^2}}{Z_0 - Z_2} = Z_0\sqrt{\frac{Z_2}{Z_0 - Z_2}} \qquad . \quad . \quad (13)$$

If Z_2 and Z_0 are known these equations give directly the value of α, L_a and C_β, in order that the network shall have image impedances respectively equal to Z_0 and Z_2 and 90° phase shift. By inverting the equations they can be made to give the values of any of three of the quantities Z_0, Z_2, α, L_a and C_β, when any other two are known.

Example. In the circuit of Network I in Fig. 6

$$Z_0 = 560\ \Omega \qquad Z_2 = 100\ \Omega$$

$$\therefore\ \alpha = \sin^{-1}\sqrt{\frac{100}{560}} = 25°$$

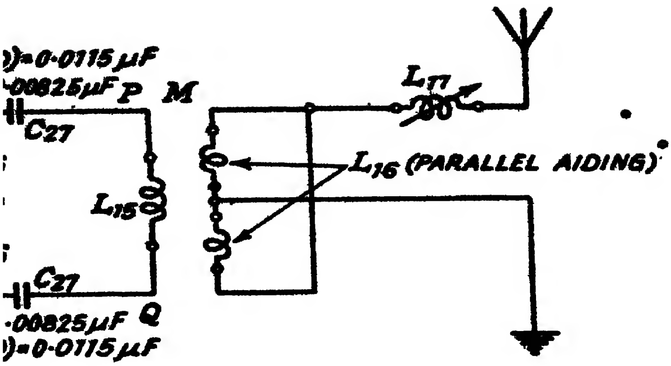
$$\therefore\ \text{Line length} = \frac{25°}{360°}\lambda = 0{\cdot}0695,\ \text{say}\ 0{\cdot}07\lambda$$

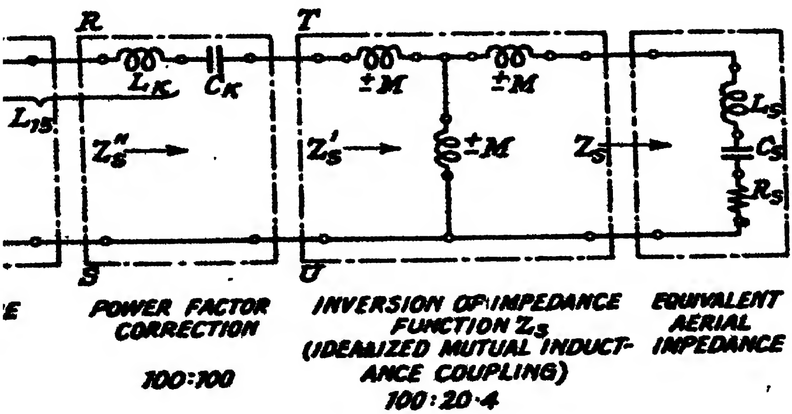
$$L_a\omega = \sqrt{56{,}000 - 10{,}000} = 214$$

$\therefore\ L_a = 170\ \mu$H, assuming a frequency of 200 kc/s.

$$\frac{1}{C_\beta\omega} = \frac{560 \times 214}{460} = 261$$

$$\therefore\ C = 3{,}050\ \mu\mu\text{F}$$





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These values may be checked as follows: the parallel reactance of $100+j214 = 214\left(1+\frac{100^2}{214^2}\right) = 261$, which is neutralized by the reactance of C_β so that the input impedance of the network is the parallel resistance $= 100\left(1+\frac{214^2}{100^2}\right) = 560\ \Omega$. The phase shift through the L network is $\tan^{-1}\frac{214}{100} = 65°$, so that the total phase shift of Network I is $65°+25° = 90°$ as required.

Network II, impedance ratio 560 : 560 ohms has 90° phase shift and is constituted partly by a π network in the transmitter building and partly a length of feeder equal to $0{\cdot}08\lambda$. The method of design of this network is derived in CIII:3.1 and is as follows:

Z_0 = feeder impedance = input and output impedance of Network II.

α = phase shift constant of line (= $0{\cdot}08 \times 360°$ in the example in Fig. 6).

$\omega = 2\pi f$, where f is the carrier frequency.

L_{14} and $\frac{1}{2}C_{25}$ are components of Network II as shown in Fig. 6.

Then
$$L_{14}\omega = Z_0 \cos \alpha \quad . \quad . \quad . \quad . \quad (14)$$
and
$$\frac{1}{\frac{1}{2}C_{25}\omega} = \frac{Z_0 \cos \alpha}{1 - \sin \alpha} \quad . \quad . \quad . \quad . \quad (15)$$

These equations give the values of L_{14}, and $\frac{1}{2}C_{25}$ in order that the network shall have 90° phase shift and image impedances equal to Z_0.

The other shunt condenser on the left of this network, undesignated, is equal to $\frac{1}{2}C_{25}$.

Example. In Network II the phase shift of the line is fixed at
$$0{\cdot}08 \times 360° = 28{\cdot}8° = 28°\ 48' \quad Z_0 = 560\ \Omega$$
Then
$$L_{14}\omega = 560 \cos 28°\ 48' = 490\ \Omega \quad \therefore\ L_{14} = 390\ \mu\text{H}$$
$$\frac{1}{\frac{1}{2}C_{25}\omega} = \frac{560 \times 0{\cdot}876}{1 - 0{\cdot}482} = 947\ \Omega$$
$$\therefore\ \tfrac{1}{2}C_{25} = 840\ \mu\mu\text{F}$$

In this case it is really simpler to use formulae (18) and (19) where α = the phase shift to be inserted $= 90° - 28°\ 48' = 61°\ 12'$.

In this case from (18)
$$L_{14}\omega = 560 \sin 61°\ 12' = 490\ \Omega$$
and
$$\frac{1}{\frac{1}{2}C_{25}\omega} = 560 \cot 30°\ 36' = 947\ \Omega$$

Check. If $L_{14}\omega = X_1$ and $\frac{1}{2}C_{25}\omega = X_2$, the input impedance of the network constituted by L_{14} and the two condensers $\frac{1}{2}C_{25}$ is

$$\frac{X_1X_2+jR(X_1-X_2)}{R\left(2-\frac{X_1}{X_2}\right)+j(X_1-X_2)}=\frac{490\times 947+j560\times -457}{560\times 1\cdot 483-j457}=560\ \Omega$$

The phase shift through the network is

$$\tan^{-1}\frac{X_1X_2}{R(X_2-X_1)}=\tan^{-1}\frac{947\times 490}{560\times 457}=61^\circ\ 12'$$

Network III is designed in accordance with Fig. 4/VII:14 to work between impedances of 25 and 560 ohms, the carrier frequency being located at $f_m=\sqrt{f_1f_2}$ where f_1 and f_2 are the cut-off frequencies. Such a network has more than 90° phase shift and therefore true impedance inversion does not occur. The effect of the extra phase shift is to introduce asymmetry of the impedance-frequency characteristic about the carrier frequency. A method of correction by adding a network or networks introducing extra phase shift is discussed below.

Network IV consists of two networks similar to Network III, stepping up from 25 Ω to 680 Ω in two stages: each stage is a network similar to Network III and designed as such. The overall phase shift of Network III is, therefore, in excess of 180°.

Owing to the excessive phase shift in networks of the type of III and IV, it may be necessary to add phase shift to build up the total phase shift to the required multiple of 90°. Before doing this the whole circuit should be examined to see whether it is possible to remove any networks which had been thought necessary before the extra phase shift was taken into account.

The phase shift through an inequality impedance ratio π network may be calculated very simply.

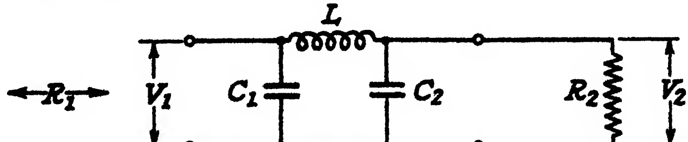


FIG. 7/XVI:12.—Determination of Phase Shift through Inequality Ratio Inductance Capacity π Network terminated in its Geometric Mid-band Image Impedance.

Fig. 7 shows an inequality ratio π network (i.e. $R_1 \neq R_2$). The ratio of output to input voltage:

$$\frac{V_2}{V_1}=\frac{\dfrac{R_2}{1+jR_2C_2\omega}}{jL\omega+\dfrac{R_2}{1+jR_2C_2\omega}}=\frac{R_2}{jL\omega-R_2LC_2\omega^2+R_2}$$

The phase shift at the geometric midband frequency f_m is then given by

$$\alpha = \tan^{-1}\frac{L\omega}{(1 - LC_2\omega^2)R_2} = \tan^{-1}\frac{L\omega}{(1 - LC_1\omega^2)R_1} \quad . \quad (16)$$

where $\omega = 2\pi f_m = 2\pi\sqrt{f_1 f_2}$ and f_1 and f_2 are the cut-off frequencies.

$$\therefore\ \alpha = \tan^{-1}\left[\frac{f_2^2 - f_1^2}{f_2^2 - 2f_1^2}\right]\frac{L\omega}{R_2} \text{ when } R_2 < R_1. \quad . \quad (17)$$

It should be remembered that negative values of the denominators in equations (16) and (17) correspond to phase shift angles between 90° and 180°. All the expressions in (16) and (17) give the same value for α, but the last is probably the simplest to use.

The easiest way to add phase shift is probably by the use of an equality ratio network of the form shown in Fig. 7 in which $C_1 = C_2$.

If Z_0 = the characteristic impedance of the circuit at the point of insertion of the network
α = the required phase shift to be introduced
$\omega = 2\pi f$ where f is the carrier frequency

It is shown in CIII:4 that

$$L\omega = X_0 \sin\alpha \quad . \quad . \quad . \quad . \quad (18)$$

$$\frac{1}{C_1\omega} = \frac{1}{C_2\omega} = Z_0 \cot\frac{\alpha}{2} \quad . \quad . \quad . \quad (19)$$

If L and C are given the values determined by these equations the network will have image impedances equal to Z_0, and a phase shift equal to α when terminated in impedance equal to Z_0.

13. Relations between Inductance and Capacity per Metre, Characteristic Impedance and Phase Velocity.

Conventions as at the beginning of XVI : 1. From equation (1)/XVI:1, where R and G are negligible,

$$Z_0 = \sqrt{\frac{L}{C}} \quad . \quad . \quad . \quad . \quad (1)$$

From (20a)/XVI:1

$$\alpha = \omega\sqrt{LC} \quad . \quad . \quad . \quad . \quad (2)$$

From (28)/XVI:1 and from (2) above

$$V = \frac{\omega}{\alpha} = \frac{1}{\sqrt{LC}} \quad . \quad . \quad . \quad (3)$$

From (1) and (3)

$$L = \frac{1}{V^2C} = CZ_0^2 \text{ and } C = \frac{1}{V^2L} = \frac{L}{Z_0^2} \quad . \quad . \quad (4)$$

$$\therefore C = \frac{1}{VZ_0} \text{ and } L = \frac{Z_0}{V} \quad . \quad . \quad . \quad (5)$$

Hence if the phase velocity V of a transmission line is known, and any one of the quantities Z_0, L or C, the other two quantities can be obtained by means of equations (4) and (5). If the phase velocity is expressed in metres per second and the characteristic impedance in ohms then the inductance and capacity are expressed respectively in Henrys and Farads per metre.

The phase velocity in an open-wire transmission line such as an aerial feeder, or in the conductors of an aerial, is very close to 300 million metres per second. It can be obtained exactly from the relation $V = \lambda f$, and the value of λ can be determined by the method given in XVI:1.9. For most practical purposes, except the adjustment of the length of dipoles, the figure of 300 million metres per second is quite accurate enough.

13.1. Inductance of Straight Vertical Wire. In an article in the *Hoch Frequenz Technik*, Vol. 41, No. 1, p. 17, Labus shows that the reactance between the lower end of a vertical round conductor and ground is

$$X = -60\left(\log_e \frac{h}{r} - 1\right) \cot \frac{2\pi h}{\lambda}. \quad . \quad . \quad (6)$$

where h is the height of the conductor and r is its radius. Comparing this with equation (35)/XVI:1, it is evident that the conductor behaves as a transmission line with a characteristic impedance

$$Z_0 = 60\left(\log_e \frac{h}{r} - 1\right) \text{ ohms}$$

$$= 60\left(2{\cdot}303 \log_{10} \frac{h}{r} - 1\right) \text{ ohms} \quad . \quad . \quad . \quad (7)$$

$$= 60 \times 10^9\left(2{\cdot}303 \log_{10} \frac{h}{r} - 1\right) \text{ abohms}$$

Therefore the inductance *per centimetre* (obtained by putting $V = 3 \times 10^{10}$) is:

$$L_m = \frac{Z_0}{V} = \frac{60 \times 10^9}{3 \times 10^{10}} \times \left(2{\cdot}303 \log_{10} \frac{h}{r} - 1\right) \text{ abhenrys}$$

$$\frac{60}{3 \times 10^{10}} \times \left(2{\cdot}303 \log_{10} \frac{h}{r} - 1\right) \text{ henrys}$$

$$= \frac{60}{3 \times 10^4} \times \left(2{\cdot}303 \log_{10} \frac{h}{r} - 1\right) \text{ microhenrys}$$

$$= 0{\cdot}002\left(2{\cdot}303 \log_{10} \frac{h}{r} - 1\right) \text{ microhenrys} \quad . \quad . \quad (8)$$

Relation (7) is plotted in Fig. 2/XVI:1 and relation (8) is plotted in Fig. 5/II:17.

Equation (8) serves no useful purpose except to show by comparison with equation (7)/II:17, that the inductance of a wire varies with change of current distribution. The above example therefore serves only to show how the necessary conversions to absolute units must be made in using equations (4) and (5). It will be appreciated that the assumption of a single value of characteristic impedance for a vertical wire is quite unwarranted since it is really a case of a tapered transmission line: its capacity and inductance per unit length vary along its length. The assumption is however a useful one since the fictitious value of characteristic impedance may be used to give the correct value of sending-end impedance.

Physicists and mathematicians will unite in condemning any attempt to represent the inductance of any part of a circuit without reference to the remainder of the circuit. The formulae for the inductance of straight conductors which are given in II:17 have, however, been included because it is considered they will be useful to engineers in assessing the approximate value of voltage drop consequent on passing a known current through a part of a circuit which consists of a straight conductor. The true value of this voltage drop depends of course on the configuration of the remainder of the circuit.

13.2. Value of Condenser required to Neutralize Reactance of Grid Lead. The concept of the inductance of the grid lead, as has already been indicated is open to the objection that it is not capable of unique determination in general terms. Further, at high frequencies, the stray capacity of the grid lead cannot be neglected. It is therefore reasonable to suppose that a closer approximation to a useful concept will be obtained by treating the grid lead as a transmission line.

As an engineering approximation, it may, however, be useful sometimes to assume that the inductance of the grid lead may be determined as the inductance of a straight conductor as given by Fig. 5/II:17. The capacity which resonates with this inductance is then an approximation to the value of capacity which "neutralizes the reactance of the grid lead". The optimum value for practical purposes can then be found by trial.

Considering the grid lead as a transmission line, two alternatives present themselves :

1. To make the input impedance to the grid lead non-reactive. (This is also the condition which makes the voltage effective on the grid a maximum.)
2. To eliminate phase shift between the driving voltage (between the input to the grid lead and ground) and the voltage between grid and ground.

An example on the first alternative is given immediately below, and the method of dealing with the second alternative requirement is given in XVI:13.3. For simplicity the example which follows has assumed no resistive component of grid input impedance : in an actual case this would have to be taken into account.

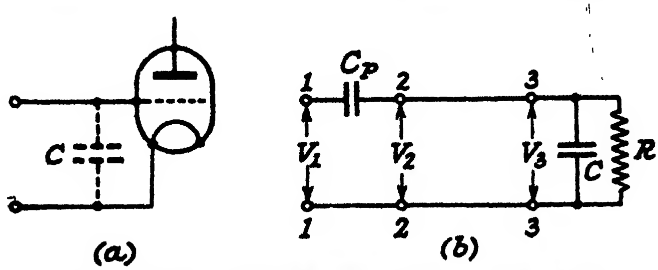


FIG. 1/XVI:13.—Neutralization of Lead Reactance.

The grid lead of the valve in Fig. 1 (*a*) is a round conductor ½ in. in diameter, 2 ft. long, and 4 ft. above ground. The grid ground capacity is 20 $\mu\mu$F. What value of neutralizing condenser is required at 20 Mc/s to make the input impedance to the grid lead non-reactive ?

It is evidently necessary to determine the characteristic impedance of the grid lead and then to calculate its sending-end impedance, when terminated in 20 $\mu\mu$F.

Entering h/d in Fig. 2/XVI:1, the value of characteristic impedance is $Z_0 = 350$ ohms.

At 20 Mc/s the wavelength is 15 metres (assuming $V = 3 \times 10^8$ metres per second) so that the line length in degrees is

$$al = \frac{2}{3{\cdot}28 \times 15} \times 360° = 14° \ 36'.$$

Also $\tan 14° \ 36' = 0{\cdot}26$.

The impedance of 20 $\mu\mu$F at 20 Mc/s is

$$-jX_c = j\frac{159{,}000}{20 \times 20} = -j398$$

From equation (46)/XVI:1, the driving-point impedance between the input of the grid lead and ground is:

$$Z_s = \frac{1 + j\dfrac{350}{-j398} \times 0{\cdot}26}{\dfrac{350}{-j398} + j0{\cdot}26} \times 350 = -j237 \qquad (9)$$

The input impedance is therefore a negative reactance so that the addition of a series condenser would reduce the drive effective on the grid instead of increasing it; from the point of view of increasing the grid drive there is no point in adding a series condenser. If the value of the grid ground capacity were large enough, or, if the grid lead were long enough, Z_s would become positive and an increase in grid drive (dependant on the impedance of the driving source, assumed resistive) would be obtained by the use of a series condenser. In the present case this condition occurs only at very high frequencies, as can be seen by examining the numerator in equation (9): Z_s is positive when this numerator is negative; i.e. when

$$\frac{350}{X_c} \times 0{\cdot}26 > 1$$

that is when $$X_c < 350 \times 0{\cdot}26 = 91$$

Such a condition also occurs when the ground grid capacity is greater than 87 $\mu\mu$F, in the case under consideration.

13.3. Value of Condenser required to Eliminate Phase Shift between Driving Voltage and Voltage Effective on Grid. When the grid input impedance is a pure capacity, there is a standing wave of voltage in the transmission line constituted by the grid lead, and the grid voltage is in phase with the voltage applied at the input to the grid lead. When the grid input impedance is not a pure capacity, but possesses resistance, there is a progressive wave in the grid lead, as well as a standing wave, and the voltage effective on the grid is out of phase with the driving voltage applied at the input to the grid lead. A parallel case of particular interest occurs in the case of an inverted amplifier (see X:35), where the cathode is driven and the input impedance to the cathode is highly resistive, in addition to having a capacity to ground. In such cases the voltage effective on the grid (or cathode) can be brought into phase with the driving voltage by inserting a series condenser.

Fig. 1 (*b*) illustrates the general case where a transmission line between 2,2 and 3,3 represents the grid or cathode lead, and C and R represent the grid or cathode input impedance in parallel form. C_p represents the phasing condenser inserted in the input of the

grid or cathode lead. It will be shown that the condenser C_p can be given such a value that the voltage V_3, which represents the voltage effective on the grid, is brought into phase with the driving voltage V_1.

Conventions.

$Z = R//-jX$ = the impedance constituted by R in parallel with C = the grid or cathode input impedance ($-jX$ = the impedance of C).

Z_0 = the characteristic impedance of the transmission line determined as in XVI:13.2.

$Z_p = -jX_p$ = the impedance of the phasing condenser C_p.

Z_2 = impedance at terminals 2,2, looking towards 3,3.

V_1, V_2, V_3 = voltages at 1,1; 2,2, and 3,3, respectively.

αl = angle of transmission line (which is non-dissipative).

Then, from (46)/XVI:1,

$$Z_2 = \frac{\cos \alpha l + j\dfrac{Z_0}{Z}\sin \alpha l}{\dfrac{Z_0}{Z}\cos \alpha l + j\sin \alpha l} \times Z_0 = \frac{AZ_0}{B}, \text{ say}$$

From (17)/XXIV:4

$$\frac{V_3}{V_2} = \frac{1}{\cos \alpha l + j\dfrac{Z_0}{Z}\sin \alpha l} = \frac{1}{A}, \text{ say}$$

$$\therefore \frac{V_3}{V_1} = \frac{V_2}{V_1} \times \frac{V_3}{V_2} = \frac{Z_2}{Z_2+Z_p} \times \frac{1}{A}$$

$$= \frac{\dfrac{AZ_0}{B}}{\dfrac{AZ_0}{B}+Z_p} \times \frac{1}{A} = \frac{Z_0}{AZ_0+BZ_p} = \frac{1}{A+\dfrac{B}{Z_0}Z_p}$$

$$= \frac{1}{\cos \alpha l + j\dfrac{Z_0}{Z}\sin \alpha l + \dfrac{Z_p}{Z}\cos \alpha l + j\dfrac{Z_p}{Z_0}\sin \alpha l} \quad . \quad . \quad (10)$$

Substituting in (10) the value of

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{-jX} = \frac{1}{R} + \frac{j}{X}$$

and ignoring the real terms in the denominator,

$$\frac{V_3}{V_1} = \frac{1}{\text{Real terms} + j\left[\dfrac{Z_0}{R}\sin \alpha l - \dfrac{X_p}{R}\cos \alpha l\right]} \quad . \quad (11)$$

V_2 is in phase with V_1 when the reactance term in the denominator of (11) is zero, that is, when:

$$X_p = Z_0 \tan \alpha l \quad . \quad . \quad . \quad . \quad (12)$$

Hence, *if a condenser of reactance equal to the short-circuit impedance of a dissipationless transmission line of less than a quarter-wave in length is placed in series with its input, the overall phase shift is zero, regardless of the magnitude or angle of the terminating impedance.*

Subject to the limitations imposed in the above statement, the principle is of quite general application, but it should be noted that the image impedances of the overall structure so obtained are no longer equal to the characteristic impedance of the unmodified transmission line. These may be obtained by constructing the matrix of the overall structure and equating it to the matrix of the general dissymmetrical fourpole. See XXIV:6 and Fig. 12/XXIV:6.

In the present case the above analysis constitutes a justification of the use of series condensers in an inverted amplifier " to neutralize the inductance of the cathode leads ".

13.4. Value of Condenser required to Neutralize the Reactance of an Earth Lead. If the condenser is placed at the top of the earth lead, which is *not* a good place to put it, it is evident that its reactance must be equal to the short-circuit impedance of the lead regarded as a transmission line. In other words, the condenser reactance is given by equation (12), although the considerations involved are different from those from which equation (12) was derived.

If the condenser is placed at the bottom of the earth lead, which is the proper place, its required reactance is derived as follows. If Z_0 is the characteristic impedance of the earth lead and $Z_n = -jX_n$ is the impedance of the neutralizing condenser, the impedance looking into the top of the lead is required to be zero.

Hence

$$\frac{\cos \alpha l + j\dfrac{Z_0}{Z_n} \sin \alpha l}{\dfrac{Z_0}{Z_n} + j \sin \alpha l} = 0$$

The numerator must therefore be zero, so that

$$jZ_0 \sin \alpha l = jX_n \cos \alpha l$$

$$\therefore\ X_n = Z_0 \tan \alpha l \quad . \quad (13)$$

which is the same as the value required at the top of the earth lead.

This gives the value of neutralizing condensers required in the filament leads of short-wave transmitters, and gives twice the value

of the grid to grid condenser in inverted amplifiers, taking the length of the lead to the grid on one side only.

In practice, owing to indeterminacy in the location of true earth and the large sizes of condensers in comparison with the lengths of lead involved, it is not possible to apply the above formulae to determine accurately the sizes of neutralizing condensers to be used in transmitters. They do however provide a reasonably close approximation to the required size of condenser, and since this is normally constituted by a variable condenser, considerable latitude is permissible in the size of condenser installed.

The final adjustments of neutralizing condensers must always be made by practical indications.

14. Stress Diagrams in Aerial Arrays.

In the design of any structure consisting of a system of wires in suspension, it is necessary to find: (1) the required shape of the structure, and (2) the various forces which are required to maintain that shape.

The various problems that arise in the mechanical design of arrays may be solved either graphically or by calculation. The first method is by far the simpler, and is the one described below.

The analysis of the various stresses which are set up in the wires of the array and its supporting structure is based upon a balance between the applied loads and the stresses set up in the system by the application of these loads.

The external loads must be so balanced that the system as a whole must not change position; this specifies the condition of "external equilibrium". Also the stresses set up internally in the system as a result of the external loads must be able to resist the latter without fracture taking place, i.e. the material must have sufficient breaking strength and safety factor. This condition is termed "internal equilibrium". It must be borne in mind that in any structure consisting of flexible members such as wires, all the members must be in tension.

In any structure of wires there are several groups of concurrent forces. Forces are said to be concurrent when their lines of action meet in a point. For example, the positions marked 1, 2, 3, etc., in Fig. 6 (*a*) each represent a point at which the lines of action of a group of concurrent forces meet. It is obvious that with such a system of forces there are no rotational tendencies, and the system may be balanced by satisfying the equations:

$$\Sigma H = 0 \quad \text{and} \quad \Sigma V = 0$$

where ΣH = the sum of the horizontal components of the forces, and ΣV = the sum of the vertical components.

14.1. Vector Conventions. In Fig. 1 (*a*) the vectors *ab*, *ac* and *ad* represent in direction and magnitude three concurrent forces acting in the direction indicated by the arrows. Resolving these three forces into their horizontal and vertical components gives horizontal components *ae*, *af* and *ag*, and vertical components *ah*, *ak* and *al*.

Forces acting upwards are considered to be positive, while those acting downwards are considered to be negative. Similarly, forces acting to the right are made positive and forces acting to the left are made negative. Using this convention, it will be seen that the horizontal component *af* of the force *ac* is positive, and the horizontal components *ae* and *ag* of the forces *ab* and *ad* are both negative. Also the vertical components *ah* and *ak* of the forces *ab* and *ac* are positive and the vertical component *al* of the force *ad* is negative.

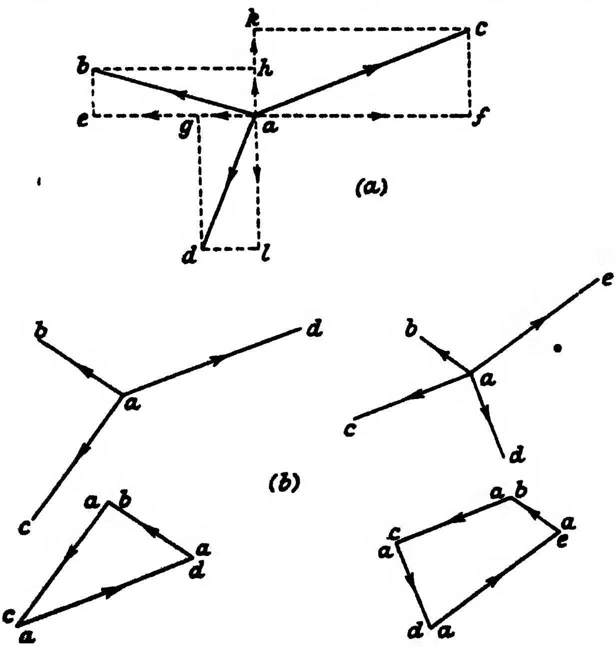


FIG. 1/XVI:14.

(*a*) Vertical and Horizontal Resolution of Concurrent Forces.
(*b*) Force Diagrams for Concurrent Forces.

The system of forces shown in Fig. 1 (*a*) will evidently be in a state of equilibrium if

$$\Sigma H = ae + af + ag = 0, \text{ and}$$
$$\Sigma V = ah + ak + al = 0$$

As an aid to numerical computation it is customary to designate the direction of forces by the order in which the letters are written. A force from a to e is written ae and a force from e to a is written ea. Hence the above equations may be written:

$$\Sigma H = -ea+af-ga = 0$$
$$\Sigma V = ah+ak-la = 0$$

14.2. Force and Space Diagrams. The above algebraic statements may be represented graphically. If any system of concurrent forces is in equilibrium, the sum of the vectors representing the forces is zero. In other words. if all the vectors are joined head to tail, the head of the last vector will coincide with the tail of the first vector. The resultant polygon is called a polygon of forces, or more simply, a force polygon. If there are only three forces the resultant polygon is a triangle. A polygon of forces which indicates the magnitude and direction of the forces will be referred to generally as a *force diagram*, while the drawing of the structure in space which determines the line of action of the forces (i.e. the shape of the structure) will be called the *space diagram.*

An example showing the construction of a force triangle and a force polygon is given in Fig. 1 (*b*), in which two systems of forces are shown. The system to the left of the diagram consists of three forces whilst that to the right consists of four forces. In each case the associated force triangle or force polygon is drawn in below. Each vector of the system has a related vector in the triangle or polygon. It will be noted that in each of these examples the force triangle or force polygon is a closed figure which indicates that both systems are in a state of equilibrium.

If the force vectors do not close the triangle or polygon, the system is not in equilibrium, and the vector which would be required to complete the polygon will represent in magnitude and direction the force which is required to be added in order to produce a state of equilibrium. Fig. 2 shows a system of five forces which are not in a state of equilibrium. The force polygon is drawn in full line and the force vector required to close the polygon, and thus produce equilibrium, is shown by the dotted line ag. This line represents in direction and magnitude the required balancing force, and it has been added (in dotted line) to the system of forces from which the polygon was constructed.

The force diagrams of array curtains which are described later in these notes are combinations of equilibrium triangles and polygons, combinations being made possible by virtue of certain forces being common to two systems of concurrent forces.

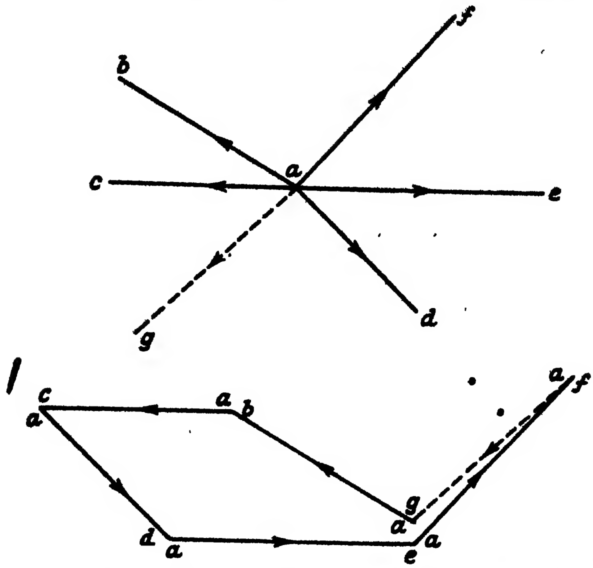


FIG. 2/XVI:14.—Concurrent Forces with Resultant *ag*.

14.3. Bow's Notation. In practice the space diagram is derived from the force diagram as described below. The work is greatly facilitated if a convenient system of notation is used to identify the related lines in each diagram. The notation most widely adopted is known as " Bow's Notation ", which represents each line of force in the force diagram by a small letter placed at each end of the line as in Figs. 1 and 2. The corresponding line in the space diagram, which represents the member in which the force acts, is represented by placing similar capital letters on either side of the line, which in practice usually amounts to placing a letter in each mesh of the space structure and certain letters outside it. See Fig. 6 (*b*). It must be remembered that in the force diagram each line represents a force in magnitude and direction, and the corresponding line in the space diagram represents only the line of action of that force. The direction of action of the force may be indicated by an arrow on the diagram or, as already stated, by the sequence of the letters when writing, i.e. a force represented by *ab* is taken as acting in a direction from *a* to *b*.

14.4. Catenary. Now consider the simple case of a single wire, free of all attachments, suspended between two masts as in Fig. 3. The shape taken by such a wire is called a " catenary ". The distance *d* between the points of suspension is termed the " span ", while the length of wire *l* is called the " bight ". The distance *S*,

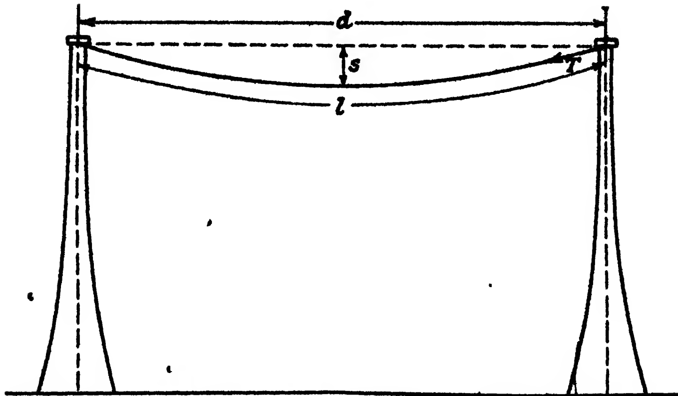


FIG. 3/XVI:14.—Catenary Form assumed by Wire Supported at Each End.

which is the distance between the apex of the curve and the straight line joining the points of suspension, is known as the "sag" or "dip". Of these three quantities only the span d would normally be known, this being the distance between the masts.

The relation between these quantities is given by:

$$S = wl^2/8T \qquad \text{(1)}$$

where S = the sag in feet

w = the weight of the wire in lbs. per foot length

l = the length of the bight in feet

T = the tension in lbs. in the wire at the middle of the bight.

The length of the bight is not normally known, neither is the tension at the middle of the bight, but when S is small compared with l, as is usually the case in practice, sufficient accuracy is obtained by taking l as being equal to the span, which is the distance between the points of suspension, and taking T as being the tension in lbs. in the wire at the point of suspension.

A problem which is often met with in practice is to determine the necessary amount of tension to be applied to a wire suspended between two masts of known height such that the maximum sag will not exceed a given amount. The amount of sag is governed by the minimum height which can be allowed at the centre of the span. The maximum tension which may be applied is determined by the maximum head pull for which the masts have been designed and this will in turn govern the maximum size of wire which will be used. It is obviously not economical to use a wire having a greater strength than the masts upon which it is to be suspended.

As an example, suppose that it is required to suspend a wire

between two 150-ft. masts, situated 300 ft. apart on a level site, such that the minimum height of the wire is not less than 145 ft., i.e. the maximum allowable sag is 5 ft. The masts are designed to withstand a maximum horizontal head pull of 1,000 lbs.

The first step is to decide whether the wire to be used is to be of steel or copper. The latter material will, normally, be used in cases where the wire is the actual conductor, and it will be assumed that this is so in the present case. Reference can now be made to standard wire tables, and a wire is chosen which is capable of withstanding a working tension within the limits of the mast design—in this case 1,000 lbs. The wire decided upon in the first instance may be far heavier than necessary, but whether or not this is so will be seen when the first calculation has been made. From Table III, a copper stranded wire of 19/0·084″ has a breaking load

TABLE III

Breaking Loads and Weights of the Various Types of Wire and Wire Ropes used in the Construction of Aerial Arrays

Material	Size	Approx. Breaking Load	Length Equivalent to 1 lb. (approx.)	Weight per ft. (approx.)	Purpose for which Used
		lbs.	ft.	lbs.	
H.D. Copper Wire	12 S.W.G.	500	30·5	0·327	Radiating elements
	6 „	1,700	9·0	0·1116	H.F. feeders
	7/18 „ (7/·048″)	750	20·0	0·049	Light M.W. and L.W. aerials
	7/16 „ (7/·064″)	1,200	11·4	0·087	Vertical feeders on array curtains
	19/16 „ (19/·064″)	3,200	4·1	0·240	M.W. and L.W. aerials
Galv. M.S. Strand	7/18 „	850	21·7	0·046	End supports for dipoles in array curtains
	7/16 „	1,500	12·5	0·080	Top supports for vertical feeders in array curtains
Steel Wire Rope, 6/7/1, 80/90 tons per sq. in.	[illegible]″ circ.	900	45·4	0·022	Light halyards
	[illegible]″ „	2,500	15·9	0·063	„ „
	[illegible]″ „	5,000	8·3	0·120	Side support wires for S.W. arrays
		tons			
Steel Wire Rope, 6/19/1, 100/110 tons per sq. in.	1[illegible]″ „	7·4	2·8	0·360	Triatics and halyards, etc.
	1[illegible]″ „	10·0	2·0	0·500	„ „ „
	2[illegible]″ „	19·4	1·1	0·920	„ „ „

of the order of 3,200 lbs., and a weight of approximately 0·24 lbs. per foot. This wire is a stock size, and allowing a safety factor of 4, it can have a working load of 800 lbs., a figure which is within the limits of the specified mast load, i.e. 1,000 lbs.

From the expression $S = wl^2/8T$, we get

$$T = wl^2/8S \text{ lbs.} \qquad (2)$$

and, substituting the values of w, l and S:

$$T = \frac{0\cdot24 \times 300^2}{8 \times 5} = 540 \text{ lbs.}$$

From this calculated tension of 540 lbs., it is noted that a lighter wire may possibly be suitable, and reference to Table III shows that a stranded copper wire of 7/0·064″ has a breaking load of 1,200 lbs., and a weight of approximately 0·087 lbs. per foot. Substituting these values in (2):

$$T = \frac{0\cdot087 \times 300^2}{8 \times 5} = 196 \text{ lbs. nearly.}$$

It will be seen from the above that the lighter and more economical wire is adequate. A safety factor of the order of 6 is obtained, and the resultant pull on the masthead is relatively small.

This reduction in the size of the wire could, of course, be taken to further limits, because a lighter wire requires less tension for a given sag; but in determining the size to be used, it is necessary to bear in mind such factors as the reduction in the cross-section of the material due to corrosion which is experienced in certain atmospheres, and shock loads which may be caused as a result of accident, etc. The size used will also be governed to a great extent by the arrangements available for suspension, the object being to maintain a nice balance between the size of wire and the type of mast from which it is being suspended.

14.5. Triatic with Single Load. In Fig. 4 (*a*) is an arrangement of a single wire suspended between two points A and B of equal height, with a load W acting at C, the centre of the span, such as would be obtained in practice in the erection of a simple T-type aerial, the load at the centre being due to the downlead. The known facts would normally be:

(1) The maximum permissible horizontal pull at the mastheads A and C *or* the maximum permissible wire tension.

(2) The distance between points of attachment A and B, and their height above ground.

(3) The required tension in the downlead to keep it reasonably taut.

It is required to know the shape the structure would take, which will also indicate the sag S at the centre of span.

In Fig. 4 (*b*) the arrangement has been re-drawn, the notation added and the direction of action of the various forces indicated by arrows. Using Bow's Notation, the load W in Fig. 4 (*a*) is indicated by the member B/C acting vertically downwards, and the

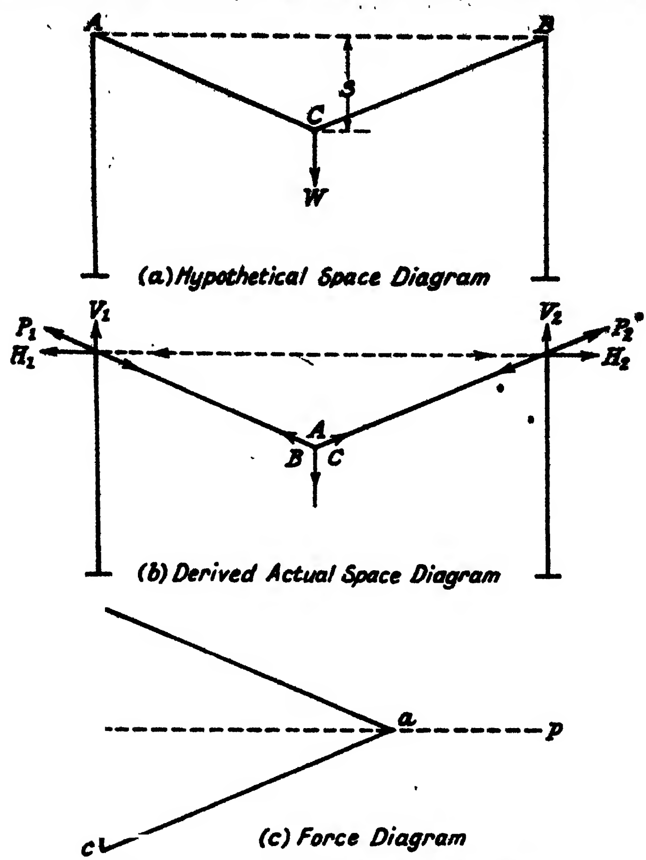


FIG. 4/XVI:14.—Wire with Single Load.

internal resistive forces resulting from B/C are indicated in the members A/B and C/A acting away from the point of attachment of the downlead. At the points of suspension are shown forces P_1 and P_2, which represent the forces required to resist the pull due to the suspension wire. The forces P_1 and P_2 may be resolved into horizontal and vertical components, H_1, V_1 and H_2, V_2. This last group of forces is, of course, theoretical ; the actual forces would, in practice, be taken up by thrust in the mast and tension in the stays (not shown). It will be obvious that the horizontal components H_1 and H_2 of P_1 and P_2 could theoretically be replaced by a strut (a member in compression) shown dotted in Fig. 4 *b*. It is convenient to assume the presence of this imaginary strut when the force diagram is being constructed.

The force diagram is shown in Fig. 4 (*c*), and the method of construction is as follows :

To a suitable scale draw *bc*, which corresponds to *B/C* in Fig. 4 (*b*), the magnitude and direction of which is known. As the load on the suspension is acting vertically downwards from its centre, it is obvious that the vertical components V_1 and V_2 at the mastheads will be equal, and their sum equal to the load *W* (Fig. 4 (*a*)), that is to say, the sum of the vertical forces V_1 and V_2 is equal to the force *bc* (Fig. 4 (*c*)). The mid-point *y* of *bc* (Fig. 4 (*c*)), may be marked and *yb* and *cy* will represent to scale the magnitude of forces V_1 and V_2. Extend *y* to any point *p* in a direction parallel to the imaginary strut shown dotted in Fig. 4 (*b*); in this particular case *yp* will be horizontal and will represent the line of action of the compressive force that would be set up in the hypothetical strut.

If the limiting condition is constituted by the horizontal pull on the masts, a distance *ya* is marked out on *yp* equal to this pull. *ab* and *ca* then give the tensions in the wire and the directions assumed by the members *A/B* and *A/C*.

If the limiting condition is the wire tension, *ab* and *ca* are drawn in by striking arcs of circles with centres at *b* and *c*, and radii equal to the wire tension. *a* is the point where these arcs cut *yp*.

bay represents the equilibrium triangle of the forces P_1, H_1 and V_1, and *cay* represents the equilibrium triangle of the forces P_2, H_2 and V_2. In the space diagram Fig. 4 (*b*), *A/B* and *C/A* are drawn parallel to *ab* and *ca* respectively in Fig. 4 (*c*), so that the shape of the construction is immediately determined; the horizontal components of the masthead load are also determined by scaling off *ya* in Fig. 4 (*c*).

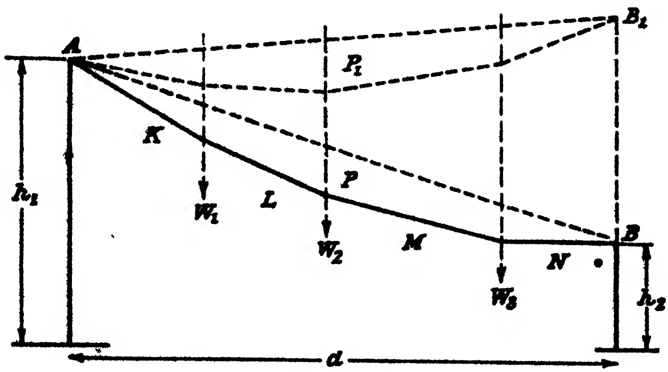
In the force diagram (Fig. 4 (*c*)) the line *bc* is called the vertical load line; the point *a* is called the pole, and the lines *ab* and *ca* are called the rays or strings.

It will be noted that in the above example no account has been taken of the weight of the suspension wire. It is permissible to neglect this without incurring great error as long as the magnitude of the load *W* is great compared with the weight of the suspended wire. If this is not the case, a fair degree of accuracy may be obtained by adding the weight of the wire to the load *W* and assuming it is concentrated at the point of attachment.

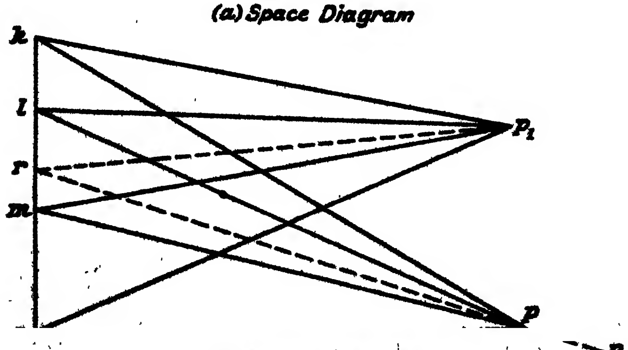
If greater accuracy is required, it is necessary to distribute the weight by taking the weight of short lengths and regarding these as so many small loads distributed at equal distance along the suspension wire, and acting vertically downwards. The greater the number of divisions, the greater will be the accuracy in the graphic

Both of the above examples represent simple design problems. The points of suspension are shown as being at the same height, and the loading of the suspension is symmetrically disposed. In practice, conditions arise where the suspension is between two points of different height, in some cases between a masthead and a ground anchorage, and where the loads are of different values and asymmetrically disposed. In these circumstances the following procedure is adopted in the construction of the graphic diagrams.

14.6. Multi-loaded Triatic. Assume that it is required to provide a suspension between two points A and B of different heights, and that this suspension is to support an array curtain which imposes various loads, W_1, W_2 and W_3, at several points along the suspension as indicated in Fig. 5 (*a*). The suspension wire supporting an array curtain is called the triatic, and from now on it will be referred to as such. Obviously, the triatic will not remain straight, as shown dotted between A and B, but will take up a definite shape, the form of which it is required to know.



(a) Space Diagram



(b) Force Diagram

FIG. 5/XVI:14.—Triatic Supporting Several Loads.

The distance d between the masts will be known, and also the mast heights h_1 and h_2. Information relating to the permissible mast loading will normally be available, and the magnitude of the loads W_1, W_2 and W_3 due to the array curtain will have been fixed (see below). The problem requires that the equilibrium polygon be made to pass through the specific points A and B. The method of solution is based upon the closed polygon which is necessary for the structure to be balanced.

The first step is to apply the notation, the letters K, L, M, N and P being used as indicated in Fig. 5 (a). Then, to a suitable scale, draw the load line of the vertical forces W_1, W_2 and W_3. This is shown in Fig. 5 (b), where kl, lm and mn represent the forces W_1, W_2 and W_3 respectively. Choose any point p_1 as the pole, and fill in the rays p_1k, p_1l, p_1m and p_1n to complete a trial force diagram.

Commencing at A in Fig. 5 (a), draw a line parallel to p_1k in Fig. 5 (b) until it cuts the line of action of K/L in Fig. 5 (a). From this point draw a second line parallel to p_1l in Fig. 5 (b), and so on with p_1m and p_1n. The last line will cut a vertical extension of B at B_1. Join A and B_1 and the trial equilibrium polygon is complete. Commencing at p_1 (Fig. 5 (b)), draw a line parallel to AB_1 (Fig. 5 (a)), cutting the load line $klmn$ at r.

The object of constructing the trial diagram is to find the point r on the load line. For a given set of conditions this point will be in the same position on the load line no matter what position is chosen for the pole p_1, and will therefore be in the correct position which is required in order to satisfy the conditions of the problem under discussion. (This is because kr must be equal to the vertical force on tower A and rn must be the vertical force on tower B. The values of kr and rn may alternatively be found by taking moments about A and B.) It is required that the equilibrium polygon passes through A and B in Fig. 5 (a), which means that the closing line must pass through these two points. To find this line, rp_2 is laid off (Fig. 5 (b)) parallel to AB in Fig. 5 (a), and if any point along rp_2 is used for the pole, the resulting equilibrium polygon will pass through A and B if it is started from either one of these points, and correctly followed through.

It will be obvious that an infinite number of force diagrams may be constructed, the poles of which will be along the line rp_2, but in practice at least one of three limiting quantities will be known: the permissible horizontal pull on the masts, the permissible wire tension or the permissible sag.

As indicated below, any one of these may be used to decide at which point along the line rp_1 the pole will be placed. The rays kp, lp, mp and np are now drawn in, and commencing at A in Fig. 5 (a), K/P, L/P, M/P and N/P are drawn parallel to these rays. If the force diagram has been correctly constructed, N/P

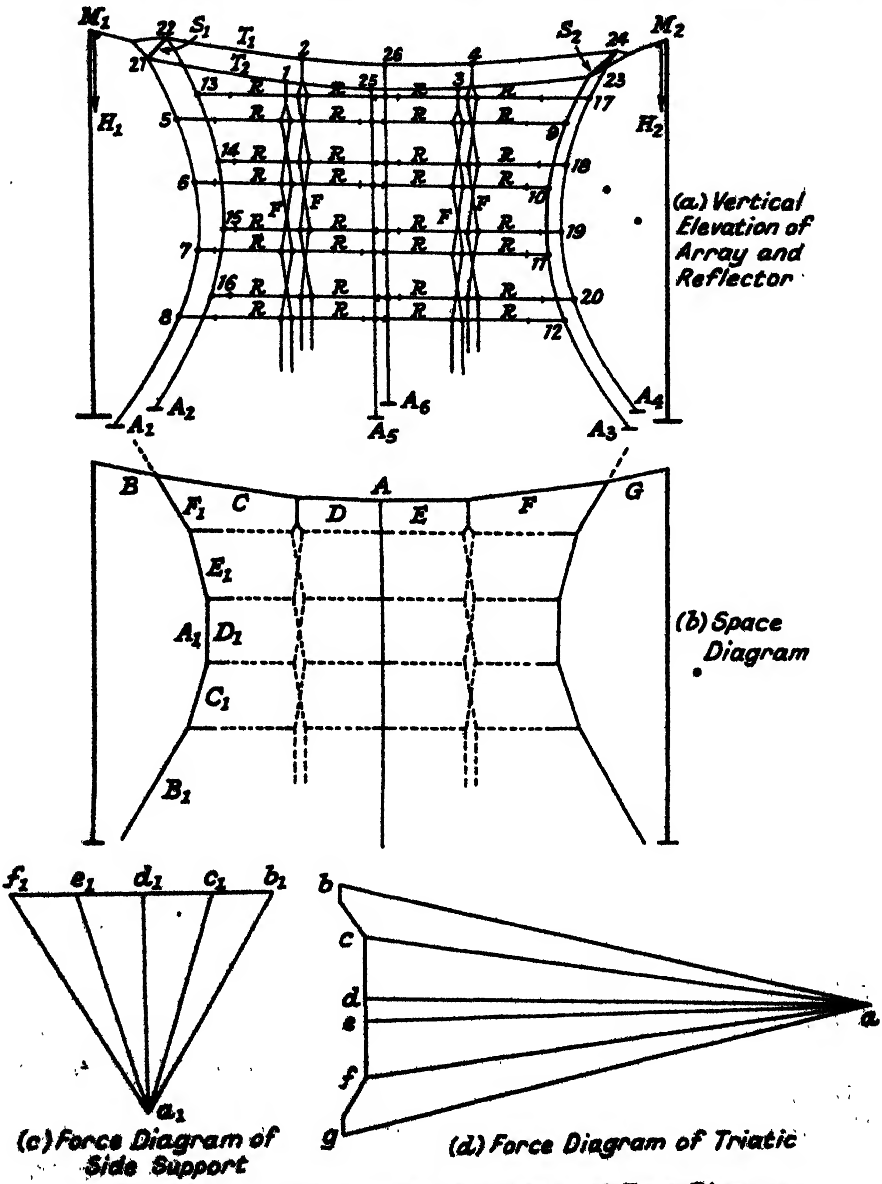


FIG. 6/XVI:14.—HR4/4 Array with Space and Force Diagrams.

(radiator and reflector elements) are acting horizontally instead of vertically.

The side support force diagram is shown on Fig. 6 (*c*), where the four forces, b_1c_1, c_1d_1, d_1e_1 and e_1f_1, comprising the load line (horizontal in this case) represent to scale the forces in the four horizontal elements, the tension of which has been decided empirically to be 50 lbs. per element. The rays a_1b_1, a_1c_1, a_1d_1, a_1e_1, and a_1f_1, represent to scale the forces acting in the various parts of the side support. The rays also give the line of action of the various forces from which the shape of the side support is obtained.

It will be seen that having drawn in the load line, an infinite number of combinations of rays having varying lengths and angles could be adopted unless the point *a* is fixed, and before this can be done it is necessary to know the lengths and/or the angles of any two of the rays. These factors are determined by trial and error, guidance being obtained from the known data with respect to the array. For instance, it is obvious that the waist of the side support (A_1/D_1 in Fig. 6 (*b*)) must not narrow to such an extent as to limit the length of the elements which will be attached at this point. Sufficient space must be allowed for insulators and end "make-offs", and in this respect it is convenient to decide upon a minimum distance of 6 ft. between the end of the radiator or reflector element and the side support at the point where the latter, by virtue of its shape, approaches nearest to the element. Further, the top section of the side support (A_1/F_1, Fig. 6 (*b*)) must run at such an angle as to ensure that it passes through a point on the triatic which will be convenient for attachment, i.e. at sufficient distance out from the masthead to clear the junction of the twin triatic with the main halyard.

Bearing in mind these points, together with the fact that it is advantageous, from the point of view of triatic loading, for the side support tensions to be kept to the minimum values which will satisfy the requirements of shape and horizontal element tensions, it will be seen that the fixing of the pole (*a* in Fig. 6 (*c*)) becomes a simple operation of trial and error following upon which the force diagram can be completed.

The side supports may now be drawn in and the notation added for reference, as shown in Fig. 6 (*b*), where all supporting members are shown in full line.

It is not known at this stage just where the side support will meet the triatic, as the shape of the latter has yet to be fixed. The top members of the side supports are therefore drawn in and extended

to a convenient length, as indicated by the dotted line in Fig. 6 (*b*). The triatic force diagram can now be drawn, and is shown in Fig. 6 (*d*), where the load line *bcdefg* represents to scale the loads on the triatic due to the various attachments, all of which are now known. The rays *ab*, *ac*, *ad*, *ae*, *af* and *ag* represent to scale the magnitude of the forces in the corresponding members (see notation) of the triatic, and also indicate their lines of action.

As the attachments are more or less evenly spaced along the triatic at five points, it is convenient to take the weight of the triatic in five sections and to include the weight of one section with the forces due to the members *bc*, *cd*, etc., of the force diagram. In cases where the weight of triatic wire is small compared with the loads due to the various attachments, it may be omitted without causing undue error in the diagrams. The vertical portion of *bc* and *fg* represents the load due to the spreader (half its weight) and the weight of triatic wire already mentioned, while the sloping section represents the force due to the side support. The forces in the vertical feeder supports C/D and E/F (Fig. 6 (*b*)) have already been discussed, and a figure of 200 lbs. decided upon. The force at D/E (Fig. 6 (*b*)), due to the bracing wire, is usually fixed at 100 lbs., this figure being arrived at empirically. The shape of the triatic may now be obtained from the force diagram (Fig. 6 (*d*)), in the manner already described, and the space diagram (Fig. 6 (*b*)) is complete.

The members comprising the supporting structure of the array may now be scaled off and dimensions added, and so all the information, necessary for the construction of such an array, is provided.

14.8. Windage and Ice. Little mention has yet been made of the factor of safety which is to be allowed when deciding the material to be used in the construction of the various types of aerial and aerial arrays. A minimum figure of 4/1 is usually considered to be satisfactory, but in assessing the various loads, the effects of wind and ice formation must not be overlooked.

Some idea of the increased loading due to windage may be gained by reference to Table IV. In this country a loading of 30 lbs. per square foot of projected area is accepted as being the maximum increase due to windage under normal circumstances. In the case of wires, however, the wind force is not perpendicular to the whole of the exposed area and the actual loading due to windage is less than the value stated above. A good working rule to adopt in allowing for windage is to take the designed load and multiply by a factor of two.

The problem with regard to ice formation is a difficult one when

TABLE IV

Wind Velocity and Pressure

Description	Miles per Hour	Feet per Min.	Forces in lbs. per sq. ft. from $P = 0{\cdot}0032\,V^2$ where V = miles/hour
Perceptible	1 – 3	88 – 264	0·0032 – 0·029
Gentle breeze	4 – 5	352 – 440	0·051 – 0·080
Breeze	10 – 15	880 – 1,320	0·320 – 0·720
Brisk wind	20 – 25	1,760 – 2,200	1·28 – 2·00
High wind	30 – 35	2,640 – 3,080	2·88 – 3·92
Very high wind	40 – 45	3,520 – 3,960	5·12 – 6·48
Storm	50	4,400	8·0
Hurricane	80 – 100	7,040 – 8,800	20·48 – 32·0

it is remembered that the weight of ice is about 60 lbs. per cubic foot, and that ice formation up to a diameter of 2 or 3 ins. on a ¼-in. diameter wire has been experienced. It will be realized, also, that in addition to the added loads due to the weight of the ice, the windage also increases as a result of the increase in projected area.

It would not be an economical proposition to erect masts with excessive safety factors designed to withstand the increased load on the triatics due to wind and ice, and protection against excessive loading is provided by taking the triatic halyard round pulleys to a balance weight system, which is designed to lift and release the tension when the maximum safe loading is reached. In the case of the top supports of an array curtain, however, the lifting of the balance weight does not entirely solve the problem of overloading. It will be appreciated that due to the weight of ice which may form on the various parts of the curtain supported by the top supports, they can be subjected to a load which is many times greater than normal, and in all cases such as this the factor of safety should be made as great as is practicable.

14.9. Factors of Safety. For triatics, halyards, etc., the factor of safety is usually about 4 to 1. For the top supports for vertical feeders 7/16 Gal. M.S. Strand wire with a breaking strain of 1,500 lbs. is used; as the load is usually of the order of 200 lbs. this gives a factor of safety greater than 7 to 1. The reason for the higher factor of safety in the top supports is that the tension in the triatics is maintained substantially constant since they run over pulleys and are tensioned by means of counter weights, while the top supports of

the vertical feeders have to carry any additional weight due to icing, which sometimes is very large. When icing occurs the balance weight rises and the triatic sags, so that its change of form maintains its tension constant. The change of tension in the top supports is, however, equal to the increase in loading due to ice less the reduction in tension resulting from the lowered triatic, and under conditions of heavy ice formation the former may greatly exceed the latter.

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NOTE: Chapters I–XVI are in VOLUME I and the remainder in VOLUME II

The numbering of sections and subsections of chapters is normally on the decimal principle, e.g. 5.1 is a subsection of section 5; subsubsection 5.14 is the fourth subdivision of subsection 5.1, etc.

When, however, it is required to divide section 5 into, say, 14 subsections, the subsection numbering after subsection 5.9 is: 5.10, 5.11, 5.12, 5.13, 5.14. Such numbering is rare, but it introduces an element of ambiguity. All ambiguity can however be quickly cleared by reference to the contents list for the chapter in question.

Example: Suppose the reference is to XVIII:5.22.11. Reference to the contents list for chapter XVIII shows that 5.22.11 occurs immediately after 5.22.10 which follows 5.229, the last being a normal decimal reference.

Any reference to a chapter evidently includes all sections; similarly, any reference to a section includes all subsections of that section.

Reference to the introductory matter at the beginning of a chapter, which sometimes is introduced before section 1, is effected by means of the symbol 'o', e.g. VI:o refers to the beginning of chapter VI.

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